09

Mechanical Properties of Fluids

TOPIC 1

Pressure and Pascal's Law

01 A barometer is constructed using a liquid (density $= 760\,kg/m^3$). What would be the height of the liquid column, when a mercury barometer reads 76 cm? (Density of mercury $= 13600\,kg/m^3$) **[NEET (Oct.) 2020]**

(a) 1.36 m
(b) 13.6 m
(c) 136 m
(d) 0.76 m

Ans. ***(b)***

Density of liquid, $\rho_l = 760\,kg/m^3$

Density of mercury, $\rho_m = 13600\,kg/m^3$

Height of liquid column in mercury barometer,

$$h_m = 76\text{ cm} = 0.76\text{ m}$$

If height of liquid in liquid column be h_l, then

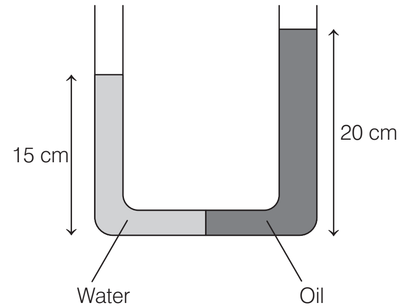
$$p_{\text{liquid}} = p_{\text{mercury}}$$

$$\Rightarrow \quad h_l \rho_l g = h_m \rho_m g$$

$$\Rightarrow \quad h_l = \frac{h_m \rho_m}{\rho_l} = \frac{0.76 \times 13600}{760}$$

$$= 13.6\text{ m}$$

02 In a U-tube as shown in a figure, water and oil are in the left side and right side of the tube respectively. The heights from the bottom for water and oil columns are 15 cm and 20 cm respectively. The density of the oil is [take $\rho_{\text{water}} = 1000\,kg/m^3$] **[NEET (Odisha) 2019]**

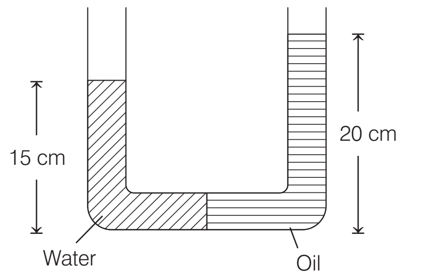


(a) $1200\,kg/m^3$ (b) $750\,kg/m^3$
(c) $1000\,kg/m^3$ (d) $1333\,kg/m^3$

Ans. ***(b)***

According to Pascal's law "Pressure applied to an enclosed fluid is transmitted undiminished to every point of the fluid and the walls of the containing vessel."

In the given situation as shown in the figure below



Pressure due to water column of height 15 cm = Pressure due to oil column of height 20 cm

$$\Rightarrow \quad h_w \rho_w g = h_0 \rho_0 g$$

$$15\rho_w = 20\rho_0$$

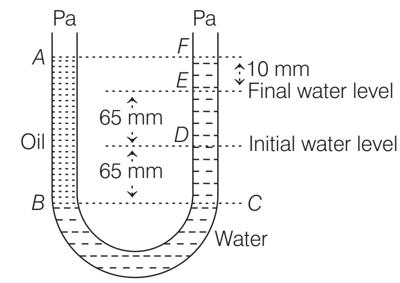
$$\Rightarrow \quad \rho_0 = \frac{15}{20}\rho_\omega$$

$$\rho_0 = \frac{15}{20} \times 1000$$

$$(\because \text{given, } \rho_w = 1000\,kg\,m^{-3})$$

$$= 750\,kgm^{-3}$$

03 A U tube with both ends open to the atmosphere, is partially filled with water. Oil, which is immiscible with water, is poured into one side until it stands at a distance of 10 mm above the water level on the other side. Meanwhile the water rises by 65 mm from its original level (see diagram). The density of the oil is **[NEET 2017]**



(a) $650\,kg\,m^{-3}$ (b) $425\,kg\,m^{-3}$
(c) $800\,kg\,m^{-3}$ (d) $928\,kg\,m^{-3}$

Ans. ***(d)***

Thinking Process Pressure of two points lie in the same horizontal level should be same and $p = hdg$

Both ends of the U tube are open, so the pressure on both the free surfaces must be equal.

i.e., $\quad p_1 = p_2$

$$h_{\text{oil}} \cdot S_{\text{oil}}\ g = h_{\text{water}} \cdot S_{\text{water}} \cdot g$$

