FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Tuesday 28th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Product of Pressure (P) and time (t) has the same dimension as that of coefficient of viscosity.

Reason R:

Coefficient of viscosity = $\dfrac{\text{Force}}{\text{Velocity gradient}}$

Question: Choose the correct answer from the options given below :

(A) Both A and R true, and R is correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Official Ans. by NTA (C)

Sol. Pressure and time

$$P : \frac{N}{m^2}, \text{Time} : \text{Sec}$$

$$Pt = \frac{N\,\text{sec}}{m^2}$$

$$\eta = \frac{F}{6\pi rv} : \frac{N}{m.m/\text{sec}} : \frac{N\,\text{sec}}{m^2}$$

2. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration (a) is varying with time t as $a = k^2rt^2$. where k is a constant. The power delivered to the particle by the force acting on it is given as

(A) zero

(B) $mk^2r^2t^2$

(C) mk^2r^2t

(D) mk^2rt

Official Ans. by NTA (C)

Sol. $a = k^2rt^2 = \dfrac{V^2}{r}$

$V = krt$

$a_t = \dfrac{dv}{dt} = kr$

$F_t = ma_t = mkr$

$P = \vec{F}.\vec{V}$

$= F\cos\theta V = F_tV = mkr(krt)$

$P = mk^2r^2t$

3. Motion of a particle in x-y plane is described by a set of following equations $x = 4\sin\left(\dfrac{\pi}{2} - \omega t\right)$ m and $y = 4\sin(\omega t)$m. The path of particle will be –

(A) circular

(B) helical

(C) parabolic

(D) elliptical

Official Ans. by NTA (A)

Sol. $x = 4\sin\left(\dfrac{\pi}{2} - \omega t\right)$ $y = 4\cos(\omega t)$

$x = 4\cos(\omega t)$ $y = 4\sin(\omega t)$

Eliminate 't' to find relation between x and y

$x^2 + y^2 = y^2\cos^2\omega t + y^2\sin^2\omega t = 4^2$

$x^2 + y^2 = 4^2$

4. Match List-I with List-II

	List-I		List-II
A	Moment of inertia of solid sphere of radius R about any tangent	I	$\frac{5}{3}MR^2$
B	Moment of inertia of hollow sphere of radius (R) about any tangent	II	$\frac{7}{5}MR^2$
C	Moment of inertia of circular ring of radius (R) about its diameter.	III	$\frac{1}{4}MR^2$
D	Moment of inertia of circular disc of radius (R) about any diameter.	IV	$\frac{1}{2}MR^2$

Question: Choose the correct answer from the options given below

(A) A-II, B-II, C-IV, D-III

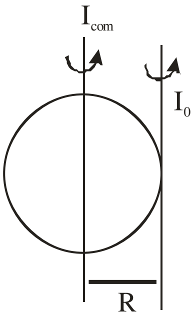
(B) A-I, B-II, C-IV, D-III

(C) A-II, B-I, C-III, D-IV

(D) A-I, B-II, C-III, D-IV

Official Ans. by NTA (A)

Sol. Solid sphere

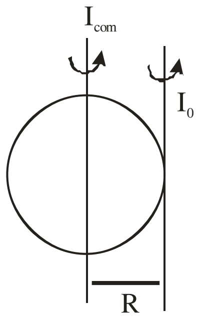


$I_0 = I_{com} + MR^2$ (Parallel Axis theorem)

$$I_0 = \frac{2}{5}MR^2 + MR^2$$

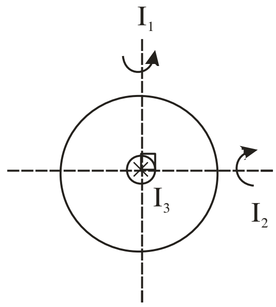
$$I_0 = \frac{7}{5}MR^2$$

Hollow sphere



$$I_0 = I_{com} + MR^2$$

$$= \frac{2}{3}MR^2 + MR^2 = \frac{5}{3}MR^2$$



$I_1 + I_2 + I_3$ (Perpendicular axis theorem)

By symmetry MOI

About 1" and 2" Axis are same i.e.

$$I_1 = I_2$$

$$\therefore\ 2I_1 = I_3 = MR^2 \quad (I_{com} = MR^2)$$

$$I_1 = \frac{MR^2}{2}$$

Similarly in disc

$$2I_1 = \frac{MR^2}{2}\left\{I_{com} = \frac{MR^2}{2}\right\}$$

$$I_1 = \frac{MR^2}{4}$$

5. Two planets A and B of equal mass are having their period of revolutions T_A and T_B such that $T_A = 2T_B$. These planets are revolving in the circular orbits of radii r_A and r_B respectively. Which out of the following would be the correct relationship of their orbits?

(A) $2r_A^2 = r_B^2$

(B) $r_A^3 = 2r_B^3$

(C) $r_A^3 = 3r_B^3$

(D) $T_A^2 - T_B^2 = \frac{\pi^2}{GM}\left(r_B^3 - 4r_A^3\right)$

Official Ans. by NTA (C)

Sol. $T = \frac{2\pi}{\sqrt{Gm_a}} r^{\frac{3}{2}}$

$T^2 \propto r^3$

$\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$

$\Rightarrow \left(\frac{2}{1}\right)^2 = \left(\frac{r_A}{r_B}\right)^3 \Rightarrow r_A^3 = 4r_B^3$

6. A water drop of diameter cm is broken into 64 equal droplets. The surface tension of water is 0.075 N/m. In this process the gain in surface energy will be :

(A) 2.8×10^{-4} J (B) 1.5×10^{-3} J

(C) 1.9×10^{-4} J (D) 9.4×10^{-5} J

Official Ans. by NTA (A)

Sol. d = 2cm; r = 1 cm; T = 0.075

$\Delta SE = T\,\Delta A$

$= 0.075\left(A_f - A_1\right)$

$A_i = 4\pi r^2$

$A_f = 4\pi r_0^2 \times 64$

By volume conservation

$\frac{4}{3}\pi r^3 = 64\cdot\frac{4}{3}\pi r_0^3$

$r_0 = \frac{r}{4}$

$A_f = 4\pi\left(\frac{r}{4}\right)^2\cdot 64 = 16\pi r^2$

$\Delta SE = 0.075\left(16\pi r^2 - 4\pi r^2\right)$

$= 0.075\left(12\pi(0.01)^2\right)$

$= 2.8\times10^{-4}$ J

7. Given below are two statement :

Statement – I : What μ amount of an ideal gas undergoes adiabatic change from state (P_1, V_1, T_1) to state (P_2, V_2, T_2), the work done is $W = \frac{1R(T_2 - T_1)}{1-\gamma}$, where $\gamma = \frac{C_P}{C_V}$ and R = universal gas constant,

Statement — II: In the above case. when work is done on the gas. the temperature of the gas would rise.

Choose the correct answer from the options given below:

(A) Both statement—I and statement-II are true.

(B) Both statement—I and statement-II are false.

(C) Statement-I is true but statement-II is false.

(D) Statement-I is false but statement-II is true.

Official Ans. by NTA (A)

Sol. $W_{adiabatic} = \dfrac{NR(T_f - T_i)}{1-\gamma} \rightarrow$ statment 1

$Q = W + \Delta U$

$0 = W + \Delta U$

$\Delta U = -W$

If work is done on the gas, i.e. work is negative $\therefore \Delta U$ is positive.

$\therefore$ Temperature will increase.

8. Given below are two statements :

Statement-I : A point charge is brought in an electric field. The value of electric field at a point near to the charge may increase if the charge is positive.

Statement-II : An electric dipole is placed in a non-uniform electric field. The net electric force on the dipole will not be zero.

Choose the correct answer from the options given below :

(A) Both statement-I and statement-II are true.

(B) Both statement-I and statement-I are false.

(C) Statement-I is true but statement-II is false.

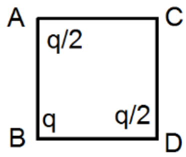
(D) Statement-I is false but statement-II is true.

Official Ans. by NTA (A)

Sol. If the electric field is in the positive direction and the positive charge is to the left of that point then the electric field will increase. But to the left of the positive charge the electric field would decrease.

If the dipole is kept at the point where the electric field is maximum then the force on it will be zero.

9. The three charges q/2, q and q/2 are placed at the corners A, B and C of a square of side 'a' as shown in figure. The magnitude of electric field (E) at the comer D of the square, is :



(A) $\dfrac{q}{4\pi \in_0 a^2}\left(\dfrac{1}{\sqrt{2}}+\dfrac{1}{2}\right)$

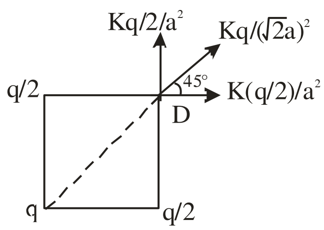
(B) $\dfrac{q}{4\pi \in_0 a^2}\left(1+\dfrac{1}{\sqrt{2}}\right)$

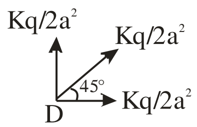
(C) $\dfrac{q}{4\pi \in_0 a^2}\left(1-\dfrac{1}{\sqrt{2}}\right)$

(D) $\dfrac{q}{4\pi \in_0 a^2}\left(\dfrac{1}{\sqrt{2}}-\dfrac{1}{2}\right)$

Official Ans. by NTA (A)

Sol.



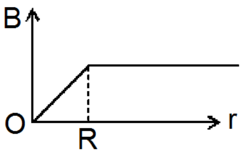


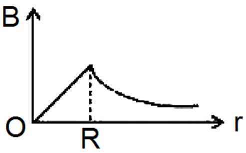
$$(E_{net})_D = \frac{kq}{2a^2} + \frac{\sqrt{2}kq}{2a^2}$$

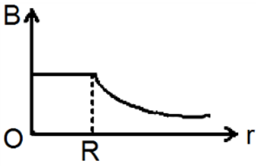
$$(E_{net})_D = \frac{kq}{a^2}\left(\frac{1}{2}+\frac{1}{\sqrt{2}}\right)$$

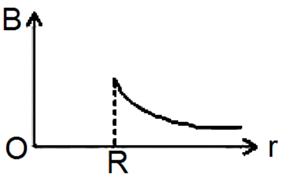
$$(E_{net})_D = \frac{q}{4\pi \in_0 a^2}\left(\frac{1}{2}+\frac{1}{\sqrt{2}}\right)$$

10. An infinitely long hollow conducting cylinder with radius R carries a uniform current along its surface. Choose the correct representation of magnetic field (B) as a function of radial distance (r) from the axis of cylinder.

