03

Motion in a Plane

TOPIC 1
Vectors

01 If the magnitude of sum of two vectors is equal to the magnitude of difference of the two vectors, the angle between these vectors is **[NEET 2016, CBSE AIPMT 1991]**

(a) 90° (b) 45°
(c) 180° (d) 0°

Ans. *(a)*

Suppose two vectors are **P** and **Q**.

It is given that

$$|\mathbf{P}+\mathbf{Q}|=|\mathbf{P}-\mathbf{Q}|$$

Let angle between **P** and **Q** is ϕ.

$$\therefore \quad P^2+Q^2+2PQ\cos\phi=P^2+Q^2-2PQ\cos\phi$$

$$\Rightarrow \quad 4PQ\cos\phi=0$$

$$\Rightarrow \quad \cos\phi=0 \quad [\because P, Q\neq 0]$$

$$\Rightarrow \quad \phi=\frac{\pi}{2}=90^\circ$$

02 If vectors $\mathbf{A}=\cos\omega t\,\hat{\mathbf{i}}+\sin\omega t\,\hat{\mathbf{j}}$ and $\mathbf{B}=\cos\frac{\omega t}{2}\hat{\mathbf{i}}+\sin\frac{\omega t}{2}\hat{\mathbf{j}}$ are functions of time, then the value of t at which they are orthogonal to each other, is **[CBSE AIPMT 2015]**

(a) $t=\frac{\pi}{4\omega}$ (b) $t=\frac{\pi}{2\omega}$
(c) $t=\frac{\pi}{\omega}$ (d) $t=0$

Ans. *(c)*

For perpendicular vector, we have

$$\mathbf{A}\cdot\mathbf{B}=0$$

$$[\cos\omega t\hat{\mathbf{i}}+\sin\omega t\hat{\mathbf{j}}]\left[\cos\frac{\omega t}{2}\hat{\mathbf{i}}+\frac{\sin\omega t}{2}\hat{\mathbf{j}}\right]=0$$

$$\Rightarrow \quad \cos\omega t\cos\frac{\omega t}{2}+\sin\omega t\sin\frac{\omega t}{2}=0$$

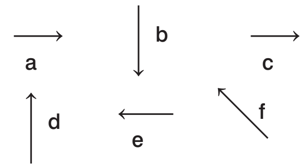
$[\because \cos(A-B)=\cos A\cos B+\sin A\sin B]$

$$\Rightarrow \quad \cos\left(\omega t-\frac{\omega t}{2}\right)=0 \Rightarrow \cos\frac{\omega t}{2}=0$$

$$\Rightarrow \quad \frac{\omega}{2}=\frac{\pi}{2} \Rightarrow t=\frac{\pi}{\omega}$$

Thus, time taken by vectors which are orthogonal to each other is $\frac{\pi}{\omega}$.

03 Six vectors **a** to **f** have the magnitudes and directions indicated in the figure. Which of the following statements is true? **[CBSE AIPMT 2010]**

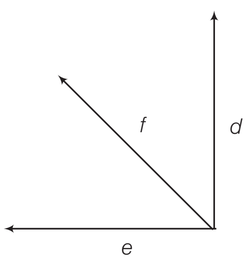


(a) $\mathbf{b}+\mathbf{c}=\mathbf{f}$ (b) $\mathbf{d}+\mathbf{c}=\mathbf{f}$
(c) $\mathbf{d}+\mathbf{e}=\mathbf{f}$ (d) $\mathbf{b}+\mathbf{e}=\mathbf{f}$

Ans. *(c)*

If two non-zero vectors are represented by the two adjacent sides of a parallelogram, then the resultant is given by the diagonal of the parallelogram passing through the point of intersection of the two vectors

$$\therefore \quad \mathbf{d}+\mathbf{e}=\mathbf{f}$$



04 A and B are two vectors and θ is the angle between them. If $|A\times B|=\sqrt{3}(A\cdot B)$, then the value of θ is **[CBSE AIPMT 2007]**

(a) 60° (b) 45° (c) 30° (d) 90°

Ans. *(a)*

Given, $|\mathbf{A}\times\mathbf{B}|=\sqrt{3}(\mathbf{A}\cdot\mathbf{B})$

$$\Rightarrow \quad AB\sin\theta=\sqrt{3}\,AB\cos\theta$$

$$\Rightarrow \quad \tan\theta=\sqrt{3}$$

$$\therefore \quad \theta=60^\circ$$

05 If a vector $2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+8\hat{\mathbf{k}}$ is perpendicular to the vector $4\hat{\mathbf{j}}-4\hat{\mathbf{i}}+\alpha\hat{\mathbf{k}}$, then the value of α is **[CBSE AIPMT 2005]**

(a) -1 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 1

Ans. *(c)*

Concept If two vectors are perpendicular to each other than their dot product is always equal to zero.

Let, $\mathbf{a}=2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+8\hat{\mathbf{k}}$

$\mathbf{b}=4\hat{\mathbf{j}}-4\hat{\mathbf{i}}+\alpha\hat{\mathbf{k}}=-4\hat{\mathbf{i}}+4\hat{\mathbf{j}}+\alpha\hat{\mathbf{k}}$

According to the above hypothesis

$$\mathbf{a}\perp\mathbf{b}$$

$$\Rightarrow \quad \mathbf{a}\cdot\mathbf{b}=0$$

$$\Rightarrow (2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+8\hat{\mathbf{k}})\cdot(-4\hat{\mathbf{i}}+4\hat{\mathbf{j}}+\alpha\hat{\mathbf{k}})=0$$

$$\Rightarrow \quad -8+12+8\alpha=0$$

$$\Rightarrow \quad 8\alpha=-4$$

$$\therefore \quad \alpha=-\frac{4}{8}=-\frac{1}{2}$$

06 If $|\mathbf{A}\times\mathbf{B}|=\sqrt{3}\,\mathbf{A}\cdot\mathbf{B}$, then the value of $|\mathbf{A}+\mathbf{B}|$ is **[CBSE AIPMT 2004]**

(a) $(A^2+B^2+AB)^{1/2}$

(b) $\left(A^2+B^2+\frac{AB}{\sqrt{3}}\right)^{1/2}$

(c) $A+B$

(d) $(A^2+B^2+\sqrt{3}\,AB)^{1/2}$

Ans. *(a)*

Given, $|\mathbf{A}\times\mathbf{B}|=\sqrt{3}\,\mathbf{A}\cdot\mathbf{B}$ ...(i)

but $|\mathbf{A}\times\mathbf{B}|=|\mathbf{A}||\mathbf{B}|\sin\theta=AB\sin\theta$

and $\mathbf{A}\cdot\mathbf{B}=|\mathbf{A}||\mathbf{B}|\cos\theta = AB\cos\theta$

Substituting these values in Eq. (i), we get

$$AB\sin\theta=\sqrt{3}\,AB\cos\theta$$

or $\tan\theta=\sqrt{3}$

$\therefore$ $\theta=60°$

The addition of vectors **A** and **B** can be given by the law of parallelogram.

$$\therefore\quad |\mathbf{A}+\mathbf{B}|=\sqrt{A^2+B^2+2AB\cos60°}$$
$$=\sqrt{A^2+B^2+2AB\times\frac{1}{2}}$$
$$=(A^2+B^2+AB)^{1/2}$$

07 The vector sum of two forces is perpendicular to their vector differences. In that case, the forces **[CBSE AIPMT 2003]**

(a) are not equal to each other in magnitude
(b) cannot be predicted
(c) are equal to each other
(d) are equal to each other in magnitude

Ans. *(d)*

Let **A** and **B** be two forces. The sum of the two forces.

$$\mathbf{F}_1=\mathbf{A}+\mathbf{B} \quad \text{...(i)}$$

The difference of the two forces,

$$\mathbf{F}_2=\mathbf{A}-\mathbf{B} \quad \text{...(ii)}$$

Since, sum of the two forces is perpendicular to their differences as given, so

$$\mathbf{F}_1\cdot\mathbf{F}_2=0$$

or $(\mathbf{A}+\mathbf{B})\cdot(\mathbf{A}-\mathbf{B})=0$

or $A^2-\mathbf{A}\cdot\mathbf{B}+\mathbf{B}\cdot\mathbf{A}-B^2=0$

or $A^2=B^2$ or $|\mathbf{A}|=|\mathbf{B}|$

Thus, the forces are equal to each other in magnitude.

