# 16

# Electrostatic Potential and Capacitance

## TOPIC 1
### Electrostatic Potential and Potential Energy

**01** Twenty seven drops of same size are charged at 220 V each. They combine to form a bigger drop. Calculate the potential of the bigger drop. **[NEET 2021]**

(a) 660 V (b) 1320 V
(c) 1520 V (d) 1980 V

**Ans.** ***(d)***

Given, the number of small drops, $n = 27$

Potential on each charged drop, $V = 220$ V

Consider $r$ be the radius of the small drops and $R$ be the radius of the bigger drop.

According to the question, 27 small drops of same size combine to form a bigger drop. So, the volume remains same, i.e.

$$V_{\text{bigger}} = 27 \times V_{\text{small}}$$

$$\Rightarrow \frac{4}{3}\pi R^3 = 27 \times \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad R^3 = 27 \times r^3 \Rightarrow R = 3r$$

Now, the potential of the small drop,

$$V = \frac{Kq}{r} \qquad \text{...(i)}$$

Here, $q$ is the charge on the small drop.

The potential of the bigger drop,

$$V' = \frac{KQ}{R}$$

Here, $Q$ is the charge on the bigger drop.

The charge on the bigger drop is 27 times the charge on the smaller drop, i.e. $Q = 27q$

So, the potential of the bigger drop,

$$V' = \frac{K \times 27q}{3r} \Rightarrow V' = 9\frac{Kq}{r}$$

$$V' = 9V \qquad \text{[from Eq. (i)]}$$

$$= 9 \times 220 = 1980 \text{ V}$$

**02** Two charged spherical conductors of radii $R_1$ and $R_2$ are connected by a wire. Then, the ratio of surface charge densities of the spheres $(\sigma_1/\sigma_2)$ is **[NEET 2021]**

(a) $\frac{R_1}{R_2}$ (b) $\frac{R_2}{R_1}$
(c) $\sqrt{\left(\frac{R_1}{R_2}\right)}$ (d) $\frac{R_1^2}{R_2^2}$

**Ans.** ***(b)***

When two charged spherical conductors are connected by a conducting wire, then the potential becomes same in both the spherical conductor, i.e. $V_1 = V_2$

$$\Rightarrow \frac{Kq_1}{R_1} = \frac{Kq_2}{R_2} \Rightarrow \frac{q_1}{q_2} = \frac{R_1}{R_2} \qquad \text{...(i)}$$

As we know that, the surface charge density of the charged spherical conductor,

$$\sigma = \frac{q}{4\pi R^2}$$

where, $q$ is the charge on the spherical conductor,

$r$ is the radius of the spherical conductor.

$$\therefore \qquad \frac{\sigma_1}{\sigma_2} = \frac{\dfrac{q_1}{4\pi R_1^2}}{\dfrac{q_2}{4\pi R_2^2}}$$

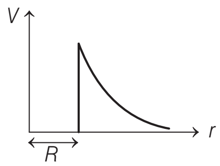
$$\Rightarrow \qquad \frac{\sigma_1}{\sigma_2} = \frac{q_1}{q_2}\frac{R_2^2}{R_1^2}$$

$$\Rightarrow \qquad \frac{\sigma_1}{\sigma_2} = \frac{R_1}{R_2}\frac{R_2^2}{R_1^2} \qquad \text{[from Eq. (i)]}$$

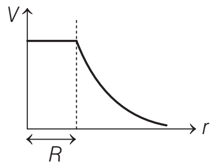
$$\Rightarrow \qquad \frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$$

**03** The variation of electrostatic potential with radial distance $r$ from the centre of a positively charged metallic thin shell of radius $R$ is given by the graph **[NEET (Oct.) 2020]**

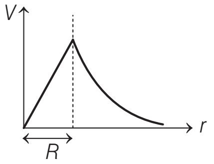
(a)



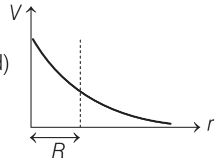

(b)




(c)




(d)

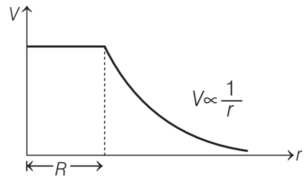


**Ans.** *(b)*

Since, electric potential remains constant inside the metallic spherical shell and same as the surface of spherical shell.

Outside the spherical shell, $V \propto \frac{1}{r}$

Hence, variation of potential $(V)$ with distance $r$ is given as




**04** In a certain region of space with volume $0.2\,\text{m}^3$, the electric potential is found to be 5 V throughout. The magnitude of electric field in this region is **[NEET (Sep.) 2020]**

(a) 0.5 N/C (b) 1 N/C
(c) 5 N/C (d) zero

**Ans.** *(d)*

Given, volume, $V = 0.2\text{m}^3$

Electric potential = 5 V = constant

Electric field = ?

We know that, for constant electric potential the value of electric field is zero.

i.e., $E = \frac{-dV}{dr} = \frac{-d(5)}{dr} = 0$

Hence, correct option is (d).

**05** A short electric dipole has a dipole moment of $16 \times 10^{-9}$ C-m. The electric potential due to the dipole at a point at a distance of 0.6 m from the centre of the dipole, situated on a line making an angle of 60° with the dipole axis is $\left(\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \text{Nm}^2/\text{C}^2\right)$ **[NEET (Sep.) 2020]**

(a) 200 V (b) 400 V (c) zero (d) 50 V

**Ans.** *(a)*

Given, electric dipole moment, $p = 16 \times 10^{-9}$ C-m

Distance, $r = 0.6$ m

Angle, $\theta = 60° \Rightarrow \cos 60° = \frac{1}{2}$

Electric potential at a point which is at a distance $r$ at some angle $\theta$ from electric dipole is

$$V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{9 \times 10^9 \times 16 \times 10^{-9} \times \frac{1}{2}}{(0.6)^2}$$

$$= 2 \times 10^2 = 200\ \text{V}$$

Hence, correct option is (a).

**06** Two metal spheres, one of radius $R$ and the other of radius $2R$ respectively have the same surface charge density $\sigma$. They are brought in contact and separated. What will be the new surface charge densities on them? **[NEET (Odisha) 2019]**

(a) $\sigma_1 = \frac{5}{6}\sigma,\ \sigma_2 = \frac{5}{2}\sigma$

(b) $\sigma_1 = \frac{5}{2}\sigma,\ \sigma_2 = \frac{5}{6}\sigma$

(c) $\sigma_1 = \frac{5}{2}\sigma,\ \sigma_2 = \frac{5}{3}\sigma$

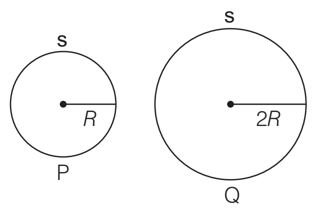
(d) $\sigma_1 = \frac{5}{3}\sigma,\ \sigma_2 = \frac{5}{6}\sigma$

**Ans.** *(d)*

The surface charge density of a closed surface area having charge $Q$ is given by

$$\sigma = \frac{\text{Charge}}{\text{Area}} = \frac{Q}{A} \quad \text{or} \quad Q = \sigma A$$

Thus, the charges on sphere $P$ and $Q$ having same charge density as shown in the figure below is given by




$$Q_P = \sigma \times 4\pi R^2 = 4\pi\sigma R^2 \quad \text{...(i)}$$

and $Q_Q = \sigma \times 4\pi(2R)^2 = 16\pi\sigma R^2$ ...(ii)

when they are brought in contact with each other, the total charge will be

$$Q_t = Q_P + Q_Q$$
$$= 4\pi\sigma R^2 + 16\pi\sigma R^2$$

[From Eq. (i) and (ii)]

$$= 20\pi\sigma R^2 \quad \text{...(iii)}$$

In connection of two charged conducting bodies, the potential will become same on both, i.e.

$$\frac{Q_P}{4\pi\epsilon_0 R} = \frac{Q_Q}{4\pi\epsilon_0 2R}$$

$$\Rightarrow \frac{Q_P}{R} = \frac{Q_Q}{2R} \Rightarrow \frac{Q_P}{Q_Q} = \frac{1}{2}$$

So, the charges on the sphere $P$ and $Q$ after separation will be distributed as

$$\Rightarrow Q'_P = \frac{1}{3}Q_t \text{ and } Q'_Q = \frac{2}{3}Q_t$$

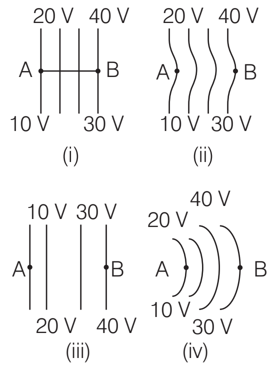
After separation, the new surface charge densities on $P$ and $Q$ will be

$$\sigma_P = \frac{Q'_P}{\text{Area}} = \frac{1}{3}\frac{Q_t}{\text{Area}} = \frac{1}{3}\frac{20\pi\sigma R^2}{4\pi R^2} = \frac{5}{3}\sigma$$

and $\sigma_Q = \frac{Q'_Q}{\text{Area}} = \frac{2}{3}\frac{Q_t}{\text{Area}} = \frac{2}{3} \times \frac{20\pi\sigma R^2}{4\pi(2R)^2}$

$$= \frac{2}{3} \times \frac{5}{4}\sigma = \frac{5}{6}\sigma$$

**07** The diagrams below show regions of equipotentials.




A positive charge is moved from $A$ to $B$ in each diagram. **[NEET 2017]**

(a) Maximum work is required to move $q$ in figure (iii)
(b) In all the four cases, the work done is the same
(c) Minimum work is required to move $q$ in figure (i)
(d) Maximum work is required to move $q$ in figure (ii)

**Ans.** *(b)*

We know that,

Work done $(W) = q\,\Delta V$

$\Delta V$ is same in all the cases. So, work done will be same in the all cases.