(A)


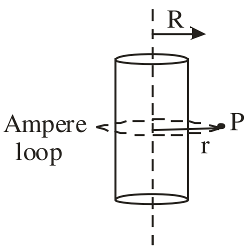
(B)


(C)


(D)


Official Ans. by NTA (D)

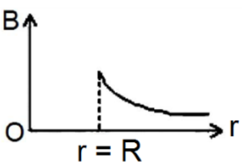
Sol.



1) $r < R, B_p = 0$

2) $r \geq R, B_p = \frac{\mu_0 I}{2\pi r}$

$B_p \propto \frac{1}{r}$



11. A radar sends an electromagnetic signal of electric field (E_0) = 2.25 V/m and magnetic field (B_0) = 1.5 x 10^{-8} T which strikes a target on line of sight at a distance of 3 km in a medium. After that, a pail of signal (echo) reflects back towards the radar vit1i same velocity and by same path. If the signal was transmitted at time t_0 from radar. then after how much time echo will reach to the radar?

(A) 2.0×10^{-5} s

(B) 4.0×10^{-5} s

(C) 1.0×10^{-5} s

(D) 8.0×10^{-5} s

Official Ans. by NTA (B)

Sol. $C = \frac{E_0}{B_0} = \frac{2.25}{1.5 \times 10^{-8}} = 1.5 \times 10^8 \, ms^{-1}$

$t = \frac{6 \times 10^3}{1.5 \times 10^8} = 4 \times 10^{-5} s$

12. The refracting angle of a prism is A and refractive index of the material of the prism is cot (A/2). Then the angle of minimum deviation will be -

(A) 180 – 2A (B) 90 – A

(C) 180 + 2A (D) 180 – 3A

Official Ans. by NTA (A)

Sol. $\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$

$\mu = \cot\frac{A}{2}$

$\Rightarrow \quad \sin\left(\frac{A+\delta_m}{2}\right) = \cos\frac{A}{2}$

$\delta_m = 180 - 2A$

13. The aperture of the objective is 24.4 cm. The resolving power of this telescope. If a light of wavelength 2440 Å is used to see the object will be

(A) 8.1×10^{6} (B) 10.0×10^{7}

(C) 8.2×10^{5} (D) 1.0×10^{-8}

Official Ans. by NTA (C)

Sol. $R.P=\frac{d}{1.22\lambda}=\frac{24.4\times10^{-2}}{1.22\times2440\times10^{-10}}=8.2\times10^{5}$

14. The de Brogue wavelengths for an electron and a photon are λ_e and λ_p respectively. For the same kinetic energy of electron and photon. which of the following presents the correct relation between the de Brogue wavelengths of two ?

(A) $\lambda_p\propto\lambda_e^2$ (B) $\lambda_p\propto\lambda_e$

(C) $\lambda_p\propto\sqrt{\lambda_e}$ (D) $\lambda_p\propto\sqrt{\frac{1}{\lambda_e}}$

Official Ans. by NTA (A)

Sol. $\lambda_e=\frac{h}{\sqrt{2mk}}$

Also for photon $k=\frac{hc}{\lambda_p}$

$\lambda_e=\frac{h\sqrt{\lambda_p}}{\sqrt{2m\,hc}}$

$\lambda_p\propto\lambda e^2$

15. The Q-value of a nuclear reaction and kinetic energy of the projectile particle, K_p are related as :

(A) $Q = K_p$ (B) $(K_p + Q) < O$

(C) $Q < K_p$ (D) $(K_p + Q) > 0$

Official Ans. by NTA (D)

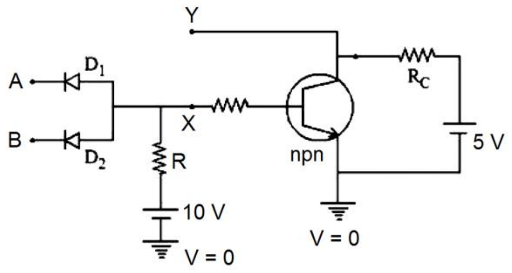
Sol. $x + p \rightarrow \gamma + b$

$Q = k_\gamma + k_b - k_p$

$Q + k_p = k_\gamma + k_b$

$Q + k_p > 0$

16. In the following circuit, the correct relation between output (Y) and inputs A and B will be :



(A) $Y=AB$ (B) $Y=A+B$

(C) $Y=\overline{AB}$ (D) $Y=\overline{A+B}$

Official Ans. by NTA (C)

Sol. This is NAND gate

A	B	Y
0	0	1
1	0	1
0	1	1
1	1	0

17. For using a multimeter to identify diode from electrical components. choose the correct statement out of the following about the diode :

(A) It is two terminal device which conducts current in both directions.

(B) It is two terminal device which conducts current in one direction only

(C) It does not conduct current gives an initial deflection which decays to zero.

(D) It is three terminal device which conducts current in ne direction only between central terminal and either of the remaining two terminals

Official Ans. by NTA (B)

Sol. In forward bias diode conducts

In revers bias it does not conducts.

18. Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : n-p-n transistor permits more current than a p-n-p transistor.

Reason R : Electrons have greater mobility as a charge carrier.

Choose the correct answer from the options given below :

(A) Both A and R true. and R is correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Official Ans. by NTA (A)

Sol. Theory

19. Match List-I with List-II

	List-I		List-II
A	Television signal	I	03 KHz
B	Radio signal	II	20 KHz
C	High Quality Music	III	02 MHz
D	Human speech	IV	06 MHz

Choose the correct answer from the options given below :

(A) A-I, B-II, C-III, D-IV

(B) A-IV, B-III, C-I, D-II

(C) A-IV, B-III, C-II, D-I

(D) A-I, B-II, C-IV, D-III

Official Ans. by NTA (C)

Sol. Theory

20. The velocity of sound in a gas. in which two wavelengths 4.08m and 4.16m produce 40 beats in 12s, will be :

(A)2.82.8 ms^{-1} (B) 175.5 ms^{-1}

(C) 353.6 ms^{-1} (D) 707.2 ms^{-1}

Official Ans. by NTA (D)

Sol. $f_b = f_1 - f_2$

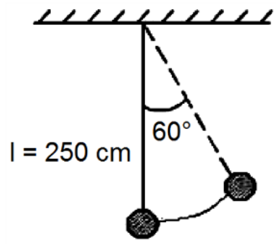
$$\frac{v}{4.08} - \frac{v}{4.16} = \frac{40}{12}$$

$$\Rightarrow v = 707.2$$

SECTION - B

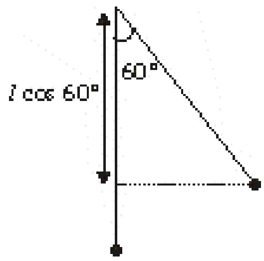
1. A pendulum is suspended by a string of length 250 cm. The mass of the bob of the pendulum is 200 g. The bob is pulled aside until the string is at 60° with vertical as shown in the figure. After releasing the bob. the maximum velocity attained by the bob will be ____________ ms^{-1}.

(if $g = 10\ m/s^2$)



Official Ans. by NTA (5)

Sol. $V_{max} = \sqrt{2gh}$

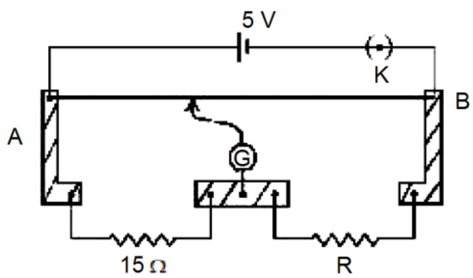


The speed will be highest at the lowest position.

$$h = (\ell - \ell\cos 60°) = \frac{\ell}{2}$$

$$V_{max} = \sqrt{2 \times g \times \frac{\ell}{2}} = \sqrt{10 \times 2.5} = 5\,m/s$$

2. A meter bridge setup is shown in the figure. It is used to determine an unknown resistance R using a given resistor of 15 Ω. The galvanometer (G) shows null deflection when tapping key is at 43 cm mark from end A. If the end correction for end A is 2 cm. then the determined value of R will be ________ Ω.



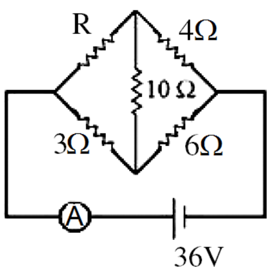
Official Ans. by NTA (19)

Sol. Using the conditions of a balanced wheat stone bridge and adding the end correction.

$$\frac{15}{(43+2)}=\frac{R}{(102-45)} \Rightarrow R=\frac{57}{45}\times 15$$

$$R=19\Omega$$

3. Current measured by the ammeter (A) in the reported circuit when no current flows through 10 Ω resistance. will be _________ A.



Official Ans. by NTA (10)

Sol. Using the condition of a balanced wheat stone bridge,

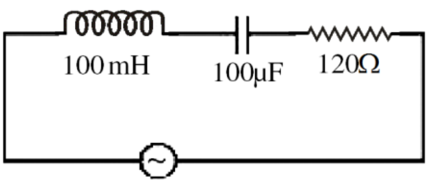
$$\Rightarrow \frac{R}{3}=\frac{4}{6} \Rightarrow R=2\Omega$$

So the effective resistance of the circuit is

$$R_{eq}=\frac{6\times 9}{6+9}=\frac{18}{5}\Omega$$

$$i=\frac{36}{R_{eq}}=10A$$

4. An AC source is connected to an inductance of 100 mH, a capacitance of 100 µ F and a resistance of 120 Ω as shown in figure. The time in which the resistance having a thermal capacity 2 J/°C will get heated by 16°C is _______s.



