FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Wednesday 29th June, 2022) **TIME : 3 : 00 PM to 06 : 00 PM**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. A small toy starts moving from the position of rest under a constant acceleration. If it travels a distance of 10m in t s,. the distance travelled by the toy in the next t s will be :

(A) 10m (B) 20m

(C) 30m (D) 40m

Official Ans. by NTA (C)

Sol. u = 0, Say acceleration is a

For t s $10 = \frac{1}{2}at^2$

For 2t s $10 + x = \frac{1}{2}a(2t)^2$

$$\frac{10+x}{10} = \frac{4}{1}$$

$$x = 30 \text{ m}$$

2. At what temperature a gold ring of diameter 6.230 cm be heated so that it can be fitted on a wooden bangle of diameter 6.241 cm? Both the diameters have been measured at room temperature (27ºC). (Given: coefficient of linear thermal expansion of gold $\alpha_L = 1.4 \times 10^{-5}\ K^{-1}$)

(A) 125.7ºC (B) 91.7ºC

(C) 425.7º (D) 152.7ºC

Official Ans. by NTA (D)

Sol. $\Delta \ell = 6.241 - 6.230 = 0.011$ cm

$\Delta \ell = \ell\ \alpha\ \Delta\ \theta$

$0.011 = 6.230 \times 1.4 \times 10^{-5}(\theta - 27)$

$$\theta - 27 = \frac{0.011\times10^5}{6.230\times1.4}$$

$\theta \approx 153.11$ nearest is 152.7°C.

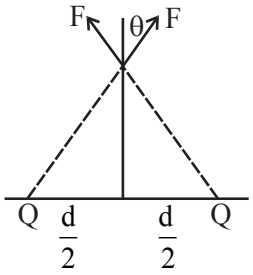
3. Two point charges Q each are placed at a distance d apart. A third point charge q is placed at a distance x from mid-point on the perpendicular bisector. The value of x at which charge q will experience the maximum Coulomb's force is :

(A) x = d (B) $x = \frac{d}{2}$

(C) $x = \frac{d}{\sqrt{2}}$ (D) $x = \frac{d}{2\sqrt{2}}$

Official Ans. by NTA (D)

Sol.



$$F = \frac{KQq}{\left(x^2 + \frac{d^2}{4}\right)}$$

Net force on g = 2 F cosθ

$$F_{net} = \frac{2KQqx}{\left(x^2 + \frac{d^2}{4}\right)^{3/2}}$$

For maximum F_{net}

$$\frac{d\ F_{net}}{dx} = 0$$

we get $x = \frac{d}{2\sqrt{2}}$

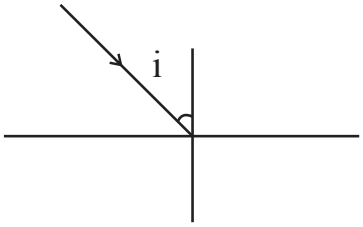
4. The speed of light in media 'A' and 'B' are 2.0×10^{10} cm/s and 1.5×10^{10} chm/s respectively. A ray of light enters from the medium B to A at an incident angle 'θ'. If the ray suffers total internal reflection, then

(A) $\theta = \sin^{-1}\left(\frac{3}{4}\right)$ (B) $\theta > \sin^{-1}\left(\frac{2}{3}\right)$

(C) $\theta < \sin^{-1}\left(\frac{3}{4}\right)$ (D) $\theta > \sin^{-1}\left(\frac{3}{4}\right)$

Official Ans. by NTA (D)

Sol. $\sin i_c = \frac{n_r}{n_d} = \frac{C_d}{C_r} = \frac{1.5\times10^{10}}{2\times10^{10}}$



$\sin i_c = \frac{3}{4}$

$i_c = \sin^{-1}\left(\frac{3}{4}\right)$

for T I R $\quad \theta > i_c$

$\theta > \sin^{-1}\left(\frac{3}{4}\right)$

5. In the following nuclear rection,

$D \xrightarrow{\alpha} D_1 \xrightarrow{\beta^-} D_2 \xrightarrow{\alpha} D_3 \xrightarrow{\gamma} D_4$

Mass number of D is 182 and atomic number is 74. Mass number and atomic number of D_4 respectively will be___.

(A) 174 and 71 (B) 174 and 69

(C) 172 and 69 (D) 172 and 71

Official Ans. by NTA (A)

Sol. Say for D_4 Atomic No = Z

Mass Number = A

A = 182 – 4 – 4 = 174

Z = 74 – 2 + 1 – 2 = 71

6. The electric field at the point associated with a light wave is given by

$E = 200\ [\sin(6\times10^{15})\ t + \sin(9\times10^{15})\ t]\ Vm^{-1}$

Given : $h = 4.14\times10^{-15}$ eVs

If this light falls on a metal surface having a work function of 2.50 eV, the maximum kinetic energy of the photoelectrons will be :

(A) 1.90 eV (B) 3.27 eV

(C) 3.60 eV (D) 3.42 eV

Official Ans. by NTA (D)

Sol. For maximum KE we will take

higher frequency $\left(f = \frac{9\times10^{15}}{2\pi}\text{Hz}\right)$

$K_{max} = hf - \phi$

$= \frac{9\times10^{15}\times4.14\times10^{-15}}{2\pi} - 2.50$

3.43 eV nearest is 3.42 eV

7. A capacitor is discharging through a resistor R. Consider in time t_1, the energy stored in the capacitor reduces to half of its initial value and in time t_2, the charge stored reduces to one eighth of its initial value. The ratio t_1/t_2 will be :

(A) 1/2 (B) 1/3

(C) 1/4 (D) 1/6

Official Ans. by NTA (D)

Sol. In t_1 time energy becomes half so charge will become $\frac{1}{\sqrt{2}}$ time

$q = Q_0\ e^{-\frac{t_1}{RC}} = \frac{Q_0}{\sqrt{2}}$

and $q = Q_0\ e^{-\frac{t_1}{RC}} = \frac{Q_0}{8} = \left(\frac{Q_0}{\sqrt{2}}\right)^6$

$t_2 = 6t_1$

$\frac{t_1}{t_2} = \frac{1}{6}$

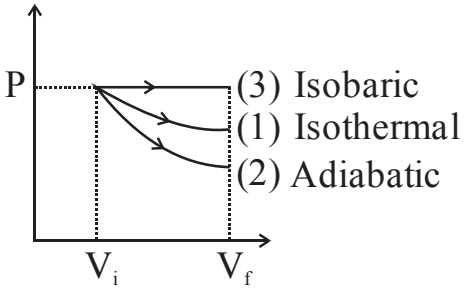
8. Starting with the same initial conditions, an ideal gas expands from volume V_1 to V_2 in three different ways. The work done by the gas is W_1 if the process is purely isothermal. W_2. if the process is purely adiabatic and W_3 if the process is purely isobaric. Then, choose the coned option

(A) $W_1 < W_2 < W_3$ (B) $W_2 < W_3 < W_1$

(C) $W_3 < W_1 < W_2$ (D) $W_2 < W_1 < W_3$

Official Ans. by NTA (D)

Sol.



Area under curve is work

$W_2 < W_1 < W_3$

9. Two long current carrying conductors are placed parallel to each other at a distance of 8 cm between them. The magnitude of magnetic field produced at mid-point between the two conductors due to current flowing in them is 300 μT. The equal current flowing in the two conductors is :

(A) 30A in the same direction.

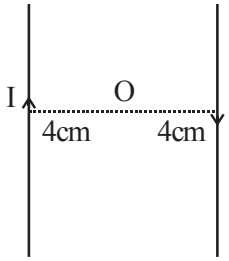
(B) 30A in the opposite direction.

(C) 60A in the opposite direction.

(D) 300A in the opposite direction.

Official Ans. by NTA (B)

Sol.



$$B \text{ at } O = 2\,\frac{\mu_0 I}{2\pi r}$$

$$\frac{2\times 4\pi\times 10^{-7} I}{2\pi\ 4\times 10^{-2}} = 3\times 10^{-4} T$$

I = 30A in opp. direction

10. The time period of a satellite revolving around earth in a given orbit is 7 hours. If the radius of orbit is increased to three times its previous value, then approximate new time period of the satellite will be :

(A) 40 hours (B) 36 hours

(C) 30 hours (D) 25 hours

Official Ans. by NTA (B)

Sol. $T = \frac{2\pi}{\sqrt{GM}}\ r^{3/2}$

$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2} = \left(\frac{1}{3}\right)^{3/2}$$

$T_2 = T_1\ 3\sqrt{3} = 21\ \sqrt{3}$ hours

$\approx$ 36 hours

11. The TV transmission tower at a particular station has a height of 125 m. For dubling the coverage of its range, the height of the tower should be increased by :

(A) 125 m (B) 250 m

(C) 375 (D) 500 m

Official Ans. by NTA (C)

Sol. Range $d = \sqrt{2Rh}$

$d_2 = 2d_1$

$\sqrt{2Rh_2} = 2\sqrt{2Rh_1}$

$h_2 = 4h_1 = 500$ m

$\Delta h = 500$ m $- 125$ m $= 375$ m

12. The motion of a simple pendulum excuting S.H.M. is represented by following equation.

$Y = A \sin(\pi t + \phi)$, where time is measured in second.