08 If a unit vector is represented by $0.5\hat{\mathbf{i}}+0.8\hat{\mathbf{j}}+c\hat{\mathbf{k}}$, then the value of c is **[CBSE AIPMT 1999]**

(a) 1 (b) $\sqrt{0.11}$ (c) $\sqrt{0.01}$ (d) 0.39

Ans. *(b)*

Concept Unit vector can be found by dividing a vector with its magnitude i.e.

$$\mathbf{A}=\frac{\mathbf{A}}{|\mathbf{B}|}$$

Let we represent the unit vector by $\hat{n}$.

We also know that the modulus of unit vector is 1 i.e., $|\hat{n}|=1$

$\therefore$ $|\hat{n}|=|0.5\hat{\mathbf{i}}+0.8\hat{\mathbf{j}}+c\hat{\mathbf{k}}|=1$

or $\sqrt{(0.5)^2+(0.8)^2+c^2}=1$

or $0.25+0.64+c^2=1$

or $0.89+c^2=1$

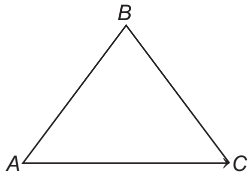
or $c^2=1-0.89=0.11 \Rightarrow c=\sqrt{0.11}$

09 Which of the following is not a vector quantity? **[CBSE AIPMT 1995]**

(a) Speed (b) Velocity
(c) Torque (d) Displacement

Ans. *(a)*

Speed is a scalar quantity. It gives no idea about the direction of motion of the object. Velocity is a vector quantity, as it has both magnitude and direction. Displacement is a vector as it possesses both magnitude and direction. When an object goes on the path *ABC*(in figure), then the displacement of the object is **AC**. The arrow head at *C* shows that the object is displaced from *A* to *C*.



Torque is turning effect of force which is a vector quantity.

10 The angle between the two vectors $\mathbf{A}=3\hat{\mathbf{i}}+4\hat{\mathbf{j}}+5\hat{\mathbf{k}}$ and $\mathbf{B}=3\hat{\mathbf{i}}+4\hat{\mathbf{j}}-5\hat{\mathbf{k}}$ will be **[CBSE AIPMT 1994]**

(a) 0° (b) 45° (c) 90° (d) 180°

Ans. *(c)*

Angle between two vectors is given as from dot product $\mathbf{A}\cdot\mathbf{B}=|\mathbf{A}||\mathbf{B}|\cos\theta$

$$\cos\theta=\frac{\mathbf{A}\cdot\mathbf{B}}{AB}$$

Here, $\mathbf{A}=3\hat{i}+4\hat{j}+5\hat{k}$

$\mathbf{B}=3\hat{i}+4\hat{j}-5\hat{k}$

$\therefore$ $A=\sqrt{(3)^2+(4)^2+(5)^2}=\sqrt{50}$

$B=\sqrt{(3)^2+(4)^2+(-5)^2}=\sqrt{50}$

and $\mathbf{A}\cdot\mathbf{B}=(3\hat{i}+4\hat{j}+5\hat{k})\cdot(3\hat{i}+4\hat{j}-5\hat{k})$

$=9+16-25=0$

$$\therefore\quad \cos\theta=\frac{0}{\sqrt{50}\cdot\sqrt{50}}=0$$

$\Rightarrow$ $\theta=90°$

11 The resultant of $\mathbf{A}\times 0$ will be equal to **[CBSE AIPMT 1992]**

(a) zero (b) A
(c) zero vector (d) unit vector

Ans. *(c)*

From the properties of vector product, the cross product of any vector with zero is a null vector or zero vector.

12 The angle between **A** and **B** is θ. The value of the triple product $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{A})$ is **[CBSE AIPMT 1989]**

(a) A^2B (b) zero
(c) $A^2B\sin\theta$ (d) $A^2B\cos\theta$

Ans. *(b)*

In scalar triple product of vectors, the positions of dot and cross can be interchanged, i.e.

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{A})=(\mathbf{A}\times\mathbf{B})\cdot\mathbf{A}=(\mathrm{A}\times\mathrm{A})\cdot\mathrm{B}$$

but $\mathbf{A}\times\mathbf{A}=0$

$\therefore$ $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{A})=0$

Alternative

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{A})$$

Let $\mathbf{A}\times\mathbf{B}=\mathbf{C}$

The direction of **C** is $\perp$ to **A** and **B** from cross product formula

So, $\mathbf{A}\cdot\mathbf{C}=0$

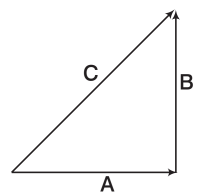
(since, **A** and **C** are $\perp$ to each other)

13 The magnitudes of vectors **A**, **B** and **C** are 3, 4 and 5 units respectively. If $\mathbf{A}+\mathbf{B}=\mathbf{C}$, the angle between **A** and **B** is **[CBSE AIPMT 1988]**

(a) $\frac{\pi}{2}$ (b) $\cos^{-1}(0.6)$
(c) $\tan^{-1}\left(\frac{7}{5}\right)$ (d) $\frac{\pi}{4}$

Ans. *(a)*

In figure shown, $\mathbf{A}+\mathbf{B}=\mathbf{C}$



Also, $|\mathbf{A}|=3$, $|\mathbf{B}|=4$, $|\mathbf{C}|=5$

As $\mathbf{A}+\mathbf{B}=\mathbf{C}$

So, $5^2=3^2+4^2+2\cdot4\cdot3\cos\theta$

$\cos\theta=0 \Rightarrow \theta=\frac{\pi}{2}$

$\Rightarrow$ **A** is perpendicular to **B**.

TOPIC 2
Motion in a Plane and Projectile Motion

14 Two bullets are fired horizontally and simultaneously towards each other from roof tops of two buildings 100 m apart and of same height of 200 m with the same velocity of 25 m/s. When and where will the two bullets collides. ($g=10\,\text{m/s}^2$) **[NEET (Odisha) 2019]**

(a) After 2s at a height 180 m

(b) After 2s at a height of 20 m
(c) After 4s at a height of 120 m
(d) They will not collide

Ans. *(a)*

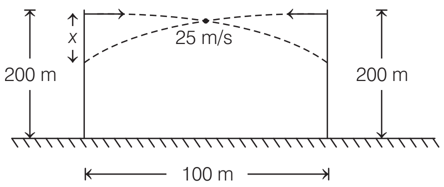
Given, distance between the two buildings

$$d = 100 \text{ m}$$

height of each tower, $h = 200$ m

speed of each bullet, $v = 25 \text{ ms}^{-1}$

The situation can be shown as below



where, x be the vertical distance travelled from the top of the building and t be the time at which they collide.

As two bullets are fired toward each other,

So, their relative velocity will be

$$v_{rel} = 25 - (-25) = 50 \text{ ms}^{-1}$$

Then, time $t = \dfrac{d}{v_{rel}} = \dfrac{100}{50} = 2 \text{ s}$

The distance or height at which they collide is calculated from equation of motion,

$$x = ut + \frac{1}{2}at^2$$

The bullet is initially a rest i.e. $u = 0$ and as it is moving under the effect of gravity $a = -g$, so

$$x = -\frac{1}{2}gt^2$$

$$x = -\frac{1}{2} \times 10(2)^2 = -20 \text{ m}$$

The negative sign shows that the bullets will collide 20 m below the top of tower i.e. at a height of $(200 - 20) = 180$ m from the ground after 2 s.

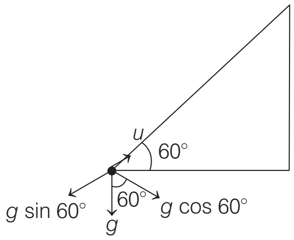
15 When an object is shot from the bottom of a long smooth inclined plane kept at an angle 60º with horizontal, it can travel a distance x_1 along the plane. But when the inclination is decreased to 30º and the same object is shot with the same velocity, it can travel x_2 distance.

