15

Electric Charges and Fields

TOPIC 1

Electric Charges and Coulomb's Law

01 The acceleration of an electron due to the mutual attraction between the electron and a proton when they are 1.6 Å apart is, ($m_e \simeq 9 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C)

$\left(\text{take, } \dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}\right)$

[NEET (Oct.) 2020]

(a) 10^{24} m/s^2 (b) 10^{23} m/s^2
(c) 10^{22} m/s^2 (d) 10^{25} m/s^2

Ans. *(c)*

Force of mutual attraction between the electron and proton.

(when, $r = 1.6$ A° $= 1.6 \times 10^{-10}$ m) is given as

$$F = 9 \times 10^9 \times \frac{e^2}{r^2}$$

$$= 9 \times 10^9 \times \frac{(1.6 \times 10^{-19})^2}{(1.6 \times 10^{-10})^2}$$

$$= 9 \times 10^{-9} \text{ N}$$

∴ Acceleration of electron

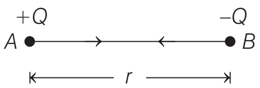
$$= \frac{F}{m_e} = \frac{9 \times 10^{-9}}{9 \times 10^{-31}} = 10^{22} \text{ m/s}^2$$

02 Two point charges A and B, having charges $+Q$ and $-Q$ respectively, are placed at certain distance apart and force acting between them is F. If 25% charge of A is transferred to B, then force between the charges becomes **[NEET (National) 2019]**

(a) $\dfrac{9F}{16}$ (b) $\dfrac{16F}{9}$ (c) $\dfrac{4F}{3}$ (d) F

Ans. *(a)*

The force between two point charges A and B having charge $+Q$ and $-Q$ respectively is given by



$$F = \frac{K\,Q_A Q_B}{r^2} = \frac{KQ(-Q)}{r^2} = -\frac{KQ^2}{r^2} \quad \text{...(i)}$$

where, K = constant $= \dfrac{1}{4\pi\varepsilon_0}$

and r = distance between two charges A and B.

When 25% charge of A is transferred to B, then new amount of charge on A and B respectively become

$$Q'_A = \frac{75}{100}(Q_A) = \frac{3}{4}Q$$

$$Q'_B = \left(\frac{25}{100}Q_A + Q_B\right) = \left(\frac{1}{4}Q - Q\right) = \frac{-3}{4}Q$$

So, the force between the two charges now becomes

$$F' = \frac{K\,Q'_A\,Q'_B}{r^2} = \frac{K\left(\frac{3Q}{4}\right)\left(-\frac{3}{4}Q\right)}{r^2}$$

$$= \frac{-9KQ^2}{16r^2} = \frac{9}{16}F \quad \text{[from Eq. (i)]}$$

Thus, the new force between the charges is 9/16 time the initial force between the charges.

03 Suppose the charge of a proton and an electron differ slightly. One of them is $-e$ and the other is $(e + \Delta e)$. If the net of electrostatic force and gravitational force between two hydrogen atoms placed at a distance d (much greater than atomic size) apart is zero, then Δe is of the order [Given mass of hydrogen, $m_h = 1.67 \times 10^{-27}$ kg] **[NEET 2017]**

(a) 10^{-20} C (b) 10^{-23} C
(c) 10^{-37} C (d) 10^{-47} C

Ans. *(c)*

Net charge on one H-atom

$$q = -e + e + \Delta e = \Delta e$$

Net electrostatic repulsive force between two H-atoms

$$F_r = \frac{Kq^2}{d^2}$$

$$= \frac{K(\Delta e)^2}{d^2}$$

Similarly, net gravitational attractive force between two H-atoms

$$F_G = \frac{Gm_r^2}{d^2}$$

It is given that, $F_r - F_G = 0$

$$\Rightarrow \quad \frac{K(\Delta e)^2}{d^2} - \frac{Gm_r^2}{d^2} = 0$$

$$\Rightarrow \quad (\Delta e)^2 = \frac{Gm_r^2}{K}$$

$$(\Delta e)^2 = \frac{(6.67 \times 10^{-11})\,(1.67 \times 10^{-27})^2}{9 \times 10^9}$$

$$\Rightarrow \quad \Delta e = 1.437 \times 10^{-37} \text{ C}$$

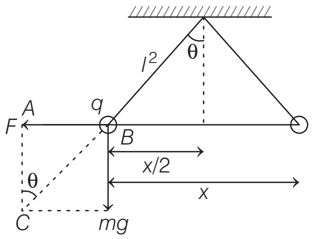
04 Two identical charged spheres suspended from a common point by two massless strings of lengths l, are initially at a distance d $(d << l)$ apart because of their mutual repulsion. The charges begin to leak from both the spheres at a constant rate. As a result, the spheres approach each other with a velocity v. Then, v varies as a function of the distance x between the sphere, as **[NEET 2016]**

(a) $v \propto x$ (b) $v \propto x^{-\frac{1}{2}}$

(c) $v \propto x^{-1}$ (d) $v \propto x^{\frac{1}{2}}$

Ans. ***(b)***

According to question, two identical charged spheres suspended from a common point by two massless strings of length L.



$\because$ In ΔABC

$$\tan\theta = \frac{F}{mg} \quad \text{or} \quad \frac{F}{mg} = \tan\theta \quad ...(i)$$

Since, the charges begins to leak from both the spheres at a constant rate. As a result, the spheres approach each other with velocity v.

Therefore, Eq. (i) can be rewritten as

$$\frac{Kq^2}{x^2 mg} = \frac{x/2}{\sqrt{l^2 - \frac{x^2}{4}}}$$

$$\Rightarrow \quad \frac{Kq^2}{x^2 mg} = \frac{x}{2l} \quad \text{or} \quad q^2 \propto x^3$$

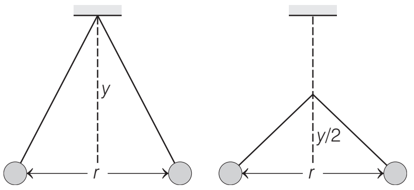
$$\Rightarrow \quad q \propto x^{3/2}$$

$$\Rightarrow \quad \frac{dq}{dt} \propto \frac{d(x^{3/2})}{dx} \cdot \frac{dx}{dt}$$

$$\Rightarrow \quad \frac{dq}{dt} \propto x^{1/2} . v$$

$$\Rightarrow \quad v \propto \frac{1}{x^{1/2}} \quad \text{or} \quad v \propto x^{-1/2}$$

