07

Gravitation

TOPIC 1

Kepler's Law and Universal Law of Gravitation

01 The time period of a geo-stationary satellite is 24 h, at a height $6R_E$ (R_E is the radius of earth) from surface of earth. The time period of another satellite whose height is $2.5\,R_E$ from surface will be **[NEET (Odisha) 2019]**

(a) $6\sqrt{2}$ h (b) $12\sqrt{2}$ h (c) $\frac{24}{2.5}$ h (d) $\frac{12}{2.5}$ h

Ans. ***(a)***

From Kepler's third law, the time period of revolution of satellite around earth is

$$T^2 \propto r^3 \quad \text{or} \quad T \propto r^{3/2} \qquad \text{...(i)}$$

where, r is the radius of satellite's orbit.

Here, $r_1 = 6R_E + R_E, T_1 = 24\,\text{h}$

$r_2 = 2.5R_E + R_E, T_2 = ?$

where R_E = radius of earth

So, from Eq. (i), we get

$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$$

$$\frac{24}{T_2} = \left(\frac{6R_E + R_E}{2.5R_E + R_E}\right)^{3/2} = \left(\frac{7}{3.5}\right)^{3/2}$$

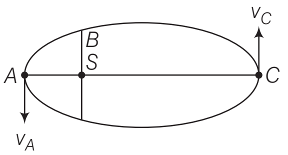
$$\Rightarrow \quad T_2 = \frac{24}{(2)^{3/2}} = \frac{24}{2\sqrt{2}} = \frac{12}{\sqrt{2}} = 6\sqrt{2}\,\text{h}$$

02 The kinetic energies of a planet in an elliptical orbit about the Sun, at positions A, B and C are K_A, K_B and K_C, respectively. AC is the major axis and SB is perpendicular to AC at the position of the Sun S as shown in the figure. Then **[NEET 2018]**

(a) $K_B < K_A < K_C$ (b) $K_A > K_B > K_C$
(c) $K_A < K_B < K_C$ (d) $K_B > K_A > K_C$

Ans. ***(b)***

According to the question,



The figure above shows an ellipse traced by a planet around the Sun, S. The closed point A is known as perihelion (perigee) and the farthest point C is known as aphelion (apogee).

Since, as per the result the Kepler's second law of area, that the planet will move slowly ($v_{\min}$) only when it is farthest from the Sun and more rapidly ($v_{\max}$) when it is nearest to the Sun.

Thus, $\quad v_A = v_{\max}, v_C = v_{\min}$

Therefore, we can write

$$v_A > v_B > v_C \qquad \text{...(i)}$$

Kinetic energy of the planet at any point is given as, $K = \frac{1}{2}mv^2$

Thus, at A, $K_A = \frac{1}{2}mv_A^2$

At B, $\quad K_B = \frac{1}{2}mv_B^2$

At C, $\quad K_C = \frac{1}{2}mv_C^2$

From Eq. (i), we can write

$$K_A > K_B > K_C$$

03 Kepler's third law states that square of period of revolution (T) of a planet around the sun, is proportional to third power of average distance r between the sun and planet i.e. $T^2 = Kr^3$, here K is constant.

If the masses of the sun and planet are M and m respectively, then as per Newton's law of gravitation force of attraction between them is **[CBSE AIPMT 2015]**

$F = \frac{GMm}{r^2}$, here G is gravitational constant. The relation between G and K is described as

(a) $GK = 4\pi^2$ (b) $GMK = 4\pi^2$
(c) $K = G$ (d) $K = \frac{I}{G}$

Ans. ***(b)***

The gravitational force of attraction between the planet and sun provide the centripetal force

i.e. $$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\Rightarrow \qquad v = \sqrt{\frac{GM}{r}}$$

The time period of planet will be

$$T = \frac{2\pi r}{v}$$

$$\Rightarrow T^2 = \frac{4\pi^2 r^2}{\frac{GM}{r}} = \frac{4\pi^2 r^3}{GM} \qquad \text{...(i)}$$

Also from Kepler's third law

$$T^2 = Kr^3 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

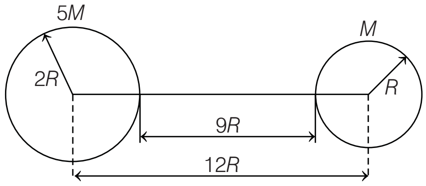
$$\frac{4\pi^2 r^3}{GM} = Kr^3 \Rightarrow GMK = 4\pi^2$$

04 Two spherical bodies of masses M and $5M$ and radii R and $2R$ are released in free space with initial separation between their centres equal to $12R$. If they attract each other due to gravitational force only, then the distance covered by the smaller body before collision is **[CBSE AIPMT 2015]**

(a) $2.5\,R$ (b) $4.5\,R$
(c) $7.5\,R$ (d) $1.5\,R$

Ans. *(c)*

Suppose, the smaller body cover a distance x before collision, then



$$Mx = 5M(9R - x)$$

or $\quad x = 45R - 5x$

or $\quad x = \dfrac{45R}{6} = 7.5\,R$

05 A geostationary satellite is orbiting the earth at a height of $5R$ above that surface of the earth, R being the radius of the earth. The time period of another satellite in hour at a height of $2R$ from the surface of the earth is **[CBSE AIPMT 2012]**

(a) 5 (b) 10
(c) $6\sqrt{2}$ (d) $6/\sqrt{2}$

Ans. *(c)*

From Kepler's third law

$$T^2 \propto r^3$$

$$\left[\begin{array}{l}\text{where, } T = \text{time period of satellite} \\ r = \text{radius of elliptical orbit (semi} \\ \hfill \text{major axis)}\end{array}\right]$$

Hence, $\quad T_1^2 \propto r_1^3$ and $T_2^2 \propto r_2^3$

So, $\quad \dfrac{T_2^2}{T_1^2} = \dfrac{r_2^3}{r_1^3} = \dfrac{(3R)^3}{(6R)^3}$ or $\dfrac{T_2^2}{T_1^2} = \dfrac{1}{8}$

$$T_2^2 = \frac{1}{8}T_1^2 \Rightarrow T_2 = \frac{24}{2\sqrt{2}} = 6\sqrt{2}\text{ h}$$

06 A planet moving along an elliptical orbit is closest to the sun at a distance r_1 and farthest away at a distance of r_2. If v_1 and v_2 are the linear velocities at these points respectively, then the ratio $\dfrac{v_1}{v_2}$ is

[CBSE AIPMT 2011]

(a) r_2/r_1 (b) $(r_2/r_1)^2$
(c) r_1/r_2 (d) $(r_1/r_2)^2$

Ans. *(a)*

Concept Apply conservation of angular momentum.

From the law of conservation of angular momentum, $L_1 = L_2$

So, $mr_1v_1 = mr_2v_2$

$$\left[\begin{array}{r}\text{where, } m = \text{mass the of planet} \\ r = \text{radius of orbit} \\ v = \text{velocity of the planet}\end{array}\right]$$

$$r_1v_1 = r_2v_2 \Rightarrow \frac{v_1}{v_2} = \frac{r_2}{r_1}$$

07 Two satellites of the earth, S_1 and S_2 are moving in the same orbit. The mass of S_1 is four times the mass of S_2. Which one of the following statements is true? **[CBSE AIPMT 2007]**

(a) The time period of S_1 is four times that of S_2
(b) The potential energies of the earth and satellite in the two cases are equal
(c) S_1 and S_2 are moving with the same speed
(d) The kinetic energies of the two satellites are equal

Ans. *(c)*

When two satellites of the earth are moving in same orbit, then time period of both are equal.