Then $x_1 : x_2$ will be **[NEET (National) 2019]**

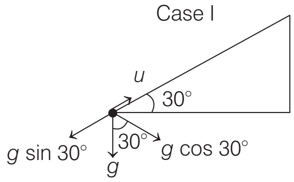
(a) $\sqrt{2}:1$ (b) $1:\sqrt{3}$
(c) $1:2\sqrt{3}$ (d) $1:\sqrt{2}$

Ans. *(b)*

The motion of object shot in two cases can be depicted as below



Case I



Case II

Using third equation of motion,

$$v^2 = u^2 - 2gh \quad \text{... (i)}$$

As the object stops finally, so

$$v = 0$$

For inclined motion,

$$g = g\sin\theta \text{ and } h = x$$

Substituting these values in Eq. (i), we get

$$\Rightarrow u^2 = 2g\sin\theta x \Rightarrow x = \frac{u^2}{2g\sin\theta}$$

For case (I), $x_1 = \dfrac{u^2}{2g\sin 60°}$

For case (II), $x_2 = \dfrac{u^2}{2g\sin 30°}$

$$\Rightarrow \frac{x_1}{x_2} = \frac{u^2}{2g\sin 60°} \times \frac{2g\sin 30°}{u^2}$$

$$= \frac{1}{2} \times \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} \text{ or } 1 : \sqrt{3}$$

16 The x and y coordinates of the particle at any time are $x = 5t - 2t^2$ and $y = 10t$ respectively, where x and y are in metres and t in seconds. The acceleration of the particle at $t = 2$s is **[NEET 2017]**

(a) 0 (b) 5 m/s^2
(c) -4 m/s^2 (d) -8 m/s^2

Ans. *(c)*

Given, $x = 5t - 2t^2$

Velocity of the particle,

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(5t - 2t^2) = 5 - 4t$$

Acceleration, $a_x = \dfrac{d}{dt}v_x = -4 \text{ ms}^{-2}$

Also, $y = 10t$

Velocity, $v_y = \dfrac{dy}{dt} = 10$

$\therefore$ Acceleration $a_y = \dfrac{dv_y}{dt} = 0$

$\therefore$ Net acceleration of the particle,

$$\mathbf{a}_{net} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} = (-4 \text{ ms}^{-2})\,\hat{\mathbf{i}}$$

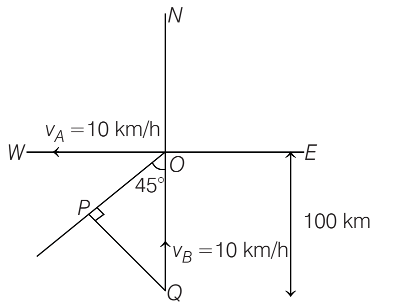
or $\mathbf{a}_{net} = -4\,\hat{\mathbf{i}} \text{ ms}^{-2}$

17 A ship A is moving Westwards with a speed of 10 km h^{-1} and a ship B 100 km South of A, is moving Northwards with a speed of 10 km h^{-1}. The time after which the distance between them becomes shortest is **[CBSE AIPMT 2015]**

(a) 0 h (b) 5 h
(c) $5\sqrt{2}$ h (d) $10\sqrt{2}$ h

Ans. *(b)*

It is clear from the diagram that the shortest distance between the ship A and B is PQ.



Here, $\sin 45º = \dfrac{PQ}{OQ} \Rightarrow PQ = 100 \times \dfrac{1}{\sqrt{2}}$

$$= 50\sqrt{2} \text{ m}$$

Also, $v_{AB} = \sqrt{v_A^2 + v_B^2} = \sqrt{10^2 + 10^2}$

$$= 10\sqrt{2} \text{ km/h}$$

So, time taken for them to reach shortest path is

$$t = \frac{PQ}{v_{AB}} = \frac{50\sqrt{2}}{10\sqrt{2}} = 5 \text{ h}$$

18 The position vector of a particle **R** as a function of time is given by

$$\mathbf{R} = 4\sin(2\pi t)\,\hat{\mathbf{i}} + 4\cos(2\pi t)\,\hat{\mathbf{j}}$$

where R is in metre, t is in seconds and $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ denote unit vectors along x and y-directions, respectively. Which one of the following statements is wrong for the motion of particle? **[CBSE AIPMT 2015]**

(a) Acceleration is along $-$R
(b) Magnitude of acceleration vector is $\dfrac{v^2}{R}$, where v is the velocity of particle

(c) Magnitude of the velocity of particle is 8 m/s
(d) Path of the particle is a circle of radius 4 m

Ans. *(c)*

(i) The position vector of a particle **R** as a function of time is given by

$$\mathbf{R} = 4\sin(2\pi t)\,\hat{\mathbf{i}} + 4\cos(2\pi t)\,\hat{\mathbf{j}}$$

x-axis component, $x = 4\sin 2\pi t$ ...(i)

y-axis component,

$y = 4\cos 2\pi t$ ...(ii)

Squaring and adding both equations, we get

$x^2 + y^2 = 4^2[\sin^2(2\pi t) + \cos^2(2\pi t)]$

i.e. $x^2 + y^2 = 4^2$ i.e. equation of circle and radius is 4 m.

(ii) Acceleration vector, $\mathbf{a} = \frac{v^2}{R}(-\mathbf{R})$, while v is velocity of a particle.

(iii) Magnitude of acceleration vector,

$$a = \frac{v^2}{R}$$

(iv) As, we have $v_x = +4(\cos 2\pi t)2\pi$ and $v_y = -4(\sin 2\pi t)2\pi$

Net resultant velocity,

$$v = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{(8\pi)^2(\cos^2 2\pi t + \sin^2 2\pi t)}$$

$v = 8\pi$ $[\because \cos^2 2\pi t + \sin^2 2\pi t = 1]$

So, option (c) is incorrect.

19 A projectile is fired from the surface of the earth with a velocity of 5 ms^{-1} at angle θ with the horizontal. Another projectile fired from another planet with a velocity of 3 ms^{-1} at the same angle follows a trajectory which is identical with the trajectory of the projectile fired from the earth. The value of the acceleration due to gravity on the planet is (in ms^{-2}) is (given, $g = 9.8$ ms^{-2}) **[CBSE AIPMT 2014]**

(a) 3.5 (b) 5.9 (c) 16.3 (d) 110.8

Ans. *(a)*

The trajectory of a projectile projected at some angle θ with the horizontal direction from ground is given by

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

For equal trajectories for same angle of projection,

$$\frac{g}{u^2} = \text{constant} \Rightarrow \frac{9.8}{5^2} = \frac{g'}{3^2}$$

$$g' = \frac{9.8 \times 9}{25} = 3.5\ ms^{-2}$$

20 A particle is moving such that its position co-ordinates (x, y) are (2m, 3m) at time $t = 0$, (6m, 7m) at time $t = 2$ s and (13m, 14m) at time $t = 5$ s. Average velocity vector (v_{av}) from $t = 0$ to $t = 5$ s is **[CBSE AIPMT 2014]**

(a) $\frac{1}{5}(13\hat{\mathbf{i}} + 14\hat{\mathbf{j}})$ (b) $\frac{7}{3}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

(c) $2(\hat{\mathbf{i}} + \hat{\mathbf{j}})$ (d) $\frac{11}{5}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

Ans. *(d)*

Given, position vector of the particle at $t = 0$ is $(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ and $t = 5$ s is $(13\hat{\mathbf{i}} + 14\hat{\mathbf{j}})$

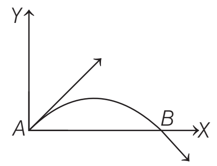
Average velocity vector

$$\mathbf{v}_{av} = \frac{\text{Net displacement}}{\text{Time taken}}$$

$$= \frac{(13-2)\hat{\mathbf{i}} + (14-3)\hat{\mathbf{j}}}{5}$$

$$= \frac{11\hat{\mathbf{i}} + 11\hat{\mathbf{j}}}{5} = \frac{11}{5}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

21 The velocity of a projectile at the initial point A is $(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ m/s. Its velocity (in m/s) at point B is **[NEET 2013]**



(a) $-2\hat{i} - 3\hat{j}$ (b) $-2\hat{i} + 3\hat{j}$

(c) $2\hat{i} - 3\hat{j}$ (d) $2\hat{i} + 3\hat{j}$

Ans. *(c)*

Concept As we know that in projectile motion only velocity of y component change, whereas velocity of x component remains constant.