05 Two pith balls carrying equal charges are suspended from a common point by strings of equal length, the equilibrium separation between them is r. Now, the strings are rigidly clamped at half the height. The equilibrium separation between the balls now becomes **[NEET 2013]**

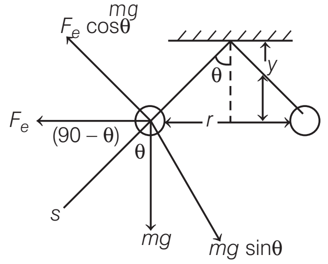


(a) $\left(\frac{1}{\sqrt{2}}\right)^2$ (b) $\left(\frac{r}{\sqrt{2}}\right)$ (c) $\left(\frac{2r}{\sqrt{3}}\right)$ (d) $\left(\frac{2r}{3}\right)$

Ans. ***(b)***

Case I $\quad F_e \cos\theta = mg \sin\theta$

$$\Rightarrow \quad \frac{F_e}{mg} = \tan\theta$$



$$\Rightarrow \quad mg = \frac{F_e}{\tan\theta} \quad ...(i)$$

Case II $\quad F_e' \cos\theta_1 = mg \sin\theta_1$

$$\Rightarrow \quad \frac{F_e'}{mg} = \tan\theta_1$$

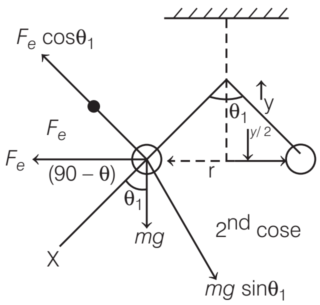
$$\Rightarrow \quad mg = \frac{F_e'}{\tan\theta_1} \quad ...(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{F_e}{\tan\theta} = \frac{F_e'}{\tan\theta_1} \Rightarrow \frac{F_e}{F_e'} = \frac{\tan\theta}{\tan\theta_1}$$

As, $\tan\theta = \dfrac{P\,(\text{perpendicular})}{b\,(\text{base})} = \dfrac{r}{2y}$

$$\tan\theta_1 = P/b = \frac{r}{2 \times y} \times 2 = \frac{r}{y}$$



$$\frac{F_e}{F_e'} = \frac{r}{2y \times r} \times y \Rightarrow F_e' = 2F_e$$

So, $\dfrac{kq^2}{r'^2} = \dfrac{2kq^2}{r^2} \Rightarrow \dfrac{1}{r'} = \dfrac{\sqrt{2}}{r}$

$$\therefore \quad r' = \frac{r}{\sqrt{2}}$$

06 Two positive ions, each carrying a charge q, are separated by a distance d. If F is the force of repulsion between the ions, the number of electrons missing from each ion will be (e being the charge on an electron) **[CBSE AIPMT 2010]**

(a) $\dfrac{4\pi\varepsilon_0 Fd^2}{e^2}$ (b) $\sqrt{\dfrac{4\pi\varepsilon_0 Fe^2}{d^2}}$

(c) $\sqrt{\dfrac{4\pi\varepsilon_0 Fd^2}{e^2}}$ (d) $\dfrac{4\pi\varepsilon_0 Fd^2}{q^2}$

Ans. ***(c)***

Two positive ions each carrying a charge q are kept at a distance d, then it is found that force of repulsion between them is

$$F = \frac{kqq}{d^2} = \frac{1}{4\pi\varepsilon_0} \frac{qq}{d^2}$$

where, $\quad q = ne$

$$\therefore \quad F = \frac{1}{4\pi\varepsilon_0} \frac{n^2 e^2}{d^2}$$

$$\Rightarrow \quad n = \sqrt{\frac{4\pi\varepsilon_0 Fd^2}{e^2}}$$

07 An electron is moving round the nucleus of a hydrogen atom in a circular orbit of radius r. The coulomb force $\mathbf{F}$ between the two is **[CBSE AIPMT 2003]**

(a) $k\dfrac{e^2}{r^3}\mathbf{r}$ (b) $-k\dfrac{e^2}{r^3}\mathbf{r}$

(c) $k\dfrac{e^2}{r^2}\hat{\mathbf{r}}$ (d) $-k\dfrac{e^2}{r^3}\hat{\mathbf{r}}$

(where, $k = \dfrac{1}{4\pi\varepsilon_0}$)

Ans.*(b)*

Let charges on an electron and hydrogen nucleus be q_1 and q_2. Then, Coulomb's force between them at a distance r is,

$$\mathbf{F} = -\frac{1}{4\pi\,\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}$$

Putting, $\quad \dfrac{1}{4\pi\varepsilon_0} = k \quad$ (given)

$$\mathbf{F} = -k\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$$

Since, the nucleus of hydrogen atom has one proton, so charge on nucleus is e i.e. $q_2 = +e$

also $q_1 = e$ for electron

So, $\quad \mathbf{F} = -k\dfrac{e.e}{r^2}\hat{\mathbf{r}} = -k\dfrac{e^2}{r^2}\hat{\mathbf{r}}$

but $\quad \hat{\mathbf{r}} = \dfrac{\mathbf{r}}{|\mathbf{r}|} = \dfrac{\mathbf{r}}{r}$.

Hence, $\quad \mathbf{F} = -k\dfrac{e^2}{r^2} \cdot \dfrac{\mathbf{r}}{r}$

$$= -k\frac{e^2}{r^3}.\mathbf{r}$$

08 When air is replaced by a dielectric medium of constant K, the maximum force of attraction between two charges, separated by a distance **[CBSE AIPMT 1999]**

(a) decreases K times

(b) increases K times

(c) remains unchanged

(d) becomes $\dfrac{1}{K^2}$ times

Ans. *(a)*

According to Coulomb's law, force between two charges is directly proportional to product of charges and inversely proportional to square of distance between them. Thus,

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \quad \text{...(i)}$$

where, $\frac{1}{4\pi\varepsilon_0}$ = proportionality constant.

If a dielectric medium of constant K is placed between them, then new force between them,

$$F_1 = \frac{1}{4\pi\varepsilon_0 K} \cdot \frac{q_1 q_2}{r^2} \quad \text{...(ii)}$$

Dividing Eq. (ii) by Eq. (i), we have

$$\frac{F_1}{F} = \frac{1}{K}$$

or $$F' = \frac{F}{K}$$

Thus, new force decreases K times.

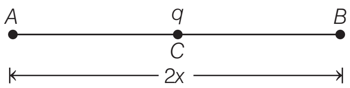
09 A charge q is placed at the centre of the line joining two exactly equal positive charges Q. The system of three charges will be in equilibrium, if q is equal to **[CBSE AIPMT 1995]**

(a) $-\frac{Q}{4}$ (b) $+Q$ (c) $-Q$ (d) $\frac{Q}{2}$

Ans. *(a)*

Let two equal charges Q each be held at A and B, where $AB = 2x$. C is the centre of AB, where charge q is held.