The length of pendulum is :

(A) 97.23 cm (B) 25.3 cm

(C) 99.4 cm (D) 406.1 cm

Official Ans. by NTA (C)

Sol. $\omega = \sqrt{\frac{g}{\ell}} = \pi$

$$\frac{g}{\ell} = \pi^2 \Rightarrow \ell = \frac{g}{\pi^2}$$

$$\ell = \frac{980}{\pi^2} \approx 99.4\,\text{cm}$$

13. A vessel contains 16g of hydrogen and 128 g of oxygen at standard temperature and pressure. The volume of the vessel in cm^3 is :

(A) 72×10^5 (B) 32×10^5

(C) 27×10^4 (D) 54×10^4

Official Ans. by NTA (C)

Sol. No of moles of H_2 = 8 moles

No of moles of O_2 = 4 moles

Total moles = 12 moles

At STP 1 mole occupy = $22.4\ell = 22.4 \times 10^3\ cm^3$

12 moles will occupy = $12 \times 22.4 \times 10^3\ cm^3$

$\approx 26.8 \times 10^4\ cm^3$

14. Given below are two statements :

Statement I: The electric force changes the speed of the charged particle and hence changes its kinetic energy: whereas the magnetic force does not change the kinetic energy of the charged particle.

Statement II: The electric force accelerates the positively charged particle perpendicular to the direction of electric field. The magnetic force accelerates the moving charged particle along the direction of magnetic field. In the light of the above statements, choose the most appropriate answer from the options given below:

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

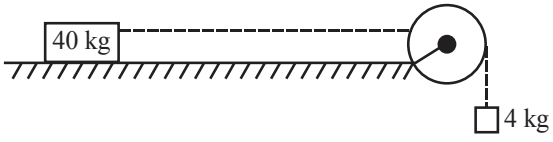
(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Official Ans. by NTA (C)

Sol. Electric field can change speed and kinetic energy but magnetic field can not change speed Δ KE. Because magnetic force is always $\perp$ to velocity.

15. A block of mass 40 kg slides over a surface, when a mass of 4 kg is suspended through an inextensible massless string passing over frictionless pulley as shown below. The coefficient of kinetic friction between the surface and block is 0.02. The acceleration of block is. (Given g = 10 ms^{-2}.)



(A) 1 ms^{-2} (B) 1/5 ms^{-2}

(C) 4/5 ms^{-2} (D) 8/11 ms^{-2}

Official Ans. by NTA (D)

Sol. For 4 kg block

$4g - T = 4a$

For 40 kg block

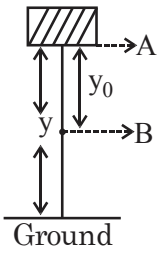
$T - 40g \times 0.02 = 40\ a$

Adding both eq.

$40 - 8 = 44a$

$a = \frac{32}{44} = \frac{8}{11} m/s^2$

16. In the given figure, the block of mass m is dropped from the point 'A'. The expression for kinetic energy of block when it reaches point 'B' is :



(A) $\frac{1}{2}mgy_0^2$ (B) $\frac{1}{2}mgy^2$

(C) $mg(y - y_0)$ (D) mgy_0

Official Ans. by NTA (D)

Sol. Work done by gravity = $K_B - K_A$

$mgy_0 = K_B - 0$

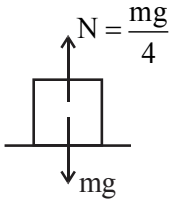
$K_B = mgy_0$

17. A block of mass M placed inside a box descends vertically with acceleration 'a'. The block exerts a force equal to one-fourth of its weight on the floor of the box. The value of 'a' will be :

(A) $\frac{g}{4}$ (B) $\frac{g}{2}$ (C) $\frac{3g}{4}$ (D) g

Official Ans. by NTA (C)

Sol.



$mg - N = ma$

$a = g - \frac{g}{4}$

$a = \frac{3g}{4}$

18. If the electric potential at any point (x, y, z)m in space is given by $V = 3x^2$ volt. The electric field at the point (1, 0, 3) m will be :

(A) 3 Vm^{-1}, directed along positive x-axis.

(B) 3 Vm^{-1}, directed along negative x-axis.

(C) 6 Vm^{-1}, directed along positive x-axis.

(D) 6 Vm^{-1}, directed along negative x-axis.

Official Ans. by NTA (D)

Sol. $E_x = -\frac{\partial V}{\partial x} = -6x$

At (1, 0, 3)

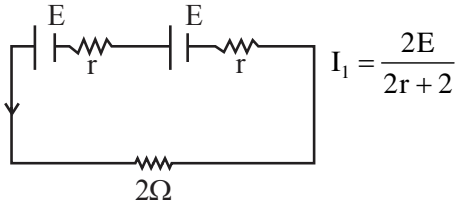
$\vec{E} = -6V/m\ \hat{i}$

19. The combination of two identical cells, whether connected in series or parallel combination provides the same current through an external resistance of 2Ω. The value of internal resistance of each cell is :

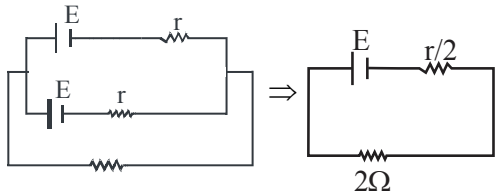
(A) 2Ω (B) 4Ω (C) 6Ω (D) 8Ω

Official Ans. by NTA (A)

Sol.



$I_1 = \frac{2E}{2r+2}$



$I_2 = \frac{E}{\frac{r}{2}+2} = \frac{2E}{r+4}$

$I_1 = I_2$

$2r + 2 = r + 4$

$2r - r = 2\Omega \Rightarrow r = 2\Omega$

20. A person can throw a ball upto a maximum range of 100 m. How high above the ground he can throw the same ball?

(A) 25 m (B) 50 m

(C) 100 m (D) 200 m

Official Ans. by NTA (B)

Sol. $\mathbf{R} = \frac{u^2 \sin 2\theta}{g}$ $R_{max} = \frac{u^2}{g} = 100$

$H_{max} = \frac{u^2}{2g} = \frac{100}{2} = 50m$

SECTION-B

1. The vernier constant of Vernier callipers is 0.1 mm and it has zero error of (–0.05) cm. While measuring diameter of a sphere, the main scale reading is 1.7 cm and coinciding vernier division is 5. The corrected diameter will be_____ $\times 10^{-2}$ cm.

Official Ans. by NTA (180)

Sol. Measured diameter = MSR + VSR × VC

= 1.7 + 0.01 × 5

= 1.75

Corrected = Measured – Error

= 1.75 – (–0.05)

= 1.80 cm

= 180 × 10^{-2} cm

180

2. A small spherical ball of radius 0.1 mm and density 10^4 kg m^{-3} falls freely under gravity through a a distance h before entering a tank of water. If after entering the water the velocity of ball does not change and it continue to fall with same constant velocity inside water, then the value of h wil be_____m.

(Given g = 10 ms^{-2}, viscosity of water = 1.0×10^{-5} N-sm^{-2}).

Official Ans. by NTA (20)

Sol. Speed after falling through height h

Should be equal to terminal velocity

$$\sqrt{2gh} = \frac{2}{9}\frac{r^2(d-\rho)g}{\eta}$$

$$\sqrt{2gh} = \frac{2}{9}\frac{10^{-8}(10000-1000)\times 10}{10^{-5}}$$

$$= \frac{2}{9}\times 10^{-8}\frac{9\times 10^4}{10^{-5}} = 20$$

$2 \times 10 \times h = 400$

$h = 20$ m

3. In an experiment to determine the velocity of sound in air at room temperature using a resonance is observed when the air column has a length of 20.0 cm for a tuning fork of frequency 400 Hz is used. The velocity of the sound at room temperature is 336 ms^{-1}. The third resonance is observed when the air column has a length of ____cm.