S_{oil} = specific density of oil

$$S_{oil} = \frac{h_{water} \cdot S_{water} \cdot g}{h_{oil} \cdot g}$$

From figure $S_{oil} = \frac{(65+65) \times 1000}{(65+65+10)}$

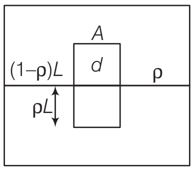
$= 928 \text{ kgm}^{-3}$

04 Two non-mixing liquids of densities ρ and $n\rho (n > 1)$ are put in a container. The height of each liquid is h. A solid cylinder of length L and density d is put in this container. The cylinder floats with its axis vertical and length pL $(p < 1)$ in the denser liquid. The density d is equal to **[NEET 2016]**

(a) $\{2+(n+1)p\}\rho$ (b) $\{2+(n-1)p\}\rho$
(c) $\{1+(n-1)p\}\rho$ (d) $\{1+(n+1)p\}\rho$

Ans. *(c)*

According to question, the situation can be drawn as following.



Applying Archemedies principle

Weight of cylinder

$= (\text{upthrust})_1 + (\text{upthrust})_2$

i.e. $ALdg = (1-P)LA\rho g + (PLA)n\rho g$

$\Rightarrow d = (1-P)\rho + Pn\rho$

$= \rho - P\rho + nP\rho$

$= \rho + (n-1)P\rho = \rho[1+(n-1)\rho]$

05 The approximate depth of an ocean is 2700 m. The compressibility of water is 45.4×10^{-11} Pa^{-1} and density of water is 10^3 kg/m^3. What fractional compression of water will be obtained at the bottom of the ocean? **[CBSE AIPMT 2015]**

(a) 0.8×10^{-2} (b) 1.0×10^{-2}
(c) 1.2×10^{-2} (d) 1.4×10^{-2}

Ans. *(c)*

Given $d = 2700$ m $\Rightarrow \rho = 10^3 \text{ kg/m}^3$

Compressibility $= 45.4 \times 10^{-11}$ per pascal

The pressure at the bottom of ocean is given by

$p = \rho g d$

$= 10^3 \times 10 \times 2700 = 27 \times 10^6$ Pa

So, fractional compression

$=$ compressibility $\times$ pressure

$= 45.4 \times 10^{-11} \times 27 \times 10^6 = 1.2 \times 10^{-2}$

TOPIC 2

Bernoullis Principle and Viscosity

06 The velocity of a small ball of mass M and density d when dropped in a container filled with glycerine becomes constant after some time. If the density of glycerine is $d/2$, then the viscous force acting on the ball will be **[NEET 2021]**

(a) $\frac{Mg}{2}$ (b) Mg
(c) $\frac{3}{2}Mg$ (d) $2\,Mg$

Ans. *(a)*

Given, the density of the small ball is d

The mass of the small ball is M

The density of the glycerine is $d/2$.

As we know that,

viscous force = weight – buoyant force

Viscous force $= Vd_1 g - Vd_2 g$

Here, V is the volume of submerged bodies,

g is the acceleration due to gravity,

d_1 is the density of the small ball,

d_2 is the density of the glycerine,

$d_1 = d$ and $d_2 = d/2$

Substituting the given values in the viscous force expression, we get

Viscous force $= Vdg - V\frac{d}{2}g$

Viscous force $= \frac{Vdg}{2} = \frac{Mg}{2}$ $(\because M = d \times V)$

07 Two small spherical metal balls, having equal masses, are made from materials of densities ρ_1 and ρ_2 $(\rho_1 = 8\rho_2)$ and have radii of 1 mm and 2 mm, respectively. They are made to fall vertically (from rest) in viscous medium whose coefficient of viscosity equals η and whose density is $0.1\rho_2$. The ratio of their terminal velocities would be **[NEET (Odisha) 2019]**

(a) $\frac{79}{72}$ (b) $\frac{19}{36}$ (c) $\frac{39}{72}$ (d) $\frac{79}{36}$

Ans. *(d)*

The terminal velocity achieved by ball in a viscous fluid is

$$v_t = \frac{2(\rho-\sigma)r^2 g}{9\eta}$$

where, $\rho =$ density of metal of ball,

$\sigma =$ density of viscous medium,

$r =$ radius of ball and

$\eta =$ coefficient of viscosity of medium

Terminal velocity of first ball,

$$v_{t_1} = \frac{2(\rho_1-\sigma)r_1^2 g}{9\eta}$$

$$= \frac{2(8\rho_2-\sigma)r_1^2 g}{9\eta} \quad \text{...(i)} [\because \rho_1 = 8\rho_2]$$