Official Ans. by NTA (15)

Sol. $|(X_L-X_C)|=|10-10^2|=90\Omega$

Z = Impendance

$$=\sqrt{(X_L-X_C)^2+R^2}=\sqrt{(90)^2+(20)^2}=150\Omega$$

$$i_{rms}=\frac{V_{rms}}{z}=\left(\frac{2}{15}\right)A$$

Now $i_{rms}^2 R\,\Delta t = ms(\Delta T)$

$$\Rightarrow \Delta t = 15 sec$$

5. The position vector of 1 kg object is $\vec{r}=(3\hat{i}-\hat{j})m$ and its velocity $\vec{v}=(3\hat{j}+k)ms^{-1}$. The magnitude of its angular momentum is $\sqrt{x}$ Nm where x is ______ .

Official Ans. by NTA (91)

Sol. Using $\vec{L}=\vec{r}\times\vec{p}=\vec{r}\times m\vec{v}, m=1kg$

$$\vec{L}=(3\hat{i}-\hat{j})\times(3\hat{j}+\hat{k})=(9\hat{k}-3\hat{j}-\hat{i})N-s$$

$$\Rightarrow |\vec{L}|=\sqrt{91}\,N-s$$

6. A man of 60 kg is running on the road and suddenly jumps into a stationary trolly car of mass 120 kg. Then. the trolly car starts moving with velocity 2 ms^{-1}. The velocity of the running man was ________________ ms^{-1}. when he jumps into the car.

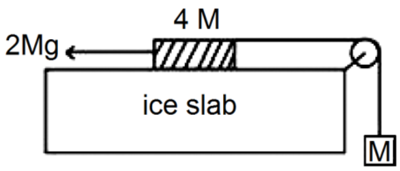
Official Ans. by NTA (6)

Sol. Taking the system as man and trolley and using conservation of linear momentum.

60 × v = (60 + 120) × 2

$\Rightarrow$ v = 6 m/s

7. A hanging mass M is connected to a four times bigger mass by using a string-pulley arrangement. as shown in the figure. The bigger mass is placed on a horizontal ice-slab and being pulled by 2 Mg force. In this situation. tension in the string is $\frac{x}{5}$ Mg for x = _________. Neglect mass of the string and friction of the block (bigger mass) with ice slab. (Given g = acceleration due to gravity)



Official Ans. by NTA (6)

Sol. Using $\vec{F}_{net} = \mu\vec{a}$,

$$\begin{aligned} 2Mg - T &= 4Ma \\ T - Mg &= Ma \\ \hline \Rightarrow a &= \frac{g}{5} \end{aligned}$$

$$T = Mg + Ma = Mg + \frac{Mg}{5} = \frac{6}{5}Mg$$

8. The total internal energy of two mole monoatomic ideal gas at temperature T = 300 K will be J. (Given R = 8.31 J/mol.K)

Official Ans. by NTA (7479)

Sol. $U = nC_vT$

$$= 2 \times \frac{3}{2} R \times 300$$

$$= 900\,R = 900 \times 8\cdot 31 = 7479\text{ J}$$

9. A sing1y ionized magnesium atom (A24) ion is accelerated to kinetic energy 5 keV and is projected perpendicularly into a magnetic field B of the magnitude 0.5 T. The radius of path formed will be _____________ cm.

Official Ans. by NTA (10)

Sol. $R = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$

10. A telegraph line of length loo km has a capacity of 0.01 μF/km and it carries an alternating current at 0.5 kilo cycle per second. If minimum impedance is required, then the value of the inductance that needs to be introduced in series is ________ mH.

$\left(\text{if } \pi = \sqrt{10}\right)$

Official Ans. by NTA (100)

Sol. For minimum impedance

$$X_L = X_C$$

$$\Rightarrow \omega L = \frac{1}{\omega C} \Rightarrow L = \frac{1}{\omega^2 C} = 10^{-1}\text{H} = 100\,\text{mH}$$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Tuesday 28th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION-A

1. The incorrect statement about the imperfections in solids is :

(A) Schottky defect decreases the density of the substance.

(B) Interstitial defect increases the density of the substance.

(C) Frenkel defect does not alter the density of the substance.

(D) Vacancy defect increases the density of the substance.

Official Ans. by NTA (D)

Sol. Due to vacancy defect density of the substance will decrease.

.

2. The Zeta potential is related to which property of colloids"

(A) Colour

(B) Tyndall effect

(C) Charge on the surface of colloidal particles

(D) Brownian movement

Official Ans. by NTA (C)

Sol. The potential difference between the fixed and diffused layer of charges in a colloidal particle is called zeta potential

3. Element "E" belongs to the period 4 and group 16 of the periodic table. The valence shell electron configuration of the element, which is just above 'E' in the group is

(A) $3s^2$. $3p^4$ (B) $3d^{10}$. $4s^2$, $4p^4$

(C) $4d^{10}$. $5s^2$, $5p^4$ (D) $2s^2$, p4

Official Ans. by NTA (A)

Sol. $E \Rightarrow [Ar]\ 3d^{10}\ 4s^2\ 4p^4$

Element above $E \Rightarrow [Ne]\ 3s^2\ 3p^4$

4. Given are two statements one is labelled as Assertion A and other is labelled as Reason R.
Assertion A :Magnesium can reduce Al_2O_3 at a temperature below 1350°C, while above 1350°C aluminium can reduce MgO.
Reason R : The melting and boiling points of magnesium are lower than those of aluminium.
In light of the above statements. choose most appropriate answer from the options given below:

(A) Both A and R are correct. and R is correct explanation of A.

(B) Both A and R are correct. but R is NOT the correct explanation of A.

(C) A is correct R is not correct.

(D) A is not correct. R is correct.

Official Ans. by NTA (B)

Sol. From Ellingham diagram given in NCERT, it can be seen that Mg, MgO line crosses Al, Al_2O_3 line after 1350ºC hence assertion is true.

Yes, Mg have lower MP and BP than aluminium but it does not explain the above fact.

5. Dihydrogen reacts with CuO to give

(A) CuH_2

(B) Cu

(C) Cu_2O

(D) $Cu(OH)_2$

Official Ans. by NTA (B)

Sol. $CuO + H_2 \rightarrow Cu + H_2O$ (under hot conditions)

6. Nitrogen gas is obtained by thermal decomposition of

(A) $Ba(NO_3)_2$ (B) $Ba(N_3)_2$

(C) $NaNO_2$ (D) $NaNO_3$

Official Ans. by NTA (B)

Sol. $Ba(N_3)_2 \rightarrow Ba + 3N_2$

7. Given below are two statements :
Statement -I :The pentavalent oxide of group- 15 element. E_2O_5. is less acidic than trivalent oxide. E_2O_3. of the same element.
Statement -II :The acidic character of trivalent oxide of group 15 elements. E_2O_3. decreases down the group.
In light of the above statements. choose most appropriate answer from the options given below:
(A) Both Statement I and Statement II are true.
(B) Both Statement I and Statement II are false.
(C) Statement I true. but statement II is false.
(D) Statement I is false but statement II is true.
Official Ans. by NTA (D)

Sol. As +ve oxidation state increases, EN of element increases hence acidic character increases. Down the group, non-metallic character decreases, acidic character decreases.

Acidic character : $E_2O_5 > E_2O_3$

Down the group, acidic character of E_2O_3 decreases

8. Which one of the lanthanoids given below is the most stable in divalent form?
(A) Ce (Atomic Number 58)
(B) Sm (Atomic Number 62)
(C) Eu (Atomic Number 63)
(D) Yb (Atomic Number 70)
Official Ans. by NTA (C)

Sol. $E^o_{M^{3+}/M^{2+}} \Rightarrow \begin{matrix} Eu & Yb \\ -0.35 & -1.05 \end{matrix}$

Hence, due to more reduction potential in Eu as compared to Yb, it can concluded that Eu^{2+} is more stable than Yb^{2+}.

9. Given below are two statements :
Statement I : $[Ni(CN)4]^{2-}$ is square planar and diamagnetic complex. with dsp^2 hybridization for Ni but $[Ni(CO)_4]$ is tetrahedral. paramagnetic and with sp^3-hybridication for Ni.
Statement II: $[NiCl_4]^{2-}$ and $[Ni(CO)_4]$ both have same d-electron configuration have same geometry and are paramagnetic.
In light the above statements. choose the correct answer form the options given below:
(A) Both Statement I and Statement II are true.
(B) Both Statement I and Statement II are false.
(C) Statement I is correct but statement II is false.
(D) Statement I is incorrect but statement II is true.
Official Ans. by NTA (B)

Sol. $[Ni(CN)_4]^{2-}$: d^8 configuration, SFL, sq. planar splitting (dsp^2), diamagnetic.

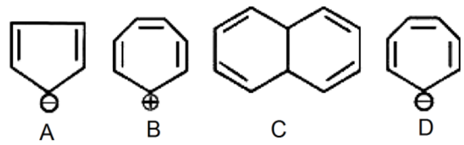
$[Ni(CO)_4]$: d^{10} config (after excitation), SFL, tetrahedral splitting (sp^3), diamagnetic.

$[NiCl_4]^{2-}$: d^8 config, WFL, tetrahedral splitting (sp^3), paramagnetic(2 unpaired e^-).

10. Which amongst the following is not a pesticide ?
(A) DDT
(B) Organophosphates
(C) Dieldrin
(D) Sodium arsenite
Official Ans. by NTA (D)

11. Which one of the following techniques is not used to spot components of a mixture separated on thin layer chromatographic plate?
(A) I_2 (Solid)
(B) U.V. Light
(C) Visualisation agent as a component of mobile phase
(D) Spraying of an appropriate reagent
Official Ans. by NTA (C)

12. Which of the following structures are aromatic in nature?

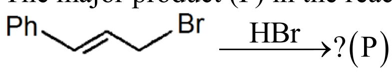


(A) A,B,C and D
(B) Only A and B
(C) Only A and C
(D) Only B, C and D
Official Ans. by NTA (B)

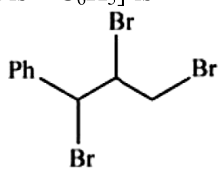
Sol. A, B aromatic

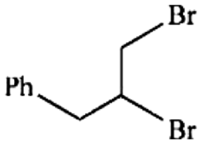
C,D is nonaromatic

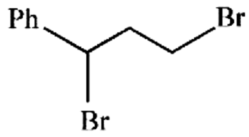
13. The major product (P) in the reaction

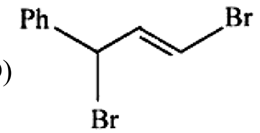


[Ph is – C_6H_5] is

(A)


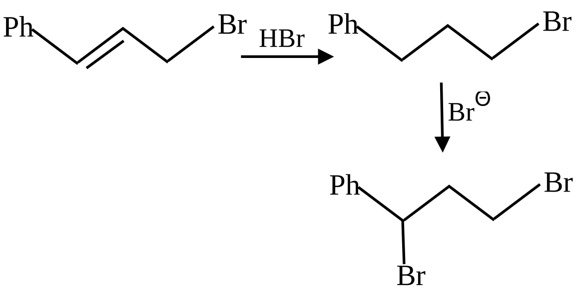
(B)


(C) 

(D) 

Official Ans. by NTA (C)

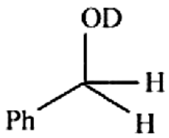
Sol.