Official Ans. by NTA (104)

Sol. For first resonance

$$\ell_1 + e = \frac{\lambda}{4}$$

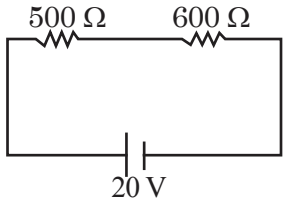
$$\lambda = \frac{336}{400}\times 100 \text{ cm} = 84 \text{ cm} \Rightarrow \frac{\lambda}{4} = 21 \text{ cm}$$

$e = 21 - 20 = 1$ cm

For third resonance

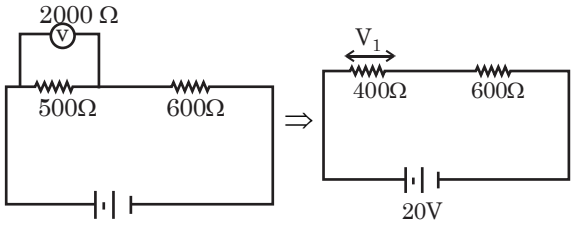
$$\ell_3 + e = \frac{5\lambda}{4} = 105 \text{ cm} \Rightarrow \ell_3 = 104 \text{ cm}$$

4. Two resistors are connected in series across a battery as shown in figure. If a voltmeter of resistance 2000 Ω is used to measure the potential difference across 500 Ω resister, the reading of the voltmeter will be____V.



Official Ans. by NTA (8)

Sol.



$$I = \frac{20}{1000}A$$

$$V_1 = I \times 400 = \frac{20}{1000}\times 400$$

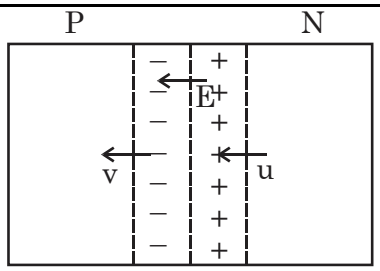
$= 8$ V

5. A potential barrier of 0.4 V exists across a p-n junction. An electron enters the junction from the n-side with a speed of 6.0×10^5 ms^{-1}. The speed with which electron enters the p side will be $\frac{x}{3}\times 10^5 ms^{-1}$ the value of x is _____.

(Given mass of electron = 9×10^{-31} kg, charge on electron = 1.6×10^{-19}C.)

Official Ans. by NTA (14)

Sol.



Work done by Electric field = $K_f - K_i$

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = -1.6 - 10^{-19} \times 0.4$$

$$\frac{1}{2}9\times10^{-31}(v^2 - u^2) = -0.64\times10^{-19}$$

$$u^2 - v^2 = \frac{2\times0.64\times10^{12}}{9}$$

$$v^2 = \left(36 - \frac{128}{9}\right)\times10^{10}$$

$$v = \frac{14}{3}\times10^5 \text{ m/s}$$

x =14

6. The displacement current of 4.425 μA is developed in the space between the plates of parallel plate capacitor when voltage is changing at a rate of 10^6 Vs^{-1}. The area of each plate of the capacitor is 40 cm^2. The distance between each plate of the capacitor is x × 10^{-3}m. The value of x is,

(Permittivity of free space, $E_0 = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2})

Official Ans. by NTA (8)

Sol. Displacement Current = Conduction Current

$$= \frac{dq}{dt}$$

$$I_d = \frac{\epsilon_0 A}{d}\frac{dV}{dt}$$

$$d = \frac{8.85\times10^{-12}\times4\times10^{-3}\times10^6}{4.425\times10^{-6}}$$

= 8 mm

X = 8

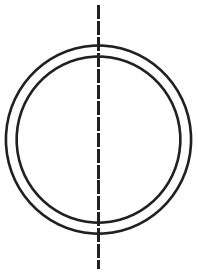
7. The moment of inertia of a uniform thin rod about a perpendicular axis passing through one end is I_1. The same rod is bent into a ring and its moment of inertia about a diameter is I_2. If $\frac{I_1}{I_2}$ is $\frac{x\pi^2}{3}$, then the value of x will be_____.

Official Ans. by NTA (8)

Sol.

$$I_1 = \frac{ml^2}{3}$$

$$\ell = 2\pi r \Rightarrow \frac{\ell}{r} = 2\pi$$



$$I_1 = \frac{mr^2}{2}$$

$$\frac{I_1}{I_2} = \frac{2}{3}\left(\frac{\ell}{r}\right)^2$$

$$= \frac{2}{3}\times4\pi^2 = \frac{8\pi^2}{3}$$

x = 8

8. The half life of a radioactive substance is 5 years. After x years a given sample of the radioactive substance gest reduced to 6.25% of its initial value of x is _____.

Official Ans. by NTA (20)

Sol. **T1/2 = 5 year**

$$N = N_0\left(\frac{1}{2}\right)^{\text{No of half lives}}$$

$$\frac{N}{N_0} = \frac{1}{16} = \left(\frac{1}{2}\right)^4$$

Time = 4 half lives = 20 years

9. In a double slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance from the plane of slits. If the screen is moved by 5×10^{-2} m towards the slits, the change in fringe width is 3×10^{-3} cm. If the distance between the slits is 1 mm, then the wavelength of the light will be _____nm.

Official Ans. by NTA (600)

Sol. $\beta = \frac{\lambda D}{d}$

$\Delta\beta = \frac{\lambda}{d}\Delta D$

$\lambda = \frac{\Delta\beta.d}{\Delta D}$

$= \frac{3\times10^{-5}\times1\times10^{-3}}{5\times10^{-2}}$

$= 60\times10^{-8} = 600 \times 10^{-9}$m

$= 600$nm

10. An inductor of 0.5 mH, a capacitor of 200 μF and a resistor of 2 Ω are connected in series with a 220 V ac source. If the current is in phase with the emf, the frequency of ac source will be___$\times 10^2$ Hz.

Official Ans. by NTA (5)

Sol. If Current is in phase with emf then the frequency of source $= \frac{1}{2\pi\sqrt{LC}}$ (Resonant frequency)

$\frac{1}{2\pi\sqrt{\frac{1}{2}\times10^{-3}\times2\times10^{-4}}}$

$= \frac{1}{2\pi}\times\sqrt{10}\times1000 = 500$ Hz

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Wednesday 29th June, 2022) **TIME : 3 : 00 PM to 06 : 00 PM**

CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION-A

1. Using the rules for significant figures, the correct answer for the expression $\frac{0.02858\times0.112}{0.5702}$ will be:

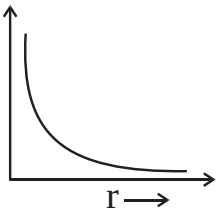
(A) 0.005613 (B) 0.00561

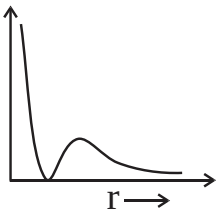
(C) 0.0056 (D) 0.006

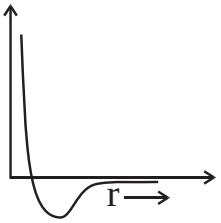
Official Ans. by NTA (B)

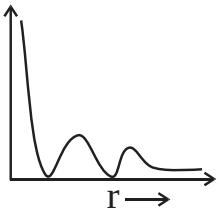
Sol. Reported answer should not be more precise than least precise term in calculations, so there should be three significant figures in reported answer.

2. Which of the following is the correct plot for the probability density $\psi^2(r)$ as a function of distance 'r' of the electron form the nucleus for 2s orbital?

(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (B)

Sol. For 2s, number of radial nodes = 2 – 0 – 1 = 1 and value of ψ^2 is always positive.

3. Consider the species CH_4, NH_4^+ and BH_4^-. Choose the correct option with respect to the there species:

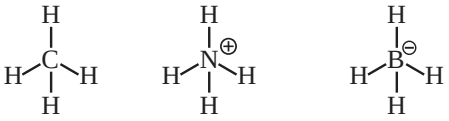
(A) They are isoelectronic and only two have tetrahedral structures

(B) They are isoelectronic and all have tetrahedral structures

(C) Only two are isoelectronic and all have tetrahedral structures

(D) Only two are isoelectronic and only two have tetrahedral structures

Official Ans. by NTA (B)

Sol. 

All are tetrahedral and each have 10 electrons.