Similarly, for second ball

$$v_{t_2} = \frac{2(\rho_2-\sigma)r_2^2 g}{9\eta} \quad \text{...(ii)}$$

From Eq. (i) and (ii), we get

$$\frac{v_{t_1}}{v_{t_2}} = \frac{2(8\rho_2-\sigma)r_1^2 g}{2(\rho_2-\sigma)r_2^2 g} \times \frac{9\eta}{9\eta}$$

$$= \left(\frac{8\rho_2 - 0.1\rho_2}{\rho_2 - 0.1\rho_2}\right)\left(\frac{r_1}{r_2}\right)^2$$

$$\text{...(iii)} [\because \sigma = 0.1\rho_2]$$

Here, $r_1 = 1$ mm and $r_2 = 2$ mm

Substituting these values in Eq. (iii), we get

$$\Rightarrow \frac{v_{t_1}}{v_{t_2}} = \left(\frac{7.9\rho_2}{0.9\rho_2}\right)\left(\frac{1}{2}\right)^2$$

$$= \frac{79}{36}$$

08 A small hole of area of cross-section 2 mm^2 is present near the bottom of a fully filled open tank of height 2 m. Taking $g = 10 \text{ m/s}^2$, the rate of flow of water through the open hole would be nearly **[NEET (National) 2019]**

(a) $8.9 \times 10^{-6} \text{ m}^3/\text{s}$
(b) $2.23 \times 10^{-6} \text{ m}^3/\text{s}$
(c) $6.4 \times 10^{-6} \text{ m}^3/\text{s}$
(d) $12.6 \times 10^{-6} \text{ m}^3/\text{s}$

Ans. *(d)*

The rate of liquid flow moving with velocity v through an area a is given by

Rate (R) = Area (a) $\times$ Velocity (v)

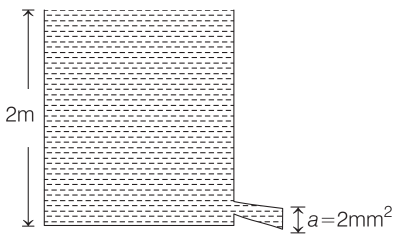
Given, area of hole,

$a = 2 \text{ mm}^2$

$= 2 \times 10^{-6} \text{ m}^2$

height of tank, $h = 2$ m.

The given situation can also be depicted as shown in the figure below.



As the velocity of liquid flow is given as $v = \sqrt{2gh}$

$\therefore R = av = a\sqrt{2gh}$

Substituting the given values, we get

$$R = 2 \times 10^{-6} \times \sqrt{2 \times 10 \times 2}$$
$$= 2 \times 10^{-6} \times 6.32 = 12.64 \times 10^{-6}\ \mathrm{m}^3/\mathrm{s}$$
$$\simeq 12.6 \times 10^{-6}\ \mathrm{m}^3/\mathrm{s}$$

09 A small sphere of radius r falls from rest in a viscous liquid. As a result, heat is produced due to viscous force. The rate of production of heat when the sphere attains its terminal velocity, is proportional to **[NEET 2018]**

(a) r^5 (b) r^2
(c) r^3 (d) r^4

Ans. *(a)*

Key Concept The rate of heat generation is equal to the rate of work done by the viscous force which in turn is equal to its power.

Rate of heat produced, $\frac{dQ}{dt} = F \times v_T$

where, F is the viscous force and v_T is the terminal velocity.

As, $F = 6\pi\eta r v_T$

$$\Rightarrow \quad \frac{dQ}{dt} = 6\pi\eta r v_T \times v_T$$
$$= 6\pi\eta r v_T^2 \quad \text{...(i)}$$

From the relation for terminal velocity,

$$v_T = \frac{2}{9}\frac{r^2(\rho - \sigma)}{\eta}g, \text{ we get}$$
$$v_T \propto r^2 \quad \text{...(ii)}$$

From Eq. (ii), we can rewrite Eq. (i) as

$$\frac{dQ}{dt} \propto r \cdot (r^2)^2$$

or $$\frac{dQ}{dt} \propto r^5$$

10 A wind with speed 40 m/s blows parallel to the roof of a house. The area of the roof is $250\,\mathrm{m}^2$. Assuming that the pressure inside the house is atmospheric pressure, the force exerted by the wind on the roof and the direction of the force will be ($p_{\mathrm{air}} = 1.2\ \mathrm{kg/m}^3$) **[CBSE AIPMT 2015]**

(a) 4.8×10^5 N, downwards
(b) 4.8×10^5 N, upwards
(c) 2.4×10^5 N, upwards
(d) 2.4×10^5 N, downwards

Ans. *(c)*

From Bernoulli's theorem

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

where, p_1, p_2 are pressure inside and outside the roof and v_1, v_2 are velocities of wind inside and outside the roof. Neglect the width of the roof.