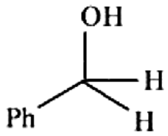


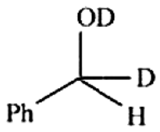
14. The correct structure of product 'A' formed in the following reaction.

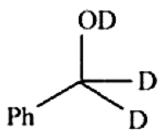
$$\text{PhCHO} + \text{Ph}\cdot\text{CHO} \xrightarrow[\text{in } D_2O]{\text{NaOD}} A + \text{Ph}-\overset{\overset{\displaystyle O}{\|}}{C}-O^-$$

$(\text{Ph is} - C_6H_5)$

(A) 

(B) 

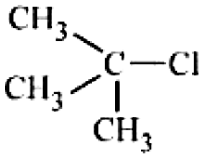
(C) 

(D) 

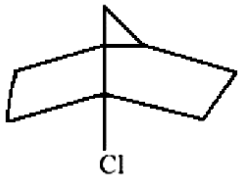
Official Ans. by NTA (A)

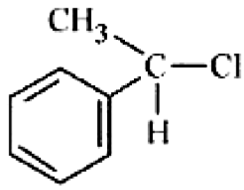
Sol. PhCH = O + PhCH = O $\xrightarrow{OD/D_2O}$ $PhCH_2OD + PhCO_2^-$

15. Which one of the following compounds is inactive towards S_N1 reaction?

(A) 

(B) 

(C) 

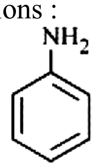
(D) 

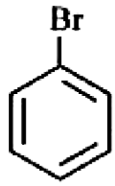
Official Ans. by NTA (C)

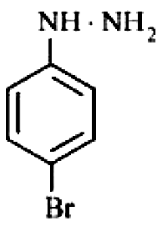
Sol. **Sol.** The carbocation fromed is very unstable.

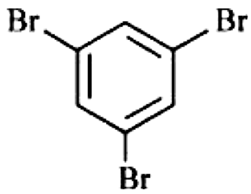
So it is inactive towards S_N1

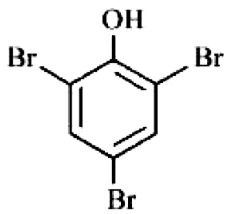
16. Identify the major product formed in the following sequence of reactions :



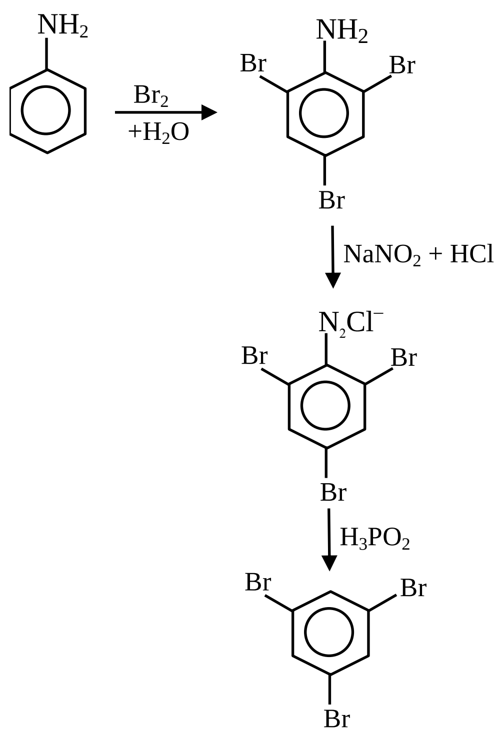
(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (C)



Sol.

17. A primary aliphatic amine on reaction with nitrous acid in cold (273 K) and there after raising temperature of reaction mixture to room temperature (298 K). Gives a/an

(A) nitrile (B) alcohol

(C) diazonium salt (D) secondary amine

Official Ans. by NTA (B)

Sol. $R-NH_2 \xrightarrow[+HCl]{NaNO_2} R-N_2^+ \rightarrow R^+ \xrightarrow[H_2O]{} R-OH$

18. Which one of the following is **NOT** a copolymer ?

(A) Buna-S (B) Neoprene

(C) PHBV (D) Butadiene-styrene

Official Ans. by NTA (B)

Sol. Buna-S, PHBr and Butadiene-styrene are copolymer. Only neoprene is namopolymer.

19. Stability of α - Helix structure of proteins depends upon

(A) dipolar interaction

(B) H-bonding interaction

(C) van der Waals forces

(D) π -stacking interaction

Official Ans. by NTA (B)

20. The formula of the purple colour formed in Laissaigne's test for sulphur using sodium nitroprusside is

(A) $NaFe[Fe(CN)_6]$ (B) $Na[Cr(NH_3)_2(NCS)_4]$

(C) $Na_2[Fe(CN)_5(NO)]$ (D) $Na_4[Fe(CN)_5(NOS)]$

Official Ans. by NTA (D)

Sol. $Na_2S + Na_2[Fe(CN)_5NO] \rightarrow Na_4[Fe(CN)_5NO_5]$

SECTION-B

1. A 2.0 g sample containing MnO_2 is treated with HCl liberating Cl_2. The Cl_2 gas is passed into a solution of KI and 60.0 mL of 0.1 M $Na_2S_2O_3$ is required to titrate the liberated iodine. The percentage of MnO_2 in the sample is __________. (Nearest integer)

[Atomic masses (in u) Mn = 55; Cl = 35.5; O = 16, I = 127, Na = 23, K = 39, S = 32]

Official Ans. by NTA (13)

Sol.

$MnO_2 + HCl \longrightarrow Cl_2 + Mn^{+2}$

6 meq 6 meq

$= 3\,m\,mol$

$Cl_2 + KI \longrightarrow Cl^- + I_2$

6 meq 6 meq

$I_2 + Na_2S_2O_3 \longrightarrow I^- + Na_2S_4O_6$

6 meq 6 m mol

= 6 meq

$\%MnO_2 = \frac{3\times10^{-3}\times87}{2}\times100$

= 13.05%

Ans. 13

2. If the work function of a metal is 6.63×10^{-19} J, the maximum wavelength of the photon required to remove a photoelectron from the metal is _______ nm. (Nearest integer)

[Given : $h = 6.63 \times 10^{-34}$ J s, and $c = 3 \times 10^8$ m s^{-1}]

Official Ans. by NTA (300)

Sol. $\phi = 6.63 \times 10^{-19} J = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{\lambda}$

$\Rightarrow \lambda = 3 \times 10^{-7} m = 300$ nm

3. The hybridization of P exhibited in PF_5 is sp^xd^y. The value of y is ________.

Official Ans. by NTA (1)

Sol. $PF_5 \Rightarrow sp^3d$ hybridisation

(5 sigma bonds, zero lone pair on central atom)

Value of y = 1

4. 4.0 L of an ideal gas is allowed to expand isothermally into vacuum until the total volume is 20 L. The amount of heat absorbed in this expansion is ________ L atm.

Official Ans. by NTA (0)

Sol. For free expansion: $P_{ext} = 0$, $w = 0$

$q = 0$, $\Delta U = 0$

Ans. 0

5. The vapour pressures of two volatile liquids A and B at 25°C are 50 Torr and 100 Torr, respectively. If the liquid mixture contains 0.3 mole fraction of A, then the mole fraction of liquid B in the vapour phase is $\frac{x}{17}$. The value of x is ________.

Official Ans. by NTA (14)

Sol. $\frac{y_B}{1-y_B} = \frac{P_B^o}{P_A^o}\left[\frac{X_B}{1-X_B}\right]$

$\Rightarrow \frac{y_B}{1-y_B} = \frac{100}{50}\left[\frac{0.7}{0.3}\right] = \frac{14}{3}$

$\Rightarrow y_B = \frac{14}{17}$

Ans. 14

6. The solubility product of a sparingly soluble salt A_2X_3 is 1.1×10^{-23}. If specific conductance of the solution is 3×10^{-5} S m^{-1}, the limiting molar conductivity of the solution is $x \times 10^{-3}$ S m^2 mol^{-1}. The value of x is ________.

Official Ans. by NTA (3)

$A_2X_{3(s)} \rightleftharpoons 2A^{+3}_{(aq)} + 3X^{-2}_{(aq)}$

solubility = sM   2s   3s

$(2s)^2(3s)^3 = 1.1 \times 10^{-23}$

$108\ s^5 = 1.1\times10^{-23}$

$s \simeq 10^{-5} M = 10^{-5}\frac{mol}{L} = 0.01\frac{mol}{m^3}$

Now $\wedge_m \simeq \wedge_m^\infty = \frac{k}{m} = \frac{k}{s}$

$\Rightarrow \wedge_m^\infty = \frac{3\times10^{-5}}{0.01} = 3\times10^{-3}$ S-m²/mol

Ans. 3

7. The quantity of electricity in Faraday needed to reduce 1 mol of $Cr_2O_7^{2-}$ to Cr^{3+} is ________.

Official Ans. by NTA (6)

Sol. $Cr_2O_7^{-2} + 6e^- \longrightarrow 2Cr^{+3}$

1mol   6mol

$\Rightarrow$ number of faradays = moles of electrons = 6

8. For a first order reaction $A \rightarrow B$, the rate constant, $k = 5.5\times10^{-14} s^{-1}$. The time required for 67% completion of reaction is $x \times 10^{-1}$ times the half life of reaction. The value of x is ________ (Nearest integer)

(Given : log 3 = 0.4771)

Official Ans. by NTA (16)

Sol. $t_{67\%} = \frac{1}{k}\ln\left(\frac{1}{1-0.67}\right) = \frac{t_{1/2}}{\ln 2}\times\ln\left(\frac{1}{1-\frac{2}{3}}\right)$

$t_{67\%} = \frac{t_{1/2}}{\log 2}\times\log 3 = \frac{t_{1/2}\times0.4771}{0.301}$

$\Rightarrow t_{67\%} = 1.585 \times t_{1/2}$

$X \times 10^{-1} = 1.585$

$\Rightarrow X = 15.85$

Ans.16

9. Number of complexes which will exhibit synergic bonding amongst, $[Cr(CO)_6]$, $[Mn(CO)_5]$ and $[Mn_2(CO)_{10}]$ is ________.