Net force on q is zero. So, q is already in equilibrium.



For the three charges to be in equilibrium, net force on each charge must be zero.

Now, total force on Q at B is

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{Qq}{x^2} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q\,Q}{(2x)^2} = 0$$

or $$\frac{1}{4\pi\varepsilon_0} \cdot \frac{Qq}{x^2} = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q^2}{4x^2}$$

or $$q = -\frac{Q}{4}$$

10 A charge q is placed at the centre of the line joining two exactly equal positive charges Q. The system of three charges will be in equilibrium, if q is equal to **[CBSE AIPMT 1995]**

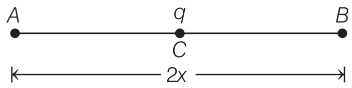
(a) $-\frac{Q}{4}$ (b) $+Q$

(c) $-Q$ (d) $\frac{Q}{2}$

Ans. *(a)*

Let two equal charges Q each be held at A and B, where $AB = 2x$. C is the centre of AB, where charge q is held.

Net force on q is zero. So, q is already in equilibrium.



For the three charges to be in equilibrium, net force on each charge must be zero.

Now, total force on Q at B is

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{Qq}{x^2} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q\,Q}{(2x)^2} = 0$$

or $$\frac{1}{4\pi\varepsilon_0} \cdot \frac{Qq}{x^2} = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q^2}{4x^2}$$

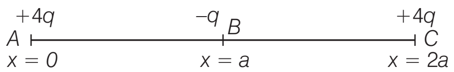
or $$q = -\frac{Q}{4}$$

11 Point charges $+4q$, $-q$ and $+4q$ are kept on the x-axis at points $x = 0$, $x = a$ and $x = 2a$, respectively. Then, **[CBSE AIPMT 1988]**

(a) only $-q$ is in stable equilibrium
(b) None of the charges is in equilibrium
(c) all the charges are in unstable equilibrium
(d) all the charges are in stable equilibrium

Ans. *(c)*

Net force on charge placed at A due to charges placed at C and B



$$F_A = \frac{1}{4\pi\varepsilon_0} \frac{(+4q)(-q)}{a^2} + \frac{1}{4\pi\varepsilon_0} \frac{(+4q)(+4q)}{(2a)^2}$$

$$= -\frac{1}{4\pi\varepsilon_0} \frac{4q^2}{a^2} + \frac{1}{4\pi\varepsilon_0} \frac{4q^2}{a^2} = 0$$

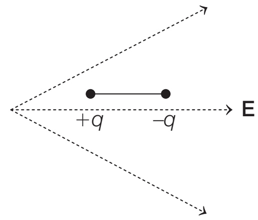
Similarly, F_B and F_C will be zero.

As net force on each charge is zero, therefore all the charges are in equilibrium. If we were to displace $-q$ to the right, (in figure), net force of attraction will be to the right which will displace it further. Therefore, equilibrium is unstable.

TOPIC 2

Electric and Field Lines

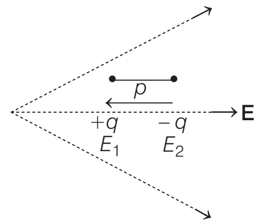
12 A dipole is placed in an electric field as shown. In which direction will it move? **[NEET 2021]**



(a) Towards the left as its potential energy will increase
(b) Towards the right as its potential energy will decrease
(c) Towards the left as its potential energy will decrease
(d) Towards the right as its potential energy will increase

Ans. *(b)*

We know that, the direction of electric dipole moment is from negative charge to positive charge.



The strength of the electric field at $+q$ is greater than that of $-q$ charge, i.e. $|E_1| > |E_2|$.

So, the net force on the dipole act towards the right side.

We know that, a system always moves to decrease in its potential energy.

So, the given dipole move towards right side as its potential energy will decreases.

13 A spherical conductor of radius 10 cm has a charge of 3.2×10^{-7} C distributed uniformly. What is the magnitude of electric field at a point 15 cm from the centre of the sphere? **[NEET (Sep.) 2020]**

$$\left(\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \ \text{Nm}^2/\text{C}^2\right)$$

(a) 1.28×10^5 N/C (b) 1.28×10^6 N/C
(c) 1.28×10^7 N/C (d) 1.28×10^4 N/C

Ans. *(a)*

Given, radius, $r = 10\text{ cm} = 10 \times 10^{-2}$ m

Charge, $q = 3.2 \times 10^{-7}$ C

Electric field, $E = ?$

Electric field at a point ($x = 15$ cm) from the centre of the sphere is

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{x^2}$$

$$= 9 \times 10^9 \times \frac{3.2 \times 10^{-7}}{(15 \times 10^{-2})^2}$$

$$= 1.28 \times 10^5 \text{ N/C}$$

Hence, correct option is (a).

14 An electron falls from rest through a vertical distance h in a uniform and vertically upward directed electric field E. The direction of electric field is now reversed, keeping its magnitude the same. A proton is allowed to fall from rest in it through the same vertical distance h. The time of fall of the electron, in comparison to the time of fall of the proton is **[NEET 2018]**

(a) 10 times greater
(b) 5 times greater
(c) smaller
(d) equal

Ans. *(c)*

Force on a charged particle in the presence of an electric field is given as

$$F = qE \quad \text{...(i)}$$

where, q is the charge on the charged particle and E is the electric field.

From Newton's second law of motion, force on a particle with mass m is given as

$$F = ma \quad \text{...(ii)}$$

where, a is the acceleration.

From Eqs. (i) and (ii), we get

$$F = ma = qE$$

$$\Rightarrow \quad a = \frac{qE}{m} \quad \text{...(iii)}$$

Now, consider that a particle falls from rest through a vertical distance h. Therefore, $u = 0$ and the second equation of motion becomes

$$s = ut + \frac{1}{2}at^2$$

or $$h = 0 \times t + \frac{1}{2}at^2$$

$$= \frac{1}{2} \times \frac{qE}{m}t^2 \quad \text{[from Eq. (iii)]}$$

$$\Rightarrow \quad t^2 = \frac{2hm}{qE}$$

or $$t = \sqrt{\frac{2hm}{qE}}$$

Since, the particles given in the question is electron and proton; and the quantity $\sqrt{\frac{2h}{qE}}$ (here, $q_p = q_e = e$) for both of them is constant. Thus, we can write

$$t = k\sqrt{m}$$

where, $$k = \sqrt{\frac{2h}{qE}}$$

or $$t \propto \sqrt{m}$$

As, mass of proton $(m_p) >>$ mass of electron (m_e).