Pressure difference is

$$p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$
$$= \frac{1}{2} \times 1.2(40^2 - 0)$$
$$= 960\ \mathrm{N/m}^2$$

Force acting on the roof is given by

$$F = (p_1 - p_2)A = 960 \times 250$$
$$= 24 \times 10^4\,\mathrm{N} = 2.4 \times 10^5\ \mathrm{N}$$

As the pressure inside the roof is more than outside to it. So the force will act in the upward direction,

i.e. $\mathbf{F} = 2.4 \times 10^5$ N- upwards.

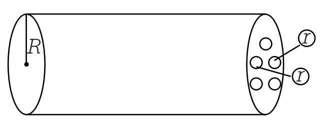
11 The cylindrical tube of a spray pump has radius R, one end of which has n fine holes, each of radius r. If the speed of the liquid in the tube is v, the speed of the ejection of the liquid through the holes is **[CBSE AIPMT 2015]**

(a) $\frac{vR^2}{n^2r^2}$ (b) $\frac{vR^2}{nr^2}$ (c) $\frac{vR^2}{n^3r^2}$ (d) $\frac{v^2R}{nr}$

Ans. *(b)*

Key Concept During the streamline flow of viscous and incompressible fluid through a pipe varying cross-section, the product of area of cross-section and normal fluid velocity (Av) remains constant throughout the flow.

Consider a cylindrical tube of a spray pump has radius R, one end having n fine holes, each of radius r and speed of liquid in the tube is v as shown in figure.



According to equation of continuity,

$$Av = \text{constant}$$

where, A is a cylindrical tube and v is velocity of liquid in a tube.

Volume in flow rate

= volume in out flow rate

$$\pi R^2 v = n\,\pi r^2 v' \Rightarrow v' = \frac{R^2 v}{nr^2}$$

Thus, speed of the ejection of the liquid through the holes is $\frac{R^2v}{nr^2}$.

TOPIC 3
Surface Tension, Excess Pressure and Capillarity

12 A liquid does not wet the solid surface if angle of contact is **[NEET (Oct.) 2020]**

(a) equal to 45°
(b) equal to 60°
(c) greater than 90°
(d) zero

Ans. *(c)*

A liquid does not wet the solid surface, if the angle of contact is obtuse i.e., $\theta > 90^\circ$.

13 A capillary tube of radius r is immersed in water and water rises in it to a height h. The mass of the water in the capillary tube is 5 g. Another capillary tube of radius $2r$ is immersed in water. The mass of water that will rise in this tube is **[NEET (Sep.) 2020]**

(a) 5.0 g (b) 10.0 g
(c) 20.0 g (d) 2.5 g

Ans. *(d)*

Relation for height of water in capillary tube is

$$h = \frac{2S\cos\theta}{\rho g r}$$
$$\Rightarrow \quad h \propto \frac{1}{r}$$
$$\Rightarrow \quad \frac{h_1}{h_2} = \frac{r_2}{r_1} = \frac{2r}{r} = 2$$

As $m = A \cdot h \cdot \rho$

$$\therefore \quad \frac{m_2}{m_1} = \frac{Ah_2\rho}{Ah_1\rho} = \frac{h_2}{h_1} = \frac{1}{2}$$
$$\Rightarrow \quad m_2 = \frac{m_1}{2} = \frac{5}{2} = 2.5\,\mathrm{g}$$

Hence, correct option is (d).

14 A soap bubble, having radius of 1 mm, is blown from a detergent solution having a surface tension of 2.5×10^{-2} N/m. The pressure inside the bubble equals at a point Z_0 below the free surface of water in a container. Taking $g = 10\ m/s^2$, density of water $= 10^3\ kg/m^3$, the value of Z_0 is **[NEET (National) 2019]**

(a) 10 cm
(b) 1 cm
(c) 0.5 cm
(d) 100 cm

Ans. *(b)*

The excess pressure inside a soap bubble of radius r is given by

$$p = \frac{4T}{r}$$

where, T = surface tension.