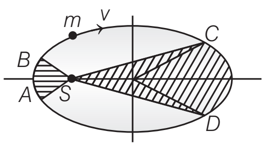
From Kepler's third law $T^2 \propto r^3$

Time period is independent of mass, hence their time periods will be equal.

The potential energy and kinetic energy are mass dependent, hence the potential energy and kinetic energy of satellites are not equal.

But, if they are orbiting in a same orbit, then they have equal orbital speed.

08 The figure shows elliptical orbit of a planet m about the sun S. The shaded area SCD is twice the shaded area SAB. If t_1 is the time for the planet to move from C to D and t_2 is the time to move from A to B, then **[CBSE AIPMT 2009]**



(a) $t_1 > t_2$ (b) $t_1 = 4\,t_2$
(c) $t_1 = 2\,t_2$ (d) $t_1 = t_2$

Ans. *(c)*

Concept Apply Kepler's second law.

The line joining the sun to the planet sweeps out equal areas in equal time interval i.e. areal velocity is constant.

$$\frac{dA}{dt} = \text{constant} \quad \text{or} \quad \frac{A_1}{t_1} = \frac{A_2}{t_2}$$

where, $A_1 =$ area under SCD

$A_2 =$ area under ABS

$$\Rightarrow \quad t_1 = \frac{A_1}{A_2}t_2$$

Given, $\quad A_1 = 2A_2$

$\therefore \quad t_1 = 2\,t_2$

09 Two spheres of masses m and M are situated in air and the gravitational force between them is F. The space around the masses is now filled with a liquid of specific gravity 3. The gravitational force will now be **[CBSE AIPMT 2003]**

(a) $\dfrac{F}{3}$ (b) $\dfrac{F}{9}$ (c) $3F$ (d) F

Ans. *(d)*

According to Newton's law of gravitation, the force between two spheres is given by,

$$F = \frac{GMm}{r^2}$$

From the relation, we can say the gravitational force does not depend on the medium between two spheres hence, it remains same, i.e. F.

10 The period of revolution of the planet A round the sun is 8 times that of B. The distance of A from the sun is how many times greater than that of B from the sun? **[CBSE AIPMT 1997]**

(a) 5 (b) 4 (c) 3 (d) 2

Ans. *(b)*

According to Kepler's third law

$$T^2 \propto r^3$$

where, $\quad T =$ Time period of revolution

$r =$ Semi major axis

$$\therefore \quad \frac{T_A^2}{T_B^2} = \frac{r_A^3}{r_B^3}$$

$$\therefore \quad \frac{r_A}{r_B} = \left(\frac{T_A}{T_B}\right)^{2/3} = (8)^{2/3} = 2^{3\times\frac{2}{3}} = 4$$

or $\quad r_A = 4r_B$

11 A satellite A of mass m is at a distance r from the surface of the earth. Another satellite B of mass $2m$ is at a distance of $2r$ from the earth's surface. Their time periods are in the ratio of **[CBSE AIPMT 1993]**

(a) 1 : 2 (b) 1 : 16
(c) 1 : 32 (d) $1:2\sqrt{2}$

Ans. *(d)*

According to Kepler's third law, the square of the time period of revolution of a planet around the sun is directly proportional to the cube of semi major-axis of its elliptical orbit i.e.

$$T^2 \propto r^3$$

where, T = time taken by the planet to go once around the sun.

r = semi-major axis of the elliptical orbit

$$\therefore \quad \frac{T_1^2}{T_2^2} = \frac{(r)^3}{(2r)^3} = \frac{1}{8} \Rightarrow \frac{T_1}{T_2} = \frac{1}{2\sqrt{2}}$$

12 If the gravitational force between two objects were proportional to $\frac{1}{R}$ (and not as $\frac{1}{R^2}$), where R is separation between them, then a particle in circular orbit under such a force would have its orbital speed v proportional to **[CBSE AIPMT 1989]**

(a) $\frac{1}{R^2}$ (b) R^0

(c) R (d) $\frac{1}{R}$

Ans. *(b)*

According to question, gravitational force between two objects

$$F = \frac{k}{R}$$

In equilibrium, the gravitational force provides the required centripetal force to the particle

$$\therefore \quad \frac{mv^2}{R} = \frac{k}{R}$$

Hence, $v \propto R^0$

13 The distances of two planets from the sun are 10^{13} and 10^{12} m respectively. The ratio of time periods of these two planets is **[CBSE AIPMT 1988]**

(a) $\frac{1}{\sqrt{10}}$ (b) 100

(c) $10\sqrt{10}$ (d) $\sqrt{10}$

Ans. *(c)*

According to Kepler's third law (or law of periods) the square of the time taken to complete the orbit (time period T) is proportional to the cube of the semi-major axis (r) of the elliptical orbit

i.e. $T^2 \propto r^3$

Here, $r_1 = 10^{13}$ m, $r_2 = 10^{12}$ m

$$\therefore \quad \frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3} = \frac{(10^{13})^3}{(10^{12})^3}$$

or $$\frac{T_1^2}{T_2^2} = \frac{10^{39}}{10^{36}} = 10^3$$

or $$\frac{T_1}{T_2} = 10\sqrt{10}$$

14 The largest and the shortest distance of the earth from the sun are r_1 and r_2. Its distance from the sun when it is perpendicular to the major axis of the orbit drawn from the sun **[CBSE AIPMT 1988]**

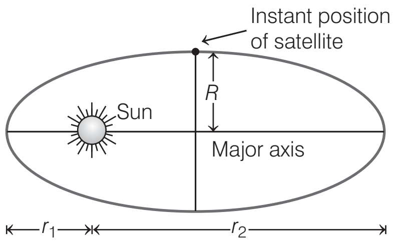
(a) $\frac{r_1 + r_2}{4}$ (b) $\frac{r_1 + r_2}{r_1 - r_2}$

(c) $\frac{2 r_1 r_2}{r_1 + r_2}$ (d) $\frac{r_1 + r_2}{3}$

Ans. *(c)*

From the property of ellipse,

$$\frac{2}{R} = \frac{1}{r_1} + \frac{1}{r_2}$$



or $$\frac{2}{R} = \frac{r_1 + r_2}{r_1 r_2} \Rightarrow R = \frac{2 r_1 r_2}{r_1 + r_2}$$

TOPIC 2

Acceleration Due to Gravity and Gravitational PE

15 What is the depth at which the value of acceleration due to gravity becomes $1/n$ times the value that the surface of earth? (Radius of earth $= R$) **[NEET (Oct.) 2020]**

(a) R/n^2 (b) $R(n-1)/n$

(c) $Rn/(n-1)$ (d) R/n

Ans. *(b)*

Radius of earth $= R$

Let at depth d, gravitational acceleration becomes $\frac{g}{n}$.

i.e., $$g_d = \frac{g}{n} \Rightarrow g\left(1 - \frac{d}{R}\right) = \frac{g}{n}$$

$$\Rightarrow \quad 1 - \frac{d}{R} = \frac{1}{n} \Rightarrow 1 - \frac{1}{n} = \frac{d}{R}$$

$$\Rightarrow \quad \frac{n-1}{n} = \frac{d}{R} \Rightarrow d = \left(\frac{n-1}{n}\right)R$$

16 A body weighs 72 N on the surface of the earth. What is the gravitational force on it, at a height equal to half of radius of the earth? **[NEET (Sep.) 2020]**

(a) 32 N (b) 30 N (c) 24 N (d) 48 N

Ans. *(a)*

Given, $w = mg = 72$ N (on the surface of earth).