From the figure, the x-component remains unchanged, while the y-component is reversed. Thus, the velocity at point B is $(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}})$ ms^{-1}.

22 The horizontal range and the maximum height of a projectile are equal. The angle of projection of the projectile is **[CBSE AIPMT 2012]**

(a) $\theta = \tan^{-1}\left(\frac{1}{4}\right)$ (b) $\theta = \tan^{-1}(4)$

(c) $\theta = \tan^{-1}(2)$ (d) $\theta = 45°$

Ans. *(b)*

Given, horizontal range

R = vertical maximum height H

Range, $R = \frac{u^2(2\sin\theta\cos\theta)}{g}$

Height, $H = \frac{u^2\sin^2\theta}{2g}$

Hence, $\frac{u^2(2\sin\theta\cos\theta)}{g} = \frac{u^2\sin^2\theta}{2g}$

$$2\cos\theta = \frac{\sin\theta}{2}$$

$$\tan\theta = 4 \Rightarrow \theta = \tan^{-1}(4)$$

23 A missile is fired for maximum range with an initial velocity of 20 m/s. If $g = 10$ m/s^2, the range of the missile is **[CBSE AIPMT 2011]**

(a) 50 m (b) 60 m (c) 20 m (d) 40 m

Ans. *(d)*

For maximum range of projectile, θ will be 45° by the law of projectile motion.

So, maximum range, $R_{max} = \frac{u^2}{g}$

Given, $u = 20$ ms^{-1} and $g = 10$ ms^{-2}

$$R_{max} = \frac{(20)^2}{10} = \frac{400}{10}$$

$$R_{max} = 40\text{ m}$$

24 A particle has initial velocity $(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$ and has acceleration $(0.4\hat{\mathbf{i}} + 0.3\hat{\mathbf{j}})$. Its speed after 10 s is **[CBSE AIPMT 2010]**

(a) 7 unit (b) $7\sqrt{2}$ unit
(c) 8.5 unit (d) 10 unit

Ans. *(b)*

Given, initial velocity $(u) = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$

Final velocity $(v) = ?$

Acceleration $(a) = (0.4\hat{\mathbf{i}} + 0.3\hat{\mathbf{j}})$

Time $(t) = 10$ s

From first equation of motion, $v = u + at$

$$v = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 10(0.4\hat{\mathbf{i}} + 0.3\hat{\mathbf{j}})$$

$$v = 7\hat{\mathbf{i}} + 7\hat{\mathbf{j}} \Rightarrow |v| = 7\sqrt{2}$$

25 A particle of mass m is projected with velocity v making an angle of 45° with the horizontal. When the particle lands on the level ground, the magnitude of the change in its momentum will be **[CBSE AIPMT 2008]**

(a) $2mv$ (b) $\frac{mv}{\sqrt{2}}$ (c) $mv\sqrt{2}$ (d) zero

Ans. *(c)*

$$\Delta\mathbf{p} = m(\mathbf{v} - \mathbf{u})$$

$$|\Delta\mathbf{p}| = |mv[\cos 45\hat{\mathbf{i}} - \sin 45\hat{\mathbf{j}}] - mv[\cos 45\hat{\mathbf{i}} + \sin 45\hat{\mathbf{j}}]|$$

$$= 2mv\sin 45° = \sqrt{2}\,mv$$

Alternative

The α component of velocity will remain constant.

$$\Rightarrow \quad \Delta v_y = \frac{v}{\sqrt{2}} - \left(\frac{-v}{\sqrt{2}}\right)$$

$$= \frac{2v}{\sqrt{2}} = \sqrt{2}v$$

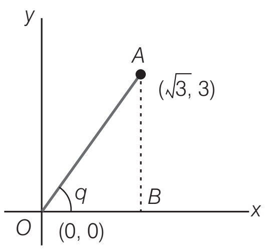
$$\Rightarrow \text{So,} \quad \Delta P_y = m\sqrt{2}v = \sqrt{2}mv$$

26 A particle starting from the origin $(0, 0)$ moves in a straight line in the (x, y) plane. Its co-ordinates at a later time are $(\sqrt{3}, 3)$. The path of the particle makes with the x-axis an angle of **[CBSE AIPMT 2007]**

(a) 30° (c) 45° (c) 60° (d) 0°

Ans. *(c)*

Draw the situation as given in questions. *OA* represents the path of the particle starting from origin $O(0, 0)$. Draw a perpendicular from point *A* to *x*-axis. Let path of the particle makes an angle θ with the *x*-axis, then



$\tan\theta$ = slope of line *OA*

= path of the particle 1 making angle θ

$$\tan\theta = \frac{3-0}{\sqrt{3}-0} = \sqrt{3}$$

$$\theta = 60°$$

27 For angles of projection of a projectile at angles $(45° - \theta)$ and $(45° + \theta)$, the horizontal ranges described by the projectile are in the ratio of **[CBSE AIPMT 2006]**

(a) 1 : 1 (b) 2 : 3
(c) 1 : 2 (d) 2 : 1

Ans. *(a)*

We know that, horizontal ranges for complementary angles of projection will be same.

The projectiles are projected at angles $(45° - \theta)$ and $(45° + \theta)$ which are complementary to each other i.e. two angles add up to give 90°. Hence, horizontal ranges will be equal. Thus, the required ratio is 1 : 1.

Alternative

Horizontal range of projectile

$$R = \frac{u^2 \sin 2\alpha}{g}$$

For $\alpha = (45° - \theta)$,

$$R_1 = \frac{u^2 \sin 2(45° - \theta)}{g}$$

$$= \frac{u^2 \sin(90° - 2\theta)}{g} = \frac{u^2 \cos 2\theta}{g}$$

For $\alpha = (45° + \theta)$,

$$R_2 = \frac{u^2 \sin 2(45° + \theta)}{g}$$

$$= \frac{u^2 \sin(90° + \theta)}{g}$$

$$= \frac{u^2 \cos 2\theta}{g}$$

Hence, $\frac{R_1}{R_2} = \frac{1}{1}$ or $R_1 : R_2 = 1 : 1$

28 Two particles are projected with same initial velocities at an angle 30° and 60° with the horizontal. Then, **[CBSE AIPMT 2000]**

(a) their heights will be equal
(b) their ranges will be equal
(c) their time of flights will be equal
(d) their ranges will be different

Ans. *(b)*

(a) Maximum height in case of projectile is given by

$$H = \frac{u^2 \sin^2\theta}{2g} \Rightarrow \frac{H_1}{H_2} = \frac{\sin^2\theta_1}{\sin^2\theta_2}$$

$$= \frac{\sin^2 30°}{\sin^2 60°} = \frac{\left(\frac{1}{2}\right)^2}{\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{1}{3}$$

(b) Range $R = \frac{u^2 \sin 2\theta}{g}$

$$\Rightarrow \frac{R_1}{R_2} = \frac{\sin(2\times 30°)}{\sin(2\times 60°)}$$

$$= \frac{\sin 60°}{\sin 120°} = \frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} = 1 \Rightarrow R_1 = R_2$$

(c) Time of flight

$$T = \frac{2u\sin\theta}{g} \Rightarrow \frac{T_1}{T_2} = \frac{\sin\theta_1}{\sin\theta_2}$$

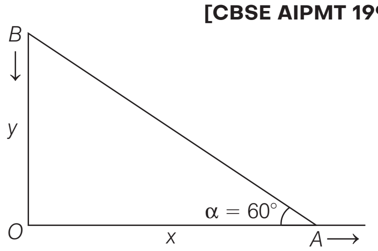
$$= \frac{\sin 30°}{\sin 60°} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$$

Hence, their horizontal ranges will be equal.

Alternative

In this problem, it is given that two particles are projected at angles 30° and 60° which are complementary angles. We know that horizontal range will be same for complementary angles. Hence, their ranges will be equal.