**08** If potential (in volts) in a region is expressed as $V(x, y, z) = 6xy - y + 2yz$, the electric field (in N/C) at point (1, 1, 0) is **[CBSE AIPMT 2015]**

(a) $-(3\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ (b) $-(6\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$
(c) $-(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}})$ (d) $-(6\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + \hat{\mathbf{k}})$

**Ans.** *(b)*

Given, potential in a region, $V = 6xy - y + 2yz$.

Electric field in a region,

$$E = -\frac{\partial V}{\partial x}\hat{\mathbf{i}} - \frac{\partial V}{\partial y}\hat{\mathbf{j}} - \frac{\partial V}{\partial z}\hat{\mathbf{k}}$$

$$\Rightarrow \quad E = -6y\hat{\mathbf{i}} - (6x - 1)\hat{\mathbf{j}} - 2y\hat{\mathbf{k}}$$

At, (1, 1, 0), electric field can be expressed,

$E = -(6 \times 1\hat{\mathbf{i}}) - (6 \times 1 - 1)\hat{\mathbf{j}} - 2 \times 1.\hat{\mathbf{k}}$

$= -(6\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ N / C

**09** A conducting sphere of radius $R$ is given a charge $Q$. The electric potential and the electric field at the centre of the sphere respectively are **[CBSE AIPMT 2014]**

(a) zero and $\frac{Q}{4\pi\varepsilon_0 R^2}$

(b) $\frac{Q}{4\pi\varepsilon_0 R}$ and zero

(c) $\frac{Q}{4\pi\varepsilon_0 R}$ and $\frac{Q}{4\pi\varepsilon_0 R^2}$

(d) Both are zero

**Ans.** ***(b)***

In a conducting sphere charge is present on the surface of the sphere. So, electric field inside will be zero and potential remains constant from centre to surface of sphere and is equal to $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$

**10** In a region, the potential is represented by $V(x, y, z) = 6x - 8xy - 8y + 6yz$, where $V$ is in volts and $x$, $y$, $z$ are in metres. The electric force experienced by a charge of 2 C situated at point (1, 1, 1) is **[CBSE AIPMT 2014]**

(a) $6\sqrt{5}$ N (b) 30 N

(c) 24 N (d) $4\sqrt{35}$ N

**Ans.** ***(d)***

As we know that relation between potential difference and electric field **E** in a particular region is given by,

$$\mathbf{E} = -\frac{dV}{dr}$$

As $V = 6x - 8xy - 8y + 6yz$

So, $\mathbf{E} = -\frac{dV}{dr}$

$= -[(6 - 8y)\hat{\mathbf{i}} + (-8x - 8 + 6z)\hat{\mathbf{j}} + 6y\hat{\mathbf{k}}]$

The value of **E** at coordinate (1, 1, 1)

$$\mathbf{E} = -[-2\hat{\mathbf{i}} - 10\hat{\mathbf{j}} + 6\hat{\mathbf{k}}]$$

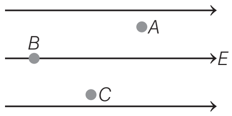
So, $\mathbf{E}_{net} = \sqrt{(-2)^2 + (-10)^2 + 6^2}$

$= 2\sqrt{35}$ N/C

and force on charge $q$ due to $\mathbf{E}_{net}$ is given by

$F = q\,\mathbf{E}_{net} = 2 \times 2\sqrt{35} = 4\sqrt{35}$ N

**11** $A, B$ and $C$ are three points in a uniform electric field. The electric potential is **[NEET 2013]**




(a) maximum at $A$

(b) maximum at $B$

(c) maximum at $C$

(d) same at all the three points $A$, $B$ and $C$

**Ans.** ***(b)***

The electric field is maximum at $B$, because electric field is directed along decreasing potential $V_B > V_C > V_A$.

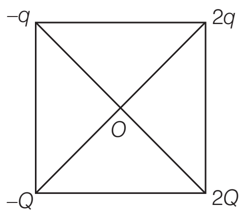
**12** Four point charges $-Q, -q, 2q$ and $2Q$ are placed, one at each corner of the square. The relation between $Q$ and $q$ for which the potential at the centre of the square is zero, is **[CBSE AIPMT 2012]**

(a) $Q = -q$ (b) $Q = -\frac{1}{q}$

(c) $Q = q$ (d) $Q = \frac{1}{q}$

**Ans.** ***(a)***

As, shown in figure




If potential at centre is zero, then

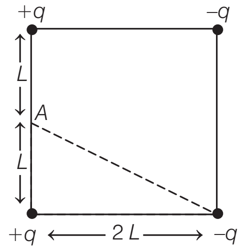
$$V_1 + V_2 + V_3 + V_4 = 0$$

$$\Rightarrow -\frac{kQ}{r} - \frac{kq}{r} + \frac{k2Q}{r} + \frac{k2q}{r} = 0$$

$$\Rightarrow \quad -Q - q + 2q + 2Q = 0$$

$$\therefore \quad Q = -q$$

**13** Four electric charges $+q, +q, -q$ and $-q$ are placed at the corners of a square of side $2L$ (see figure). The electric potential at point $A$, mid-way between the two charges $+q$ and $+q$ is **[CBSE AIPMT 2011]**




(a) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{L}\left(1 + \frac{1}{\sqrt{5}}\right)$

(b) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{L}\left(1 - \frac{1}{\sqrt{5}}\right)$

(c) zero

(d) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{L}(1 + \sqrt{5})$

**Ans.** ***(b)***

Potential at any distance $r$ due to a point charge is given by,

$$V = \frac{kq}{r} \qquad \left[k = \frac{1}{4\pi\varepsilon_0}\right]$$

Given, $V = 2V_{positive} + 2V_{negative}$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{2q}{L} - \frac{2q}{L\sqrt{5}}\right]$$

$$V = \frac{2q}{4\pi\varepsilon_0 L}\left(1 - \frac{1}{\sqrt{5}}\right)$$

**14** The electric potential at a point $(x, y, z)$ is given by $V = -x^2y - xz^3 + 4$. The electric field **E** at that point is **[CBSE AIPMT 2009]**

(a) $\mathbf{E} = (2xy + z^3)\hat{\mathbf{i}} + x^2\hat{\mathbf{j}} + 3xz^2\hat{\mathbf{k}}$

(b) $\mathrm{E} = 2xy\hat{\mathbf{i}} + (x^2 + y^2)\hat{\mathbf{j}} + (3xz - y^2)\hat{\mathbf{k}}$

(c) $\mathbf{E} = z^3\hat{\mathbf{i}} + xyz\hat{\mathbf{j}} + z^2\hat{\mathbf{k}}$

(d) $\mathbf{E} = (2xy - z^3)\hat{\mathbf{i}} + xy^2\hat{\mathbf{j}} + 3z^2x\hat{\mathbf{k}}$

**Ans.** ***(a)***

Potential gradient relates with electric field according to the following relation,

$$E = \frac{-dV}{dr}$$

$$\mathrm{E} = -\frac{dV}{dr}$$

As $V = -x^2y - xz^3 + 4$

So, $\mathbf{E} = -\frac{dV}{dx}\hat{\mathbf{i}} - \frac{dV}{dy}\hat{\mathbf{j}} - \frac{dV}{dz}\hat{\mathbf{k}}$

$$\mathbf{E} = (2xy + z^3)\hat{\mathbf{i}} + x^2\hat{\mathbf{j}} + 3xz^2\hat{\mathbf{k}}$$

**15** Three concentric spherical shells have radii $a$, $b$ and $c$ $(a < b < c)$ and have surface charge densities $\sigma, -\sigma$ and $\sigma$ respectively. If