At height equal to half of radius of the earth, $\left(\text{i.e.}\, h = \frac{R}{2}\right)$

Acceleration due to gravity,

$$g' = g\left(\frac{R}{R+h}\right)^2$$

$$= g\left(\frac{R}{R + R/2}\right)^2 = g\left(\frac{4R^2}{9R^2}\right)$$

$$\Rightarrow \quad g' = \frac{4}{9} g$$

$$\Rightarrow \quad mg' = \frac{4}{9} mg = \frac{4}{9} \times 72 = 32 \text{ N}$$

$$w' = 32 \text{ N}$$

Hence, correct option is (a).

17 Assuming that the gravitational potential energy of an object at infinity is zero, the change in potential energy (final – initial) of an object of mass m, when taken to a height h from the surface of earth (of radius R) is given by, **[NEET (Odisha) 2019]**

(a) $-\frac{GMm}{R+h}$ (b) $\frac{GMmh}{R(R+h)}$

(c) mgh (d) $\frac{GMm}{R+h}$

Ans. *(b)*

The gravitational potential energy of an object placed at earth's surface is

$$U_1 = -\frac{GMm}{R} \quad \text{...(i)}$$

where, G = gravitational constant,

M = Mass of earth, m = mass of object and R = radius of the earth

The negative sign in the above relation indicates that it is the work done in bringing the object from infinity to a distance R.

The gravitational potential energy of object at a height h above the surface of earth is

$$U_2 = -\frac{GMm}{(R+h)} \quad ...(ii)$$

So, the change in potential energy is

$$\Delta U = U_2 - U_1 = -\frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right)$$

[From Eq. (i) and (ii)]

$$= -GMm\left[\frac{1}{(R+h)} - \frac{1}{R}\right]$$

$$= -GMm\left(-\frac{h}{R(R+h)}\right)$$

$$= \frac{GMmh}{R(R+h)}$$

18 A body weighs 200 N on the surface of the earth. How much will it weigh half way down to the centre of the earth ?

[NEET (National) 2019]

(a) 200 N (b) 250 N (c) 100 N (d) 150 N

Ans. ***(c)***

Given, weight of the body, $w = 200$ N

As we know,

$w = mg$, where m is the mass of the body and g ($\approx 10\ m/s^2$) is acceleration due to gravity of the body at the surface of the earth.

Since, mass m remains constant irrespective of the position of the body on the earth. However, g is not constant and its value at a depth d below the earth's surface is given as

$$g' = g\left(1 - \frac{d}{R}\right) \quad ...(i)$$

where, R is the radius of the earth.

Multiplying m on the both sides of Eq. (i), we get

$$mg' = mg\left(1 - \frac{d}{R}\right)$$

Thus, the weight of the body at half way down $\left(\text{i.e. } d = \frac{R}{2}\right)$ to the centre of the earth is

$$mg' = 200 \times \left(1 - \frac{R/2}{R}\right)$$

$$= 200\left(1 - \frac{1}{2}\right) = 200 \times \frac{1}{2} = 100$$

$\therefore$ The body will weigh 100 N half way down to the centre of the earth.

19 The work done to raise a mass m from the surface of the earth to a height h, which is equal to the radius of the earth, is

[NEET (National) 2019]

(a) $2mgR$

(b) $\frac{1}{2}mgR$

(c) $\frac{3}{2}mgR$

(d) mgR

Ans. ***(b)***

Key Idea Amount of work done in moving the given body from one point to another against the gravitational force is equal to the change in potential energy of the body.

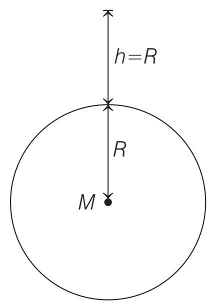
As we know, the potential energy of body of mass m of the surface of earth is

$$U_1 = -\frac{GMm}{R} \quad ...(i)$$

where, G = gravitational constant,

M = mass of earth and R = radius of earth.

When the mass is raised to a height h from the surface of the earth, then the potential energy of the body becomes



$$U_2 = -\frac{GMm}{(R+h)}$$

Here, $h = R$ (given)

$$\Rightarrow \quad U_2 = -\frac{GMm}{2R} \quad ...(ii)$$

Thus, the change in potential energy, $\Delta U = U_2 - U_1$

Substituting the values from Eqs. (i) and (ii), we get

$$\Delta U = -\frac{GMm}{2R} + \frac{GMm}{R}$$

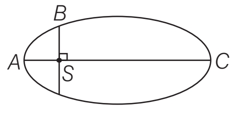
$$= \frac{GMm}{2R} = \frac{gR^2 m}{2R} \quad \left[\because g = \frac{GM}{R^2}\right]$$

$$= \frac{mgR}{2}$$

Thus, the work done in raising the mass to a height R is equals to $\frac{mgR}{2}$.

20 If the mass of the Sun were ten times smaller and the universal gravitational constant were ten times larger in magnitude, which of the following is not correct?

[NEET 2018]



(a) Time period of a simple pendulum on the Earth would decrease

(b) Walking on the ground would become more difficult

(c) Raindrops will fall faster

(d) 'g' on the Earth will not change

Ans. ***(d)***

Let the original mass of Sun was M_s and gravitational constant G'.

According to the question,

New mass of Sun, $M_s' = \frac{M_s}{10}$

New gravitational constant, $G' = 10G$

As, the acceleration due to gravity is given as

$$g = \frac{GM_E}{R^2} \quad ...(i)$$

where, M_E is the mass of Earth and R is the radius of the Earth.

Now, new acceleration due to gravity,

$$g' = \frac{G'M_e}{R^2}$$

$$= \frac{10\,M_e G}{R^2} \quad ...(ii)$$

$\therefore \quad g' = 10\,g$ [from Eqs. (i) and (ii)]

This means the acceleration due to gravity has been increased. Hence, force of gravity acting on a body placed on or surface of the Earth increases.

Due to this, rain drops will fall faster, walking on ground would become more difficult.

As, time period of the simple pendulum is

$$T = 2\pi\sqrt{\frac{l}{g}} \text{ Or } T \propto \frac{1}{\sqrt{g}}$$

Thus, time period of the pendulum also decreases with the increase in g.

21 The acceleration due to gravity at a height 1 km above the earth is the same as at a depth d below the surface of earth. Then **[NEET 2017]**

(a) $d = \frac{1}{2}$ km (b) $d = 1$ km

(c) $d = \frac{3}{2}$ km (d) $d = 2$ km

Ans. *(d)*

Thinking Process g_h = Acceleration due to gravity at height h above earth's surface

$$= g\left(\frac{R}{R+h}\right)^2 = g\left(1-\frac{2h}{R}\right)$$

g_d = Acceleration at depth d below earth's surface

$$= g\left(1-\frac{d}{R}\right)$$

Given, when $h = 1$ km, $g_d = g_h$

or $\quad g\left(1-\frac{d}{R}\right) = g\left(1-\frac{2h}{R}\right)$

$\Rightarrow \quad d = 2h \;$ or $\; d = 2$ km

22 Two astronauts are floating in gravitational free space after having lost contact with their spaceship. The two will **[NEET 2017]**

(a) keep floating at the same distance between them
(b) move towards each other
(c) move away from each other
(d) will become stationary

Ans. *(b)*

In the space, there is no external gravity. Due to masses of the astronauts, there will be small gravitational attractive force between them. Thus, these astronauts will move towards each other.