29 Two particles *A* and *B* are connected by a rigid rod *AB*. The rod slides along perpendicular rails as shown here. The velocity of *A* to the right is 10m/s. What is the velocity of *B* when angle $\alpha = 60°$? **[CBSE AIPMT 1998]**



(a) 9.8 m/s (b) 10 m/s
(c) 5.8 m/s (d) 17.3 m/s

Ans. *(d)*

Let the velocity along *x* and *y*-axes be v_x and v_y respectively.

$$\therefore \quad v_x = \frac{dx}{dt} \text{ and } v_y = \frac{dy}{dt}$$

From figure,

$$\tan\alpha = \frac{y}{x} \Rightarrow y = x\tan\alpha$$

Differentiating Eq. (i), w.r.t. t, we get

$$\frac{dy}{dt} = \frac{dx}{dt}\tan\alpha$$

$$\Rightarrow \quad v_y = v_x \tan\alpha$$

Here, $v_x = 10$ m/s, $\alpha = 60°$

$$\therefore \quad v_y = 10\tan 60°$$

$$= 10\sqrt{3} = 17.3 \text{ m/s}$$

30 A bullet is fired from a gun with a speed of 1000 m/s in order to hit a target 100 m away. At what height above the target should the gun be aimed? (The resistance of air is negligible and $g = 10\,\text{m/s}^2$) **[CBSE AIPMT 1995]**

(a) 5 cm (b) 10 cm (c) 15 cm (d) 20 cm

Ans. *(a)*

Horizontal distance of the target is 100 m. Speed of bullet = 1000 m/s

Time taken by bullet to cover the horizontal distance,

$$t = \frac{100}{1000} = \frac{1}{10}\text{ s}$$

During $\frac{1}{10}$ s, the bullet will fall down vertically due to gravitational acceleration.
Therefore, height above the target, so that the bullet hit the target is

$$h = ut + \frac{1}{2}gt^2 = \left(0 \times \frac{1}{10}\right) + \frac{1}{2} \times 10 \times (0.1)^2$$
$$= 0.05\ \text{m} = 5\ \text{cm}$$

31 The position vector of a particle is $\mathbf{r} = (a\cos\omega t)\hat{\mathbf{i}} + (a\sin\omega t)\hat{\mathbf{j}}$. The velocity of the particle is **[CBSE AIPMT 1995]**

(a) directed towards the origin
(b) directed away from the origin
(c) parallel to the position vector
(d) perpendicular to the position vector

Ans. *(d)*
Velocity is rate of change of position vector, i.e.,

$$\mathbf{v} = \frac{d\mathbf{r}}{dt}$$

where, **r** is the position vector.

$$\therefore\ \mathbf{v} = \frac{d}{dt}[(a\cos\omega t)\hat{\mathbf{i}} + (a\sin\omega t)\hat{\mathbf{j}}]$$
$$= (-a\omega\sin\omega t)\hat{\mathbf{i}} + (a\omega\cos\omega t)\hat{\mathbf{j}}$$
$$= \omega[(-a\sin\omega t)\hat{\mathbf{i}} + (a\cos\omega t)\hat{\mathbf{j}}]$$

Slope of position vector

$$= \frac{a\sin\omega t}{a\cos\omega t} = \tan\omega t$$

and slope of velocity vector $= \frac{-a\cos\omega t}{a\sin\omega t} = -\frac{1}{\tan\omega t}$

$\therefore$ Velocity is perpendicular to the displacement.

Alternative
Velocity vector and position vector are perpendicular to each other if their scalar product is zero, i.e.

$$\mathbf{v}\cdot\mathbf{r} = 0$$

Now, $\mathbf{v}\cdot\mathbf{r} = [(-a\omega\sin\omega t)\hat{i} + (a\omega\cos\omega t)\hat{j}]$
$[(a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}]$
$= -a^2\omega\sin\omega t\cos\omega t + a^2\omega\sin\omega t\cos\omega t = 0$

$\Rightarrow$ Velocity vector is perpendicular to position vector.

32 Two bodies of same mass are projected with the same velocity at an angle 30° and 60° respectively. The ratio of their horizontal ranges will be **[CBSE AIPMT 1990]**

(a) 1 : 1 (b) 1 : 2 (c) 1 : 3 (d) $2:\sqrt{2}$

Ans. *(a)*
When an object is projected with velocity *u* making an angle θ with the horizontal direction, then horizontal range will be

$$R_1 = \frac{u^2\sin 2\theta}{g} \quad \text{...(i)}$$

when an object is projected with velocity *u* making an angle $(90° - \theta)$ with the horizontal direction, then horizontal range will be

$$R_2 = \frac{u^2\sin 2(90° - \theta)}{g}$$
$$= \frac{u^2}{g}\sin(180° - 2\theta)$$
$$= \frac{u^2}{g}\sin 2\theta \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we note that
$R_1 = R_2$
Here, the projection angle is 30° and $60° = (90° - 30°)$, so horizontal range is same for both angles.

$$\therefore \quad \frac{R_1}{R_2} = 1$$

33 The maximum range of a gun of horizontal terrain is 16 km. If $g = 10\ \text{ms}^{-2}$, then muzzle velocity of a shell must be **[CBSE AIPMT 1990]**

(a) $160\ \text{ms}^{-1}$ (b) $200\sqrt{2}\ \text{ms}^{-1}$
(c) $400\ \text{ms}^{-1}$ (d) $800\ \text{ms}^{-1}$

Ans. *(c)*
Range of projectile is given by

$$R = \frac{u^2\sin 2\theta}{g}$$

For range to be maximum, angle θ should be of 45°.

$$\therefore \quad R_{max} = \frac{u^2\sin 2\times 45°}{g} = \frac{u^2\sin 90°}{g}$$

or $R_{max} = \frac{u^2}{g}$

Here, $R_{max} = \frac{u^2}{g} = 16\ \text{km} = 16000\ \text{m}$

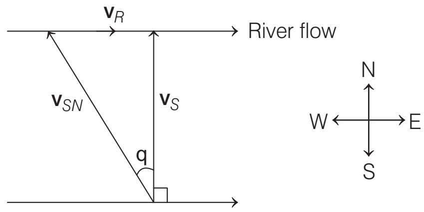
or $u = \sqrt{16000\,g} = \sqrt{16000 \times 10}$
$= 400\ \text{ms}^{-1}$

TOPIC 3
Relative Velocity

34 The speed of a swimmer in still water is 20 m/s. The speed of river water is 10 m/s and is flowing due east. If he is standing on the south bank and wishes to cross the river along the shortest path the angle at which he should make his strokes w.r.t. north is given by **[NEET (National) 2019]**

(a) 0º (b) 60° west
(c) 45º west (d) 30º west

Ans. *(d)*
Given,
speed of river, $v_R = 10$ m/s
speed of swimmer in still water and $v_{SN} = 20$ m/s.



For the shortest path to cross the river, he should swim at an angle $(90° + \theta)$ with the stream flow. From the figure above,
$\mathbf{v}_{SN} = \mathbf{v}_R + \mathbf{v}_S$
So, angle θ is given by

$$\sin\theta = \left|\frac{\mathbf{v}_R}{\mathbf{v}_{SN}}\right| = \frac{10}{20} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 30°$$

As the river is flowing in East direction, so he should swim towards West.

35 Two particles *A* and *B*, move with constant velocities $\mathbf{v}_1$ and $\mathbf{v}_2$. At the initial moment, their position vectors are $\mathbf{r}_1$ and $\mathbf{r}_2$ respectively. The condition for particles *A* and *B* for their collision is **[CBSE AIPMT 2015]**

(a) $\frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{|\mathbf{v}_2 - \mathbf{v}_1|}$
(b) $\mathbf{r}_1 \cdot \mathbf{v}_1 = \mathbf{r}_2 \cdot \mathbf{v}_2$
(c) $\mathbf{r}_1 \times \mathbf{v}_1 = \mathbf{r}_2 \times \mathbf{v}_2$
(d) $\mathbf{r}_1 - \mathbf{r}_2 = \mathbf{v}_1 - \mathbf{v}_2$

Ans. *(a)*
For two particles *A* and *B* move with constant velocities $\mathbf{v}_1$ and $\mathbf{v}_2$. Such that two particles to collide, the direction of the relative velocity of one with respect to other should be directed towards the relative position of the other particle.