Official Ans. by NTA (3)

Sol. Carbonyl complex compounds have tendency to show synergic bonding.

10. In the estimation of bromine, 0.5 g of an organic compound gave 0.40 g of silver bromide. The percentage of bromine in the given compound is _________% (nearest integer)

(Relative atomic masses of Ag and Br are 108u and 80u, respectively).

Official Ans. by NTA (34)

Sol

$$\underset{0.5\,g}{O.C} \longrightarrow \underset{0.4\,g}{AgBr}$$

$$\text{mol of Br} = \text{mol of AgBr} = \frac{0.4}{188}$$

$$\%\ Br = \%Br = \frac{\frac{0.4}{188}\times 80}{0.5}\times 100$$

$= 34.04\%$

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Tuesday 28th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. If

$$\sum_{k=1}^{31}\left({}^{31}C_k\right)\left({}^{31}C_{k-1}\right)-\sum_{k=1}^{30}\left({}^{30}C_k\right)\left({}^{30}C_{k-1}\right)=\frac{\alpha(60!)}{(30!)(31!)},$$

Where $\alpha \in R$, then the value of 16α is equal to

(A) 1411 (B) 1320

(C) 1615 (D) 1855

Official Ans. by NTA (A)

Sol. $\sum_{R=1}^{31} {}^{31}C_R \cdot {}^{31}C_{R-1}$

$= {}^{31}C_1 \cdot {}^{31}C_0 + {}^{31}C_2 \cdot {}^{31}C_1 + ... + {}^{31}C_{31} \cdot {}^{31}C_{30}$

$= {}^{31}C_0 \cdot {}^{31}C_{30} + {}^{31}C_1 \cdot {}^{31}C_{29} + ... + {}^{31}C_{30} \cdot {}^{31}C_0$

$= {}^{62}C_{30}$.

Similarly

$$\sum_{R=1}^{30}\left({}^{30}C_R \cdot {}^{30}C_{R-1}\right) = {}^{60}C_{29}$$

$${}^{62}C_{30} - {}^{60}C_{29} = \frac{62!}{30!32!} - \frac{60!}{29!31!}$$

$$= \frac{60!}{29!\,31!}\left\{\frac{62\cdot 61}{30\cdot 32} - 1\right\}$$

$$= \frac{60!}{30!31!}\left(\frac{2822}{32}\right)$$

$$\therefore\ 16\alpha = 16 \times \frac{2822}{32} = 1411$$

2. Let a function $f:\mathbb{N}\to\mathbb{N}$ be defined by

$$f(n) = \begin{cases} 2n, & n = 2,4,6,8,..... \\ n-1, & n = 3,7,11,15,..... \\ \dfrac{n+1}{2}, & n = 1,5,9,13,..... \end{cases}$$

then, f is

(A) one-one but not onto

(B) onto but not one-one

(C) neither one-one nor onto

(D) one-one and onto

Official Ans. by NTA (D)

Sol. $f(x) = \begin{cases} 4R & ; \quad n = 2R \\ 4R-2 & ; \quad n = 4R-1 \\ 2R-1 & ; \quad n = 4R-3 \end{cases}$

$(R \in N)$

Note that for any element, it will fall into exactly. one of these sets.

$\{y : y = 4R; y \in N\}$

$\{y : y = 4R-2; y \in N\}$

$\{y : y = 2R-1; y \in N\}$

Corresponding to that y, we will get exactly one value of n.

Thus, f is one – one & onto.

3. If the system of linear equations

$2x + 3y - z = -2$

$x + y + z = 4$

$x - y + |\lambda| z = 4\lambda - 4$

where $\lambda \in \mathbb{R}$, has no solution, then

(A) $\lambda = 7$ (B) $\lambda = -7$

(C) $\lambda = 8$ (D) $\lambda^2 = 1$

Official Ans. by NTA (B)

Sol. $\begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & |\lambda| \end{vmatrix} = 0$

$\Rightarrow |\lambda| = 7 \Rightarrow \lambda = \pm 7$ ...(1)

System :

$2x + 3y - z = -2$ ...(2)

$x + y + z = 4$ ...(3)

$x - y + |\lambda| z = 4\lambda - 4$ ...(4)

Eliminating y from equal (2) & (3) we get

$x + 4z = 14$ ...(5)

$(3) + (4) \Rightarrow x + \left(\frac{|\lambda|+1}{2}\right) z = 2\lambda$ ...(6)

Clearly for $\lambda = -7$, system is inconsistent.

4. Let A be a matrix of order 3 × 3 and det (A) = 2. Then det (det (A) adj (5 adj (A^3))) is equal to _____.

(A) 512×10^6 (B) 256×10^6

(C) 1024×10^6 (D) 256×10^{11}

Official Ans. by NTA (A)

Sol. |(det (A)) adj (5 adj (A))|

$= |2\text{adj}(5\text{adj}(A^3))|$

$= 2^3 |\text{adj}(5\text{ adj}(A^3))|$

$= 2^3 . |5\text{adj}(A^3)|^2$

$= 2^3 (5^3 .|\text{adj}(A^3)|)^2$

$= 2^3 . 5^6 . |\text{adj}A^3|^2$

$= 2^3 . 5^6 ((|A|^3)^2)^2$

$= 2^3 . 5^6 . 2^{12} = 2^{15} \times 5^6$

$= 2^9 \times 10^6$

$= 512 \times 10^6$.

5. The total number of 5-digit numbers, formed by using the digits 1, 2, 3, 5, 6, 7 without repetition, which are multiple of 6, is

(A) 36 (B) 48

(C) 60 (D) 72

Official Ans. by NTA (D)

Sol. To make a no. divisible by 3 we can use the digits 1,2,5,6,7 or 1,2,3,5,7.

Using 1,2,5,6,7, number of even numbers is

$= 4 \times 3 \times 2 \times 1 \times 2 = 48$

Using 1,2,3,5,7, number of even numbers is

$= 4 \times 3 \times 2 \times 1 \times 1 = 24$

Required answer is 72.

6. Let $A_1, A_2, A_3, \ldots\ldots$ be an increasing geometric progression of positive real numbers. If $A_1A_3A_5A_7 = \frac{1}{1296}$ and $A_2 + A_4 = \frac{7}{36}$, then, the value of $A_6 + A_8 + A_{10}$ is equal to

(A) 33 (B) 37

(C) 43 (D) 47

Official Ans. by NTA (C)

Sol. $A_1 \cdot A_3 \cdot A_5 \cdot A_7 = \frac{1}{1296}$

$(A_4)^4 = \frac{1}{1296}$

$A_4 = \frac{1}{6}$ ...(1)

$A_2 + A_4 = \frac{7}{36}$

$A_2 = \frac{1}{36}$ ...(2)

$A_6 = 1$

$A_8 = 6$

$A_{10} = 36$

$A_6 + A_8 + A_{10} = 43$

7. Let [t] denote the greatest integer less than or equal to t. Then, the value of the integral $\int_0^1 [-8x^2 + 6x - 1]dx$ is equal to

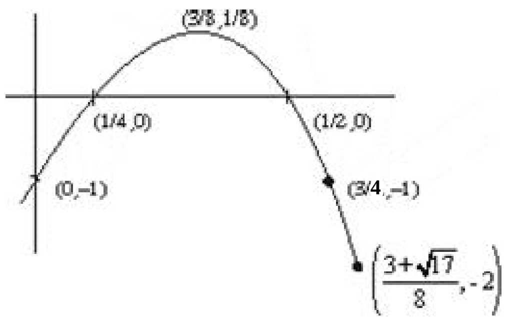
(A) -1 (B) $-\frac{5}{4}$

(C) $\frac{\sqrt{17}-13}{8}$ (D) $\frac{\sqrt{17}-16}{8}$

Official Ans. by NTA (C)

Sol. $\int_0^1 \left[-8x^2 + 6x - 1\right]dx$

$= \int_0^{1/4} -1\,dx + \int_{1/4}^{1/2} 0\,dx + \int_{1/2}^{3/4} -1\,dx$



$+ \int_{3/4}^{\frac{3+\sqrt{17}}{8}} -2\,dx + \int_{\frac{3+\sqrt{17}}{8}}^{1} -3\,dx$

$= -[x]_0^{1/4} + 0 - [x]_{1/2}^{3/4} + -2[x]_{3/4}^{\frac{3+\sqrt{17}}{8}} - 3[x]_{\frac{3+\sqrt{17}}{8}}^{1}$

$$=-\left(\frac{1}{4}-0\right)-\left(\frac{3}{4}-\frac{1}{2}\right)-2\left(\frac{3+\sqrt{17}}{8}-\frac{3}{4}\right)-3\left(1-\frac{3+\sqrt{17}}{8}\right)$$

$$=-\frac{1}{4}-\frac{1}{4}+\frac{-6-2\sqrt{17}}{8}+\frac{3}{2}-3+\frac{9+3\sqrt{17}}{8}$$

$$=\frac{\sqrt{17}-13}{8}$$

8. Let $f:\mathbb{R}\to\mathbb{R}$ be defined as

$$f(x)=\begin{cases}[e^x], & x<0\\ ae^x+[x-1], & 0\le x<1\\ b+[\sin(\pi x)], & 1\le x<2\\ [e^{-x}]-c, & x\ge 2\end{cases}$$

where $a,b,c\in\mathbb{R}$ and [t] denotes greatest integer less than or equal to t. Then, which of the following statements is true ?

(A) There exists $a,b,c\in\mathbb{R}$ such that f is continuous of $\mathbb{R}$.