23 At what height from the surface of earth the gravitation potential and the value of g are -5.4×10^7 J kg^{-2} and 6.0 ms^{-2} respectively? Take, the radius of earth as 6400 km. **[NEET 2016]**

(a) 1600 km (b) 1400 km
(c) 2000 km (d) 2600 km

Ans. *(d)*

Gravitational potential at some height h from the surface of the earth is given by

$$V = -\frac{GM}{R+h} \qquad \text{...(i)}$$

And acceleration due to gravity at some height h from the earth surface can be given as

$$g' = \frac{GM}{(R+h)^2} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{|V|}{g'} = \frac{GM}{(R+h)} \times \frac{(R+h)^2}{GM}$$

$$\Rightarrow \frac{|V|}{g'} = R+h \qquad \text{(iii)}$$

$\because \; V = -5.4\times10^7$ J kg^{-2} and $g' = 6.0$ ms^{-2}

Radius of earth, $R = 6400$ km.

Substitute these values in Eq. (iii), we get

$$\frac{5.4\times10^7}{6.0} = R+h \Rightarrow 9\times10^6 = R+h$$

$$\Rightarrow \quad h = (9-6.4)\times10^6 = 2.6\times10^6 \text{ m}$$

$$\Rightarrow \quad h = 2600 \text{ km}$$

24 Starting from the centre of the earth having radius R, the variation of g (acceleration due to gravity) is shown by **[NEET 2016]**

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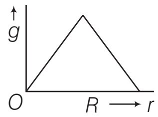

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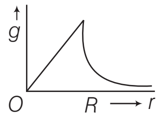

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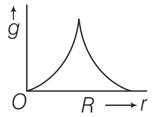

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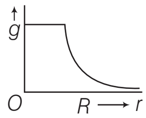


Ans. *(b)*

Acceleration due to gravity at a depth d below the surface of the earth is given by

$$g_{\text{depth}} = g_{\text{surface}}\left(1-\frac{d}{R}\right)$$

Also, for a point at height h above surface,

$$g_{\text{height}} = g_{\text{surface}}\left[\frac{R^2}{(R+h)^2}\right]$$

Therefore, we can say that value of g increases from centre of maximum at the surface and then decreases as depicted in graph (b).

25 Infinite number of bodies, each of mass 2 kg are situated on X-axis at distances 1m, 2 m, 4 m and 8 m, respectively from the origin. The resulting gravitational potential due to this system at the origin will be **[NEET 2013]**

(a) $-G$ (b) $-\frac{8}{3}G$
(c) $-\frac{4}{3}G$ (d) $-4G$

Ans. *(d)*

The resulting gravitational potential,

$$V = -2G\left[\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots\right]$$

$$\Rightarrow \; V = -2G\left[1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}\dots\right]$$

$$\Rightarrow \; V = -2G\left(1+\frac{1}{2}\right)^{-1}$$

$$\Rightarrow \; V = -\frac{2G}{\left(1-\frac{1}{2}\right)} = -\frac{2G}{\left(\frac{1}{2}\right)} = -4G$$

26 A body of mass m taken from the earth's surface to the height equal to twice the radius (R) of the earth. The change in potential energy of body will be **[NEET 2013]**

(a) $mg2R$ (b) $\frac{2}{3}mgR$
(c) $3mgR$ (d) $\frac{1}{3}mgR$

Ans. *(b)*

Change in potential energy

So, $\quad \Delta U = -\frac{GMm}{R+2R} - \left(-\frac{GMm}{R}\right)$

where, $U_{\text{Final}} = \frac{GMm}{R+2R}$

$$U_{\text{initial}} = \frac{-GMm}{R}$$

$$\therefore \; \Delta U = -\frac{GMm}{3R} + \frac{GMm}{R}$$

$$= \frac{2GMm}{3R} = \frac{2}{3}mgR \qquad \left[\because g = \frac{GM}{R^2}\right]$$

27 The height at which the weight of a body becomes 1/16th, its weight on the surface of the earth (radius R), is **[CBSE AIPMT 2012]**

(a) $5R$ (b) $15R$
(c) $3R$ (d) $4R$

Ans. *(c)*

According to the question,

$$\frac{GMm}{(R+h)^2} = \frac{1}{16}\frac{GMm}{R^2}$$

where, m = mass of the body

and $\frac{GM}{R^2}$ = gravitational acceleration

$$\frac{1}{(R+h)^2} = \frac{1}{16R^2} \;\text{ or }\; \frac{R}{R+h} = \frac{1}{4}$$

or $\frac{R+h}{R} = 4$

$$h = 3R$$

28 A compass needle which is allowed to move in a horizontal plane is taken to a geomagnetic pole. It **[CBSE AIPMT 2012]**

(a) will become rigid showing no movement
(b) will stay in any position
(c) will stay in North-South direction only
(d) will stay in East-West direction only

Ans. *(c)*

It will stay in North-South direction only at geomagnetic North and South poles.

29 A spherical planet has a mass M_p and diameter D_p. A particle of mass m falling freely near the surface of this planet will experience an acceleration due to gravity, equal to **[CBSE AIPMT 2012]**

(a) $4GM_p/D_p^2$ (b) $GM_p m/D_p^2$
(c) GM_p/D_p^2 (d) $4GM_p m/D_p^2$

Ans. *(a)*

Concept Apply Newton's gravitation law.

According to Newton's law of gravitation force,

$$F = \frac{GMm}{R^2}$$

Force on planet of mass M_P and body of mass m is given by

$$F = \frac{GM_P m}{(D_P/2)^2}$$

$$\left[\begin{array}{l}\text{where, } D_P = \text{diameter of planet} \\ \text{and } R_P = \text{radius of planet} = \dfrac{D_P}{2}\end{array}\right]$$

$$F = \frac{4GM_P m}{D_P^2}$$

As we know that, $F = ma$

So, acceleration due to gravity

$$a = \frac{F}{m} = \frac{4GM_P}{D_P^2}$$

30 A body projected vertically from the earth reaches a height equal to earth's radius before returning to the earth. The power exerted by the gravitational force is greatest **[CBSE AIPMT 2011]**

(a) at the instant just before the body hits the earth
(b) it remains constant all through
(c) at the instant just after the body is projected
(d) at the highest position of the body

Ans. *(a)*

We know that, Power, $P = \mathbf{F} \cdot \mathbf{V} = FV\cos\theta$

So, just before hitting, θ is zero, power will be maximum.

31 A particle of mass M is situated at the centre of a spherical shell of same mass and radius a. The gravitational potential at a point situated at $a/2$ distance from the centre, will be **[CBSE AIPMT 2010]**

(a) $-\frac{3GM}{a}$ (b) $-\frac{2GM}{a}$
(c) $-\frac{GM}{a}$ (d) $-\frac{4GM}{a}$

Ans. *(a)*

Gravitational potential at point $a/2$ distance from centre is given by,

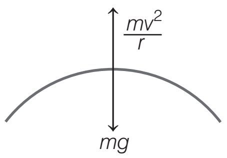
$$V = -\frac{GM}{a} - \frac{GM}{a/2} = -\frac{3GM}{a}$$

32 A roller coaster is designed such that riders experience "weightlessness" as they go round the top of a hill whose radius of curvature is 20 m. The speed of the car at the top of the hill is between **[CBSE AIPMT 2008]**

(a) 14 m/s and 15 m/s
(b) 15 m/s and 16 m/s
(c) 16 m/s and 17 m/s
(d) 13 m/s and 14 m/s

Ans. *(a)*

The appearance of weightlessness occurs in space when the gravitational attraction of the earth on a body in space is equal to the centripetal force.



$$\therefore \quad \frac{mv^2}{r} = mg$$

or $\quad v = \sqrt{rg} = \sqrt{20 \times 10} = 14.14$ m/s

33 Imagine a new planet having the same density as that of the earth but it is 3 times bigger than the earth in size. If the acceleration due to gravity on the surface of the earth is g and that on the surface of the new planet is g', then **[CBSE AIPMT 2005]**

(a) $g' = 3g$ (b) $g' = \frac{g}{9}$
(c) $g' = 9g$ (d) $g' = 27g$

Ans. *(a)*

The acceleration due to gravity on the new planet can be found using the relation

$$g = \frac{GM}{R^2} \qquad \text{...(i)}$$

but $M = \frac{4}{3}\pi R^3\rho$, ρ being density.