Thus, the time of fall of an electrons would be smaller than the time of fall of a protons.

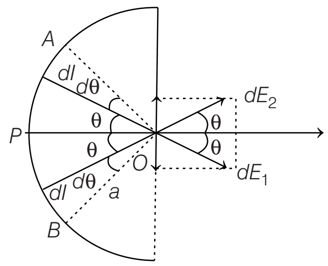
15 A charged wire is bent in the form of a semicircular arc of radius a. If charge per unit length is λ coulomb/metre, the electric field at the centre O is **[CBSE AIPMT 2000]**

(a) $\frac{\lambda}{2\pi a^2 \varepsilon_0}$ (b) $\frac{\lambda}{4\pi^2 \varepsilon_0 a}$

(c) $\frac{\lambda}{2\pi\varepsilon_0 a}$ (d) zero

Ans. *(c)*

Considering symmetric elements each of length dl at A and B, we know that electric fields perpendicular to PO are cancelled and those along PO are added. The electric field due to an element of length $dl (= a\,d\theta)$ along PO.



$$dE = \frac{1}{4\pi\varepsilon_0} \frac{d\theta}{a^2} \cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\lambda dl}{a^2} \cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\lambda(a\,d\theta)}{a^2} \cos\theta \quad (\because dl = a\,d\theta)$$

Net electric field at O

$$E = \int_{-\pi/2}^{\pi/2} dE = 2\int_0^{\pi/2} \frac{1}{4\pi\varepsilon_0} \frac{\lambda\, a \cos\theta\, d\theta}{a^2}$$

$$= 2 \cdot \frac{1}{4\pi\varepsilon_0} \frac{\lambda}{a} [\sin\theta]_0^{\pi/2}$$

$$= 2 \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda}{a} \cdot 1 = \frac{\lambda}{2\pi\varepsilon_0\, a}$$

16 A particle of mass m and charge q is placed at rest in a uniform electric field E and then released. The kinetic energy attained by the particle after moving a distance y is **[CBSE AIPMT 1998]**

(a) qEy^2 (b) qE^2y (c) qEy (d) q^2Ey

Ans. *(c)*

Electric force on charged particle is given by

$$F = qE$$

Kinetic energy attained by particle

$$= \text{work done}$$
$$= \text{force} \times \text{displacement}$$
$$= qE \times y$$

Alternative

Force on charged particle in a uniform electric field is

$$F = ma = Eq$$

or $$a = \frac{Eq}{m} \quad \text{...(i)}$$

From the equation of motion, we have

$$v^2 = u^2 + 2ay$$

$$= 0 + 2 \times \frac{Eq}{m} \times y \quad [u = 0]$$

$$= \frac{2Eqy}{m}$$

Now, kinetic energy of the particle

$$K = \frac{1}{2}mv^2$$

$$= \frac{m}{2} \times \frac{2Eqy}{m} = qEy$$

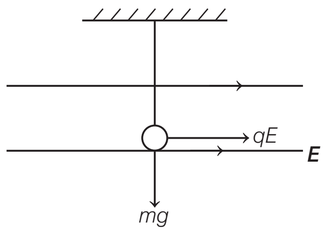
17 A pendulum bob of mass 30.7×10^{-6} kg and carrying a charge 2×10^{-8} C is at rest in a horizontal uniform electric field of 20000 V/m. The tension in the thread of the pendulum is ($g = 9.8\text{ m/s}^2$) **[CBSE AIPMT 1990]**

(a) 3×10^4 N (b) 4×10^{-4} N

(c) 5×10^{-4} N (d) 6×10^{-4} N

Ans. *(c)*

There are 2 forces acting on pendulum (i) its weight



(i) Weight,

$$mg = 30.7 \times 10^{-6} \times 9.8$$
$$= 3 \times 10^{-4} \text{ N}$$

vertically downward, and

(ii) Force, $F = qE$

$= 2 \times 10^{-8} \times 20000$

$= 4 \times 10^{-4}$ N

In the horizontal direction.

So, net tension, $T = \sqrt{(mg)^2 + F^2}$

$= \sqrt{(3 \times 10^{-4})^2 + (4 \times 10^{-4})^2}$

$= 5 \times 10^{-4}$ N

TOPIC 3
Electric Dipole

18 Polar molecules are the molecules **[NEET 2021]**

(a) having zero dipole moment
(b) acquire a dipole moment only in the presence of electric field due to displacement of charges.
(c) acquire a dipole moment only when magnetic field is absent
(d) having a permanent electric dipole moment

Ans. ***(d)***

In polar molecules, the positive charge and negative charge do not overlap each other. So, the total charge on the polar molecule is zero. Thus, these types of molecules have permanent electric dipole moment. e.g. Water is polar molecule.

19 The electric field at a point on the equatorial plane at a distance r from the centre of a dipole having dipole moment $\vec{P}$ is given by ($r >>$ separation of two charges forming the dipole, ε_0 = permittivity of free space) **[NEET (Oct.) 2020]**

(a) $E = \frac{P}{4\pi\varepsilon_0 r^3}$ (b) $E = \frac{2P}{4\pi\varepsilon_0 r^3}$

(c) $E = -\frac{P}{4\pi\varepsilon_0 r^2}$ (d) $E = -\frac{P}{4\pi\varepsilon_0 r^3}$

Ans. ***(a)***

Electric field due to electric dipole on equatorial plane at a distance r from the centre of dipole is given as

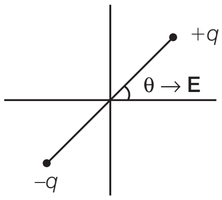
$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^3}$$

20 An electric dipole of moment p is placed in an electric field of intensity E. The dipole acquires a position such that the axis of the dipole makes an angle θ with the direction of the field. Assuming that the potential energy of the dipole to be zero when $\theta = 90°$, the torque and the potential energy of the dipole will respectively be **[CBSE AIPMT 2012]**

(a) $pE \sin\theta, -pE\cos\theta$
(b) $pE\sin\theta, -2pE\cos\theta$
(c) $pE\sin\theta, 2pE\cos\theta$
(d) $pE\cos\theta, -pE\sin\theta$