4. 4.0 moles of argon and 5.0 moles of PCl_5 are introduced into an evacuated flask of 100 litre capacity at 610 K. The system is allowed to equilibrate. At equilibrium, the total pressure of mixture was found to be 6.0 atm. The K_p for the reaction is [Given : R = 0.082L atm K^{-1} mol^{-1}]

(A) 2.25 (B) 6.24

(C) 12.13 (D) 15.24

Official Ans. by NTA (A)

Sol. $PCl_5 = 5$ mole

Ar = 4 mole

$$P_{Total} = \frac{9\times0.82\times610}{100} = 4.5\,atm$$

$$P_{PCl_5} = \frac{5\times4.5}{9} = 2.5\ ;\ P_{Ar} = \frac{4\times4.5}{9} = 2$$

$$PCl_5 \rightleftharpoons PCl_3 + Cl_2$$

$2.5 - P \quad P \quad P$

$P_{total} = 2.5 - P + P + P + P_{Ar} = 6$

$P = 1.5$

$$K_p = \frac{1.5\times1.5}{1} = 2.25$$

5. A 42.12% (w/v) solution of NaCl causes precipitation of a certain sol in 10 hours. The coagulating value of NaCl for the sol is

[Given : Molar mass : Na = 23.0 g mol^{-1}; Cl = 35.5 g mol^{-1}]

(A) 36 mmol L^{-1}

(B) 36 mol L^{-1}

(C) 1440 mol L^{-1}

(D) 1440 mmol L^{-1}

Official Ans. by NTA (D)

Sol. Data insufficient.

6. Given below are two statements. One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : The first ionization enthalpy for oxygen is lower than that of nitrogen.

Reason R : The four electrons in 2p orbitals of oxygen experience more electron-electron repulsion.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both A and R are correct and R is the correct explanation of A.

(B) Both A and R are correct but R is NOT the correct explanation of A.

(C) A is correct but R is not correct.

(D) A is not correct but R is correct

Official Ans. by NTA (A)

Sol. Ionisation energy = N > O.

In oxygen atom, 2 of the 4 2p electrons must occupy the same 2p orbital resulting in an increased electron electron-repulsion.

7. Match List I with List II.

List I Ore	List II Composition
A. Siderite	I. $FeCO_3$
B. Malachite	II. $CuCO_3.Cu(OH)_2$
C. Sphalerite	III. ZnS
D. Calamine	IV. $ZnCO_3$

Choose the correct answer from the options given below:

(A) A-I, B-II, C-III, D-IV

(B) A-III, B-IV, C-II, D-I

(C) A-IV, B-III, C-I, D-II

(D) A-I, B-II, C-IV, D-III

Official Ans. by NTA (A)

Sol. Siderite – $FeCO_3$

Malachite – $CuCO_3.Cu(OH)_2$

Calamine – $ZnCO_3$

Sphalerite – ZnS

8. Given below are two statements .

Statement I : In $CuSO_4.5H_2O$, Cu–O bonds are present.

Statement II : In $CuSO_4.5H_2O$, ligands coordinating with Cu(II) ion are O-and S-based ligands.

In the light of the above statements, choose the correct answer from the options given below

(A) Both Statement I and Statement II are correct

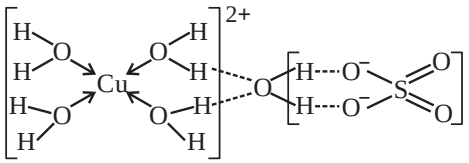
(B) Both Statement I and Statement II are incorrect

(C) Statement I is correct but Statement II is incorrect

(D) Statement I is incorrect but Statement II is correct

Official Ans. by NTA (C)

Sol.



9. Amongst baking soda, caustic soda and washing soda carbonate anion is present in :

(A) washing soda only.

(B) washing soda and caustic soda only.

(C) washing soda and baking soda only.

(D) baking soda, caustic soda and washing soda.

Official Ans. by NTA (A)

Sol. Baking soda $\rightarrow NaHCO_3$

Washing soda $\rightarrow Na_2CO_3.10H_2O$

Caustic soda $\rightarrow NaOH$

10. Number of lone pair (s) of electrons on central atom and the shape of BrF_3 molecule respectively, are :

(A) 0, triangular planar.

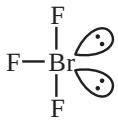
(B) 1, pyramidal.

(C) 2, bent T-shape.

(D) 1, bent T-shape

Official Ans. by NTA (C)

Sol.



Steric no. = 5 (sp^3d), lone pair = 2

Bent T shape.

11. Aqueous solution of which of the following boron compounds will be strongly basic in nature?

(A) $NaBH_4$ (B) $LiBH_4$

(C) B_2H_6 (D) $Na_2B_4O_7$

Official Ans. by NTA (D)

Sol. $Na_2B_4O_7$ gives H_3BO_3 and NaOH (strong base) in water.

12. Sulphur dioxide is one of the components of polluted air. SO_2 is also a major contributor to acid rain. The correct and complete reaction to represent acid rain caused by SO_2 is :

(A) $2\ SO_2 + O_2 \rightarrow 2\ SO_3$

(B) $SO_2 + O_3 \rightarrow SO_3 + O_2$

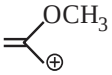
(C) $SO_2 + H_2O_2 \rightarrow H_2SO_4$

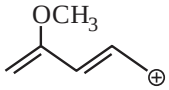
(D) $2\ SO_2 + O_2 + 2\ H_2O \rightarrow 2\ H_2SO_4$

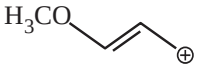
Official Ans. by NTA (D)

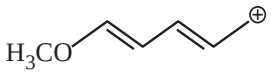
Sol. $2SO_2 + O_2 + 2H_2O \rightarrow 2H_2SO_4$ (Acid rain)

13. Which of the following carbocations is most stable :

(A) 

(B) 

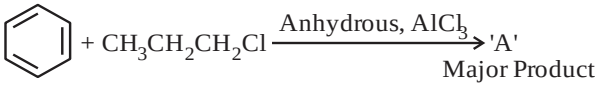
(C) 

(D) 

Official Ans. by NTA (D)

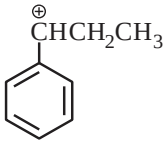
Sol. 

Is most stable carbocation

14. 

The stable carbocation formed in the above reaction is :

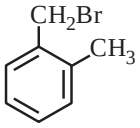
(A) $CH_3CH_2\overset{\oplus}{C}H_2$ (B) $CH_3\overset{\oplus}{C}H_2$

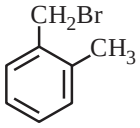
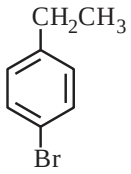
(C) $CH_3-\overset{\oplus}{C}H-CH_3$ (D) 

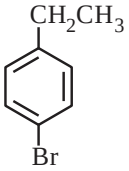
Official Ans. by NTA (C)

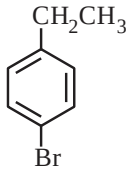
Sol. $CH_3\overset{\oplus}{C}H-CH_3$ is formed in the above reaction

15. Two isomers (A) and (B) with Molar mass 184 g/mol and elemental composition C, 52.2%; H, 4.9% and Br 42.9% gave benzoic acid and p-bromobenzoic acid, respectively on oxidation with $KMnO_4$. Isomer 'A' is optically active and gives a pale yellow precipitate when warmed with alcoholic $AgNO_3$. Isomer 'A' and 'B' are, respectively :

(A) $H_3C–CHBr–C_6H_5$ and 

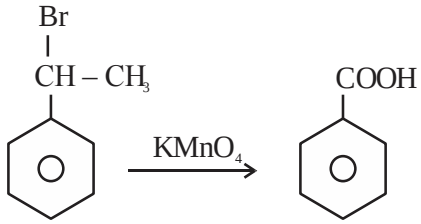
(B)  and 

(C) $H_3C–CHBr–C_6H_5$ and 

(D)  and $H_3C–CHBr–C_6H_5$

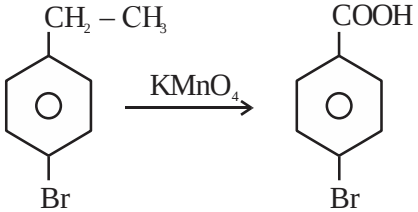
Official Ans. by NTA (C)

Sol.



(A)

Optically Active



16. In Friedel-Crafts alkylation of aniline, one gets :

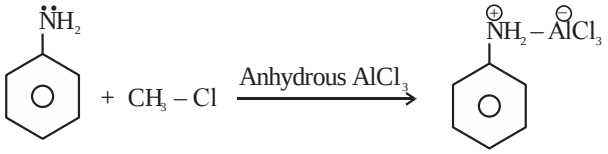
(A) alkylated product with ortho and para substitution.

(B) secondary amine after acidic treatment.

(C) an amide product.

(D) positively charged nitrogen at benzene ring.

Official Ans. by NTA (D)

Sol. 

17. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: Dacron is an example of polyester polymer.

Reason R: Dacron is made up of ethylene glycol and terephthalic acid monomers.