Thus, Eq. (i) becomes

$$g = \frac{G \times \frac{4}{3}\pi R^3 \rho}{R^2} = G \times \frac{4}{3}\pi R\rho$$

$$\Rightarrow \quad g \propto R$$

$$\therefore \quad \frac{g'}{g} = \frac{R'}{R}$$

$$\Rightarrow \quad \frac{g'}{g} = \frac{3R}{R} = 3 \Rightarrow g' = 3g$$

34 The density of newly discovered planet is twice that of the earth. The acceleration due to gravity at the surface of the planet is equal to that at the surface of the earth. If the radius of the earth is R, the radius of the planet would be **[CBSE AIPMT 2004]**

(a) $2R$ (b) $4R$ (c) $\frac{1}{4}R$ (d) $\frac{1}{2}R$

Ans. *(d)*

The acceleration due to gravity on an object of mass m

$$g = \frac{F}{m}$$

but from Newton's law of gravitation

$$F = \frac{GMm}{R^2}$$

where, M is the mass of the earth and R is the radius of the earth.

$$\therefore \quad g = \frac{GMm/R^2}{m} = \frac{GM}{R^2}$$

$$\left[\because M_p = \frac{4}{3}\pi R_p^3 P_p \text{ and } M_e = \frac{4}{3}\pi R_e^3 P_e\right]$$

Given, $\quad \rho_{\text{planet}} = 2\rho_{\text{earth}}$

Also, $\quad g_{\text{planet}} = g_{\text{earth}}$

$$\frac{GM_p}{R_p^2} = \frac{GM_e}{R_e^2}$$

As, $\quad \text{Density}(\rho) = \frac{\text{Mass}(M)}{\text{Volume}(V)}$

So, $$\frac{G \times \frac{4}{3}\pi R_p^3 \rho_p}{R_p^2} = \frac{G \times \frac{4}{3}\pi R_e^3 \rho_e}{R_e^2}$$

or $\quad R_p \rho_p = R_e \rho_e$

or $\quad R_p \times 2\rho_e = R_e \rho_e$

or $\quad R_p = \dfrac{R_e}{2} = \dfrac{R}{2}$

35 The acceleration due to gravity on the planet A is 9 times the acceleration due to gravity on the planet B. A man jumps to a height of 2m on the surface of A. What is the height of jump by the same person on the planet B?

[CBSE AIPMT 2003]

(a) 6 m $\quad$ (b) $\dfrac{2}{3}$ m

(c) $\dfrac{2}{9}$ m $\quad$ (d) 18 m

Ans. *(d)*

It is given that, acceleration due to gravity on planet A is 9 times the acceleration due to gravity on planet B i.e.

$$g_A = 9g_B \qquad \text{...(i)}$$

From third equation of motion,

$$v^2 = 2gh$$

At planet A, $h_A = \dfrac{v^2}{2g_A}$ ...(ii)

At planet B, $h_B = \dfrac{v^2}{2g_B}$ ...(iii)

Dividing Eq. (ii) by Eq. (i), we have

$$\frac{h_A}{h_B} = \frac{g_B}{g_A}$$

From Eq. (i), $g_A = 9g_B$

$$\therefore \quad \frac{h_A}{h_B} = \frac{g_B}{9g_B} = \frac{1}{9}$$

or $\quad h_B = 9h_A = 9 \times 2 = 18\text{ m} \; (\because h_A = 2\text{ m})$

36 A body of mass m is placed on the earth's surface. It is then taken from the earth's surface to a height $h = 3R$, then the change in gravitational potential energy is

[CBSE AIPMT 2002]

(a) $\dfrac{mgh}{R}$ $\quad$ (b) $\dfrac{2}{3}\,mgR$

(c) $\dfrac{3}{4}\,mgR$ $\quad$ (d) $\dfrac{mgR}{2}$

Ans. *(c)*

Potential energy, $U = -\dfrac{GMm}{r}$

At the earth's surface, $r = R$

$$\therefore \quad U_e = -\frac{GMm}{R}$$

Now, if a body is taken to height $h = 3R$, then the potential energy is given by

$$U_h = -\frac{GMm}{R+h} \qquad (\because r = h + R)$$

$$= -\frac{GMm}{4R}$$

Thus, change in gravitational potential energy,

$$\Delta U = U_h - U_e$$

$$= -\frac{GMm}{4R} - \left(-\frac{GMm}{R}\right)$$

$$= -\frac{GMm}{4R} + \frac{GMm}{R} = \frac{3}{4}\frac{GMm}{R}$$

$$\therefore \quad \Delta U = \frac{3}{4}\frac{gR^2 m}{R} \qquad (\because GM = gR^2)$$

$$= \frac{3}{4}\,mgR$$

37 A body attains a height equal to the radius of the earth. The velocity of the body with which it was projected is **[CBSE AIPMT 2001]**

(a) $\sqrt{\dfrac{GM}{R}}$ $\quad$ (b) $\sqrt{\dfrac{2GM}{R}}$

(c) $\sqrt{\dfrac{5}{4}\dfrac{GM}{R}}$ $\quad$ (d) $\sqrt{\dfrac{3GM}{R}}$

Ans. *(a)*

Energy at surface of the earth

= energy at maximum height

or $(K + U)$ at the earth's surface

$= (K + U)$ at maximum height

$$\therefore \quad \frac{1}{2}mu^2 - \frac{GMm}{R} = \frac{1}{2}m \times (0)^2 - \frac{GMm}{R+h}$$

At maximum height it has only potential energy

or $\quad \dfrac{1}{2}mu^2 = \dfrac{GMm}{R} - \dfrac{GMm}{R+R} \qquad (\because h = R)$

or $\quad u^2 = \dfrac{2GM}{R} - \dfrac{2GM}{2R}$

or $\quad u^2 = \dfrac{GM}{R} \quad \therefore \; u = \sqrt{\dfrac{GM}{R}}$

Alternative

The expression for the speed with which a body should be projected so as to reach a height h is

$$u = \sqrt{\frac{2gh}{1 + (h/R)}}$$

Here, $h = R$ (given)

$$u = \sqrt{\frac{2gR}{1 + (R/R)}} = \sqrt{\frac{2 \times \frac{GM}{R^2} \times R}{2}} = \sqrt{\frac{GM}{R}}$$

38 What will be the formula of the mass in terms of g, R and G? (R = radius of the earth)

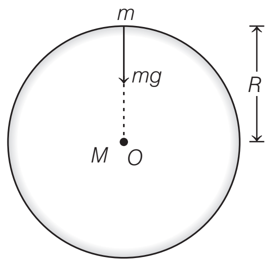
[CBSE AIPMT 1996]

(a) $g^2\,\dfrac{R}{G}$ $\quad$ (b) $G\,\dfrac{R^2}{g}$

(c) $G\,\dfrac{R}{g}$ $\quad$ (d) $g\,\dfrac{R^2}{G}$

Ans. *(d)*

Let m be the mass of body, it is placed on spherical body of mass M, radius R and centre O. If acceleration due to gravity is g and density of spherical body is uniform such that its mass can be supposed to be concentrated at its centre O.



Let F be the force of attraction between body of mass m and spherical body of mass M.