Ans. ***(a)***

Here, torque, $\tau = pE \sin\theta$



Potential energy of the dipole,

$$U = \int_0^{\theta} \tau\, d\theta$$

$$= \int_{\pi/2}^{\theta} pE \sin\theta d\theta = -pE[\cos\theta]_{\pi/2}^{\theta}$$

$$= -pE\cos\theta$$

21 An electric dipole is placed at an angle of 30° with an electric field intensity 2×10^5 N/C. It experiences a torque equal to 4 Nm. The charge on the dipole, if the dipole length is 2 cm, is **[NEET 2016]**

(a) 8 mC (b) 2 mC (c) 5 mC (d) 7 μC

Ans. ***(b)***

$\because$ Torque on an electric dipole in an electric field,

$$\tau = \mathbf{p} \times \mathbf{E} \Rightarrow |\tau| = pE\sin\theta$$

where, θ is angle between E and p

$\Rightarrow \quad 4 = p \times 2 \times 10^5 \times \sin 30$

$\Rightarrow \quad p = 4 \times 10^{-5}$ cm

$\Rightarrow \quad p = q2l$

$\therefore \quad q2l = 4 \times 10^{-5}$

where, $2l = 2\text{ cm} = 2 \times 10^{-4}$ m

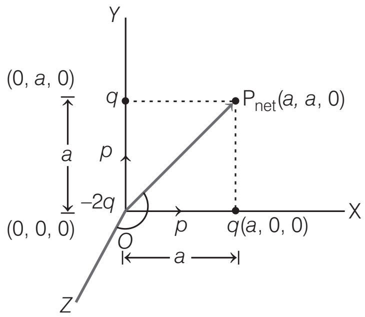
$\therefore \quad q = \frac{4 \times 10^{-5}}{2 \times 10^{-2}} \Rightarrow 2 \times 10^{-3}\text{ C} = 2\text{ mC}$

22 Three point charges $+q$, $-2q$ and $+q$ are placed at points $(x = 0, y = a, z = 0)$, $(x = 0, y = 0, z = 0)$ and $(x = a, y = 0, z = 0)$, respectively. The magnitude and direction of the electric dipole moment vector of this charge assembly are **[CBSE AIPMT 2007]**

(a) $\sqrt{2}\, qa$ along $+y$ direction
(b) $\sqrt{2}\, aq$ along the line joining points $(x = 0, y = 0, z = 0)$ and $(x = a, y = a, z = 0)$
(c) qa along the line joining points $(x = 0, y = 0, z = 0)$ and $(x = a, y = a, z = 0)$
(d) $\sqrt{2}\, aq$ along $+x$ direction

Ans. ***(b)***

Choose the three coordinate axes as X, Y and Z and plot the charges with the given coordinates as shown. O is the origin at which $-2q$ charge is placed. The system is equivalent to two dipoles along x and y-directions respectively. The dipole moments of two dipoles are shown in figure.



The resultant dipole moment will be directed along OP where coordinate of $\mathbf{p}_{net}$ is given by $(a, a, 0)$. The magnitude of resultant dipole moment is

$$\mathbf{p}_{net} = \sqrt{p^2 + p^2}$$

$$= \sqrt{(qa)^2 + (qa)^2} = \sqrt{2}\, qa$$

23 An electric dipole of moment $\mathbf{p}$ is lying along a uniform electric field $\mathbf{E}$. The work done in rotating the dipole by 90° is **[CBSE AIPMT 2006]**

(a) $\sqrt{2}\, pE$ (b) $\frac{pE}{2}$

(c) $2pE$ (d) pE

Ans. ***(d)***

When an electric dipole is placed in an electric field $\mathbf{E}$, a torque $\tau = \mathbf{p} \times \mathbf{E}$ acts on it. This torque tries to rotate the dipole.

If the dipole is rotated from an angle θ_1 to θ_2, then work done by external force is given by

$$W = pE(\cos\theta_1 - \cos\theta_2) \quad \text{...(i)}$$

Putting $\theta_1 = 0°, \theta_2 = 90°$ in the Eq. (i), we get

$$W = pE(\cos 0° - \cos 90°)$$

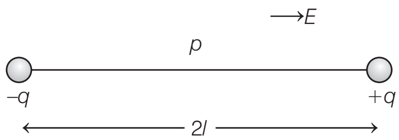
$$= pE(1 - 0) = pE$$

24 An electric dipole has the magnitude of its charge as q and its dipole moment is p. It is placed in a uniform electric field E. If its dipole moment is along the direction of the field, the force on it and its potential energy are respectively **[CBSE AIPMT 2004]**

(a) $2qE$ and minimum
(b) qE and pE
(c) zero and minimum
(d) qE and maximum

Ans. ***(c)***

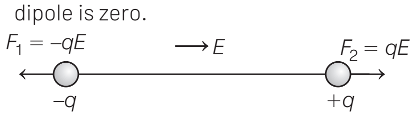
An electric dipole is an arrangement of two equal and opposite charges placed at a distance $2l$. The dipole is placed in electric field as such its dipole moment is in direction of electric field as shown in figure. Direction of dipole moment is from negative to positive charge.



Now, force on the charge q is $F_2 = qE$ along the direction of E and force on charge $-q$ is

$F_1 = -qE$ in the direction opposite to E.

Since, forces on the dipole are equal and opposite, so net force on the electric dipole is zero.



Now, potential energy of the dipole.

$$U = -pE\cos\theta$$

where, θ is the angle between direction of electric field and direction of dipole moment.

$\therefore \quad \theta = 0°$

Hence, $U = -pE\cos 0° = -pE$ (minimum)

25 A point Q lies on the perpendicular bisector of an electric dipole of dipole moment p. If the distance of Q from the dipole is r, (much larger than the size of the dipole) then electric field at Q is proportional to **[CBSE AIPMT 1998]**

(a) p^{-1} and r^2 (b) p and r^{-2}
(c) p^2 and r^{-3} (d) p and r^{-3}

Ans. ***(d)***

Electric field due to a dipole at bisector or at a point on its broad side on position or equatorial position is given by

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^3} \text{ or } E \propto \frac{p}{r^3} \quad \text{...(i)}$$

where, r is the distance of that point from centre of dipole and p is dipole moment.

So, from Eq. (i)

$$E \propto p \text{ and } E \propto r^{-3}$$

26 The formation of a dipole is due to two equal and dissimilar point charges placed at a **[CBSE AIPMT 1996]**

(a) short distance
(b) long distance
(c) above each other
(d) None of these

Ans. ***(a)***

An electric dipole consists of a pair of equal and opposite point charges separated by a very small distance. Atoms or molecules of ammonia, water, alcohol, carbon dioxide, HCl etc are some of the examples of electric dipoles, because in these cases, the centres of positive and negative charge distributions are separated by some small distance.