$V_A, V_B$ and $V_C$ denote the potentials of the three shells, then for $c = a + b$, we have

**[CBSE AIPMT 2009]**

(a) $V_C = V_A \neq V_B$ (b) $V_C = V_B \neq V_A$
(c) $V_C \neq V_B \neq V_A$ (d) $V_C = V_B = V_A$

**Ans.** *(d)*

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{\sigma 4\pi a^2}{a} - \frac{1}{4\pi\varepsilon_0}\frac{\sigma 4\pi b^2}{b} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{\sigma 4\pi c^2}{c}$$

$$= \frac{\sigma}{\varepsilon_0}(a - b + c) = \frac{\sigma}{\varepsilon_0}(2a) \quad (\because c = a + b)$$

$$V_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\sigma 4\pi a^2}{c} - \frac{1}{4\pi\varepsilon_0}\frac{\sigma 4\pi b^2}{b} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{\sigma 4\pi c^2}{c}$$

$$= \frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{c} - b + c\right) = \frac{\sigma}{\varepsilon_0}(2a) \quad (\because c = a + b)$$

and $$V_C = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\sigma 4\pi a^2}{c} - \frac{1}{4\pi\varepsilon_0}\frac{\sigma 4\pi b^2}{c} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{\sigma 4\pi c^2}{c}$$

$$= \frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{c} - \frac{b^2}{c} + c\right) = \frac{\sigma}{\varepsilon_0}(2a) \quad (\because c = a + b)$$

Hence, $V_A = V_C = V_B$

**16** The electric potential at a point in free space due to a charge $Q$ coulomb is $Q \times 10^{11}$ V. The electric field at that point is

**[CBSE AIPMT 2008]**

(a) $4\pi\varepsilon_0 Q \times 10^{22}$ V/m
(b) $12\pi\varepsilon_0 Q \times 10^{20}$ V/m
(c) $4\pi\varepsilon_0 Q \times 10^{20}$ V/m
(d) $12\pi\varepsilon_0 Q \times 10^{22}$ V/m

**Ans.** *(a)*

As potential at any point due to a point charge is given by,

$$V = \frac{kQ}{r} \qquad \left[k = \frac{1}{4\pi\varepsilon_0}\right]$$

$$V = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{r}$$

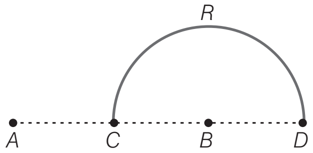
$$E = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{r^2}$$

$$E = \frac{4\pi\varepsilon_0 V^2}{Q} = 4\pi\varepsilon_0 \times \frac{Q^2 \times 10^{22}}{Q}$$

$$E = 4\pi\varepsilon_0 Q \times 10^{22} \text{ V/m}$$

**17** Charges $+q$ and $-q$ are placed at points $A$ and $B$ respectively which are a distance $2L$ apart, $C$ is the midpoint between $A$ and $B$. The work done in moving a charge $+Q$ along the semicircle $CRD$ is

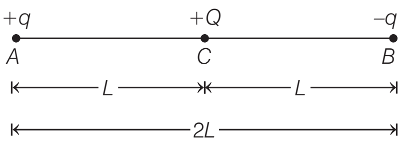
**[CBSE AIPMT 2007]**




(a) $\frac{qQ}{4\pi\varepsilon_0 L}$ (b) $\frac{qQ}{2\pi\varepsilon_0 L}$
(c) $\frac{qQ}{6\pi\varepsilon_0 L}$ (d) $-\frac{qQ}{6\pi\varepsilon_0 L}$

**Ans.** *(d)*

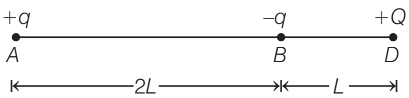
In case I, when charge $+Q$ is situated at $C$.




Electric potential energy of system,

$$U_1 = \frac{1}{4\pi\varepsilon_0}\frac{(q)(-q)}{2L} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)Q}{L} + \frac{1}{4\pi\varepsilon_0}\frac{qQ}{L}$$

In case II, when charge $+Q$ is moved from $C$ to $D$.




Electric potential energy of system in that case,

$$U_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(q)(-q)}{2L} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ}{3L} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)(Q)}{L}$$

As we know that work done in moving a charge is equal to change in potential energy between the points it has been moved.

Work done, $\Delta U = U_2 - U_1$

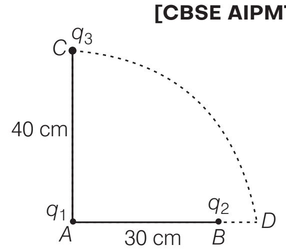
$$= \left(-\frac{1}{4\pi\varepsilon_0}\frac{q^2}{2L} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ}{3L} - \frac{1}{4\pi\varepsilon_0}\frac{qQ}{L}\right) - \left(-\frac{1}{4\pi\varepsilon_0}\frac{q^2}{2L} - \frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ}{L} + \frac{1}{4\pi\varepsilon_0}\frac{qQ}{L}\right)$$

$$= \frac{qQ}{4\pi\varepsilon_0}\cdot\left(\frac{1}{3L} - \frac{1}{L}\right) = \frac{qQ}{4\pi\varepsilon_0}\frac{(1-3)}{3L}$$

$$= \frac{-2qQ}{12\pi\varepsilon_0 L} = -\frac{qQ}{6\pi\varepsilon_0 L}$$

**18** Two charges $q_1$ and $q_2$ are placed 30 cm apart, as shown in the figure. A third charge $q_3$ is moved along the arc of a circle of radius 40 cm from $C$ to $D$. The change in the potential energy of the system is $\frac{q_3}{4\pi\varepsilon_0}k$, where $k$ is

**[CBSE AIPMT 2005]**




(a) $8q_2$ (b) $8q_1$
(c) $6q_2$ (d) $6q_1$

**Ans.** *(a)*

When charge $q_3$ is at $C$, then its potential energy is

$$U_C = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{0.4} + \frac{q_2 q_3}{0.5}\right)$$

When charge $q_3$ is at $D$, then potential energy is

$$U_D = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{0.4} + \frac{q_2 q_3}{0.1}\right)$$

Hence, change in potential energy

$$\Delta U = U_D - U_C = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_2 q_3}{0.1} - \frac{q_2 q_3}{0.5}\right)$$

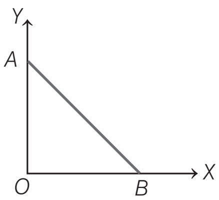
but $$\Delta U = \frac{q_3}{4\pi\varepsilon_0}k$$

$$\frac{q_3}{4\pi\varepsilon_0}k = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_2 q_3}{0.1} - \frac{q_2 q_3}{0.5}\right)$$

$$k = q_2(10 - 2) = 8q_2$$

**19** As per this diagram a point charge $+q$ is placed at the origin $O$. Work done in taking another point charge $-Q$ from the point $A$ [coordinates $(0, a)$] to another point $B$ [coordinates $(a, 0)$] along the straight path $AB$ is

**[CBSE AIPMT 2005]**



(a) zero

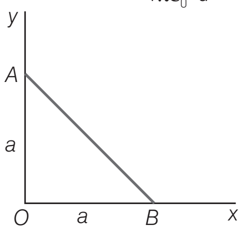
(b) $\left(\dfrac{-qQ}{4\pi\varepsilon_0}\dfrac{1}{a^2}\right)\sqrt{2}a$

(c) $\left(\dfrac{qQ}{4\pi\varepsilon_0}\dfrac{1}{a^2}\right).\dfrac{a}{\sqrt{2}}$

(d) $\left(\dfrac{qQ}{4\pi\varepsilon_0}\dfrac{1}{a^2}\right)\sqrt{2}a$

**Ans.** ***(a)***

Potential at $A$, $V_A = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{a}$




Potential at $B$,

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{a}$$

Thus, work done in carrying a test charge $-Q$ from $A$ to $B$

$$W = (V_A - V_B)(-Q) = 0$$

**20** A bullet of mass 2 g is having a charge of 2 $\mu$C. Through what potential difference must it be accelerated, starting from rest, to acquire a speed of 10 m/s? **[CBSE AIPMT 2004]**

(a) 5 kV (b) 50 kV (c) 5 V (d) 50 V

**Ans.** ***(b)***

Kinetic energy of bullet $= qV$

$$\begin{bmatrix} q = \text{charge on bullet} \\ V = \text{potential difference} \end{bmatrix}$$

From energy conservation,

$$\frac{1}{2}mv^2 = qV \Rightarrow V = \frac{mv^2}{2q}$$

Given, $m = 2\text{ g} = 2\times 10^{-3}\text{ kg}, v = 10\text{ m/s}$,

$q = 2\mu\text{C} = 2\times 10^{-6}\text{ C}$

Substituting the values in relation for $V$, we obtain

$$V = \frac{2\times 10^{-3}\times(10)^2}{2\times 2\times 10^{-6}}$$

$$= 50\times 10^3\text{ V} = 50\text{ kV}$$

**21** Some charge is being given to a conductor, then its potential is **[CBSE AIPMT 2002]**

(a) maximum at surface

(b) maximum at centre

(c) same throughout the conductor

(d) maximum somewhere between surface and centre

**Ans.** ***(c)***

**Concept** An equipotential surface has a constant value of potential at all points on its surface. The surface and interior of a charged conductor is equipotential. Therefore, the potential is same throughout the charged conductor.