(B) If f is discontinuous at exactly one point, then $a+b+c=1$.

(C) If f is discontinuous at exactly one point, then $a+b+c\neq 1$.

(D) f is discontinuous at atleast two points, for any values of a, b and c.

Official Ans. by NTA (C)

Sol. $f(x)$ is discontinuous at $x = 1$

For continuous at $x = 0$; $a = 1$

For continuous at $x = 2$; $b + c = 1$

$a + b + c = 2$

9. The area of the region

$S=\{(x,y):y^2\le 8x, y\ge\sqrt{2}x, x\ge 1\}$ is

(A) $\frac{13\sqrt{2}}{6}$ (B) $\frac{11\sqrt{2}}{6}$

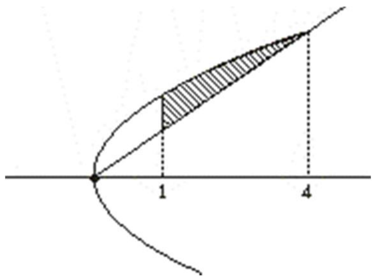
(C) $\frac{5\sqrt{2}}{6}$ (D) $\frac{19\sqrt{2}}{6}$

Official Ans. by NTA (B)

Sol. $y^2=8x$ ...(1)

$y=\sqrt{2}x$ ...(2)

$y^2=2x^2$



$\Rightarrow 8x = 2x^2$

$\Rightarrow x = 0 \;\&\; 4$

Area : $=\int_1^4 2\sqrt{2}\sqrt{x}-\sqrt{2}x\,dx$

$$=2\sqrt{2}\left(\frac{x^{\frac{3}{2}}}{3/2}\right)_1^4-\sqrt{2}\left(\frac{x^2}{2}\right)_1^4$$

$$=\frac{4\sqrt{2}}{3}(8-1)-\frac{\sqrt{2}}{3}(16-1)$$

$$=\frac{28\sqrt{2}}{3}-\frac{15\sqrt{2}}{2}=\frac{11\sqrt{2}}{6}$$

10. Let the solution curve $y = y(x)$ of the differential equation,

$$\left[\frac{x}{\sqrt{x^2-y^2}}+e^{\frac{y}{x}}\right]x\frac{dy}{dx}=x+\left[\frac{x}{\sqrt{x^2-y^2}}+e^{\frac{y}{x}}\right]y$$

pass through the points (1, 0) and $(2\alpha,\alpha),\alpha>0$. Then α is equal to

(A) $\frac{1}{2}\exp\left(\frac{\pi}{6}+\sqrt{e}-1\right)$ (B) $\frac{1}{2}\exp\left(\frac{\pi}{3}+\sqrt{e}-1\right)$

(C) $\exp\left(\frac{\pi}{6}+\sqrt{e}+1\right)$ (D) $2\exp\left(\frac{\pi}{3}+\sqrt{e}-1\right)$

Official Ans. by NTA (A)

Sol. $$\left(\frac{x}{\sqrt{x^2-y^2}}+e^{\frac{y}{x}}\right)x\frac{dy}{dx}=x+\left(\frac{x}{\sqrt{x^2y^2}}+e^{\frac{y}{x}}\right)y$$

$$\Rightarrow\quad e^{\frac{y}{x}}(x\,dy-y\,dx)+\frac{x}{\sqrt{x^2-y^2}}(xdy-ydx)=xdx$$

Dividing both side by x^2

$$\Rightarrow \quad e^{\frac{y}{x}}\left(\frac{xdy-y\,dx}{x^2}\right)+\frac{1}{\sqrt{1-\left(\frac{y}{x}\right)^2}}\left(\frac{xdy-ydx}{x^2}\right)=\frac{dx}{x}$$

$$\Rightarrow \quad e^{\frac{y}{x}}\,|\,d\left(\frac{t}{x}\right)+\frac{1}{\sqrt{1-\left(\frac{y}{x}\right)^2}}d\left(\frac{y}{x}\right)=\frac{dy}{x}$$

Integrate both side.

$$\int e^{\frac{y}{x}}d\left(\frac{y}{x}\right)+\int\frac{1}{\sqrt{1-\left(\frac{y}{x}\right)^2}}d\left(\frac{y}{x}\right)=\int\frac{dx}{x}$$

$$\Rightarrow e^{\frac{y}{x}}+\sin^{-1}\left(\frac{y}{x}\right)=\ln x+c$$

It passes through (1, 0)

$1 + 0 = 0 + c \Rightarrow c = 1$

It passes through $(2\alpha,\alpha)$

$$e^{\frac{1}{2}}+\sin^{-1}\frac{1}{2}=\ln 2\alpha+1$$

$$\Rightarrow \ln 2\alpha=\sqrt{e}+\frac{\pi}{6}-1$$

$$\Rightarrow 2\alpha=e^{\left(\sqrt{e}+\frac{\pi}{6}-1\right)}$$

$$\Rightarrow \alpha=\frac{1}{2}e^{\left(\frac{\pi}{6}+\sqrt{e}-1\right)}$$

11. Let y = y(x) be the solution of the differential equation $x(1-x^2)\frac{dy}{dx}+(3x^2y-y-4x^3)=0, x>1,$ with y(2) = –2. Then y(3) is equal to

(A) –18 (B) –12
(C) –6 (D) –3

Official Ans. by NTA (A)

Sol. $$x(1-x^2)\frac{dy}{dx}+(3x^2y-y-4x^3)=0$$

$$x(1-x^2)\frac{dy}{dx}+(3x^2-1)y=4x^3$$

$$\frac{dy}{dx}+\frac{(3x^2-1)}{(x-x^3)}y=\frac{4x^3}{(x-x^3)}$$

$$\frac{dy}{dx}+Py=Q$$

$$IF=e^{\int P dx}=e^{\int\frac{3x^2-1}{x-x^3}dx}$$

$$x-x^3=t \Rightarrow IF=e^{\int\frac{-dt}{t}}$$

$$=e^{-\ell nt}=\frac{1}{t}$$

$$\therefore\ IF=\frac{1}{x-x^3}$$

$$y\times IF=\int Q\times IF\,dx$$

$$y\left(\frac{1}{x-x^3}\right)=\int\frac{4x^3}{x-x^3}\times\frac{1}{(x-x^3)}dx$$

$$=\int\frac{4x^3}{(x-x^3)^2}dx$$

$$=\int\frac{4x}{(1-x^2)^2}dx \qquad 1-x^2=K$$

$$=2\int\frac{-dK}{K^2} \qquad -2xdx=dK$$

$$=-2\left(-\frac{1}{K}\right)+c$$

$$\frac{y}{x-x^3}=\frac{2}{K}+c$$

$$\frac{y}{x-x^3}=\frac{2}{1-x^2}+c$$

At x = 2 , y = – 2

$$\frac{-2}{2-8}=\frac{2}{1-4}+c$$

$$\frac{1}{3}=\frac{-2}{3}+c$$

$\therefore$ C = 1

$$\frac{y}{x-x^3}=\frac{2}{1-x^2}+1$$

Put x = 3

$$\frac{y}{3-27}=\frac{2}{1-9}+1$$

$$\frac{y}{-24}=-\frac{1}{4}+1$$

$$\frac{y}{-24} = \frac{3}{4}$$

$$y = \frac{3}{4}(-24) = -18$$

12. The number of real solutions of $x^7 + 5x^3 + 3x + 1 = 0$ is equal to ________.

(A) 0 (B) 1

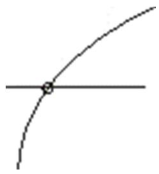
(C) 3 (D) 5

Official Ans. by NTA (B)

Sol. $f(x) = x^7 + 5x^3 + 3x + 1$

$f'(x) = 7x^6 + 15x^2 + 3 > 0$

$\therefore$ $f(x)$ is strictly increasing function



$x \to -\infty$, $y \to -\infty$

$x \to \infty$, $y \to \infty$

$\therefore$ no. of real solution = 1

13. Let the eccentricity of the hyperbola $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be $\sqrt{\frac{5}{2}}$ and length of its latus rectum be $6\sqrt{2}$, If $y = 2x + c$ is a tangent to the hyperbola H, then the value of c^2 is equal to

(A) 18 (B) 20

(C) 24 (D) 32

Official Ans. by NTA (B)

Sol. $y = mx \pm \sqrt{a^2m^2 - b^2}$

$m = 2,\ c^2 = a^2m^2 - b^2$

$c^2 = 4a^2 - b^2$

$$e^2 = 1 + \frac{b^2}{a^2}$$

$$\frac{5}{2} = 1 + \frac{b^2}{a^2}$$

$$\frac{3}{2} = \frac{b^2}{a^2} \Rightarrow b^2 = \frac{3a^2}{2}$$

$$\frac{2b^2}{a} = 6\sqrt{2}$$

$$\frac{2}{a} \times \frac{3a^2}{2} = 6\sqrt{2}$$

$$3a = 6\sqrt{2}$$

$$\boxed{a = 2\sqrt{2}}$$

$$b^2 = \frac{3}{2} \times 8 = 12$$

$$b = 2\sqrt{3}$$

$$\therefore\ c^2 = 4 \times 8 - 12$$

$$c^2 = 20$$

14. If the tangents drawn at the point O(0, 0) and $P(1+\sqrt{5}, 2)$ on the circle $x^2 + y^2 - 2x - 4y = 0$ intersect at the point Q, then the area of the triangle OPQ is equal to

(A) $\frac{3+\sqrt{5}}{2}$ (B) $\frac{4+2\sqrt{5}}{2}$

(C) $\frac{5+3\sqrt{5}}{2}$ (D) $\frac{7+3\sqrt{5}}{2}$

Official Ans. by NTA (C)

Sol. Tangent at O

$-(x + 0) - 2(y + 0) = 0$

$\Rightarrow \boxed{x + 2y = 0}$

Tangent at P

$x(1 + \sqrt{5}) + y.2 - (x + 1 + \sqrt{5}) - 2(y + 2 = 0)$

Put $x = -2y$

$-2y(1 + \sqrt{5}) + 2y + 2y - 1 - \sqrt{5} - 2y - 4 = 0$

$-2\sqrt{5}\,y = 5 + \sqrt{5} \Rightarrow y = \left(\frac{\sqrt{5}+1}{2}\right)$

$Q\left(\sqrt{5}+1, -\frac{\sqrt{5}+1}{2}\right)$

Length of tangent OQ = $\frac{5+\sqrt{5}}{2}$

Area = $\frac{RL^3}{R^2+L^2}$

$R=\sqrt{5}$

$$=\frac{\sqrt{5}\times\left(\frac{5+\sqrt{5}}{2}\right)^3}{5+\left(\frac{5+\sqrt{5}}{2}\right)^2}$$

$$=\frac{\sqrt{5}}{2}\times\frac{4\times\left(125+75+75\sqrt{5}+5\sqrt{5}\right)}{\left(20+25+10\sqrt{5}+5\right)}$$

$$=\frac{5+3\sqrt{5}}{2}$$

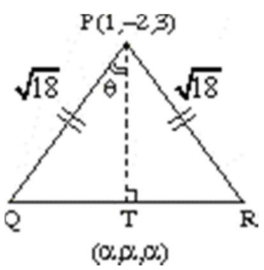
15. If two distinct point Q, R lie on the line of intersection of the planes –x + 2y – z = 0 and $3x-5y+2z=0$ and $PQ=PR=\sqrt{18}$ where the point P is (1, –2, 3), then the area of the triangle PQR is equal to

(A) $\frac{2}{3}\sqrt{38}$ (B) $\frac{4}{3}\sqrt{38}$

(C) $\frac{8}{3}\sqrt{38}$ (D) $\sqrt{\frac{152}{3}}$

Official Ans. by NTA (B)

Sol.