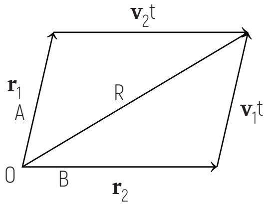
i.e. $\frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|} \longrightarrow$ direction of relative position of 1 w.r.t. 2.

Similarly, $\frac{\mathbf{v}_1 - \mathbf{v}_2}{|\mathbf{v}_1 - \mathbf{v}_2|} \longrightarrow$ direction of velocity of 2 w.r.t. 1.

So, for collision of A and B, we get

$$\frac{\mathbf{r}_1-\mathbf{r}_2}{|\mathbf{r}_1-\mathbf{r}_2|}=\frac{\mathbf{v}_2-\mathbf{v}_1}{|\mathbf{v}_2-\mathbf{v}_1|}$$

Alternate Method As resultant displacement of a particle,



$$\mathbf{R}=\mathbf{r}_1-\mathbf{v}_1 t=\mathbf{r}_2-\mathbf{v}_2\, t$$

i.e. $\mathbf{r}_1-\mathbf{r}_2=(\mathbf{v}_2-\mathbf{v}_1)\,t$

So, $\dfrac{\mathbf{r}_1-\mathbf{r}_2}{|\mathbf{r}_1-\mathbf{r}_2|}=\dfrac{(\mathbf{v}_2-\mathbf{v}_1)\,t}{|\mathbf{v}_2-\mathbf{v}_1|\,t}$

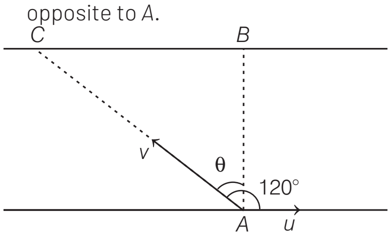
$$\frac{\mathbf{r}_1-\mathbf{r}_2}{|\mathbf{r}_1-\mathbf{r}_2|}=\frac{(\mathbf{v}_2-\mathbf{v}_1)}{|\mathbf{v}_2-\mathbf{v}_1|}$$

36 A person swims in a river aiming to reach exactly opposite point on the bank of a river. His speed of swimming is 0.5 m/s at an angle 120° with the direction of flow of water. The speed of water in stream is **[CBSE AIPMT 1999]**

(a) 1.0 m/s (b) 0.5 m/s
(c) 0.25 m/s (d) 0.43 m/s

Ans. ***(c)***

Let u be the speed of stream and v be the speed of person started from A. He wants to reach at point B directed opposite to A.



As given, v makes an angle of 120° with direction of flow u, the resultant of v and u is along AB. From figure

$$u=v\sin\theta=v\sin 30^\circ$$

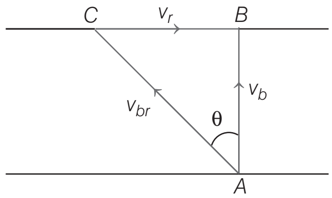
$\therefore\quad u=\dfrac{v}{2}=\dfrac{0.5}{2}\qquad(\because v=0.5\text{ m/s})$

$$=0.25\text{ m/s}$$

37 The speed of a boat is 5 km/h in still water. It crosses a river of width 1.0 km along the shortest possible path in 15 min. The velocity of the river water is (in km/h) **[CBSE AIPMT 1998]**

(a) 5 (b) 1 (c) 3 (d) 4

Ans. ***(c)***



Let v_r = velocity of river
v_{br} = velocity of boat in still water and
w = width of river

Time taken to cross the river = 15 min

$$=\frac{15}{60}\text{ h}=\frac{1}{4}\text{h}$$

Shortest path is taken when v_b is along AB. In this case,

$$v_{br}^2=v_r^2+v_b^2$$

Now, $\quad t=\dfrac{w}{v_b}=\dfrac{w}{\sqrt{v_{br}^2-v_r^2}}$

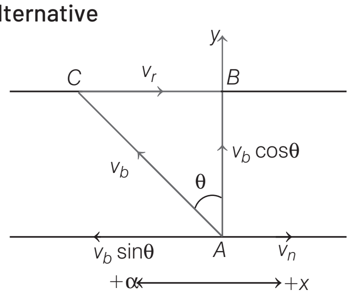
$\therefore\quad \dfrac{1}{4}=\dfrac{1}{\sqrt{5^2-v_r^2}}$

$\Rightarrow\quad 5^2-v_r^2=16$

$\Rightarrow\quad v_r^2=25-16=9$

$\therefore\quad v_r=\sqrt{9}=3$ km/h

Alternative



$$t=\frac{15}{60}=\frac{1}{4}\lambda\text{ s}$$

$\Rightarrow$ Motion along the Y-axis $t=\dfrac{y}{v_b\cos\theta}$

$\Rightarrow\quad \dfrac{1}{4}=\dfrac{1}{5\cos\theta}\quad\Rightarrow\quad\cos\theta=\dfrac{4}{5}$

So , $\sin\theta=\dfrac{3}{5}$

$\Rightarrow$ Motion is along the x-axis.

For the boat to reach at B

$$v_b\sin\theta=v_r\quad\Rightarrow\quad 5\times\frac{3}{5}=v_r$$

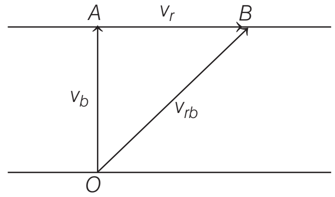
$\Rightarrow\quad v_r=3$ km/h

38 A boat is sent across a river with a velocity of 8 km h^{-1}. If the resultant velocity of boat is 10 km h^{-1}, then velocity of river is **[CBSE AIPMT 1993]**

(a) 12.8 km h^{-1} (b) 6 km h^{-1}
(c) 8 km h^{-1} (d) 10 km h^{-1}

Ans. ***(b)***

The situation is depicted in figure.



Let v_b be the velocity of boat, v_r be the velocity of river and v_{rb} be the resultant velocity of boat.

From figure and concept of relative velocity

$$v_{rb}^2=v_r^2+v_b^2$$

$\therefore\quad v_r=\sqrt{v_{rb}^2-v_b^2}$

$$=\sqrt{10^2-8^2}=6\text{ km h}^{-1}$$

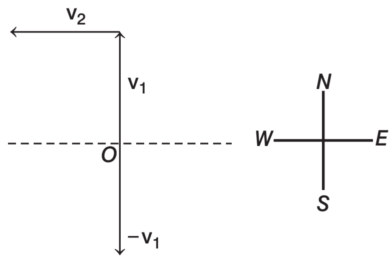
39 A bus is moving on a straight road towards North with a uniform speed of 50 km/h.

If the speed remains unchanged after turning through 90°, the increase in the velocity of bus in the turning process is **[CBSE AIPMT 1989]**

(a) 70.7 km/h along South-West direction
(b) zero
(c) 50 km/h along West
(d) 70.7 km/h along North-West direction

Ans. ***(a)***

The situation is depicted in figure.



Here, $\quad\mathbf{v}_1$ = 50 km/h due North
$\mathbf{v}_2$ = 50 km/h due West

From figure it indicates that angle between $\mathbf{v}_1$ and $\mathbf{v}_2$ is 90°.