**22** Identical charges $(-q)$ are placed at each corners of a cube of side $b$, then the electrostatic potential energy of charge $(+q)$ placed at the centre of the cube will be **[CBSE AIPMT 2002]**

(a) $-\dfrac{4\sqrt{2}q^2}{\pi\varepsilon_0}$ (b) $\dfrac{8\sqrt{2}q^2}{\pi\varepsilon_0 b}$

(c) $-\dfrac{4q^2}{\sqrt{3}\pi\varepsilon_0 b}$ (d) $\dfrac{8\sqrt{2}q^2}{4\pi\varepsilon_0 b}$

**Ans.** ***(c)***

Electrostatic potential energy of charge $+q$ placed at the centre of cube is

$$U = 8\times\frac{1}{4\pi\varepsilon_0}\times\frac{q(-q)}{\text{half - diagonal distance}}$$

$$= 8\times\frac{1}{4\pi\varepsilon_0}\frac{-q^2}{b\frac{\sqrt{3}}{2}}$$

$$\begin{bmatrix}\text{diagonal of cube} = \sqrt{3}b \\ \text{where, } b = \text{ side of cube}\end{bmatrix}$$

$$= \frac{-4q^2}{\sqrt{3}\pi\varepsilon_0 b}$$

**23** In bringing an electron towards another electron, the electrostatic potential energy of the system **[CBSE AIPMT 1999]**

(a) decreases (b) increases

(c) remains same (d) becomes zero

**Ans.** ***(b)***

The electron has negative charge. When an electron is brought towards another electron, then due to same negative charges repulsive force is produced between them. So, to bring them closer a work is done against this repulsive force. This work is stored in the form of electrostatic potential energy. Thus, electrostatic potential energy of system increases.

**Alternative**

Electrostatic potential energy of system of two electrons

$$U = \frac{1}{4\pi\varepsilon_0}\frac{(-e)(-e)}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

Thus, as $r$ decreases, potential energy $U$ increases.

**24** An electron of mass $m$ and charge $e$ is accelerated from rest through a potential difference of $V$ volt in vacuum. Its final speed will be **[CBSE AIPMT 1996]**

(a) $\dfrac{eV}{2m}$ (b) $\dfrac{eV}{m}$

(c) $\sqrt{\dfrac{2eV}{m}}$ (d) $\sqrt{\dfrac{eV}{2m}}$

**Ans.** ***(c)***

In J J Thomson's method, as the electron beam is accelerated from cathode to anode, its potential energy at the cathode appears as gain in kinetic energy at the anode. If $V$ is the potential difference between cathode and anode, then potential energy of electron at cathode

$= \text{charge}\times\text{potential difference} = eV$

Gain in kinetic energy of electron at anode

$$= \frac{1}{2}mv^2$$

According to conservation of energy, we have

$$eV = \frac{1}{2}mv^2$$

$$\therefore \quad v = \sqrt{\left(\frac{2eV}{m}\right)}$$

**25** There is an electric field $E$ in $x$-direction. If the work done on moving a charge of 0.2 C through a distance of 2 m along a line making an angle 60° with $x$-axis is 4 J, then what is the value of $E$? **[CBSE AIPMT 1995]**

(a) 3 N/C (b) 4 N/C

(c) 5 N/C (d) 20 N/C

**Ans.** ***(d)***

Work done in moving the charge,

$$W = Fd\cos\theta$$

As $F = qE$

$\therefore$ $W = qEd\cos\theta$

or $E = \dfrac{W}{qd\cos\theta}$

Here, $q = 0.2\text{ C}, d = 2\text{ m}$

$\theta = 60°, W = 4\text{ J}$

$$\therefore \quad E = \frac{4}{0.2\times 2\times\cos 60°}$$

$$= 20\text{ N/C}$$

**Alternative**

As we know that potential at any point in the direction of $\theta$ and electric field $\mathbf{E}$ is given by $dV = -\mathbf{E}\cdot d\mathbf{r}$

$\left(\begin{array}{l}\text{negative sign indicates decreasing} \\ \text{potential in direction of electric field}\end{array}\right)$

So, for the given situation

$$dr = d\cos\theta$$

So, $\quad dV = Ed\cos\theta$

Now, work done for a charge moving in potential difference $dV$ is given by

$$W = q\,dV$$

$\Rightarrow \quad W = qEd\cos\theta$

Given, $q = 0.2$ C, $d = 2$ m, $\theta = 60°$, $W = 4$ J

So, $\quad 4\text{ J} = 0.2 \times E \times 2 \times \cos 60°$

$\Rightarrow \quad E = \dfrac{4}{0.2 \times 2} \times 2 = 20$ J

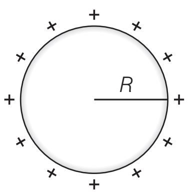
**26** A hollow metal sphere of radius 10 cm is charged such that the potential on its surface is 80 V. The potential at the centre of the sphere is **[CBSE AIPMT 1994]**

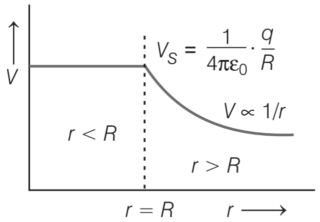
(a) zero (b) 80 V

(c) 800 V (d) 8 V

**Ans.** ***(b)***

In case of spherical metal conductor hollow or solid for an internal point (i.e. $r < R$) potential everywhere inside is same. It is maximum at the surface of sphere and further going out of sphere its value decreases.




So, according to above graph

$$V_{in} = V_{centre} = V_{surface}$$

$$= \frac{1}{4\pi\varepsilon_0} \times \frac{q}{R} = 80\text{ V}$$

$[\because V_{surface} = 80\text{ V}]$

**27** Two concentric spheres of radii $R$ and $r$ have similar charges with equal surface charge densities ($\sigma$). What is the electric potential at their common centre ? **[CBSE AIPMT 1991]**

(a) $\dfrac{\sigma}{\varepsilon_0}$ (b) $\dfrac{\sigma}{\varepsilon_0}(R - r)$

(c) $\dfrac{\sigma}{\varepsilon_0}(R + r)$ (d) None of these

**Ans.** ***(c)***

Let $Q$ and $q$ be the charges on the spheres. The potential at the common centre will be

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R}\right) + \frac{4}{4\pi\varepsilon_0}\left(\frac{q}{r}\right)$$

$$= \frac{1}{\varepsilon_0}\left[\frac{Q}{4\pi R^2} \times R + \frac{q}{4\pi r^2} \times r\right]$$

But, $\dfrac{Q}{4\pi R^2} = \dfrac{q}{4\pi r^2} = \sigma$

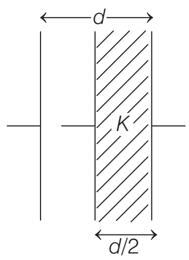
$\therefore \quad V = \dfrac{1}{\varepsilon_0}[\sigma R + \sigma r]$

$$= \frac{\sigma}{\varepsilon_0}(R + r)$$

# TOPIC 2

## Capacitors and Capacitance

**28** A parallel plate capacitor having cross-sectional area $A$ and separation $d$ has air in between the plates. Now, an insulating slab of same area but thickness $d/2$ is inserted between the plates as shown in figure having dielectric constant $K(=4)$. The ratio of new capacitance to its original capacitance will be **[NEET (Oct.) 2020]**




(a) 2 : 1 (b) 8 : 5

(c) 6 : 5 (d) 4 : 1

**Ans.** ***(b)***

Capacitance of parallel plate capacitor when medium is air

$$C_0 = \frac{\varepsilon_0 A}{d} \quad \text{...(i)}$$

According to second condition,

$$A' = A, t = d/2, k = 4$$

$\therefore$ Capacitance, $C = \dfrac{\varepsilon_0 A}{(d - t) + \dfrac{t}{k}}$

$$= \frac{\varepsilon_0 A}{\left(d - \frac{d}{2}\right) + \frac{d/2}{4}}$$

$$= \frac{\varepsilon_0 A}{\frac{d}{2} + \frac{d}{8}} = \frac{8}{5}\cdot\frac{\varepsilon_0 A}{d}$$