In the light of the above statements, choose the ***most appropriate*** answer from the options given below.

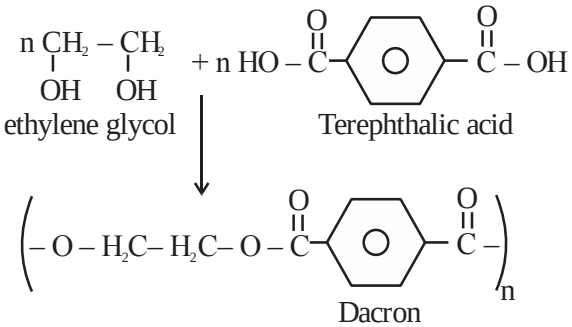
(A) Both **A** and **B** are correct and **R** is the correct explanation of **A**.

(B) Both **A** and **B** are correct but **R** is NOT the correct explanation of **A**.

(C) **A** is correct but **R** is not correct.

(D) **A** is not correct but **R** is correct.

Official Ans. by NTA (A)

Sol. 

18. The structure of protein that is unaffected by heating is :

(A) secondary structure (B) tertiary structure

(C) primary structure (D) quaternary structure

Official Ans. by NTA (C)

Sol. Primary structure of protein is unaffected by physical 'or' chemical changes.

19. The mixture of chloroxylenol and terpineol is an example of :

(A) antiseptic (B) pesticide

(C) disinfectant (D) narcotic analgesic

Official Ans. by NTA (A)

Sol. Antiseptic Dettol is mixture of chloroxylenol and terpineol.

20. A white precipitate was formed when $BaCl_2$ was added to water extract of an inorganic salt. Further, a gas 'X' with characteristic odour was released when the formed white precipitate was dissolved in dilute HCl. The anion present in the inorganic salt is :

(A) I^- (B) SO_3^{2-}

(C) S^{2-} (D) NO_2^-

Official Ans. by NTA (B)

Sol. $BaCl_2 + SO_3^{2-} \rightarrow \underset{\text{white}}{BaSO_3 \downarrow} \xrightarrow{\text{dil.HCl}} \underset{\text{burning sulphur like smell}}{SO_2 \uparrow}$

SECTION-B

1. A box contains 0.90 g of liquid water in equilibrium with water vapour at 27°C. The equilibrium vapour pressure of water at 27°C 32.0 Torr. When the volume of the box is increased, some of the liquid water evaporates to maintain the equilibrium pressure. If all the liquid water evaporates, then the volume of the box must be____ litre. [nearest integer]

(Given: R = 0.082 L atm K^{-1} mol^{-1})

(Ignore the volume of the liquid water and assume water vapours behave as an ideal gas.)

Official Ans. by NTA (29)

)

Sol. $V = \frac{nRT}{P} = \frac{0.90 \times 0.82 \times 300 \times 760}{18 \times 32} = 29.21$

2. 2.2 g of nitrous oxide (N_2O) gas is cooled at a constant pressure of 1 atm from 310 K to 270 K causing the compression of the gas from 217.1 mL to 167.75 mL. The change in internal energy of the process, ΔU is '–x' J. The value of 'x' is __.

[nearest integer]

(Given: atomic mass of N = 14 g mol^{-1} and of O = 16 g mol^{-1}.

Molar heat capacity of N_2O is 100 JK^{-1} mol^{-1})

Official Ans. by NTA (195)

Sol. $N_2O \text{ moles} = \frac{2.2}{44} = \frac{1}{20}$

$\Delta H = nC_p\Delta T = \frac{1}{20} \times 100(-40) = -200J$

$\Delta U = q_p + w$

$w = -P_{ext.}\,\Delta V$

$W = -1\frac{(167.75 - 217.1)}{1000} \times 101.3J$

$w = +5J$

$\Delta U = -200 + 5 = -195J$

3. Elevation in boiling point for 1.5 molal solution of glucose in water is 4K. The depression in freezing point for 4.5 molal solution of glucose in water is 4K. The ratio of molal elevation constant to molal depression constant (K_b/K_f) is____.

Official Ans. by NTA (3)

Sol. $\Delta T_b = iK_b m$

$\Delta T_f = iK_f m$

$\frac{4}{4} = \frac{K_b 1.5}{K_f 4.5}$

$\frac{K_b}{K_f} = 3$

4. The cell potential for the given cell at 298 K

$Pt \mid H_2(g,1\ bar) \mid H^+(aq) \parallel Cu^{2+}(aq) \mid Cu(s)$

is 0.31V. The pH of the acidic solution is found to be 3, whereas the concentration of Cu^{2+} is 10^{-x} M. The value of x is _____.

(Given: $E^{\Theta}_{Cu^{2+}/Cu} = 0.34$ V and $\frac{2.303RT}{F} = 0.06V$)

Official Ans. by NTA (7)

Sol. $H_2(g) + Cu^{2+}(aq.) \rightarrow 2H^+(aq.) + Cu(s)$

$0.31 = 0.34 - \frac{0.06}{2}\log\frac{[H^+]^2}{[Cu^{2+}]}$

$[Cu^{2+}] = 10^{-7}$ M

x = 7

5. The equation

$k = (6.5 \times 10^{12}\ s^{-1})\ e^{-26000K/T}$

is followed for the decomposition of compound A. The activation energy for the reaction is ____ kJ mol^{-1}. [nearest integer]

(Given: R = 8.314 J K^{-1} mol^{-1})

Official Ans. by NTA (216)

)

Sol. $K = Ae^{-Ea/RT} = (6.5 \times 10^{12}\ s^{-1})\ e^{-26000K/T}$

$\frac{Ea}{8.314} = 26000$

Ea = 216.164 kJ/mol.

6. Spin only magnetic moment of $[MnBr_6]^{4-}$ is____ B.M. (round off to the closest integer)

Official Ans. by NTA (6)

Sol. $Mn^{2+} \rightarrow t_{2g^{111}} e_{g^{11}}$

$\mu_s = \sqrt{35}$

= 5.91

= 6

7. For the reaction given below:

$CoCl_3 \,.\, xNH_3 + AgNO_3\,(aq) \rightarrow$

If two equivalents of AgCl precipitate out, then the value of x will be _____.

Official Ans. by NTA (5)

Sol. $CoCl_3.xNH_3 + AgNO_3 \rightarrow AgCl\downarrow$

2 mol

$[Co(NH_3)_5Cl]Cl_2 + AgNO_3 \rightarrow AgCl\downarrow$

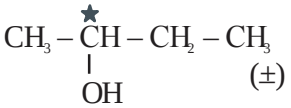
2 mol

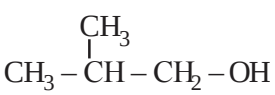
x = 5

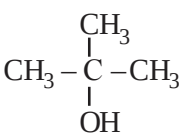
8. The number of chiral alcohol(s) with molecular formula $C_4H_{10}O$ is_____.

Official Ans. by NTA (1)

Sol. $CH_3 - CH_2 - CH_2 - CH_2 - OH$

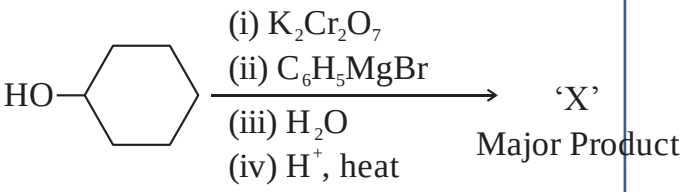






Out of which only two are chiral

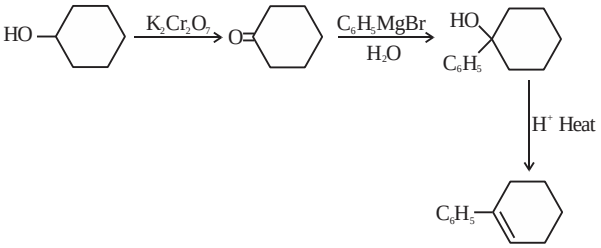
9. In the given reaction



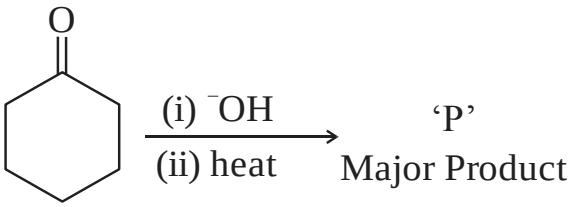
the number of sp^2 hybridised carbon (s) in compound 'X' is_____.

Official Ans. by NTA (8)

Sol.