Now, $-\mathbf{v}_1$ = 50 km/h due South

$\therefore$ Change in velocity

$$=|\mathbf{v}_2-\mathbf{v}_1|=|\mathbf{v}_2+(-\mathbf{v}_1)|$$

$$=\sqrt{v_2^2+v_1^2}$$

$$=\sqrt{50^2+50^2}$$

$$=70.7\text{ km/h}$$

TOPIC 4
Uniform Circular Motion

40 A particle moving in a circle of radius R with a uniform speed takes a time T to complete one revolution. If this particle were projected with the same speed at an angle θ to the horizontal, the maximum height attained by it equals $4R$. The angle of projection θ is then given by **[NEET 2021]**

(a) $\theta = \cos^{-1}\left(\frac{gT^2}{\pi^2 R}\right)^{\frac{1}{2}}$

(b) $\theta = \cos^{-1}\left(\frac{\pi^2 R}{gT^2}\right)^{\frac{1}{2}}$

(c) $\theta = \sin^{-1}\left(\frac{\pi^2 R}{gT^2}\right)^{\frac{1}{2}}$

(d) $\theta = \sin^{-1}\left(\frac{2gT^2}{\pi^2 R}\right)^{\frac{1}{2}}$

Ans. *(d)*

Given, the radius of the circular path $= R$

The time taken by the particle to complete one revolution $= T$

When the particle is projected with the same speed (by which it is moving in circular orbit) at angle θ to the horizontal, the maximum height attained it is given as

$$H_{max} = \frac{u^2 \sin^2 \theta}{2g} \quad ...(i)$$

$$H_{max} = 4R \quad \text{(given)}$$

Also, we know that,
speed of the particle in circular path,

$$u = \frac{2\pi R}{T}$$

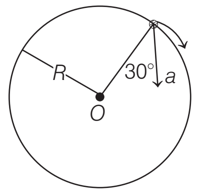
Substituting the values in the Eq. (i), we get

$$4R = \frac{\left(\frac{2\pi R}{T}\right)^2 \sin^2 \theta}{2g}$$

$$\Rightarrow \quad \sin\theta = \left(\frac{2gT^2}{\pi^2 R}\right)^{1/2}$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{2gT^2}{\pi^2 R}\right)^{1/2}$$

41 In the given figure, $a = 15\ \text{m/s}^2$ represents the total acceleration of a particle moving in the clockwise direction in a circle of radius $R = 2.5$ m at a given instant of time. The speed of the particle is **[NEET 2016]**



(a) 4.5 m/s (b) 5.0 m/s
(c) 5.7 m/s (d) 6.2 m/s

Ans. *(c)*

Centripetal acceleration of a particle moving on a circular path is given by

$$a_c = \frac{v^2}{R}$$

In the given figure,

$a_c = a \cos 30° = 15 \cos 30°\ \text{m/s}^2$

$$\Rightarrow \quad \frac{v^2}{R} = 15 \cos 30°$$

$$\Rightarrow \quad v^2 = R \times 15 \times \frac{\sqrt{3}}{2} = 2.5 \times 15 \times \frac{\sqrt{3}}{2}$$

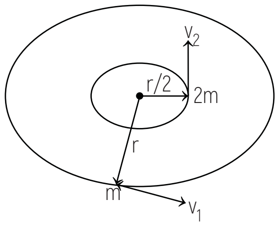
$$\therefore \quad v = 5.7\ \text{m/s}$$

42 Two stones of masses m and $2m$ are whirled in horizontal circles, the heavier one in a radius $\frac{r}{2}$ and the lighter one in radius r. The tangential speed of lighter stone is n times that of the value of heavier stone when they experience same centripetal forces. The value of n is **[CBSE AIPMT 2015]**

(a) 2 (b) 3 (c) 4 (d) 1

Ans. *(a)*

Given, that two stones of masses m and $2\,m$ are whirled in horizontal circles, the heavier one in a radius $\frac{r}{2}$ and lighter one in radius r as shown in figure.



As, lighter stone is n times that of the value of heavier stone when they experience same centripetal forces, we get

$$(F_c)_{heavier} = (F_c)_{lighter}$$

$$\Rightarrow \quad \frac{2m\,(v)^2}{(r/2)} = \frac{m\,(nv)^2}{r}$$

$$\Rightarrow \quad n^2 = 4 \Rightarrow n = 2$$

43 A particle moves in a circle of radius 5 cm with constant speed and time period 0.2πs. The acceleration of the particle is **[CBSE AIPMT 2011]**

(a) $25\ \text{m/s}^2$ (b) $36\ \text{m/s}^2$
(c) $5\ \text{m/s}^2$ (d) $15\ \text{m/s}^2$

Ans. *(c)*

Given, $r = 5\,\text{cm} = 5 \times 10^{-2}\ \text{m}$

$T = 0.2\ \pi\text{s}$

We know that acceleration is given by

$$a = r\omega^2 = \frac{4\pi^2}{T^2} r = \frac{4 \times \pi^2 \times 5 \times 10^{-2}}{(0.2\ \pi)^2}$$

$$= 5\ \text{m/s}^2$$

44 A car runs at a constant speed on a circular track of radius 100 m, taking 62.8 s for every circular lap. The average velocity and average speed for each circular lap respectively is **[CBSE AIPMT 2006]**

(a) 0, 0 (b) 0, 10 m/s
(c) 10 m/s, 10 m/s (d) 10 m/s, 0

Ans. *(b)*

Concept Average velocity is defined as the ratio of displacement to time taken while the average speed of a particle in a given interval of time is defined as the ratio of distance travelled to the time taken. On a circular path in completing one turn, the distance travelled is $2\pi r$ while displacement is zero.

Hence, average velocity $= \frac{\text{Displacement}}{\text{Time - interval}}$

$$= \frac{0}{t} = 0$$

Average speed $= \frac{\text{Distance}}{\text{Time - interval}}$

$$= \frac{2\pi r}{t} = \frac{2 \times 3.14 \times 100}{62.8}$$

$$= 10\ \text{ms}^{-1}$$

45 A stone tied to the end of a string of 1 m long is whirled in a horizontal circle with a constant speed. If the stone makes 22 revolutions in 44 s, what is the magnitude and direction of acceleration of the stone? **[CBSE AIPMT 2005]**

(a) $\frac{\pi^2}{4}\ \text{ms}^{-2}$ and direction along the radius towards the centre

(b) π^2 ms^{-2} and direction along the radius away from centre
(c) π^2 ms^{-2} and direction along the radius towards the centre
(d) π^2 ms^{-2} and direction along the tangent to the circle

Ans. *(c)*

Since, speed is constant throughout the motion, so, it is a uniform circular motion. Therefore, its radial acceleration is given by

$$a_r = r\omega^2$$
$$= r\left(\frac{2\pi n}{t}\right)^2 = r \times \frac{4\pi^2 n^2}{t^2}$$
$$= \frac{1 \times 4 \times \pi^2 \times (22)^2}{(44)^2}$$
$$= \pi^2 \text{ m/s}^2$$

This acceleration is directed along radius of circle.

46 The circular motion of a particle with constant speed is **[CBSE AIPMT 2005]**

(a) simple harmonic but not periodic
(b) periodic and simple harmonic
(c) neither periodic nor simple harmonic
(d) periodic but not simple harmonic

Ans. *(d)*

In a circular motion, particle repeats its motion after equal intervals of time. So, particle moving on a circular path is periodic but not simple harmonic as it does not execute to and fro motion about a fixed point.

47 A particle moves along a circle of radius $\left(\frac{20}{\pi}\right)$ m with constant tangential acceleration. If the velocity of the particle is 80 m/s at the end of the second revolution after motion has begin, the tangential acceleration is **[CBSE AIPMT 2003]**

(a) $160\,\pi$ m/s^2 (b) 40 m/s^2
(c) $40\,\pi$ m/s^2 (d) $640\,\pi$ m/s^2

Ans. *(b)*

The tangential acceleration is given by

$$a_T = r\alpha \quad \text{...(i)}$$

From 2nd equation of motion for rotational motion,

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

Here, initial angular velocity, $\omega_0 = 0$,
Final angular velocity,

$$\omega = \frac{v}{r} = \frac{80}{20/\pi}$$
$$= 4\pi \text{ rad/s}$$
$$\theta = 2 \times 2\pi \text{ rad}$$

So, angular acceleration

$$\alpha = \frac{\omega^2}{2\theta} = \frac{(4\pi)^2}{2 \times (2 \times 2\pi)} = \frac{16\pi^2}{8\pi} = 2\pi$$

Hence, from Eq. (i), we have

$$a_T = r\alpha = \frac{20}{\pi} \times 2\pi = 40 \text{ m/s}^2$$

Alternative

Initial velocity, $u = 0$
Final velocity, $v = 80$ m/s
Radius of circle $r = \left(\frac{20}{11}\right)$ m
Distance travelled, $S = 2 \times (2\pi r)$

$$= 2 \times \left(2\pi \times \frac{20}{\pi}\right)$$
$$= 80 \text{ m}$$

Now, by applying third equation of motion,

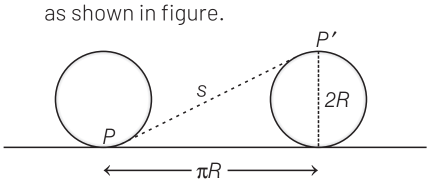
$$v^2 = u^2 + 2as$$
$$(80)^2 = 0 + 2 \times a_f \times 80$$
$$a_f = 40 \text{ m/s}^2$$

48 P is the point of contact of a wheel and the ground. The radius of wheel is 1m. The wheel rolls on the ground without slipping. The displacement of point P when wheel completes half rotation is **[CBSE AIPMT 2002]**

(a) 2 m (b) $\sqrt{\pi^2 + 4}$ m
(c) π m (d) $\sqrt{\pi^2 + 2}$ m

Ans. *(b)*

When the wheel rolls on the ground without slipping and completes half rotation, point P takes new position as P' as shown in figure.