27 Intensity of an electric field (E) depends on distance r due to a dipole, is related as **[CBSE AIPMT 1996]**

(a) $E \propto \frac{1}{r}$ (b) $E \propto \frac{1}{r^2}$ (c) $E \propto \frac{1}{r^3}$ (d) $E \propto \frac{1}{r^4}$

Ans. ***(c)***

Field intensity on axial line of electric dipole is given by

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3} \quad \text{...(i)}$$

and electric field at equatorial position is given by

$$E = \frac{1}{4\pi\varepsilon_0} \times \frac{p}{r^3} \quad \text{...(ii)}$$

where, p is electric dipole moment.

From Eqs. (i) and (ii), we get

$$E \propto \frac{1}{r^3}$$

28 An electric dipole, consisting of two opposite charges of 2×10^{-6} C each separated by a distance 3 cm is placed in an electric field of 2×10^5 N/C. Torque on the dipole is **[CBSE AIPMT 1995]**

(a) 12×10^{-1} N-m (b) 12×10^{-2} N-m
(c) 12×10^{-3} N-m (d) 12×10^{-4} N-m

Ans. ***(c)***

When a dipole of dipole moment $\mathbf{p}$ is placed in a uniform external electric field $\mathbf{E}$, then torque acting on dipole is

$$\tau = \mathbf{p} \times \mathbf{E} = |p| \times |E| \sin\theta$$

where, $\mathbf{p} = q \times 2\mathbf{l}$ ($2l$ is length of dipole)

Given, $q = 2 \times 10^{-6}$ C

$2l = 3 \text{ cm} = 3 \times 10^{-2}$ m

$E = 2 \times 10^5$ N/C

As, $\tau = pE\sin\theta \quad (\theta = 90°\ p \perp \mathbf{E})$

So, $\tau = pE$

$\therefore \quad \tau = (2 \times 10^{-6}) \times (3 \times 10^{-2}) \times (2 \times 10^5)$

$= 12 \times 10^{-3}$ N-m

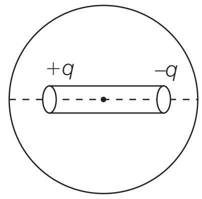
TOPIC 4
Electric Flux and Gauss's Laws

29 A sphere encloses an electric dipole with charge $\pm 3 \times 10^{-6}$ C. What is the total electric flux across the sphere? **[NEET (Odisha) 2019]**

(a) -3×10^{-6} N-m^2/C
(b) zero
(c) 3×10^6 N-m^2/C
(d) 6×10^{-6} N-m^2/C

Ans. ***(b)***

When a sphere encloses a charged dipole,



Here, $q = \pm 3 \times 10^{-6}$ C

Thus, according to Gauss's law, the net electric flux across the closed surface is equal to the net charge enclosed by it divided by ε_0, i.e.

$$\phi_E = \frac{q_{\text{in}}}{\varepsilon_0} = \frac{+3 \times 10^{-6} - 3 \times 10^{-6}}{\varepsilon_0} = 0$$

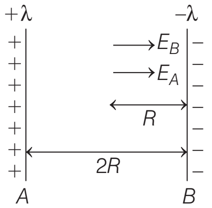
Hence, electric flux across the sphere is zero.

30 Two parallel infinite line charges with linear charge densities $+\lambda$ C/m and $-\lambda$ C/m are placed at a distance of $2R$ in free space. What is the electric field mid-way between the two line charges? **[NEET (National) 2019]**

(a) $\frac{2\lambda}{\pi\varepsilon_0 R}$ N/C (b) $\frac{\lambda}{\pi\varepsilon_0 R}$ N/C
(c) $\frac{\lambda}{2\pi\varepsilon_0 R}$ N/C (d) Zero

Ans. *(b)*

Consider two infinite line charges with linear charge densities $+\lambda$ C/m and $-\lambda$ C/m respectively, which are lying in y-direction as shown in the figure below.



Then, the electric field due to line A at the mid way between the two line charges, i.e. at R is

$$|\mathbf{E}_A| = \frac{\lambda}{2\pi\varepsilon_0 R} \text{ N / C} \quad \text{...(i)}$$

which lies along +ve x-axis (outward), i.e. from A to B.

Similarly, the electric field due to line B at the mid-way between the two line charges, i.e. at R is

$$|\mathbf{E}_B| = \frac{\lambda}{2\pi\varepsilon_0 R} \text{ N / C} \quad \text{...(ii)}$$

Due to negative charge on B, $\mathbf{E}_B$ also lies along +ve x-axis (inward), i.e. from A to B.

So, the resultant electric field at R is given as

$$|\mathbf{E}_R| = |\mathbf{E}_A| + |\mathbf{E}_B|$$

Substituting the values from Eqs. (i) and (ii), we get

$$|\mathbf{E}_R| = \frac{\lambda}{2\pi\varepsilon_0 R} + \frac{\lambda}{2\pi\varepsilon_0 R} = \frac{\lambda}{\pi\varepsilon_0 R} \text{ N / C}$$

Which also lies along the + ve x-axis, i.e. from A to B.

31 A hollow metal sphere of radius R is uniformly charged.The electric field due to the sphere at a distance r from the centre **[NEET (National) 2019]**

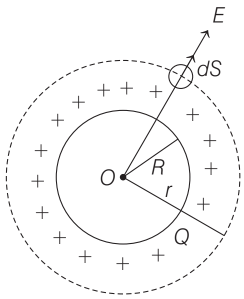
(a) zero as r increases for $r < R$, decreases as r increases for $r > R$
(b) zero as r increases for $r < R$, increases as r increases for $r > R$
(c) decreases as r increases for $r < R$ and for $r > R$
(d) increases as r increases for $r < R$ and for $r > R$

Ans. *(a)*

As the hollow sphere is uniformly charged, so the net charge will appear on the surface of the sphere.