$\therefore \quad \dfrac{C}{C_0} = \dfrac{\frac{8}{5}\cdot\frac{\varepsilon_0 A}{d}}{\frac{\varepsilon_0 A}{d}} \Rightarrow \dfrac{C}{C_0} = \dfrac{8}{5}$

$\Rightarrow \quad C : C_0 = 8 : 5$

**29** The capacitance of a parallel plate capacitor with air as medium is 6 $\mu$F. With the introduction of a dielectric medium, the capacitance becomes 30 $\mu$F. The permittivity of the medium is ($\varepsilon_0 = 8.85 \times 10^{-12}\text{ C}^2\text{N}^{-1}\text{m}^{-2}$) **[NEET (Sep.) 2020]**

(a) $1.77 \times 10^{-12}\text{ C}^2\text{N}^{-1}\text{m}^{-2}$

(b) $0.44 \times 10^{-10}\text{ C}^2\text{N}^{-1}\text{m}^{-2}$

(c) $5.00\text{ C}^2\text{N}^{-1}\text{m}^{-2}$

(d) $0.44 \times 10^{-13}\text{ C}^2\text{N}^{-1}\text{m}^{-2}$

**Ans.** ***(b)***

Given, $C_0 = 6\mu$F

$$C_m = 30\mu\text{F}$$

$\therefore$ As, dielectric constant

$$K = \varepsilon_r = \frac{C_m}{C_0} = \frac{30}{6} = 5$$

Permittivity of the medium,

$$\varepsilon_m = K \times \varepsilon_0 = 5 \times \varepsilon_0$$

$$= 5 \times 8.85 \times 10^{-12}$$

$$= 0.44 \times 10^{-10}\text{ C}^2\text{N}^{-1}\text{m}^{-2}$$

Hence, correct option is (b).

**30** The electrostatic force between the metal plates of an isolated parallel plate capacitor $C$ having a charge $Q$ and area $A$, is **[NEET 2018]**

(a) proportional to the square root of the distance between the plates

(b) linearly proportional to the distance between the plates

(c) independent of the distance between the plates

(d) inversely proportional to the distance between the plates

**Ans.** ***(c)***

As we know that, the total work done in transferring a charge to a parallel plate capacitor is given as

$$W=\frac{Q^2}{2C} \quad \text{...(i)}$$

where, $C$ is the capacitance of the capacitor.

We can also wirte a relation for work done as,

$$W=F\cdot d \quad \text{...(ii)}$$

where, $F$ is the electrostatic force between the plates of capacitor and $d$ is the distance between the plates.

From Eqs. (i) and (ii), we get

$$W=\frac{Q^2}{2C}=Fd$$

$$\Rightarrow \quad F=\frac{Q^2}{2Cd} \quad \text{...(iii)}$$

As, the capacitance of a parallel plate is given as

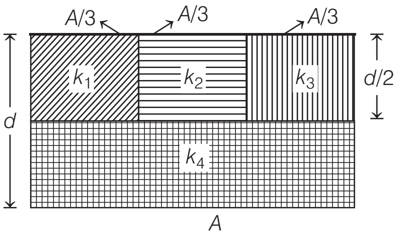
$$C=\frac{\varepsilon_0 A}{d}$$

Substituting the value of $C$ in Eq. (iii), we get

$$F=\frac{Q^2 d}{2\varepsilon_0 A d}=\frac{Q^2}{2\varepsilon_0 A}$$

This means, electrostatic force is independent of the distance between the plates.

**31** A parallel-plate capacitor of area $A$, plate separation $d$ and capacitance $C$ is filled with four dielectric materials having dielectric constants $k_1, k_2, k_3$ and $k_4$ as shown in the figure below. If a single dielectric material is to be used to have the same capacitance $C$ in this capacitor, then its dielectric constant $k$ is given by **[NEET 2016]**




(a) $k=k_1+k_2+k_3+3k_4$

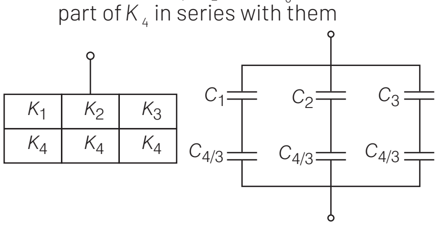
(b) $k=\frac{2}{3}(k_1+k_2+k_3)+2k_4$

(c) $\frac{2}{k}=\frac{3}{k_1+k_2+k_3}+\frac{1}{k_4}$

(d) $\frac{1}{k}=\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{k_3}+\frac{3}{2k_4}$

**Ans.** ***(*)***

Given capacitor is equivalent to capacitors $K_1, K_2$ and $K_3$ in parallel and part of $K_4$ in series with them




$$\frac{1}{C_1}+\frac{3}{C_4}=\frac{3d}{2K_1\varepsilon_0 A}+\frac{3d}{2K_4\varepsilon_0 A}$$

$$=\frac{3d}{2\varepsilon_0 A}\left[\frac{1}{K_1}+\frac{1}{K_4}\right]$$

$$\Rightarrow \quad C_{eq}=\frac{K\varepsilon_0 A}{d}$$

$$=\frac{2\varepsilon_0 A}{3d}\left[\frac{K_1K_4}{K_1+K_4}+\frac{K_2K_4}{K_2+K_4}+\frac{K_3K_4}{K_3+K_4}\right]$$

$$K=\frac{2}{3}\left[\frac{K_1K_4}{K_1+K_4}+\frac{K_2K_4}{K_2+K_4}+\frac{K_3K_4}{K_3+K_4}\right]$$

No option is matching.

**32** A parallel plate air capacitor has capacity $C$, distance of separation between plates is $d$ and potential difference $V$ is applied between the plates. Force of attraction between the plates of the parallel plate air capacitor is **[CBSE AIPMT 2015]**

(a) $\frac{C^2V^2}{2d}$ (b) $\frac{CV^2}{2d}$ (c) $\frac{CV^2}{d}$ (d) $\frac{C^2V^2}{2d^2}$

**Ans.** ***(b)***

Force between plates of parallel capacitor,

$$F=q\,E=q\left[\frac{\sigma}{2\varepsilon_0}\right]$$

$\because$ Surface charge density $\sigma=\frac{q}{A}$

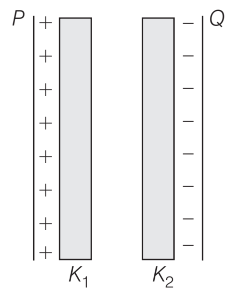
$$\therefore \quad F=q\left[\frac{q}{2A\varepsilon_0}\right] \Rightarrow F=\frac{q^2}{2A\varepsilon_0}$$

So, net charge across a capacitor, $q=CV$

$$F=\frac{C^2V^2}{2A\varepsilon_0} \qquad \left[C=\frac{A\varepsilon_0}{d}\right]$$

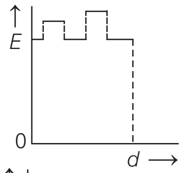
$$\Rightarrow \quad F=\frac{\left(\frac{A\varepsilon_0}{d}\right)\times CV^2}{2\,A\,\varepsilon_0}=\frac{CV^2}{2d}$$

**33** Two thin dielectric slabs of dielectric constants $K_1$ and $K_2$ $(K_1<K_2)$ are inserted between plates of a parallel plate capacitor, as shown in the figure.

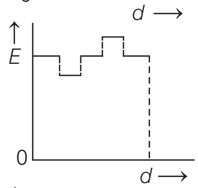



The variation of electric field $E$ between the plates with distance $d$ as measured from plate $P$ is correctly shown by **[CBSE AIPMT 2014]**

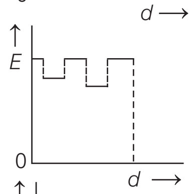
(a)



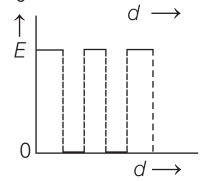

(b)




(c)

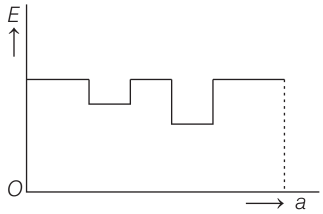



(d)




**Ans.** ***(c)***

Graph (c) will be the right graph, the electric field inside the dielectrics will be less than the electric field outside the dielectrics. The electric field inside the dielectrics could not be zero.




As $K_2>K_1$, the drop in electric field for $K_2$ dielectric must be more than $K_1$.

**34** A parallel plate air capacitor is charged to a potential difference of $V$ volts. After disconnecting the charging battery the distance between the plates of the capacitor is increased using an insulating handle. As a result the potential difference between the plates **[CBSE AIPMT 2006]**

(a) decreases (b) does not change
(c) becomes zero (d) increases

**Ans.** *(d)*

If the battery is removed after charging, then the charge stored in the capacitor remains constant.

$$q = \text{constant}$$

Change in capacitance,

$$C' = \frac{\varepsilon_0 A}{d'}$$

As $d' > d$

Hence, $C' < C$

As, potential difference between the plates of capacitor is given by

$$V = \frac{q}{C}$$

So, for the new capacitor formed

$$V' \propto \frac{1}{C'} \quad [q = \text{constant}]$$

As capacitance decreases, so potential difference increases.