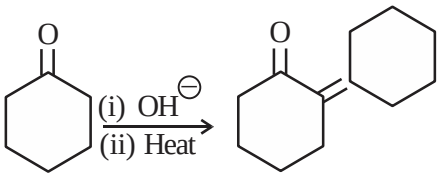
10. In the given reaction,



The number of π electrons present in the product 'P' is _____.

Official Ans. by NTA (4s)

Sol.



FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Wednesday 29th June, 2022) **TIME : 3 : 00 PM to 06 : 00 PM**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Let α be a root of the equation $1 + x^2 + x^4 = 0$. Then the value of $\alpha^{1011} + \alpha^{2022} - \alpha^{3033}$ is equal to:

(A) 1 (B) α

(C) $1 + \alpha$ (D) $1 + 2\alpha$

Official Ans. by NTA (A)

Sol. $x^4 + x^2 + 1 = 0$

$\Rightarrow (x^2 + x + 1)(x^2 - x + 1) = 0$

$\Rightarrow x = \pm\omega, \pm\omega^2$ where $\omega = 1^{1/3}$ and imaginary.

So $\alpha^{1011} + \alpha^{2022} - \alpha^{3033} = 1 + 1 - 1 = 1$

2. Let arg (z) represent the principal argument of the complex number z. The, $|z| = 3$ and $\arg(z - 1) - \arg(z + 1) = \frac{\pi}{4}$ intersect:

(A) Exactly at one point

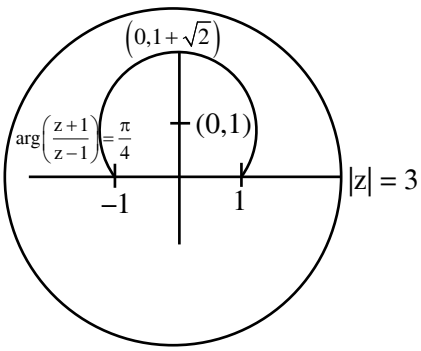
(B) Exactly at two points

(C) Nowhere

(D) At infinitely many points.

Official Ans. by NTA (C)

Sol.



3. Let $A = \begin{pmatrix} 2 & -1 \\ 0 & 2 \end{pmatrix}$. If $B = I - {}^5C_1 (adjA) + {}^5C_2 (adjA)^2 - \ldots - {}^5C_5 (adjA)^5$, then the sum of all elements of the matrix B is:

(A) –5 (B) –6

(C) –7 (D) –8

Official Ans. by NTA (C)

Sol. $B = (I - adjA)^5 = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}^5 = \begin{bmatrix} -1 & -5 \\ 0 & -1 \end{bmatrix}$

Sum of its all elements = –7.

4. The sum of the infinite series $1 + \frac{5}{6} + \frac{12}{6^2} + \frac{22}{6^3} + \frac{35}{6^4} + \frac{51}{6^5} + \frac{70}{6^6} + \ldots.$ is equal to:

(A) $\frac{425}{216}$ (B) $\frac{429}{216}$

(C) $\frac{288}{125}$ (D) $\frac{280}{125}$

Official Ans. by NTA (C)

Sol. $S = 1 + \frac{5}{6} + \frac{12}{6^2} + \frac{22}{6^3} + \frac{35}{6^4} + \ldots.$

$\frac{S}{6} = \quad \frac{1}{6} + \frac{5}{6^2} + \frac{12}{6^3} + \frac{22}{6^4} + \ldots.$

---

on subtraction

$\frac{5}{6}S = 1 + \frac{4}{6} + \frac{7}{6^2} + \frac{10}{6^3} + \frac{13}{6^4} + \ldots.$

$\frac{5}{36}S = \quad 1 + \frac{4}{6^2} + \frac{7}{6^3} + \frac{10}{6^4} + \frac{13}{6^5} + \ldots$

---

on subtraction

$\frac{25}{36}S = 1 + \frac{3}{6} + \frac{3}{6^2} + \frac{3}{6^3} + \ldots. = \frac{8}{5}$

$S = \frac{288}{125}$

5. The value of $\lim\limits_{x\to 1} \frac{\left(x^2-1\right)\sin^2(\pi x)}{x^4-2x^3+2x-1}$ is equal to:

(A) $\frac{\pi^2}{6}$ (B) $\frac{\pi^2}{3}$

(C) $\frac{\pi^2}{2}$ (D) π^2

Official Ans. by NTA (D)

Sol. $\lim\limits_{x\to 1} \frac{(x^2-1)\sin^2 \pi x}{(x^2-1)(x-1)^2} = \lim\limits_{x\to 1}\left(\frac{\sin((1-x)\pi))}{\pi(1-x)}\right)^2 \pi^2 = \pi^2$

6. Let $f:\mathbb{R}\to\mathbb{R}$ be a function defined by $f(x) = (x-3)^{n_1}(x-5)^{n_2}$, $n_1, n_2 \in N$. The, which of the following is <u>NOT</u> true?

(A) For $n_1 = 3$, $n_2 = 4$, there exists $\alpha \in (3,5)$ where f attains local maxima.

(B) For $n_1 = 4$, $n_2 = 3$, there exists $\alpha \in (3,5)$ where f attains local manima.

(C) For $n_1 = 3$, $n_2 = 5$, there exists $\alpha \in (3,5)$ where f attains local maxima.

(D) For $n_1 = 4$, $n_2 = 6$, there exists $\alpha \in (3,5)$ where f attains local maxima.

Official Ans. by NTA (C)

Sol. $f'(x) = (x-3)^{n_1-1}(x-5)^{n_2-1}(n_1+n_2)\left(x - \frac{5n_1+3n_2}{n_1+n_2}\right)$

Option (3) is incorrect since

for $n_1 = 3$, $n_2 = 5$

$f'(x) = 8(x-3)^2(x-5)^4\left(x-\frac{30}{8}\right)$

minima at $x = \frac{30}{8}$

7. Let f be a real valued continuous function on [0,1] and $f(x) = x + \int_0^1 (x-t)f(t)dt$. Then which of the following points (x,y) lies on the curve $y = f(x)$?

(A) (2, 4) (B) (1, 2)

(C) (4, 17) (D) (6, 8)

Official Ans. by NTA (D)

Sol. $f(x) = \left(1 + \int_0^1 f(t)dt\right)x - \int_0^1 tf(t)dt$

$f(x) = Ax - B$ ...(i)

$A = 1 + \int_0^1 f(t)dt = 1 + \int_0^1 (At - B)dt$

$\Rightarrow A = 2(1-B)$ ...(ii)

Also $B = \int_0^1 tf(t)dt = \int_0^1 (At^2 - Bt)dt$

$A = \frac{9}{2}B$ ...(iii)

From (2), (3)

$A = \frac{18}{13}$, $B = \frac{4}{13}$

so f(6) = 8

8. If $\int_0^2 \left(\sqrt{2x} - \sqrt{2x-x^2}\right)dx =$

$\int_0^1 \left(1 - \sqrt{1-y^2} - \frac{y^2}{2}\right)dy + \int_1^2 \left(2 - \frac{y^2}{2}\right)dy + I$

(A) $\int_0^1 \left(1+\sqrt{1-y^2}\right)dy$

(B) $\int_0^1 \left(\frac{y^2}{2} - \sqrt{1-y^2} + 1\right)dy$

(C) $\int_0^1 \left(1-\sqrt{1-y^2}\right)dy$

(D) $\int_0^1 \left(\frac{y^2}{2} + \sqrt{1-y^2} + 1\right)dy$

Official Ans. by NTA (C)

Sol. $LHS = \int_0^2 \left(\sqrt{2x} - \sqrt{2x-x^2}\right)dx = \frac{8}{3} - \frac{\pi}{2}$

$RHS = \int_0^1 \left(1 - \sqrt{1-y^2} - \frac{y^2}{2}\right)dy + \int_1^2 \left(2 - \frac{y^2}{2}\right)dy + I$

$I+\frac{5}{3}-\frac{\pi}{4}$

So, $I = 1-\frac{\pi}{4} = \int_0^1\left(1-\sqrt{1-y^2}\right)dy$

9. If y = y (x) is the solution of the differential equation $\left(1+e^{2x}\right)\frac{dy}{dx}+2\left(1+y^2\right)e^x=0$ and y(0)= 0, then $6\left(y'(0)+\left(y\left(\log_e\sqrt{3}\right)\right)^2\right)$ is equal to:

(A) 2 (B) –2

(C) –4 (D) –1

Official Ans. by NTA (C)

Sol. $\frac{dy}{1+y^2}+\frac{2e^x}{1+e^{2x}}dx=0$ (i)

on integration

$\tan^{-1}y+2\tan^{-1}e^x=c$

$\because y(0)=0$

so, $C=\frac{\pi}{2}\Rightarrow\tan^{-1}y+2\tan^{-1}e^x=\frac{\pi}{4}$

from eq.(i), $\left(\frac{dy}{dx}\right)_{x=0}=-1$

$\arg y(\ln\sqrt{3})=-\frac{1}{\sqrt{3}}$

$6\left[y'(0)+(y(\ln\sqrt{3})^2\right]=6\left[-1+\frac{1}{3}\right]=-4$

10. Let P : $y^2 = 4ax$, $a > 0$ be a parabola with focus S.Let the tangents to the parabola P make an angle of $\frac{\pi}{4}$ with the line y = 3x + 5 touch the parabola P at A and B. Then the value of *a* for which A,B and S are collinear is:

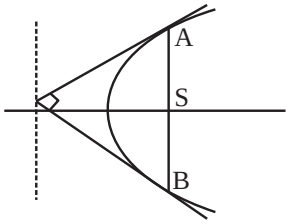
(A) 8 only (B) 2 only

(C) $\frac{1}{4}$ only (D) any a > 0

Official Ans. by NTA (D)

Sol. Lines making angle $\frac{\pi}{4}$ with y = 3x + 5 have slope –2 & 1/2.