(i) The electric field at a point outside the hollow sphere is

$$\phi = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{Q_{\text{enclosed}}}{\varepsilon_0} \quad \text{[from Gauss' law]}$$



$$\Rightarrow \quad E(4\pi r^2) = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$\Rightarrow \quad E \propto \frac{1}{r^2}$$

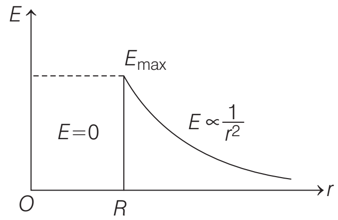
(ii) The electric field at the surface $(r = R)$,

$$E = \frac{Q}{4\pi\varepsilon_0 R}$$

(iii) The electric field inside hollow sphere is

$$E = \frac{Q}{4\pi\varepsilon_0 r} = 0 \quad [\because Q_{\text{inside}} = 0]$$

Thus, the electric field is zero inside the hollow sphere maximum at the surface and decreases as r increases for $r > R$. This can be shown graphically as



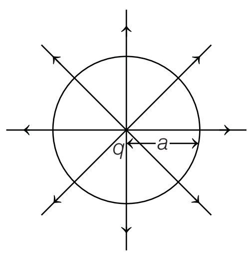
32 The electric field in a certain region is acting radially outward and is given by $E = Ar$. A charge contained in a sphere of radius 'a' centred at the origin of the field' will be given by **[CBSE AIPMT 2015]**

(a) $4\pi\varepsilon_0 Aa^2$ (b) $A\varepsilon_0 a^2$
(c) $4\pi\varepsilon_0 Aa^3$ (d) $\varepsilon_0 Aa^3$

Ans. *(c)*

Given,

$$E = Ar \quad \text{...(i)}$$



Here, $r = a \Rightarrow E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{a^2}$

From Eq. (i),

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{a^2} = Aa \Rightarrow q = 4\pi\varepsilon_0 Aa^3$$

33 What is the flux through a cube of side a if a point charge of q is at one of its corner? **[CBSE AIPMT 2012]**

(a) $\frac{2q}{\varepsilon_0}$ (b) $\frac{q}{8\varepsilon_0}$ (c) $\frac{q}{\varepsilon_0}$ (d) $\frac{q}{2\varepsilon_0} 6a^2$

Ans. *(b)*

Concept According to Gauss's law, the electric flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface.

As the charge enclosed $= q/8$

So, electric flux $= \frac{q_{\text{enclosed}}}{\varepsilon_0} \Rightarrow \phi = \frac{q}{8\varepsilon_0}$

34 A charge Q is enclosed by a Gaussian spherical surface of radius R. If the radius is doubled, then the outward electric flux will **[CBSE AIPMT 2011]**

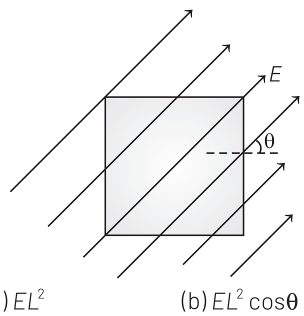
(a) be reduced to half
(b) remain the same
(c) be doubled
(d) increase four times

Ans. *(b)*

Total flux through any enclosed surface is given by, $\phi_{\text{net}} = \frac{\text{net charge enclosed}}{\varepsilon_0}$

Hence, we can say that the electric flux depends only on net enclosed charge by surface.

35 A square surface of side L metre in the plane of the paper is placed in a uniform electric field E (V/m) acting along the same place at an angle θ with the horizontal side of the square as shown in figure. The electric flux linked to the surface in unit of V-m, is **[CBSE AIPMT 2010]**

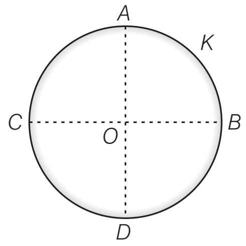


(a) EL^2 (b) $EL^2 \cos\theta$
(c) $EL^2 \sin\theta$ (d) 0

Ans. *(d)*

Flux of electric field $\mathbf{E}$ through any area $\mathbf{A}$ is defined as $\phi = E \cdot A \cos\theta$ or $\phi = \mathbf{E} \cdot \mathbf{A} = 0$, the lines are parallel to the surface.

36



A thin conducting ring of radius R is given a charge $+Q$. The electric field at the centre O of the ring due to the charge on the part AKB of the ring is E. The electric field at the centre due to the charge on the part $ACDB$ of the ring is **[CBSE AIPMT 2008]**

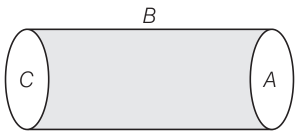
(a) $3E$ along KO (b) E along OK
(c) E along KO (d) $3E$ along OK

Ans. *(b)*

Concept Net electric field at the centre of thin conducting ring is zero.

$$\mathbf{E}_{\text{total}} = 0$$
$$\mathbf{E}_{AKB} + \mathbf{E}_{ACDB} = 0$$
$$\mathbf{E}_{ACDB} = -\mathbf{E}_{AKB}$$
$$\mathbf{E}_{ACDB} = E \text{ along } OK$$

37 A hollow cylinder has a charge q coulomb within it. If ϕ is the electric flux in unit of voltmeter associated with the curved surface B, the flux linked with the plane surface A in unit of voltmeter will be **[CBSE AIPMT 2007]**



(a) $\frac{1}{2}\left(\frac{q}{\varepsilon_0} - \phi\right)$ (b) $\frac{q}{2\varepsilon_0}$

(c) $\frac{\phi}{3}$ (d) $\frac{q}{\varepsilon_0} - \phi$

Ans. *(a)*

Gauss's law states that the net electric flux through any closed surface is equal to the net charge inside the surface divided by ε_0.

i.e. $$\phi_{\text{total}} = \frac{q_{\text{inside}}}{\varepsilon_0}$$

Let electric flux linked with surfaces A, B and C are ϕ_A, ϕ_B and ϕ_C respectively, i.e.

$$\phi_{\text{total}} = \phi_A + \phi_B + \phi_C$$

Since, $\phi_C = \phi_A$

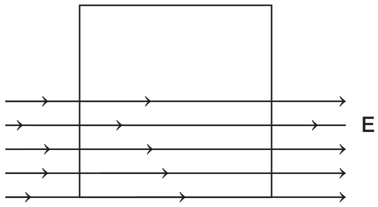
$\therefore$ $$2\phi_A + \phi_B = \phi_{\text{total}} = \frac{q}{\varepsilon_0}$$

or $$\phi_A = \frac{1}{2}\left(\frac{q}{\varepsilon_0} - \phi_B\right)$$

But $\phi_B = \phi$ (given)

Hence, $$\phi_A = \frac{1}{2}\left(\frac{q}{\varepsilon_0} - \phi\right)$$

38 A square surface of side L metre is in the plane of the paper. A uniform electric field **E** (V/m), also in the plane of the paper, is limited only to the lower half of the square surface, (see figure). The electric flux in SI units associated with the surface is **[CBSE AIPMT 2006]**