Horizontal displacement, $x = \pi R$
Vertical displacement, $y = 2R$
Thus, displacement of the point P when wheel completes half rotation,

$$s = \sqrt{x^2 + y^2} = \sqrt{(\pi R)^2 + (2R)^2}$$
$$= \sqrt{\pi^2 R^2 + 4R^2}$$

but $R = 1$ m (given)

$$\therefore \quad s = \sqrt{\pi^2 (1)^2 + 4(1)^2} = \sqrt{\pi^2 + 4} \text{ m}$$

49 A particle of mass M is revolving along a circle of radius R and another particle of mass m is revolving in a circle of radius r. If time periods of both particles are same, then the ratio of their angular velocities is **[CBSE AIPMT 2001]**

(a) 1 (b) $\frac{R}{r}$
(c) $\frac{r}{R}$ (d) $\sqrt{\frac{R}{r}}$

Ans. *(a)*

Angular velocity of particle is given by

$$\omega = \frac{2\pi}{T}$$

or $$\omega \propto \frac{1}{T}$$

[T = time period of the particle]
It simply implies that ω does not depend on mass of the body and radius of the circle.

$$\therefore \quad \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$$

but given time period is same, i.e. $T_1 = T_2$

Hence, $$\frac{\omega_1}{\omega_2} = \frac{1}{1}$$

50 What is the linear velocity, if angular velocity vector $\boldsymbol{\omega} = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and position vector $\mathbf{r} = 5\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$? **[CBSE AIPMT 1999]**

(a) $6\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$
(b) $-18\hat{\mathbf{i}} - 13\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$
(c) $18\hat{\mathbf{i}} + 13\hat{\mathbf{k}} - 2\hat{\mathbf{k}}$
(d) $6\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$

Ans. *(b)*

The relation between linear velocity **v**, angular velocity ω and position vector **r** is given by

$$\mathbf{v} = \omega \times \mathbf{r}$$
$$= (3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + \hat{\mathbf{k}}) \times (5\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$$
$$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix}$$
$$= \hat{\mathbf{i}}\begin{vmatrix} -4 & 1 \\ -6 & 6 \end{vmatrix} - \hat{\mathbf{j}}\begin{vmatrix} 3 & 1 \\ 5 & 6 \end{vmatrix} + \hat{\mathbf{k}}\begin{vmatrix} 3 & -4 \\ 5 & -6 \end{vmatrix}$$
$$= (-24 + 6)\,\hat{\mathbf{i}} - (18 - 5)\,\hat{\mathbf{j}} + (-18 + 20)\,\hat{\mathbf{k}}$$
$$= -18\hat{\mathbf{i}} - 13\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$$

Alternative

$$\mathbf{v} = \omega \times \mathbf{r}$$
$$= (3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + \hat{\mathbf{k}}) \times (5\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$$
$$= (3\times 5)(\hat{\mathbf{i}}\times\hat{\mathbf{i}}) + [3\times(-6)](\hat{\mathbf{i}}\times\hat{\mathbf{j}})$$
$$+ (3\times 6)(\hat{\mathbf{i}}\times\hat{\mathbf{k}}) + (-4\times 5)(\hat{\mathbf{j}}\times\hat{\mathbf{i}})$$
$$+ (-4\times -6)(\hat{\mathbf{j}}\times\hat{\mathbf{j}}) + (-4\times 6)(\hat{\mathbf{j}}\times\hat{\mathbf{k}})$$
$$+ (1\times 5)(\hat{\mathbf{k}}\times\hat{\mathbf{i}}) + (1\times -6)(\hat{\mathbf{k}}\times\hat{\mathbf{j}})$$
$$+ (1\times 6)(\hat{\mathbf{k}}\times\hat{\mathbf{k}})$$

Use $\hat{\mathbf{i}}\times\hat{\mathbf{j}} = -\hat{\mathbf{j}}\times\hat{\mathbf{i}} = \hat{\mathbf{k}}$

$\hat{\mathbf{j}}\times\hat{\mathbf{k}} = -\hat{\mathbf{k}}\times\hat{\mathbf{j}} = \hat{\mathbf{i}}$

and $\hat{\mathbf{k}}\times\hat{\mathbf{i}} = -\hat{\mathbf{i}}\times\hat{\mathbf{k}} = \hat{\mathbf{j}}$

Thus, $\mathbf{v} = 0 + (-18)(\hat{\mathbf{k}}) + (18)(-\hat{\mathbf{j}})$

$$+ (-20)(-\hat{\mathbf{k}}) + 0 + (-24)(\hat{\mathbf{i}}) + (5)(\hat{\mathbf{j}})$$
$$+ (-6)(-\hat{\mathbf{i}}) + 0$$
$$= -18\hat{\mathbf{k}} - 18\hat{\mathbf{j}} + 20\hat{\mathbf{k}} - 24\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 6\hat{\mathbf{i}}$$
$$= -18\hat{\mathbf{i}} - 13\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$$

51 A body is whirled in a horizontal circle of radius 20 cm. It has an angular velocity of 10 rad/s. What is its linear velocity at any point on circular path? **[CBSE AIPMT 1996]**

(a) $\sqrt{2}$ m/s
(b) 2 m/s
(c) 10 m/s
(d) 20 m/s

Ans. *(b)*

Linear speed = radius × angular speed

or $v = r\omega$

Here, $r = 20\text{ cm} = 0.20\text{ m}$

$\omega = 10\text{ rad/s}$

$\therefore$ $v = 0.20 \times 10$

$\Rightarrow$ $v = 2\text{ m/s}$

52 When milk is churned, cream gets separated due to **[CBSE AIPMT 1991]**

(a) centripetal force
(b) centrifugal force
(c) frictional force
(d) gravitational force

Ans. *(b)*

By the concept of centrifugal force cream is separated from milk. A mass *m* of milk revolving at a distance *r* from the axis of rotation of the centrifuge requires a centripetal force $mr\omega^2$, where ω is the angular speed of the centrifuge. If in place of this mass of milk, lighter particles of mass (cream) $m'(m' < m)$ are present, then the centripetal force $(mr\omega^2)$ on the milk will be greater than the centripetal force $(m' r\omega^2)$ on the cream.

As a result, cream move towards the axis of rotation under the effect of the net force $(m - m')\, r\omega^2$. When the centrifuge is stopped , the cream is found at the top and milk at the bottom.

53 An electric fan has blades of length 30 cm measured from the axis of rotation. If the fan is rotating at 120 rev/min, the acceleration of a point on the tip of the blade is **[CBSE AIPMT 1990]**

(a) 1600 ms^{-2} (b) 47.4 ms^{-2}
(c) 23.7 ms^{-2} (d) 50.55 ms^{-2}

Ans. *(b)*

Centripetal acceleration of rotating body is given by

$$a_c = \frac{v^2}{r} = \frac{r^2\omega^2}{r} = r\omega^2 \qquad (\text{as } v = r\omega)$$

Where, ω is angular frequency, but $\omega = 2\pi\nu$, where ν is frequency of rotation.

$\therefore$ $a_c = r(2\pi\nu)^2 = r4\pi^2\nu^2$

Here, $r = 30\text{ cm} = 30\times 10^{-2}\text{ m} = 0.30\text{ m}$

$$v = 120\text{ rev/m} = \frac{120}{60}\text{ rev/s} = 2\text{ rev/s}$$

$\therefore$ $a_c = (0.30\times 4\times 3.14\times 3.14\times 2\times 2)$
$= 47.4\,ms^{-2}$