According to Newton's law of gravitation

$$F = \frac{GMm}{R^2}$$

From gravity pull $F = mg$

$$\therefore \quad mg = \frac{GMm}{R^2} \quad \text{or} \quad g = \frac{GM}{R^2}$$

$$\therefore \quad M = \frac{gR^2}{G}$$

39 A seconds pendulum is mounted in a rocket. Its period of oscillation decreases when the rocket **[CBSE AIPMT 1991]**

(a) comes down with uniform acceleration

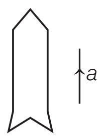
(b) moves round the earth in a geostationary orbit

(c) moves up with a uniform velocity

(d) moves up with uniform acceleration

Ans. *(d)*

When rocket accelerates upward with acceleration a, then effective acceleration of rocket is $(g + a)$.



As, $T = 2\pi\sqrt{\dfrac{l}{g}} = 2\pi\sqrt{\left(\dfrac{l}{g+a}\right)}$

Hence, period of oscillation of seconds pendulum decreases when the rocket moves up with uniform acceleration.

40 A planet is moving in an elliptical orbit around the sun. If T, U, E and L stand for its kinetic energy, gravitational potential energy, total energy and magnitude of angular momentum about the centre of force, which of the following is correct? **[CBSE AIPMT 1990]**

(a) T is conserved
(b) U is always positive
(c) E is always negative
(d) L is conserved but direction of vector L changes continuously

Ans. ***(c)***

When the planet moves in circular or elliptical orbit, then torque is always acting parallel to displacement or velocity. So, angular momentum is conserved. When the field is attractive, then potential energy is given by

$$U = -\frac{GMm}{R}$$

Negative sign shows that the potential energy is due to attractive gravitational force. Kinetic energy changes as velocity increases when distance is less. Hence, option (c) is correct.

TOPIC 3
Escape Speed and Motion of Satellites

41 The escape velocity from the Earth's surface is v. The escape velocity from the surface of another planet having a radius, four times that of Earth and same mass density is **[NEET 2021]**

(a) v (b) $2v$ (c) $3v$ (d) $4v$

Ans. ***(d)***

We know that,
escape velocity from Earth's surface,

$$v_e = \sqrt{\frac{2GM_e}{R_e}}$$

where, G is the gravitational constant,
R_e is the radius of the Earth,
M_e is the mass of the Earth.

Also, density $= \frac{\text{mass}}{\text{volume}}$

$\Rightarrow$ Mass = Density × Volume

or $M_e = \rho \times V$

or $M_e = \rho \times \frac{4}{3}\pi R_e^3 \quad \left(\because V = \frac{4}{3}\pi R_e^3\right)$

Substituting the values in the escape velocity expression, we get

$$v_e = \sqrt{\frac{2G\rho \times \frac{4}{3}\pi R_e^3}{R_e}}$$

$$v_e = \sqrt{2G\rho \times \frac{4}{3}\pi R_e^2} \quad \text{...(i)}$$

Now, we shall determine the escape velocity for another planet.

Given, radius of another planet is 4 times the radius of the Earth,

$$\Rightarrow \quad v_p = \sqrt{2G\rho \times \frac{4}{3}\pi R_p^2}$$

$$v_p = \sqrt{2G\rho \times \frac{4}{3}\pi (4R_e)^2} \quad (\because R_p = 4R_e)$$

$$v_p = 4\sqrt{2G\rho \times \frac{4}{3}\pi (R_e)^2} \quad \text{...(ii)}$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{v_p}{v_e} = \frac{4\sqrt{2G\rho \times \frac{4}{3}\pi (R_e)^2}}{\sqrt{2G\rho \times \frac{4}{3}\pi R_e^2}}$$

$$\Rightarrow \quad v_p = 4v_e = 4v$$

42 A particle of mass m is projected with a velocity $v = kv_e$ $(k < 1)$ from the surface of the Earth.
(Here, v_e = escape velocity)
The maximum height above the surface reached by the particle is **[NEET 2021]**

(a) $R\left(\frac{k}{1-k}\right)^2$ (b) $R\left(\frac{k}{1+k}\right)^2$
(c) $\frac{R^2k}{1+k}$ (d) $\frac{Rk^2}{1-k^2}$

Ans. ***(d)***

Given, the mass of the particle $= m$

The velocity of the projected particle, $v = kv_e$

Initial energy at the time of projection

$$= \frac{1}{2}mv^2 - \frac{GMm}{R}$$

Final energy at the maximum height h from surface of the Earth

$$= \frac{1}{2}mv^2 - \frac{GMm}{R+h}$$

$$= \frac{1}{2}m(0)^2 - \frac{GMm}{R+h}$$

($\because$ final velocity is zero at height h from the surface of the Earth)

$$= -\frac{GMm}{R+h}$$

Using the law of conservation of energy, initial energy at the time of projection = final energy at the maximum height h from the surface of the Earth

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

Substituting the values in the above equation, we get

$$\frac{1}{2}m(kv_e)^2 - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

$$\Rightarrow \frac{1}{2}mk^2\left(2\frac{GM}{R}\right) - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

$$\left(\because v_e = \sqrt{\frac{2GM}{R}}\right)$$

$$\Rightarrow \quad \frac{k^2}{R} - \frac{1}{R} = -\frac{1}{R+h}$$

$$\Rightarrow \quad \frac{1}{R+h} = \frac{1}{R} - \frac{k^2}{R}$$

$$\Rightarrow \quad \frac{R}{1-k^2} = R + h \Rightarrow h = \frac{R}{1-k^2} - R$$

$$\Rightarrow \quad h = R\left(\frac{1}{1-k^2} - 1\right)$$

$$\Rightarrow \quad h = R\left(\frac{1-1+k^2}{1-k^2}\right)$$

$$\Rightarrow \quad h = R\left(\frac{k^2}{1-k^2}\right)$$

43 The ratio of escape velocity at earth (v_e) to the escape velocity at a planet (v_p) whose radius and mean density are twice as that of earth is **[NEET 2016]**

(a) $1:2\sqrt{2}$ (b) $1:4$
(c) $1:\sqrt{2}$ (d) $1:2$

Ans. ***(a)***

Since, the escape velocity of earth can be given as

$$v_e = \sqrt{2gR} = R\sqrt{\frac{8}{3}\pi G\rho}$$

[ρ = density of earth]

$$\Rightarrow \quad v_e = R\sqrt{\frac{8}{3}\pi G\rho} \quad \text{...(i)}$$

As it is given that the radius and mean density of planet are twice as that of earth. So, escape velocity at planet will be

$$v_p = 2R\sqrt{\frac{8}{3}\pi G2\rho} \quad \text{...(ii)}$$

Divide, Eq. (i) by Eq. (ii), we get

$$\frac{v_e}{v_p} = \frac{R\sqrt{\frac{8}{3}\pi G\rho}}{2R\sqrt{\frac{8}{3}\pi G2\rho}}$$

$$\Rightarrow \frac{v_e}{v_p} = \frac{1}{2\sqrt{2}}$$

44 A satellite of mass m is orbiting the earth (of radius R) at a height h from its surface. The total energy of the satellite in terms of g_0, the value of acceleration due to gravity at the earth's surface is **[NEET 2016]**

(a) $\frac{mg_0R^2}{2(R+h)}$ (b) $-\frac{mg_0R^2}{2(R+h)}$

(c) $\frac{2mg_0R^2}{R+h}$ (d) $-\frac{2mg_0R^2}{R+h}$

Ans. *(b)*

$\because$ Total energy of a satellite at height h is

$$= \text{KE} + \text{PE} = \frac{GMm}{2(R+h)} - \frac{GMm}{(R+h)}$$

$$= \frac{-GMm}{2(R+h)} = \frac{-GMmR^2}{2R^2(R+h)} = \frac{-mg_0R^2}{2(R+h)}$$

$$\left(\because g_0 = \frac{GM}{R^2}\right)$$

45 A remote sensing satellite of earth revolves in a circular orbit at a height of 0.25×10^6 m above the surface of earth. If earth's radius is 6.38×10^6 m and $g = 9.8$ ms^{-2}, then the orbital speed of the satellite is **[CBSE AIPMT 2015]**