NOTE

If the battery remains connected, the charge stored increases. Also, the potential difference $V$ becomes constant.

**35** A parallel plate condenser with oil (dielectric constant 2) between the plates has capacitance $C$. If oil is removed, the capacitance of capacitor becomes **[CBSE AIPMT 1999]**

(a) $\sqrt{2}C$ (b) $2C$
(c) $\frac{C}{\sqrt{2}}$ (d) $\frac{C}{2}$

**Ans.** *(d)*

The capacitance of a parallel plate capacitor with dielectric (oil) between its plates is

$$C = \frac{K\varepsilon_0 A}{d} \quad \text{...(i)}$$

where, $\varepsilon_0$ = electric permittivity of free space

$K$ = dielectric constant of oil

$A$ = area of each plate of capacitor

$d$ = distance between two plates

When dielectric (oil) is removed, so capacitance of capacitor becomes,

$$C_0 = \frac{\varepsilon_0 A}{d} \quad \text{...(ii)}$$

Comparing Eqs. (i) and (ii), we get

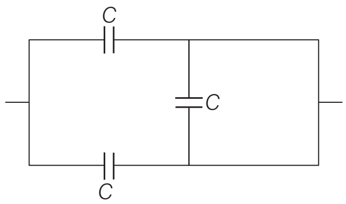
$$C = KC_0$$

$$\Rightarrow \quad C_0 = \frac{C}{K} = \frac{C}{2} \quad (K = 2)$$

# TOPIC 3

## Combination of Capacitors and Energy Stored in a Capacitor

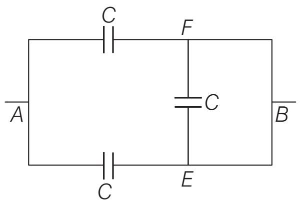
**36** The equivalent capacitance of the combination shown in the figure is **[NEET 2021]**




(a) $3C$ (b) $2C$
(c) $\frac{C}{2}$ (d) $\frac{3C}{2}$

**Ans.** *(b)*

Consider the nodes be $A$, $B$, $E$ and $F$ as shown in the figure

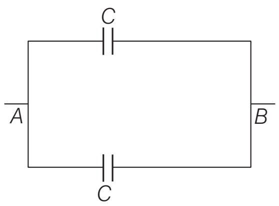



The potential at node $A$ is equal to the potential at node $B$.

Similarly, the potential at $F$ is equal to the potential at node $E$.

So, there is no potential difference between the $EF$ arms, thus, no current (hence, charge) will flow in the circuit.

So, $EF$ behave as open circuit.




Now, the two capacitors are arranged in parallel.

The net equivalent capacitors,

$$C_{eq} = C_1 + C_2$$

$$\Rightarrow \quad C_{eq} = C + C$$

$$\Rightarrow \quad C_{eq} = 2C$$

**37** A parallel plate capacitor has a uniform electric field $\mathbf{E}$ in the space between the plates. If the distance between the plates is $d$ and the area of each plate is $A$, the energy stored in the capacitor is ($\varepsilon_0$ = permittivity of free space). **[NEET 2021]**

(a) $\frac{1}{2}\varepsilon_0 E^2$ (b) $\varepsilon_0 EAd$
(c) $\frac{1}{2}\varepsilon_0 E^2 Ad$ (d) $\frac{E^2 Ad}{\varepsilon_0}$

**Ans.** *(c)*

We know that, capacitance of a parallel plate capacitor,

$$C = \frac{A\varepsilon_0}{d} \quad \text{...(i)}$$

The relation between the potential difference ($V$) and electric field ($E$),

$$V = Ed \quad \text{...(ii)}$$

The energy stored in the capacitor,

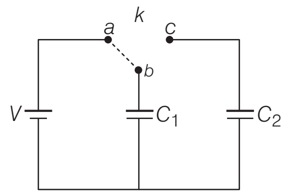
$$U = \frac{1}{2}CV^2$$

$$U = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2$$

[$\because$ from Eqs. (i) and (ii)]

$$U = \frac{1}{2}(\varepsilon_0 A)(E)^2 d = \frac{1}{2}\varepsilon_0 E^2 Ad$$

**38** Two identical capacitors $C_1$ and $C_2$ of equal capacitance are connected as shown in the circuit. Terminals $a$ and $b$ of the key $k$ are connected to charge capacitor $C_1$ using battery of emf $V$ volt. Now, disconnecting $a$ and $b$ the terminals $b$ and $c$ are connected. Due to this, what will be the percentage loss of energy? **[NEET (Odisha) 2019]**




(a) 75% (b) 0% (c) 50% (d) 25%

**Ans.** *(c)*

When $C_1$ is connected to voltage source, it is charged to a potential $V$ and this will be stored as a potential energy in the capacitor given by

$$U = \frac{1}{2}CV^2$$

When key is disconnected from battery and $b$ and $c$ are connected, the charge will be transformed from the capacitor $C_1$ to capacitor $C_2$,
then

The loss of energy due to redistribution of charge is given by

$$\Delta U = \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$$

$$= \frac{C \times C}{2(C + C)}(V - 0)^2 = \frac{1}{4}CV^2$$

$$[\because C_1 = C_2]$$

$$\therefore \text{Percentage loss} = \frac{\Delta U}{U} \times 100$$

$$= \frac{\frac{1}{4}CV^2}{\frac{1}{2}CV^2} \times 100 = 50\%$$

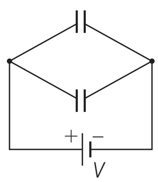
**39** A capacitor is charged by a battery. The battery is removed and another identical uncharged capacitor is connected in parallel. The total electrostatic energy of resulting system **[NEET 2017]**

(a) increases by a factor of 4
(b) decreases by a factor of 2
(c) remains the same
(d) increases by a factor of 2

**Ans.** *(d)*

Thinking Process Energy stored in a system of capacitors

$$= \Sigma \frac{1}{2}CV^2$$




Also, potential drop remains same in parallel across both capacitors.
Initially stored energy

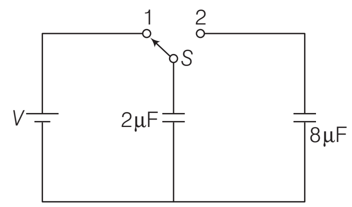
$$U_1 = \frac{1}{2}CV^2$$

Finally, potential drop across each capacitor will be still $V$.
So, finally stored energy

$$U_2 = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = \frac{1}{2}(2C)V^2$$

$$= 2\left(\frac{1}{2}CV^2\right) = 2U_1$$

**40** A capacitor of $2\mu F$ is charged as shown in the figure. When the switch $S$ is turned to position 2, the percentage of its stored energy dissipated is **[NEET 2016]**




(a) 20% (b) 75%
(c) 80% (d) 0%

**Ans.** *(c)*

Consider the given figure,
When the switch $S$ is connected to point 1, then initial energy stored in the capacitor can be given as $= \frac{1}{2}(2\mu F) \times V^2$.

When the switch $S$ is connected to point 2, energy dissipated on connection across $8\mu F$ will be

$$= \frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right).V^2$$

$$= \frac{1}{2} \times \frac{2\mu F \times 8\mu F}{10\mu F} \times V^2$$

$$= \frac{1}{2} \times (1.6\mu F) \times V^2$$

Therefore, % loss of energy

$$= \frac{1.6}{2} \times 100 = 80\%.$$

**41** A parallel plate air capacitor of capacitance $C$ is connected to a cell of emf $V$ and then disconnected from it. A dielectric slab of dielectric constant $K$, which can just fill the air gap of the capacitor, is now inserted in it. Which of the following is incorrect? **[CBSE AIPMT 2015]**

(a) The potential difference between the plates decreases K times
(b) The energy stored in the capacitor decreases $K$ times
(c) The change in energy stored is $\frac{1}{2}CV^2\left(\frac{1}{K} - 1\right)$
(d) The charge on the capacitor is not conserved

**Ans.** *(d)*

When a parallel plate air capacitor connected to a cell of emf $V$, then charge stored will be

$$q = CV \Rightarrow V = \frac{q}{C}$$

Also energy stored is $U = \frac{1}{2}CV^2 = \frac{q^2}{2C}$

As the battery is disconnected from the capacitor the charge will not be destroyed, i.e. $q' = q$ with the introduction of dielectric in the gap of capacitor the new capacitance will be

$$C' = CK \Rightarrow V' = \frac{q}{C'} = \frac{q}{CK}$$

The new energy stored will be

$$U' = \frac{q^2}{2CK} \Rightarrow \Delta U = U' - U$$

$$= \frac{q^2}{2C}\left(\frac{1}{K} - 1\right) = \frac{1}{2}CV^2\left(\frac{1}{K} - 1\right)$$