$-x+2y-z=0$

$3x-5y+2z=0$

$$\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 2 & -1\\ 3 & -5 & 2\end{vmatrix}$$

$$=\hat{i}(-1)-\hat{j}(1)+\hat{k}(-1)$$

$$\vec{n}=-\hat{i}-\hat{j}-\hat{k}$$

Equation of LOI is $\frac{x}{1}=\frac{y}{1}=\frac{z}{1}$

DR: of PT $\rightarrow \alpha-1, \alpha+2, \alpha-3$

DR: of QR $\rightarrow$ 1, 1, 1

$\Rightarrow (\alpha-1)\times1+(\alpha+2)\times1+(\alpha-3)\times1=0$

$3\alpha=2$

$\boxed{\alpha=\frac{2}{3}}$

$PT^2=\frac{1}{9}+\frac{64}{9}+\frac{49}{9}$

$PT^2=\frac{114}{9}$

$\boxed{PT=\frac{\sqrt{114}}{3}}$

$$\cos\theta=\frac{\sqrt{114}}{3}\times\frac{1}{3\sqrt{2}}=\frac{\sqrt{57}}{9}=\frac{\sqrt{19\times3}}{3\times3}$$

$$=\frac{\sqrt{19}}{3\sqrt{3}}$$

$$\cos2\theta=\frac{2\times19}{27}-1=\frac{11}{27}$$

$$\sin2\theta=\sqrt{1-\left(\frac{11}{27}\right)^2}=\frac{\sqrt{38}\sqrt{16}}{27}$$

$$=\frac{4}{27}\sqrt{38}$$

Area $=\frac{1}{2}\times\sqrt{18}\sqrt{18}\times\frac{4}{27}\sqrt{38}$

$$=\frac{18}{2}\times\frac{4}{27}\sqrt{38}=\frac{36}{27}\sqrt{38} \quad =\frac{4}{3}\sqrt{38}$$

16. The acute angle between the planes P_1 and P_2, when P_1 and P_2 are the planes passing through the intersection of the planes 5x + 8y + 13z – 29 = 0 and 8x – 7y + z – 20 = 0 and the points (2, 1, 3) and (0, 1, 2), respectively, is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{12}$

Official Ans. by NTA (A)

Sol. Equation of plane passing through the intersection of planes 5x + 8y + 13z –29 = 0 and 8x – 7y + z – 20 = 0 is

$5x+8y+3z-29+\lambda(8x-7y+z-20)=0$ and

if it is passing through (2,1,3) then $\lambda=\frac{7}{2}$

P_1: Equation of plane through intersection of $5x+8y+13z-29=0$ and $8x-7y+z-20=0$ and the point (2, 1, 3) is

$5x+8y+3z-29+\frac{7}{2}(8x-7y+z-20)=0$

$\Rightarrow 2x-y+z=6$

Similarly P_2 : Equation of plane through intersection of

5x + 8y + 13z – 29 = 0 and 8x – 7y + z – 20 = 0 and the point (0,1,2) is

$\Rightarrow x+y+2z=5$

Angle between planes $=\theta=\cos^{-1}\left(\frac{3}{\sqrt{6}\sqrt{6}}\right)=\frac{\pi}{3}$

17. Let the plane $P:\vec{r}\cdot\vec{a}=d$ contain the line of intersection of two planes $\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k})=6$ and $\vec{r}\cdot(-6\hat{i}+5\hat{j}-\hat{k})=7$. If the plane P passes through the point $\left(2,3,\frac{1}{2}\right)$, then the value of $\frac{|13\vec{a}|^2}{d^2}$ is equal to

(A) 90 (B) 93

(C) 95 (D) 97

Official Ans. by NTA (B)

Sol. Equation of plane passing through line of intersection of planes $P_1:\vec{r}((\hat{i}+3\hat{j}-\hat{k})=6$ and $P_2:\vec{r}\cdot(-6\hat{i}+5\hat{j}-\hat{k})=7$ is

$P_1+\lambda P_2=0$

$\left(\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k})-6\right)+\lambda\left(\vec{r}\cdot(-6\hat{i}+5\hat{j}-\hat{k})-7\right)=0$

and it passes through point $\left(2,3,\frac{1}{2}\right)$

$\Rightarrow\left(2+9-\frac{1}{2}-6\right)+\lambda\left(-12+15-\frac{1}{2}-7\right)=0$

$\Rightarrow \lambda=1$

Equation of plane is $\vec{r}\cdot(-5\hat{i}+8\hat{j}-2\hat{k})=13$

$|\vec{a}|^2=25+64+4=93$; d = 13

Value of $\frac{|13\vec{a}|^2}{d^2}=93$

18. The probability, that in a randomly selected 3-digit number at least two digits are odd, is

(A) $\frac{19}{36}$ (B) $\frac{15}{36}$

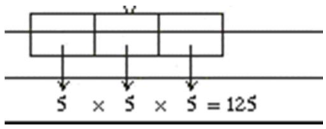
(C) $\frac{13}{36}$ (D) $\frac{23}{36}$

Official Ans. by NTA (A)

Sol. Atleast two digits are odd

= exactly two digits are odd + exactly there 3 digits are odd

For exactly three digits are odd



For exactly two digits odd :

If 0 is used then : $2 \times 5 \times 5 = 50$

If 0 is not used then : ${}^3C_1 \times 4 \times 5 \times 5 = 300$

Required Probability $= \frac{475}{900} = \frac{19}{36}$

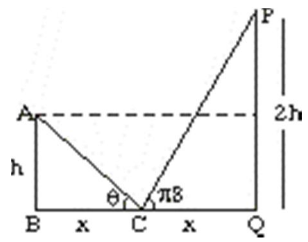
19. Let AB and PQ be two vertical poles, 160 m apart from each other. Let C be the middle point of B and Q, which are feet of these two poles. Let $\frac{\pi}{8}$ and θ be the angles of elevation from C to P and A, respectively. If the height of pole PQ is twice the height of pole AB, then $\tan^2\theta$ is equal to

(A) $\frac{3-2\sqrt{2}}{2}$ (B) $\frac{3+\sqrt{2}}{2}$

(C) $\frac{3-2\sqrt{2}}{4}$ (D) $\frac{3-\sqrt{2}}{4}$

Official Ans. by NTA (C)

Sol.



Let BC = CQ = x & AB = h and PQ = 2 h

$\tan\theta = \frac{h}{x}, \tan\frac{\pi}{8} = \frac{2h}{x}$

$\frac{\tan\theta}{\tan\left(\frac{\pi}{8}\right)} = \frac{1}{2}$

$\tan\theta = \frac{1}{2}\tan\left(\frac{\pi}{8}\right) = \frac{1}{2}\left(\sqrt{2}-1\right)$

$\tan^2\theta = \frac{1}{4}\left(3-2\sqrt{2}\right)$

20. Let p, q, r be three logical statements. Consider the compound statements

$S_1 : ((\sim p) \vee q) \vee ((\sim p) \vee r)$ and

$S_2 : p \rightarrow (q \vee r)$

Then, which of the following is **NOT** true ?

(A) If S_2 is True, then S_1 is True

(B) If S_2 is False, then S_1 is False

(C) If S_2 is False, then S_1 is True

(D) If S_1 is False, then S_2 is False

Official Ans. by NTA (C)

Sol. $s_1 : (\sim p \vee q) \vee (\sim p \vee r)$

$\equiv \sim p \vee (q \vee r)$

$s_2 : p \rightarrow (q \vee r)$

$\equiv \sim p \vee (q \vee r) \rightarrow$ By conditional law

$s_1 \equiv s_2$

SECTION-B

1. Let R_1 and R_2 be relations on the set {1, 2, …, 50} such that

$R_1 = \{(p, p^n) : p \text{ is a prime and } n \geq 0 \text{ is an integer}\}$

and $R_2 = \{(p, p^n) : p \text{ is a prime and } n = 0 \text{ or } 1\}$.

Then, the number of elements in $R_1 - R_2$ is ______.

Official Ans. by NTA (8)

Sol. Here, $p, p^n \in \{1, 2, ...50\}$

Now p can take values

2,3,5,7,11,13,17,23,29,31,37,41,43 and 47.