If p_0 be the outside pressure from the water, then total pressure inside the bubble becomes

$$p_1 = p_0 + \frac{4T}{r} \quad \text{...(i)}$$

The pressure at the depth Z_0 below the water surface is

$$p_2 = p_0 + Z_0 \rho g \quad \text{...(ii)}$$

As it is given that the pressure inside the bubble is same as the pressure at depth Z_0, then equating

Eqs. (i) and (ii), we get

$$p_0 + \frac{4T}{r} = p_0 + Z_0 \rho g$$

$$\Rightarrow \quad Z_0 = \frac{4T}{r\rho g} \quad \text{...(iii)}$$

Here, $T = 2.5 \times 10^{-2}$ N/m, $\rho = 10^3\ kg/m^3$, $g = 10\ ms^{-2}$ and $r = 1\ mm = 1 \times 10^{-3}$ m

Substituting these values in Eq. (iii), we get

$$Z_0 = \frac{4 \times 2.5 \times 10^{-2}}{1 \times 10^{-3} \times 10^3 \times 10}$$

$$= 10 \times 10^{-3}\ m$$

$$= 1\ cm$$

15 A rectangular film of liquid is extended from $(4\ cm \times 2\ cm)$ to $(5\ cm \times 4 \times cm)$. If the work done is 3×10^{-4} J, the value of the surface tension of the liquid is **[NEET 2016]**

(a) $0.250\ Nm^{-1}$ (b) $0.125\ Nm^{-1}$
(c) $0.2\ Nm^{-1}$ (d) $8.0\ Nm^{-1}$

Ans. *(b)*

Key Idea Increase in surface energy = work done in area $\times$ surface tension

$\because$ Increase in surface area,

$\Delta A = (5 \times 4 - 4 \times 2) \times 2$

($\because$ film has two surfaces)

$= (20 - 8) \times 2\ cm^2 = 24\ cm^2$

$= 24 \times 10^{-4}\ m^2$

So, work done, $W = T \cdot \Delta A$

$$3 \times 10^{-4} = T \times 24 \times 10^{-4}$$

$$\therefore \quad T = \frac{1}{8} = 0.125\ N/m$$

16 Three liquids of densities ρ_1, ρ_2 and ρ_3 (with $\rho_1 > \rho_2 > \rho_3$), having the same value of surface tension T, rise to the same height in three identical capillaries. The angles of contact θ_1, θ_2 and θ_3 obey **[NEET 2016]**

(a) $\frac{\pi}{2} > \theta_1 > \theta_2 > \theta_3 \geq 0$
(b) $0 \leq \theta_1 < \theta_2 < \theta_3 < \frac{\pi}{2}$
(c) $\frac{\pi}{2} < \theta_1 < \theta_2 < \theta_3 < \pi$
(d) $\pi > \theta_1 > \theta_2 > \theta_3 > \frac{\pi}{2}$

Ans. *(b)*

According to ascent formula for capillary tube,

$$h = \frac{2T \cos\theta}{\rho g r}$$

$$\therefore \quad \frac{\cos\theta_1}{\rho_1} = \frac{\cos\theta_2}{\rho_2} = \frac{\cos\theta_3}{\rho_3}$$

Thus, $\cos\theta \propto \rho$

$\therefore \quad \rho_1 > \rho_2 > \rho_3$

$\therefore \quad \cos\theta_1 > \cos\theta_2 > \cos\theta_3$

$$0 \leq \theta_1 < \theta_2 < \theta_3 < \frac{\pi}{2}$$

17 A certain number of spherical drops of a liquid of radius r coalesce to form a single drop of radius R and volume V. If T is the surface tension of the liquid, then **[CBSE AIPMT 2014]**

(a) energy $= 4VT\left(\frac{1}{r} - \frac{1}{R}\right)$ is released
(b) energy $= 3VT\left(\frac{1}{r} + \frac{1}{R}\right)$ is absorbed
(c) energy $= 3VT\left(\frac{1}{r} - \frac{1}{R}\right)$ is released
(d) energy is neither released nor absorbed

Ans. *(c)*

As energy released $= (A_f - A_i)T$

where, $A_i = 4\pi R^2 = \frac{3}{3} \times 4\pi \frac{R^3}{R} = \frac{3V}{R}$

and $A_f = 4\pi r^2 = \frac{V}{\frac{4}{3}\pi r^2} 4\pi r^2 = \frac{3V}{r}$

$\therefore$ Energy released $= 3VT\left[\frac{1}{r} - \frac{1}{R}\right]$

18 The wettability of a surface by a liquid depends primarily on **[NEET 2013]**

(a) viscosity
(b) surface tension
(c) density
(d) angle of contact between the surface and the liquid

Ans. *(d)*

The wettability of a surface by a liquid depends primarily on angle of contact between the surface and the liquid.