Hence, option (d) is incorrect

**42** A parallel plate condenser has a uniform electric field $E$ (V/m) in the space between the plates. If the distance between the plates is $d$(m) and area of each plate is $A(\mathrm{m}^2)$, the energy (joule) stored in the condenser is **[CBSE AIPMT 2011]**

(a) $\frac{1}{2}\varepsilon_0 E^2$ (b) $\varepsilon_0 EAd$ (c) $\frac{1}{2}\varepsilon_0 E^2 Ad$ (d) $E^2 Ad / \varepsilon_0$

**Ans.** *(c)*

As we know that the energy stored in the capacitor is given by,

$$U = \frac{1}{2}CV^2$$

$$\begin{bmatrix} C = \text{capacitance of capacitor} \\ V = \text{voltage across the plate} \end{bmatrix}$$

$$U = \frac{1}{2}\left(\frac{A\varepsilon_0}{d}\right)(Ed)^2$$

$$\left(\because C = \frac{\varepsilon_0 A}{d} \text{ and } V = Ed\right)$$

$$U = \frac{1}{2}\frac{\varepsilon_0 A}{d}(Ed)^2, \ U = \frac{1}{2}\varepsilon_0 E^2 Ad$$

**43** A series combination of $n_1$ capacitors, each of value $C_1$, is charged by a source of potential difference $4V$. When another parallel combination of $n_2$ capacitors, each of value $C_2$, is charged by a source of potential difference $V$, it has the same (total) energy stored in it, as the first combination has. The value of $C_2$, in terms of $C_1$, is then **[CBSE AIPMT 2010]**

(a) $\frac{2C_1}{n_1 n_2}$ (b) $16\frac{n_2}{n_1}C_1$

(c) $2\frac{n_2}{n_1}C_1$ (d) $\frac{16C_1}{n_1 n_2}$

**Ans.** ***(d)***

**Case I** When the capacitors are joined in series,

$$U_{\text{series}} = \frac{1}{2}\frac{C_1}{n_1}(4V)^2$$

**Case II** When the capacitors are joined in parallel,

$$U_{\text{parallel}} = \frac{1}{2}(n_2 C_2)V^2$$

Given, $U_{\text{series}} = U_{\text{parallel}}$

So, $\frac{1}{2}\frac{C_1}{n_1}(4V)^2 = \frac{1}{2}(n_2 C_2)V^2$

$\Rightarrow$ $C_2 = \frac{16C_1}{n_2 n_1}$

**44** Three capacitors each of capacitance $C$ and of breakdown voltage $V$ are joined in series. The capacitance and breakdown voltage of the combination will be **[CBSE AIPMT 2009]**

(a) $\frac{C}{3}, \frac{V}{3}$ (b) $3C, \frac{V}{3}$

(c) $\frac{C}{3}, 3V$ (d) $3C, 3V$

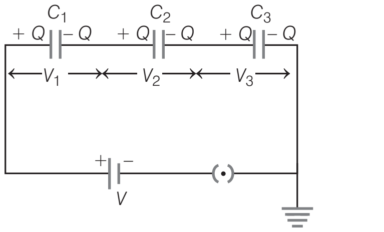
**Ans.** ***(c)***

In series arrangement charge on each plate of all the capacitors have same magnitude. The potential difference is distributed inversely in the ratio of capacitors, i.e.

$$V = V_1 + V_2 + V_3 \quad [\because V_1 = V_2 = V_3 = V]$$

Here, $V = 3V$

The equivalent capacitance $C_s$ is given by




$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad [\because C_1 = C_2 = C_3 = C]$$

$$C_s = \frac{C}{3}$$

**45** The energy required to charge a parallel plate condenser of plate separation $d$ and plate area of cross-section $A$ such that the uniform electric field between the plates $E$, is **[CBSE AIPMT 2008]**

(a) $\frac{1}{2}\frac{\varepsilon_0 E^2}{Ad}$ (b) $\frac{\varepsilon_0 E^2}{Ad}$

(c) $\varepsilon_0 E^2 Ad$ (d) $\frac{1}{2}\frac{\varepsilon_0 E^2}{Ad}$

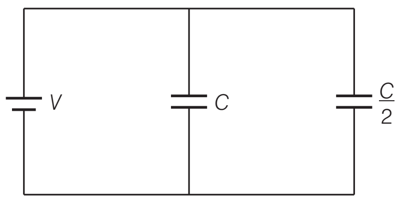
**Ans.** ***(c)***

Energy of charged capacitor $= \frac{1}{2}CV^2$

Energy given by cell $= CV^2 = \frac{A\varepsilon_0}{d} \times (Ed)^2$

As, $V = Ed = A\varepsilon_0 E^2 d$

**46** Two condensers, one of capacity $C$ and the other of capacity $\frac{C}{2}$, are connected to a $V$ volt battery, as shown.




The work done in charging fully both the condensers is **[CBSE AIPMT 2007]**

(a) $2CV^2$ (b) $\frac{1}{4}CV^2$

(c) $\frac{3}{4}CV^2$ (d) $\frac{1}{2}CV^2$

**Ans.** ***(c)***

The two condensers in the circuit are in parallel order, hence

$$C' = C + \frac{C}{2} = \frac{3C}{2}$$

The work done in charging the equivalent capacitor is stored in the form of potential energy.

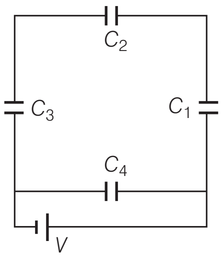
Hence, $W = U = \frac{1}{2}CV^2$

So, for the equivalent capacitor $C'$

$$= \frac{1}{2}\left(\frac{3C}{2}\right)V^2 \qquad \left(C' = \frac{3C}{2}\right)$$

$$= \frac{3}{4}CV^2$$

**47** A network of four capacitors of capacity equal to $C_1 = C, C_2 = 2C$, $C_3 = 3C$ and $C_4 = 4C$ are connected to a battery as shown in the figure. The ratio of the charges on $C_2$ and $C_4$ is **[CBSE AIPMT 2005]**




(a) $\frac{22}{3}$ (b) $\frac{3}{22}$

(c) $\frac{7}{4}$ (d) $\frac{4}{7}$

**Ans.** ***(a)***

The charge flowing through $C_4$ is

$$q_4 = C_4 \times V = 4CV$$

The series combination of $C_1, C_2$ and $C_3$ gives

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{3C}$$

$$= \frac{6+3+2}{6C} = \frac{11}{6C} \Rightarrow C_{\text{eq}} = \frac{6C}{11}$$

Charge flowing through capacitors $C_1$, $C_2$ and $C_3$ will be same as they are in series.

So, $q_1$ flowing through $C_1, C_2$ and $C_3$ is given by

$$q_1 = C_{\text{eq}}V = \frac{6C}{11} \times V$$

Now, ratio of charge on $C_4$ and $C_2$ is given by,

$$\frac{q_4}{q_1} = \frac{4CV \times 11}{6CV} = \frac{22}{3}$$

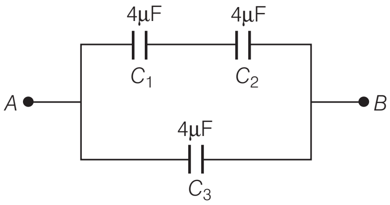
**48** Three capacitors each of capacity $4\mu F$ are to be connected in such a way that the effective capacitance is $6\mu F$. This can be done by **[CBSE AIPMT 2003]**

(a) connecting two in series and one in parallel
(b) connecting two in parallel and one in series
(c) connecting all of them in series
(d) connecting all of them in parallel

**Ans.** *(a)*

Given, $C_1 = C_2 = C_3 = 4\,\mu F$

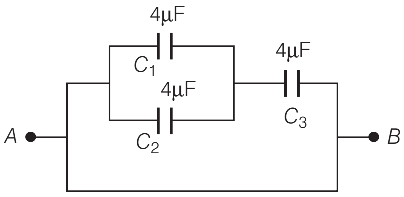
(a) The network of three capacitors can be shown as




Here, $C_1$ and $C_2$ are in series and the combination of two is in parallel with $C_3$.

$$C_{net} = \frac{C_1 C_2}{C_1 + C_2} + C_3 = \left(\frac{4\times4}{4+4}\right) + 4$$
$$= 2 + 4 = 6\mu F$$

(b) The corresponding network is shown in figure below




Here, $C_1$ and $C_2$ are in parallel and this combination is in series with $C_3$.