(a) 7.76 kms^{-1} (b) 8.56 kms^{-1}

(c) 9.13 kms^{-1} (d) 6.67 kms^{-1}

Ans. *(a)*

Given, height of a satellite

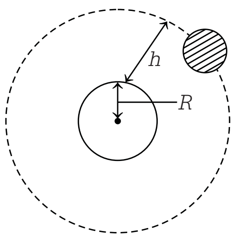
$h = 0.25 \times 10^6$ m

Earth's radius, $R_e = 6.38 \times 10^6$ m

For the satellite revolving around the earth, orbital velocity of the satellite

$$v_0 = \sqrt{\frac{GM_e}{R_e}} = \sqrt{\frac{GM_e}{R_e\left[1+\frac{h}{R_e}\right]}}$$

$$\Rightarrow v_0 = \sqrt{\frac{gR_e}{1+\frac{h}{R_e}}}$$



Substitutes the values of g, R_e and h, we get

$$v_0 = \sqrt{60 \times 10^6} \text{ m/s}$$

$$v_0 = 7.76 \times 10^3 = 7.76 \text{ km/s}$$

46 A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small as compared to the mass of the earth. Then, **[CBSE AIPMT 2015]**

(a) the angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant

(b) the total mechanical energy of S varies periodically with time

(c) the linear momentum of S remains constant in magnitude

(d) the acceleration of S is always directed towards the centre of the earth

Ans. *(d)*

As we know that, force on satellite is only gravitational force which will always be towards the centre of earth. Thus, the acceleration of S is always directed towards the centre of the earth.

47 A black hole is an object whose gravitational field is so strong that even light cannot escape from it. To what approximate radius would earth (mass $= 5.98 \times 10^{24}$ kg) have to be compressed to be a black hole? **[CBSE AIPMT 2014]**

(a) 10^{-9} m (b) 10^{-6} m

(c) 10^{-2} m (d) 10^{-7} m

Ans. *(c)*

Problem Solving Strategy For the black hole, the escape speed is more than c (speed of light). We should compare the escape speed with the c (Note that the escape speed should be at least just greater than c).

$$v_e = \sqrt{\frac{2GM}{R'}}$$

[R' = New radius of the earth]

$$c = \sqrt{\frac{2GM}{R'}} \, [v_e \approx c] \Rightarrow c^2 = 2\frac{GM}{R'}$$

$$R' = \frac{2GM}{c^2} = \frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{9 \times 10^{16}}$$

$$= \frac{4 \times 6.67}{3} \times 10^{-3} = 8.89 \times 10^{-3}$$

$$= 0.889 \times 10^{-2} \simeq 10^{-2} \text{ m}$$

48 The radii of circular orbits of two satellites A and B of the earth are $4R$ and R, respectively. If the speed of satellite A is $3v$, then the speed of satellite B will be **[CBSE AIPMT 2010]**

(a) $3v/4$ (b) $6v$ (c) $12v$ (d) $3v/2$

Ans. *(b)*

Orbital velocity of satellite is given by,

$$v = \sqrt{\frac{GM}{r}}$$

Ratio of orbital velocities of A and B is given by,

$$\Rightarrow \frac{v_A}{v_B} = \sqrt{\frac{r_B}{r_A}} = \sqrt{\frac{R}{4R}} = \frac{1}{2}$$

$$\therefore \frac{v_A}{v_B} = \frac{3v}{v_B} = \frac{1}{2}$$

$$\therefore v_B = 6v$$

49 The earth is assumed to be a sphere of radius R. A platform is arranged at a height R from the surface of the earth. The escape velocity of a body from this platform is fv_e, where v_e is its escape velocity from the surface of the earth. The value of f is **[CBSE AIPMT 2006]**

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

Ans. *(b)*

At a platform at a height h,

Escape energy = binding energy of sphere

or $\frac{1}{2}m(fv_e)^2 = \frac{GMm}{R+h}$

or $fv_e = \sqrt{\frac{2GM}{R+h}} = \sqrt{\frac{2GM}{2R}}$ ...(i) $(\because h = R)$

But at surface of the earth,

$$v_e = \sqrt{\frac{2GM}{R}} \quad \text{...(ii)}$$

Dividing Eq. (ii) by Eq. (i).

Hence, $\frac{fv_e}{v_e} = \frac{\sqrt{\frac{GM}{R}}}{\sqrt{\frac{2GM}{R}}} \Rightarrow f = \frac{1}{\sqrt{2}}$

50 For a satellite moving in an orbit around the earth, the ratio of kinetic energy to potential energy is **[CBSE AIPMT 2005]**

(a) 2 (b) 1/2 (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

Ans. *(b)*

Potential energy, $U = -\frac{GM_em}{R_e}$

where,

M_e = mass of the earth

m = mass of satellite

R_e = radius of the earth

G = gravitational constant

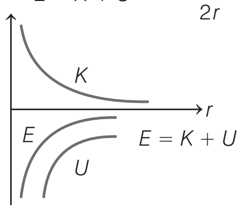
or $\quad |U| = \dfrac{GM_e m}{R_e}$

Kineticenergy, $K = \dfrac{1}{2}\dfrac{GM_e m}{R_e}$

Thus, $\quad \dfrac{K}{|U|} = \dfrac{1}{2}\dfrac{GM_e m}{R_e} \times \dfrac{R_e}{GM_e m} = \dfrac{1}{2}$

The total energy,

$$E = K + U = -\frac{GM_e m}{2r}$$



This energy is constant and negative, i.e. the system is closed. To farther the satellite from the earth, the greater is its total energy.

51 Escape velocity from the earth is 11.2 km/s. Another planet of same mass has radius 1/4 times that of the earth. What is the escape velocity from another planet? **[CBSE AIPMT 2000]**

(a) 11.2 km/s (b) 44.8 km/s
(c) 22.4 km/s (d) 5.6 km/s

Ans. *(c)*

Problem Solving Strategy Compare the equation of escape velocity of earth and planet.

Escape velocity is given by,

$$v_{es} = \sqrt{\frac{2GM_e}{R_e}}$$

From a planet, $v'_{es} = \sqrt{\dfrac{2GM_p}{R_p}}$

Therefore, $\quad \dfrac{v'_{es}}{v_{es}} = \sqrt{\dfrac{2GM_p}{R_p}} \times \sqrt{\dfrac{R_e}{2GM_e}}$

It is given that,

mass of the planet = mass of the earth

i.e. $M_p = M_e$

So, $\dfrac{v'_{es}}{v_{es}} = \sqrt{\dfrac{R_e}{R_p}}$ $\quad$ ...(i)

Given, $R_p = \dfrac{R_e}{4} \Rightarrow \dfrac{R_p}{R_e} = \dfrac{1}{4}$ and

$v_{es} = 11.2$ km/s

Substituting in Eq. (i), we have

$$\frac{v'_{es}}{11.2} = \sqrt{\frac{4}{1}} = 2,$$

$$v'_{es} = 11.2 \times 2 = 22.4 \text{ km/s}$$

52 The escape velocity of a sphere of mass m is given by (G = universal gravitational constant, M_e = mass of the earth and R_e = radius of the earth) **[CBSE AIPMT 1999]**

(a) $\sqrt{\dfrac{GM_e}{R_e}}$ (b) $\sqrt{\dfrac{2GM_e}{R_e}}$

(c) $\sqrt{\dfrac{2Gm}{R_e}}$ (d) $\dfrac{GM_e}{R_e^2}$

Ans. *(b)*

The binding energy of sphere of mass m (say) on the surface of the earth kept at rest is $\dfrac{GM_e m}{R_e}$. To escape it from the earth's surface, this much energy in the form of kinetic energy is supplied to it.