∴ we can calculate no. of elements in R_1 as

$(2, 2^0), (2, 2^1) .. (2, 2^5)$

$(3, 3^0), \dots (3, 3^3)$

$(5, 5^0), \dots (5, 5^2)$

$(7, 7^0), \dots (7, 7^2)$

$(11, 11^0), \dots (11, 11^1)$

And rest for all other two elements each

$\therefore \quad n(R_1) = 6+4+3+3+(2\times 10) = 36$

Similarly for R_2

(2, 2°), (2,2[1])

(47, 47°), (47, 47[1])

$\therefore \quad n(R_2) = 2\times 14 = 28$

$\therefore \quad n(R_1) - n(R_2) = 36 - 28 = 8$

2. The number of real solutions of the equation $e^{4x} + 4e^{3x} - 58e^{2x} + 4e^{x} + 1 = 0$ is ______.

Official Ans. by NTA (2)

Sol. $e^{4x} + 4e^{3x} - 58e^{2x} + 4e^{x} + 1 = 0$

Let f(x) = $e^{2x}\left(e^{2x} + \frac{1}{e^{2x}} + 4\left(e^{x} + \frac{1}{e^{x}}\right) - 58\right)$

$e^x + \frac{1}{e^x}$

Let h(t) = $t^2 + 4t - 58 = 0$

$t = \frac{-4 \pm \sqrt{16+4.58}}{2}$

$\frac{-4\pm 2\sqrt{62}}{2}$

$t_1 = -2 + 2\sqrt{62}$

$t_2 = -2 - 2\sqrt{62}$ (not possible)

$t \geq 2$

$e^x + \frac{1}{e^x} = -2 + 2\sqrt{62}$

$e^{2x} - (-2 + 2\sqrt{62})\, e^x + 1 = 0$

$(-2 + 2\sqrt{62}) - 4$

$4 + 4.62 - 8\sqrt{62} - 4$

$248 - 8\sqrt{62} > 0$

$\frac{-b}{2a} > 0$

both roots are positive

2 real roots

3. The mean and standard deviation of 15 observations are found to be 8 and 3 respectively. On rechecking it was found that, in the observations, 20 was misread as 5. Then, the correct variance is equal to ______.

Official Ans. by NTA (17)

Sol. We have

$$\text{Variance} = \frac{\sum_{r=1}^{15} x_r^2}{15} - \left(\frac{\sum_{r=1}^{15} x_r}{15}\right)^2$$

Now, as per information given in equation

$\frac{\sum x_r^2}{15} - 8^2 = 3^2 \Rightarrow \sum x_r^2 = \log 5$

Now, the new $\sum x_r^2 = \log 5 - 5^2 + 20^2 = 1470$

And, new $\sum x_r = (15\times 8) - 5 + (20) = 135$

$\therefore$ Variance $= \frac{1470}{15} - \left(\frac{135}{15}\right)^2 = 98 - 81 = 17$

4. If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ are coplanar vectors and $\vec{a}\cdot\vec{c} = 5$, $\vec{b} \perp \vec{c}$, then 122 (c_1 + c_2 + c_3) is equal to ______.

Official Ans. by NTA (150)

Sol. $\vec{a}\cdot\vec{c} = 5 \Rightarrow 2c_1 + c_2 + 3c_3 = 5$ **...(1)**

$\vec{b}\cdot\vec{c} = 0 \Rightarrow 3c_1 + 3c_2 + c_3 = 0$ **...(2)**

And $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0 \Rightarrow \begin{vmatrix} c_1 & c_2 & c_3 \\ 2 & 1 & 3 \\ 3 & 3 & 1 \end{vmatrix} = 0$

$\Rightarrow 8c_1 - 7c_2 - 3c_3 = 0$ ...(3)

By solving (1), (2), (3) we get

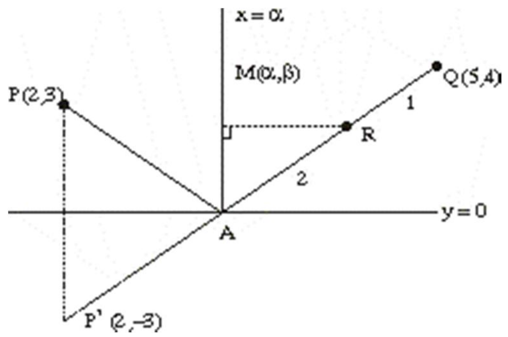
$c_1 = \frac{10}{122}, c_2 = \frac{-85}{122}, c_3 = \frac{225}{122}$

$\therefore 122(c_1 + c_2 + c_3) = 150$

5. A ray of light passing through the point P(2, 3) reflects on the x-axis at point A and the reflected ray passes through the point Q(5, 4). Let R be the point that divides the line segment AQ internally into the ratio 2 : 1. Let the co-ordinates of the foot of the perpendicular M from R on the bisector of the angle PAQ be (α, β). Then, the value of $7\alpha + 3\beta$ is equal to ______.

Official Ans. by NTA (31)

Sol.



By observation we see that $A(\alpha, 0)$.

And $\beta = y$ -cordinate of R

$$= \frac{2\times4+1\times0}{2+1} = \frac{8}{3} \quad \ldots(1)$$

Now P' is image of P in y = 0 which will be $P'(2,-3)$

∴ Equation of P'Q is $(y+3) = \frac{4+3}{5-2}(x-2)$

i.e. $3y+9 = 7x-14$

$A \equiv \left(\frac{23}{7}, 0\right)$ by solving with y = 0

$$\therefore\ \alpha = \frac{23}{7} \quad \ldots(2)$$

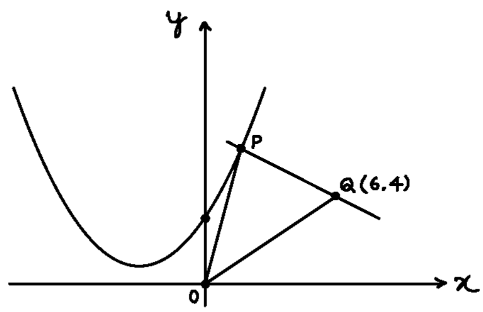
By (1), (2)

$$7\alpha + 3\beta = 23 + 8 = 31$$

6. Let ℓ be a line which is normal to the curve $y = 2x^2 + x + 2$ at a point P on the curve. If the point Q(6, 4) lies on the line ℓ and O is origin, then the area of the triangle OPQ is equal to ______.

Official Ans. by NTA (13)

Sol. $y = 2x^2 + x + 2$



$$\frac{dy}{dx} = 4x + 1$$

Let P be (h, k), then normal at P is

$$y - k = -\frac{1}{4h+1}(x-h)$$

This passes through Q (6,4)

$$\therefore 4 - k = -\frac{1}{4h+1}(6-h)$$

$$\Rightarrow (4h+1)(4-k) + 6 - h = 0$$

Also $k = 2h^2 + h + 2$

$$\therefore (4h+1)(4-2h^2-h-2) + 6 + h = 0$$

$$\Rightarrow 4h^3 - 3h^2 + 3h - 8 = 0$$

$$\Rightarrow h = 1, k = 5$$

Now area of ΔOPQ will be $= \frac{1}{2}\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 5 \\ 1 & 6 & 4 \end{vmatrix} = 13$

7. Let A = {1, a_1, a_2…… a_{18}, 77} be a set of integers with $1 < a_1 < a_2 < \ldots < a_{18} < 77$. Let the set $A + A = \{x + y : x, y \in A\}$ contain exactly 39 elements. Then, the value of $a_1 + a_2 + \ldots + a_{18}$ is equal to ________.

Official Ans. by NTA (702)

Sol. $a_1, a_2, a_3, \ldots, a_{18}, 77$

are in AP i.e. 1, 5, 9, 13, ..., 77.

Hence $a_1 + a_2 + a_3 + \ldots + a_{18}$ = 5+9+13+...18 terms

=702

8. The number of positive integers k such that the constant term in the binomial expansion of $\left(2x^3 + \frac{3}{x^k}\right)^{12}$, $x \neq 0$ is $2^8 \cdot \ell$, where ℓ is an odd integer, is ________.

Official Ans. by NTA (2)

Sol. $\left(2x^3 + \frac{3}{x^k}\right)^{12}$

$t_{r+1} = {}^{12}C_r \left(2x^3\right)^r \left(\frac{3}{x^k}\right)^{12-r}$

$x^{3r-(12-r)k} \to \text{constant}$

$\therefore 3r - 12k + rk = 0$

$\Rightarrow k = \frac{3r}{12-r}$

$\therefore$ possible values of r are 3,6,8,9,10 and corresponding values of k are 1,3,6,9,15

Now $^{12}C_r = 220, 924, 495, 220, 66$

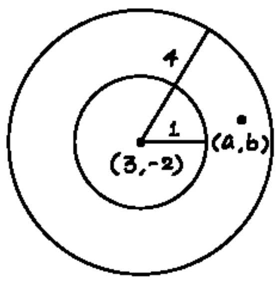
$\therefore$ possible values of k for which we will get 2^8 are 3, 6

9. The number of elements in the set

$\{z = a + ib \in \mathbb{C} : a, b \in \mathbb{Z} \text{ and } 1 < |z - 3 + 2i| < 4\}$ is ______.

Official Ans. by NTA (40)

Sol. $1 < |Z - 3 + 2i| < 4$



$1 < (a-3)^2 + (b+2)^2 < 16$

$(0,\pm 2),(\pm 2,0),(\pm 1,\pm 2),(\pm 2,\pm 1)$

$(\pm 2,\pm 3),(3\pm,\pm 2),(\pm 1,\pm 1),(2\pm,\pm 2)$

$(\pm 3,0),(0,\pm 3),(\pm 3 \pm 1),(\pm 1,\pm 3)$

Total 40 points

10. Let the lines $y + 2x = \sqrt{11} + 7\sqrt{7}$ and $2y + x = 2\sqrt{11} + 6\sqrt{7}$ be normal to a circle $C : (x-h)^2 + (y-k)^2 = r^2$. If the line $\sqrt{11}y - 3x = \frac{5\sqrt{77}}{3} + 11$ is tangent to the circle C, then the value of $(5h - 8k)^2 + 5r^2$ is equal to ____.

Official Ans. by NTA (816)

Sol. Normal are

$y + 2x = \sqrt{11} + 7\sqrt{7},$

$2y + x = 2\sqrt{11} + 6\sqrt{7}$

Center of the circle is point of intersection of normals i.e.

$\left(\frac{8\sqrt{7}}{3}, \sqrt{11} + \frac{5\sqrt{7}}{3}\right)$

Tangent is $\sqrt{11}y - 3x = \frac{5\sqrt{77}}{3} + 11$

Radius will be $\perp$ distance of tangent from center i.e. $4\sqrt{\frac{7}{5}}$

Now $(5h - 8k)^2 + 5r^2 = 816$