So, $$C_{net} = \frac{(C_1 + C_2)\times C_3}{(C_1 + C_2) + C_3} = \frac{(4+4)\times4}{(4+4)+4}$$
$$= \frac{32}{12} = \frac{8}{3}\mu F$$

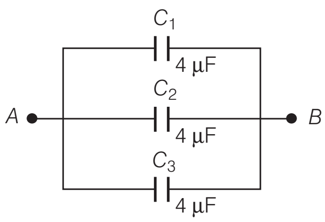
(c) The corresponding network is shown below.

All of three capacitors are in series.

So, $$\frac{1}{C_{net}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

$\therefore \quad C = \frac{4}{3}\mu F$

(d) The corresponding network is shown below.




All of them are in parallel.

So, $\quad C_{net} = C_1 + C_2 + C_3$
$= 4 + 4 + 4 = 12\mu F$

Thus, options (a) is correct.

**49** A capacitor of capacity $C_1$ is charged upto potential $V$ volt and then connected in parallel to an uncharged capacitor of capacity $C_2$. The final potential difference across each capacitor will be **[CBSE AIPMT 2002]**

(a) $\frac{C_2 V}{C_1 + C_2}$ (b) $\frac{C_1 V}{C_1 + C_2}$

(c) $\left(1 + \frac{C_2}{C_1}\right)V$ (d) $\left(1 - \frac{C_2}{C_1}\right)V$

**Ans.** *(b)*

The common potential difference across two capacitors connected in parallel.

$$V_{eq} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

Here, potential of charged capacitor

$V_1 = V,$

potential of uncharged capacitor $V_2 = 0$

$$\therefore \quad V_{eq} = \frac{C_1 V}{C_1 + C_2}$$

**50** In a parallel plate capacitor, the distance between the plates is $d$ and potential difference across plates is $V$. Energy stored per unit volume between the plates of capacitor is **[CBSE AIPMT 2001]**

(a) $\frac{Q^2}{2V^2}$ (b) $\frac{1}{2}\frac{\varepsilon_0 V^2}{d^2}$

(c) $\frac{1}{2}\frac{V^2}{\varepsilon_0 d^2}$ (d) $\frac{1}{2}\varepsilon_0\frac{V^2}{d}$

**Ans.** *(b)*

Energy stored, in parallel plate capacitor is given by

$$U = \frac{1}{2}\frac{q^2}{C}$$

but $\quad \sigma = \frac{q}{A} \quad$ and $\quad C = \frac{\varepsilon_0 A}{d}$

$$\therefore \quad U = \frac{1}{2}\frac{(\sigma A)^2}{\left(\frac{\varepsilon_0 A}{d}\right)} = \frac{A\sigma^2 d}{2\varepsilon_0}$$

or $$= \frac{1}{2}\left(\frac{\sigma}{\varepsilon_0}\right)^2 \times \varepsilon_0 A d$$

or $$U = \frac{1}{2}E^2 \varepsilon_0 A d$$

Energy stored per unit volume i.e. energy density is thus given by

$$u = \frac{U}{V} = \frac{U}{Ad} = \frac{1}{2}\varepsilon_0 E^2$$

$$\Rightarrow \quad = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2 = \frac{1}{2}\varepsilon_0\frac{V^2}{d^2}$$

**51** A capacitor is charged by connecting a battery across its plates. It stores energy $U$. Now the battery is disconnected and another identical capacitor is connected across it, then the energy stored by both capacitors of the system will be **[CBSE AIPMT 2000]**

(a) $U$ (b) $\frac{U}{2}$ (c) $2U$ (d) $\frac{3}{2}U$

**Ans.** *(b)*

When a capacitor is charged by connecting a battery across its plates, the initial energy stored,

$$U = \frac{q^2}{2C}$$

When the battery is disconnected, then the charge remains constant i.e. $q$ = constant. Now, another identical capacitor is connected across it i.e. the capacitors are connected in parallel, so the equivalent capacitance

$$C_{eq} = C_1 + C_2$$
$$= C + C = 2C$$

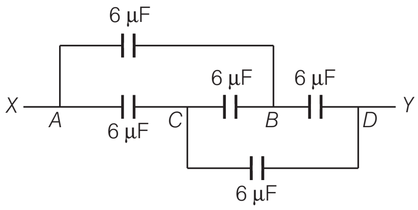
Thus, final energy stored by the system of capacitors,

$$U' = \frac{q^2}{2C_{eq}} \Rightarrow U' = \frac{q^2}{2(2C)}$$

$$U' = \frac{1}{2}U$$

$$\therefore \quad U' = \frac{U}{2}$$

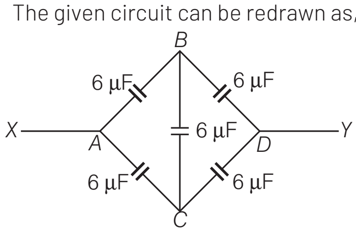
**52** The effective capacitance between points $X$ and $Y$ of figure shown is **[CBSE AIPMT 1999]**




(a) 6μF (b) 12μF (c) 18μF (d) 24μF

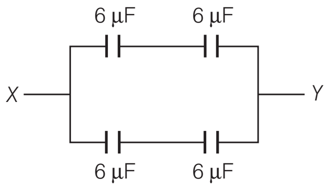
**Ans.** *(a)*

The given circuit can be redrawn as,



It is a balanced Wheatstone's bridge $\left(\text{as } \frac{C_{AB}}{C_{BD}} = \frac{C_{AC}}{C_{CD}} = \frac{6}{6}\right)$.

So, potential of B and C are equal and a 6 μF capacitor between $B$ and $C$ is ineffective. The simplified circuit is shown as below.




Capacitors of 6μF and 6μF in upper arms are in series order, so

$$C'_{eq_1} = \frac{6 \times 6}{6+6} = \frac{36}{12} = 3\mu F$$

Similarly, 6μF and 6μF in lower arms are in series order, so

$$C''_{eq_2} = \frac{6 \times 6}{6+6} = 3\mu F$$

Now, $C'_{eq_1}$ and $C''_{eq_2}$ are in parallel, hence

$$C_{net} = C'_{eq_1} + C''_{eq_2} = 3 + 3 = 6\mu F$$

**53** If the potential of a capacitor having capacity 6μF is increased from 10 V to 20 V, then increase in its energy will be **[CBSE AIPMT 1995]**

(a) $4 \times 10^{-4}$ J (b) $4 \times 10^{-14}$ J
(c) $9 \times 10^{-4}$ J (d) $12 \times 10^{-6}$ J

**Ans.** ***(c)***

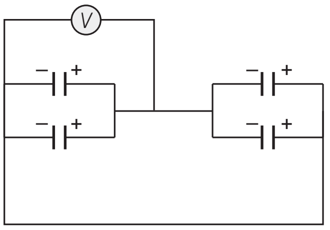
Energy stored in a charged capacitor is in the form of electric field energy and it resides in the dielectric medium between the plates. This energy stored in the capacitor is given by

$$U = \frac{1}{2}CV^2$$

If initial potential is $V_1$ and final potential is $V_2$, then increase in energy $(\Delta U)$

$$\Delta U = \frac{1}{2}C(V_2^2 - V_1^2)$$
$$= \frac{1}{2} \times (6 \times 10^{-6}) \times [(20)^2 - (10)^2]$$
$$= (3 \times 10^{-6}) \times 300 = 9 \times 10^{-4} \text{ J}$$

**54** The four capacitors, each of 25μF are connected as shown in figure. The DC voltmeter reads 200 V. The charge on each plate of capacitor is **[CBSE AIPMT 1994]**




(a) $\pm 2 \times 10^{-3}$ C (b) $\pm 5 \times 10^{-3}$ C
(c) $\pm 2 \times 10^{-2}$ C (d) $\pm 5 \times 10^{-2}$ C

**Ans.** ***(b)***

As from the given diagram, potential difference across each capacitor is 200 V.

So, charge on each plate of capacitor is given by

$$Q = \pm CV$$

[$C$ = capacitance of capacitor
$V$ = voltage or potential difference across the capacitor]

$$= \pm 25 \times 10^{-6} \times 200 = \pm 5 \times 10^{-3} \text{ C}$$

**55** A 4μF capacitor is charged to 400 V and then its plates are joined through a resistance of 1kΩ. The heat produced in the resistance is **[CBSE AIPMT 1989]**

(a) 0.16 J (b) 1.28 J
(c) 0.64 J (d) 0.32 J

**Ans.** ***(d)***

The energy stored in the capacitor $= \frac{1}{2}CV^2$.

This energy will be converted into heat in the resistor.

∴ Heat produced = energy stored

$$= \frac{1}{2}CV^2$$
$$= \frac{1}{2} \times 4 \times 10^{-6} \times (400)^2$$
$$= 0.32 \text{ J}$$