So, $\dfrac{1}{2}mv_e^2 = \dfrac{GM_e m}{R_e}$

or $\quad v_e$ = escape velocity $= \sqrt{\dfrac{2GM_e}{R_e}}$

where, R_e = radius of earth,

M_e = mass of the earth.

53 The escape velocity of a body on the surface of the earth is 11.2 km/s. If the earth's mass increases to twice its present value and the radius of the earth becomes half, the escape velocity would become **[CBSE AIPMT 1997]**

(a) 44.8 km/s
(b) 22.4 km/s
(c) 11.2 km/s (remain unchanged)
(d) 5.6 km/s

Ans. *(b)*

Escape velocity on the earth's surface is given by

$$v_{es} = \sqrt{\frac{2GM_e}{R_e}}$$

where, G is gravitational constant, M_e and R_e are the mass and radius of the earth respectively. By taking the ratios of two different cases

$$\therefore \quad \frac{v'_{es}}{v_{es}} = \sqrt{\frac{M'_e}{M_e} \times \frac{R_e}{R'_e}}$$

but $M'_e = 2M_e$

and $R'_e = \dfrac{R_e}{2}$

$v_{es} = 11.2$ km/s.

$$\therefore \quad \frac{v'_{es}}{v_{es}} = \sqrt{\frac{2M_e}{M_e} \times \frac{R_e}{R_e/2}} = \sqrt{4} = 2$$

$$\therefore \quad v'_{es} = 2v_{es} = 2 \times 11.2 = 22.4 \text{ km/s}$$

The escape velocity on moon's surface is only 2.5 km/s. This is the basic fundamental on which, absence of atmosphere on moon can be explained.

54 A ball is dropped from a satellite revolving around the earth at a height of 120 km. The ball will **[CBSE AIPMT 1996]**

(a) continue to move with same speed along a straight line tangentially to the satellite at that time
(b) continue to move with the same speed along the original orbit of satellite
(c) fall down to the earth gradually
(d) go far away in space

Ans. *(b)*

The ball, when dropped from the orbiting satellite will not reach the surface of the earth. When ball is dropped from the satellite, the ball also starts moving with the same speed due to inertia. As the orbit of a satellite does not depend upon its mass, the ball continues to move along with the satellite in the same orbit.

55 The escape velocity from the surface of the earth is v_e. The escape velocity from the surface of a planet whose mass and radius are three times those of the earth, will be **[CBSE AIPMT 1995]**

(a) v_e (b) $3v_e$ (c) $9v_e$ (d) $\dfrac{1}{3v_e}$

Ans. *(a)*

Escape velocity on surface of the earth is given by

$$v_e = \sqrt{2gR_e} = \sqrt{\frac{2GM_e}{R_e}} \quad \left(\because g = \frac{GM_e}{R_e^2}\right)$$

where, M_e = mass of earth

R_e = radius of the earth

G = gravitational constant

$$\therefore \quad v_e \propto \sqrt{\frac{M_e}{R_e}}$$

If v_p is escape velocity from the surface of the planet, then $\quad \dfrac{v_e}{v_p} = \sqrt{\dfrac{M_e}{R_e}} \times \sqrt{\dfrac{R_p}{M_p}}$

where, M_p is mass of the planet and R_p is radius of the planet.

but $R_p = 3R_e$ $\quad$ (given)

and $M_p = 3M_e$

$$\therefore \quad \frac{v_e}{v_p} = \sqrt{\frac{M_e}{R_e}} \times \sqrt{\frac{3R_e}{3M_e}} = \frac{1}{1} = 1 \text{ or } v_p = v_e$$

56 The escape velocity from the earth is 11.2 km/s. If a body is to be projected in a direction making an angle 45° to the vertical, then the escape velocity is

[CBSE AIPMT 1993]

(a) 11.2×2 km/s (b) 11.2 km/s

(c) $\frac{11.2}{\sqrt{2}}$ km/s (d) $11.2\sqrt{2}$ km/s

Ans. *(b)*

As, $v_e = \sqrt{2gR} = \sqrt{\frac{2GM}{R}}$

Hence, escape velocity does not depend on the angle of projection. Escape velocity will remain same.

57 The mean radius of the earth is R, its angular speed on its own axis is ω and the acceleration due to gravity at the earth's surface is g. What will be the radius of the orbit of a geostationary satellite?

[CBSE AIPMT 1992]

(a) $\left(\frac{R^2 g}{\omega^2}\right)^{1/3}$ (b) $\left(\frac{Rg}{\omega^2}\right)^{1/3}$

(c) $\left(\frac{R^2\omega^2}{g}\right)^{1/3}$ (d) $\left(\frac{R^2 g}{\omega}\right)^{1/3}$

Ans. *(a)*

Let v_o be orbital speed and be is the radius of orbit of a geostationary satellite. So, time period of satellite

$$T = \frac{2\pi r}{v_o}$$

As, $v_o = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR^2}{r}}$ $\left(\because g = \frac{GM}{R^2}\right)$

$\therefore \quad T = \frac{2\pi r}{(gR^2/r)^{1/2}} z = \frac{2\pi r^{3/2}}{\sqrt{gR^2}}$

but $T = \frac{2\pi}{\omega} \Rightarrow T = \frac{2\pi r^{3/2}}{\sqrt{gR^2}} = \frac{2\pi}{\omega}$

Hence, $r^{3/2} = \frac{\sqrt{gR^2}}{\omega}$ or $r^3 = \frac{gR^2}{\omega^2}$

or $r = \left(\frac{gR^2}{\omega^2}\right)^{1/3}$

58 The satellite of mass m is orbiting around the earth in a circular orbit with a velocity v. What will be its total energy? **[CBSE AIPMT 1991]**

(a) $\frac{3}{4}mv^2$ (b) $\frac{1}{2}mv^2$

(c) mv^2 (d) $-\left(\frac{1}{2}\right)mv^2$

Ans. *(d)*

Let satellite of mass m be revolving closely around the earth of mass M and radius R.

Total energy of satellite

$$= \text{PE} + \text{KE} = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$= -\frac{GMm}{R} + \frac{m}{2}\frac{GM}{R} \left[\text{as } v = \sqrt{\frac{GM}{R}}\right]$$

$$= -\frac{GMm}{2R}$$

$\therefore$ Total energy of satellite $= -\frac{1}{2}mv^2$

59 For a satellite escape velocity is 11 km/s. If the satellite is launched at an angle of 60° with the vertical, then escape velocity will be

[CBSE AIPMT 1989]

(a) 11 km/s (b) $11\sqrt{3}$ km/s

(c) $\frac{11}{\sqrt{3}}$ km/s (d) 33 km/s

Ans. *(a)*

Escape velocity on earth (or any other planet) is defined as the minimum velocity with which the body has to be projected vertically upwards from the surface of the earth (or any other planet). So, that it just crosses the gravitational field of earth and never returns on its own. The escape velocity of the earth is given by

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

$$= \sqrt{\frac{8\pi\rho GR^2}{3}}$$

From above equation it is clear that value of escape velocity of a body does not depend upon the mass (m) of the body and its angle of projection from the surface of the earth or the planet. So, escape velocity remains same.