(a) $\frac{EL^2}{(2\varepsilon_0)}$ (b) $\frac{EL^2}{2}$

(c) zero (d) EL^2

Ans. *(c)*

As we know, the electric flux(ϕ) through any surface area is given by,

$$\phi = \mathbf{E} \cdot \mathbf{ds} = |E|\,|ds|\cos\theta$$

As according to question, surface area is in plane of paper and **E** is also in plane of paper. So, angle between area vector and **E** is 90°

So, $\phi = |E|\,|ds|\cos 90° = 0°$

39 A charge q is located at the centre of a cube. The electric flux through any face is **[CBSE AIPMT 2003]**

(a) $\frac{\pi q}{6(4\pi\varepsilon_0)}$ (b) $\frac{q}{6(4\pi\varepsilon_0)}$

(c) $\frac{2\pi q}{6(4\pi\varepsilon_0)}$ (d) $\frac{4\pi q}{6(4\pi\varepsilon_0)}$

Ans. *(d)*

As we know expression of Gauss's law is

$$\oint \mathbf{E} \cdot \mathbf{ds} = \frac{q_{\text{inside}}}{\varepsilon_0}$$

So, $$\phi = \frac{q}{\varepsilon_0} \qquad \left[\phi = \oint \mathbf{E} \cdot \mathbf{ds}\right]$$

This is the net flux coming out from cube.

Since, a cube has 6 sides so electric flux through any face is $\phi' = \frac{\phi}{6} = \frac{q}{6\varepsilon_0}$

40 A charge $q\,\mu$C is placed at the centre of a cube of a side 0.1 m, then the electric flux diverging from each face of the cube is **[CBSE AIPMT 2001]**

(a) $\frac{q \times 10^{-6}}{24\varepsilon_0}$ (b) $\frac{q \times 10^{-4}}{\varepsilon_0}$

(c) $\frac{q \times 10^{-6}}{6\varepsilon_0}$ (d) $\frac{q \times 10^{-4}}{12\varepsilon_0}$

Ans. *(c)*

The electric flux emerging from the cube is

$$\phi = \frac{1}{\varepsilon_0} \times \text{charge enclosed} \qquad (q_{\text{inside}})$$
$$= \frac{1}{\varepsilon_0} \times q_{\text{inside}} \times 10^{-6}$$
$$= \frac{1}{\varepsilon_0} \times q_{\text{inside}} \times 10^{-6} \quad [q_{\text{inside}} = q \times 10^{-6}]$$

Since, a cube has six faces, so electric flux through each face is,

$$\phi' = \frac{\phi}{6} = \frac{1}{6\varepsilon_0} \times q \times 10^{-6} = \frac{q \times 10^{-6}}{6\varepsilon_0}$$

41 A charge q is placed at the corner of a cube of side a. The electric flux through the cube is **[CBSE AIPMT 2000]**

(a) $\frac{q}{\varepsilon_0}$ (b) $\frac{q}{3\varepsilon_0}$ (c) $\frac{q}{6\varepsilon_0}$ (d) $\frac{q}{8\varepsilon_0}$

Ans. *(d)*

According to Gauss's law, the electric flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface.

Since, q is the charge enclosed by the surface, then electric flux, $\phi = \frac{q}{\varepsilon_0}$

If charge q is placed at a corner of cube, it will be divided into 8 such cubes. Therefore, electric flux through the cube is

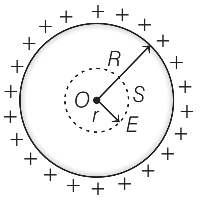
$$\phi' = \frac{1}{8}\left(\frac{q}{\varepsilon_0}\right)$$

42 A hollow insulated conducting sphere is given a positive charge of 10μC. What will be the electric field at the centre of the sphere if its radius is 2 m? **[CBSE AIPMT 1998]**

(a) Zero (b) $5\,\mu\text{Cm}^{-2}$
(c) $20\,\mu\text{Cm}^{-2}$ (d) $8\,\mu\text{Cm}^{-2}$

Ans. *(a)*

Charge resides on the outer surface of a conducting hollow or solid sphere of radius R (say). We consider a spherical surface of radius $r < R$.



By Gauss' theorem,

$$\sum \mathbf{E} \cdot \mathbf{ds} = \frac{q_{\text{inside}}}{\varepsilon_0}$$

or $\quad E \times 4\pi r^2 = \frac{1}{\varepsilon_0} \times q_{\text{inside}}$

and we know that $q_{\text{inside}} = 0$

So, $\quad E = 0$

i.e. electric field inside a hollow sphere is zero.

43 A point charge $+q$ is placed at mid-point of a cube of side L. The electric flux emerging from the cube is **[CBSE AIPMT 1996]**

(a) $\frac{q}{\varepsilon_0}$ (b) $\frac{6qL^2}{\varepsilon_0}$ (c) $\frac{q}{6L^2\varepsilon_0}$ (d) zero

Ans. *(a)*

By Gauss's theorem, total electric flux over the closed surface is $\frac{1}{\varepsilon_0}$ times the total charges contained inside surface.

$\therefore$ Total electric flux

$$= \frac{\text{total charge inside cube}}{\varepsilon_0}$$

or $\quad \phi = \frac{q}{\varepsilon_0}$

44 The electric field strength in air at NTP is 3×10^6 V / m. The maximum charge that can be given to a spherical conductor of radius 3 m is **[CBSE AIPMT 1993]**

(a) 3×10^4 C (b) 3×10^{-3} C
(c) 3×10^{-2} C (d) 3×10^{-1} C

Ans. *(b)*

Given, $E_{max} = 3 \times 10^6$ V/m and $R = 3$ m

We know that,

$$E = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{R^2} \Rightarrow$$

$$Q_{max} = 4\pi\varepsilon_0 R^2 E_{max} \quad = \frac{3 \times 3 \times 3 \times 10^6}{9 \times 10^9}$$

$$= 3 \times 10^{-3} \text{C}$$

45 Two spherical conductors A and B of radii 1 mm and 2 mm are separated by a distance of 5 cm and are uniformly charged. If the spheres are connected by a conducting wire then in equilibrium condition, the ratio of the magnitude of the electric fields at the surfaces of spheres A and B is **[CBSE AIPMT 1992]**

(a) 4 : 1 (b) 1 : 2 (c) 2 : 1 (d) 1 : 4

Ans. *(a)*

Concept The electric field at any point, outside or inside, the conducting sphere can depend only on r (the radial distance from the centre of the sphere to the point).

When the two conducting spheres are connected by a conducting wire, charge will flow from one sphere (having higher potential) to other (having lower potential) till both acquire the same potential.

As, $\quad E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$

So, for different cases, $\frac{E_1}{E_2} = \left(\frac{r_2}{r_1}\right)^2 = 4 : 1$