Which are perpendicular to each-other so, A, S, B are collinear for all a > 0.



11. Let a triangle *ABC* be inscribed in the circle $x^2 + \sqrt{2}(x+y)+y^2=0$ such that $\angle BAC = \frac{\pi}{2}$. If the length of side AB is $\sqrt{2}$, then the area of the ΔABC is equal to:

(A) $\left(\sqrt{2}+\sqrt{6}\right)/3$ (B) $\left(\sqrt{6}+\sqrt{3}\right)/2$

(C) $\left(3+\sqrt{3}\right)/4$ (D) $\left(\sqrt{6}+2\sqrt{3}\right)/4$

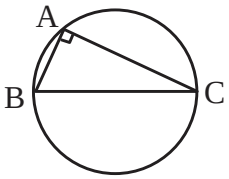
Official Ans. by NTA (Dropped)

Sol. Radius of given circle is 1.

BC = diameter = 2, $AB=\sqrt{2}$

$AC=\sqrt{BC^2-AB^2}=\sqrt{2}$

$\Delta ABC=\frac{1}{2}AB.AC=1$



12. Let $\frac{x-2}{3}=\frac{y+1}{-2}=\frac{z+3}{-1}$ lie on the plane px – qy + z = 5, for some p, $q\in\mathbb{R}$. The shortest distance of the plane from the origin is:

(A) $\sqrt{\frac{3}{109}}$ (B) $\sqrt{\frac{5}{142}}$

(C) $\sqrt{\frac{5}{71}}$ (D) $\sqrt{\frac{1}{142}}$

Official Ans. by NTA (B)

Sol. (2, –1, –3) satisfy the given plane.

So 2p + q = 8 .... (i)

Also given line is perpendicular to normal plane so

3p + 2q – 1 = 0 .... (ii)

$\Rightarrow p = 15, q = -22$

Eq. of plane $15x - 22y + z - 5 = 0$

its distance from origin $= \frac{6}{\sqrt{710}} = \sqrt{\frac{5}{142}}$

13. The distance of the origin from the centroid of the triangle whose two sides have the equations $x - 2y + 1 = 0$ and $2x - y - 1 = 0$ and whose orthocenter is $\left(\frac{7}{3}, \frac{7}{3}\right)$ is:

(A) $\sqrt{2}$ (B) 2

(C) $2\sqrt{2}$ (D) 4

Official Ans. by NTA (C)

Sol. $AB \equiv x - 2y + 1 = 0$

$AC \equiv 2x - y - 1 = 0$

So A(1, 1)

Altitude from B is $BH = x + 2y - 7 = 0 \Rightarrow B(3, 2)$

Altitude from C is $CH = 2x + y - 7 = 0 \Rightarrow C(2, 3)$

Centroid of $\Delta ABC = E(2, 2)$ $OE = 2\sqrt{2}$

14. Let Q be the mirror image of the point P(1, 2, 1) with respect to the plane $x + 2y + 2z = 16$. Let T be a plane passing through the point Q and contains the line $\vec{r} = -\hat{k} + \lambda\left(\hat{i} + \hat{j} + 2\hat{k}\right), \lambda \in \mathbb{R}$. Then, which of the following points lies on T?

(A) (2, 1, 0) (B) (1, 2, 1)

(C) (1, 2, 2) (D) (1, 3, 2)

Official Ans. by NTA (B)

Sol. Image of P(1, 2, 1) in $x + 2y + 2z - 16 = 0$ is given by Q(4, 8, 7)

Eq. of plane $T = \begin{vmatrix} x & y & z+1 \\ 4 & 8 & 6 \\ 1 & 1 & 2 \end{vmatrix} = 0$

$\Rightarrow 2x - z = 1$ so B(1, 2, 1) lies on it.

15. Let A, B, C be three points whose position vectors respectively are:

$\vec{a} = \hat{i} + 4\hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} + \alpha\hat{j} + 4\hat{k}, \alpha \in \mathbb{R}$

$\vec{c} = 3\hat{i} - 2\hat{j} + 5\hat{k}$

If α is the smallest positive integer for which $\vec{a}, \vec{b}, \vec{c}$ are non-collinear, then the length of the median, in ΔABC, through A is:

(A) $\frac{\sqrt{82}}{2}$ (B) $\frac{\sqrt{62}}{2}$

(C) $\frac{\sqrt{69}}{2}$ (D) $\frac{\sqrt{66}}{2}$

Official Ans. by NTA (A)

Sol. $\overrightarrow{AB} \parallel \overrightarrow{AC}$ if $\frac{1}{2} = \frac{\alpha - 4}{-6} = \frac{1}{2} \Rightarrow \alpha = 1$

$\vec{a}, \vec{b}, \vec{c}$ are non-collinear for $\alpha = 2$ (smallest positive integer)

Mid-point of BC = $M\left(\frac{5}{2}, 0, \frac{9}{2}\right)$

$AM = \sqrt{\frac{9}{4} + 16 + \frac{9}{4}} = \frac{\sqrt{82}}{2}$

16. The probability that a relation R from {x,y} to {x,y} is both symmetric and transitive, is equal to:

(A) $\frac{5}{16}$ (B) $\frac{9}{16}$

(C) $\frac{11}{16}$ (D) $\frac{13}{16}$

Official Ans. by NTA (A)

Sol. Total no. of relations = $2^{2\times2} = 16$

Fav. relation = ϕ, {(x, x)}, {(y, y)}, {(x, x)(y, y)}

{(x, x), (y, y), (x, y)(y, x)}

Prob. = $\frac{5}{16}$

17. The number of values of $a \in \mathbb{N}$ such that the variance of 3, 7, 12 a, 43 – a is a natural number is:

(A) 0 (B) 2

(C) 5 (D) infinite

Official Ans. by NTA (A)

Sol. Mean =13

Variance $= \frac{9+49+144+a^2+(43-a)^2}{5} - 13^2 \in N$

$\Rightarrow \frac{2a^2-a+1}{5} \in N$

$\Rightarrow$ $2a^2 - a + 1 - 5n = 0$ must have solution as natural numbers

its D = 40n – 7 always has 3 at unit place

$\Rightarrow$D can't be perfect square

So, a can't be integer.

18. From the base of a pole of height 20 meter, the angle of elevation of the top of a tower is 60°. The pole subtends an angle 30° at the top of the tower. Then the height of the tower is:

(A) $15\sqrt{3}$ (B) $20\sqrt{3}$

(C) $20+10\sqrt{3}$ (D) 30

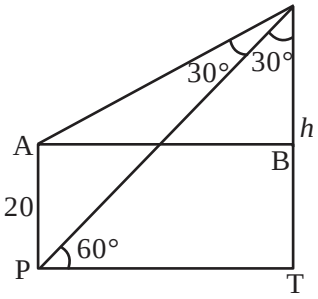
Official Ans. by NTA (4)

Sol. $PT = \frac{h}{\sqrt{3}} = AB$

$\frac{AB}{h-20} = \sqrt{3}$

h = 3(h – 20)

h = 30



19. Negation of the Boolean statement $(p \vee q) \Rightarrow ((\sim r) \vee p)$ is equivalent to:

(A) $p \wedge (\sim q) \wedge r$ (B) $(\sim p) \wedge (\sim q) \wedge r$

(C) $(\sim p) \wedge q \wedge r$ (D) $p \wedge q \wedge (\sim r)$

Official Ans. by NTA (C)

Sol. $P \vee q \Rightarrow (\sim r \vee p)$

$\equiv \sim (p \vee q) \vee (\sim r \vee p)$

$\equiv (\sim p \wedge \sim q) \vee (p \vee \sim r)$

$\equiv [\sim p \vee p) \wedge (\sim q \vee p)] \vee \sim r$

$\equiv [\sim q \vee p) \vee \sim r$

Its negation is $\sim p \wedge q \wedge r$

20. Let $n \geq 5$ be an integer. If $9^n - 8n - 1 = 64\ \alpha$ and $6^n - 5n - 1 = 25\ \beta$, then $\alpha - \beta$ is equal to:

(A) $1 + {}^nC_2(8-5) + {}^nC_3(8^2-5^2) + ... + {}^nC_n(8^{n-1} - 5^{n-1})$

(B) $1 + {}^nC_3(8-5) + {}^nC_4(8^2-5^2) + ... + {}^nC_n(8^{n-2} - 5^{n-2})$

(C) ${}^nC_3(8-5) + {}^nC_4(8^2-5^2) + ... + {}^nC_n(8^{n-2} - 5^{n-2})$

(D) ${}^nC_4(8-5) + {}^nC_5(8^2-5^2) + ... + {}^nC_n(8^{n-3} - 5^{n-3})$

Official Ans. by NTA (C)

Sol. $\alpha = \frac{(1+8)^n - 8n - 1}{64} = {}^nC_2 + {}^nC_3 8 + {}^nC_4 8^2 + ...$

$\beta = {}^nC_2 + {}^nC_3 5 + {}^nC_4 5^2 + ...$

option (3) will be the answer.

SECTION-B

1. Let $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}, \vec{b}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}$ be a vector such that $\vec{a}+\left(\vec{b}\times\vec{c}\right)=\vec{0}$ and $\vec{b}.\vec{c}=5$. Then, the value of $3\left(\vec{c}.\vec{a}\right)$ is equal to____.

Official Ans. by NTA (DROP)

Sol. $\vec{a}+\vec{b}\times\vec{c}=0$

$\vec{a}\times\vec{b}+|\vec{b}|^2\vec{c}-5\vec{b}=0$

It gives $\vec{c}=\frac{1}{3}(10\hat{i}+3\hat{j}+2\hat{k})$

so $3\vec{a}.\vec{c}=10$

But it does not satisfy $\vec{a}+\vec{b}\times\vec{c}=0$.

This question has data error.

Alternate (Explanation) :

According to given $\vec{a}$ & $\vec{b}$

$\vec{a}\cdot\vec{b}=1-2+3=2$ …. (i)

but given equation

$\vec{a}=-(\vec{b}\times\vec{c})$

$\Rightarrow \vec{a}\perp\vec{b} \Rightarrow \vec{a}\cdot\vec{b}=0$

which contradicts.

2. Let y = y(x), x > 1, be the solution of the differential equation $(x-1)\frac{dy}{dx}+2xy=\frac{1}{x-1}$, with $y(2)=\frac{1+e^4}{2e^4}$. If $y(3)=\frac{e^{\alpha}+1}{\beta e^{\alpha}}$. then the value of $\alpha+\beta$ is equal to_____.

Official Ans. by NTA (14)

Sol. $\frac{dy}{dx}+\frac{2x}{x-1}.y=\frac{1}{(x-1)^2}$

$y=\frac{1}{(x-1)^2}\left[\frac{e^{2x}+1}{2e^{2x}}\right]$

$y(3)=\frac{e^6+1}{8e^6}$

$\alpha+\beta=14$

3. Let 3, 6, 9, 12,... upto 78 terms and 5, 9, 13, 17,... upto 59 terms be two series. Then, the sum of the terms common to both the series is equal to___.

Official Ans. by NTA (2223)

Sol. For series of common terms

a=9, d=12, n=19

$S_{19}=\frac{19}{2}[2(9)+18(12)]=2223$

4. The number of solutions of the equation sin x = $\cos^2$ x in the interval (0,10) is__.

Official Ans. by NTA (4)

Sol. $\sin^2x+\sin x-1=0$

$\sin x=\frac{-1+\sqrt{5}}{2}=+ve$

Only 4 roots

5. For real numbers *a*,b ($a>b>0$), let

$$Area\left\{(x,y):x^2+y^2\le a^2 \text{ and } \frac{x^2}{a^2}+\frac{y^2}{b^2}\ge 1\right\}=30\pi$$

and

$$Area\left\{(x,y):x^2+y^2\ge b^2 \text{ and } \frac{x^2}{a^2}+\frac{y^2}{b^2}\le 1\right\}=18\pi$$

Then the value of $(a-b)^2$ is equal to__.

Official Ans. by NTA (12)

Sol. given $\pi a^2-\pi ab=30\pi$ and $\pi ab-\pi b^2=18\pi$

on subtracting, we get $(a-b)^2=a^2-2ab+b^2=12$

6. Let *f* and g be twice differentiable even functions on (–2, 2) such that $f\left(\frac{1}{4}\right)=0, f\left(\frac{1}{2}\right)=0, f(1)=1$ and $g\left(\frac{3}{4}\right)=0, g(1)=2$ Then, the minimum number

of solutions of $f(x)\, g''(x) + f'(x)g'(x) = 0$ in $(-2,2)$ is equal to__.

Official Ans. by NTA (4)

Sol. Let $h(x) = f(x)\, g'(x) \rightarrow 5$ roots

$\because$ f (x) is even $\Rightarrow$

$$f\left(\frac{1}{4}\right)=f\left(\frac{1}{2}\right)=f\left(-\frac{1}{2}\right)=f\left(\frac{1}{4}\right)=0$$

$$g(x) \text{ is even} \Rightarrow g\left(\frac{3}{4}\right)=g\left(-\frac{3}{4}\right)=0$$

g'(x) = 0 has minimum one root

h'(x) has at last 4 roots

7. Let the coefficients of x^{-1} and x^{-3} in the expansion of $\left(2x^{\frac{1}{5}}-\frac{1}{x^{\frac{1}{5}}}\right)^{15}$, x > 0, be *m* and *n* respectively. If r is a positive integer such $mn^2 = {}^{15}C_r.\, 2^r$, then the value of r is equal to__.

Official Ans. by NTA (5)

Sol. $T_{r+1} = (-1)^r.{}^{15}C_r.2^{15-r}\, x^{\frac{15-2r}{5}}$

$m = {}^{15}C_{10}2^5$

$n = -1$

so $mn^2 = {}^{15}C_5 2^5$

8. The total number of four digit numbers such that each of the first three digits is divisible by the last digit, is equal to_____.

Official Ans. by NTA (1086)

Sol. Let the number is abcd, where a,b,c are divisible by d.

	No. of such numbers
d = 1,	9 × 10 × 10 = 900
d = 2	4 × 5 × 5 = 100
d = 3	3 × 4 × 4 = 48
d = 4	2 × 3 × 3 = 18
d = 5	1 × 2 × 2 = 4
d = 6, 7, 8, 9	4 × 4 = 16
	1086

9. Let $M=\begin{bmatrix}0 & -\alpha\\ \alpha & 0\end{bmatrix}$, where α is a non-zero real number an $N=\sum_{k=1}^{49}M^{2k}$. If $(I - M^2)N = -2I$, then the positive integral value of α is ____.

Official Ans. by NTA (1)

Sol. $M=\begin{bmatrix}0 & -\alpha\\ \alpha & 0\end{bmatrix}$; $M^2=\begin{bmatrix}-\alpha^2 & 0\\ 0 & -\alpha^2\end{bmatrix}=-\alpha^2 I$

$N = M^2 + M^4 + ... + M^{98} = [-\alpha^2 + \alpha^4 - \alpha^6 + ...]I$

$$= -\alpha^2\frac{(1-(-\alpha^2)^{49})}{1+\alpha^2}.I$$

$I - M^2 = (1 + \alpha^2)\, I$

$(I - M^2)N = -\alpha^2(\alpha^{98} + 1) = -2$

$\alpha = 1$

10. Let f(x) and g(x) be two real polynomials of degree 2 and 1 respectively. If $f(g(x)) = 8x^2 - 2x$, and $g(f(x)) = 4x^2 + 6x + 1$, then the value of f(2) + g(2) is_____.

Official Ans. by NTA (18)

Sol. $f(g(x) = 8x^2 - 2x$

$g(f(x) = 4x^2 + 6x + 1$

So, $g(x) = 2x - 1$ $\qquad g(2) = 3$

& $f(x) = 2x^2 + 3x + 1$

f(2) = 8 + 6 + 1 = 15

Ans. 18