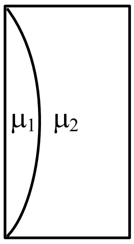
FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Friday 27th August, 2021) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Curved surfaces of a plano-convex lens of refractive index μ_1 and a plano-concave lens of refractive index μ_2 have equal radius of curvature as shown in figure. Find the ratio of radius of curvature to the focal length of the combined lenses.

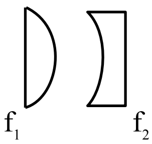


(1) $\frac{1}{\mu_2-\mu_1}$ (2) $\mu_1-\mu_2$

(3) $\frac{1}{\mu_1-\mu_2}$ (4) $\mu_2-\mu_1$

Official Ans. by NTA (2)

Sol.



$$\frac{1}{f_1}=(\mu_1-1)\left(\frac{1}{R}\right)$$

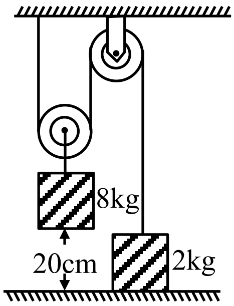
$$\frac{1}{f_2}=(\mu_2-1)\left(-\frac{1}{R}\right)$$

$$\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{f_{eq}}=\frac{(\mu_1-1)-(\mu_2-1)}{R}$$

$$\frac{1}{f_{eq}}=\frac{(\mu_1-\mu_2)}{R}$$

$$\frac{R}{f_{eq}}=(\mu_1-\mu_2)$$

2. The boxes of masses 2 kg and 8 kg are connected by a massless string passing over smooth pulleys. Calculate the time taken by box of mass 8 kg to strike the ground starting from rest. (use $g = 10\ m/s^2$)

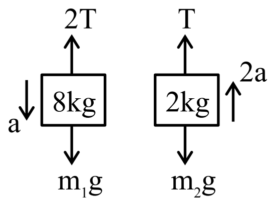


(1) 0.34 s (2) 0.2 s

(3) 0.25 s (4) 0.4 s

Official Ans. by NTA (4)

Sol.



$(m_1g - 2T) = m_1a$ – (1)

$T - m_2g = m_2(2a)$

$2T - 2m_2g = 4m_2\,a$ – (2)

$m_1g - 2m_2g = (m_1 + 4m_2)\,a$

$$a=\frac{(8-4)g}{(8+8)}=\frac{4}{16}g=\frac{g}{4}$$

$$a=\frac{10}{4}\ m/s^2$$

$$S=\frac{1}{2}at^2$$

$$\frac{0.2\times 2\times 4}{10}=t^2$$

$t = 0.4$ sec

3. For a transistor α and β are given as $\alpha = \frac{I_C}{I_E}$ and $\beta = \frac{I_C}{I_B}$. Then the correct relation between α and β will be :

(1) $\alpha = \frac{1-\beta}{\beta}$

(2) $\beta = \frac{\alpha}{1-\alpha}$

(3) $\alpha\beta = 1$

(4) $\alpha = \frac{\beta}{1-\beta}$

Official Ans. by NTA (2)

Sol. $\alpha = \frac{I_C}{I_E}$, $\beta = \frac{I_C}{I_B}$; $I_E = I_C + I_B$

$$\alpha = \frac{I_C}{I_C + I_B} = \frac{I_C / I_B}{\frac{I_C}{I_B} + 1} = \frac{\beta}{\beta + 1} +$$

$$1 + \frac{1}{\beta} = \frac{1}{\alpha}$$

$$\frac{1}{\beta} = \frac{1-\alpha}{\alpha}$$

$$\beta = \frac{\alpha}{1-\alpha}$$

4. Water drops are falling from a nozzle of a shower onto the floor, from a height of 9.8 m. The drops fall at a regular interval of time. When the first drop strikes the floor, at that instant, the third drop begins to fall. Locate the position of second drop from the floor when the first drop strikes the floor.

(1) 4.18 m

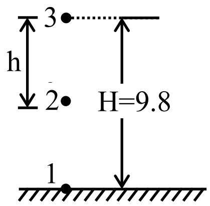
(2) 2.94 m

(3) 2.45 m

(4) 7.35 m

Official Ans. by NTA (4)

Sol.



$$H = \frac{1}{2}gt^2$$

$$\frac{9.8 \times 2}{9.8} = t^2$$

$t = \sqrt{2}$ sec

Δt: time interval between drops

$$h = \frac{1}{2}g(\sqrt{2} - \Delta t)^2$$

$$0 = \frac{1}{2}g(\sqrt{2} - 2\Delta t)^2$$

$$\Delta t = \frac{1}{\sqrt{2}}$$

$$h = \frac{1}{2}g\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} \times 9.8 \times \frac{1}{2} = \frac{9.8}{4} = 2.45\text{m}$$

$H - h = 9.8 - 2.45$

$= 7.35$ m

5. Two discs have moments of intertia I_1 and I_2 about their respective axes perpendicular to the plane and passing through the centre. They are rotating with angular speeds, ω_1 and ω_2 respectively and are brought into contact face to face with their axes of rotation coaxial. The loss in kinetic energy of the system in the process is given by :

(1) $\frac{I_1 I_2}{(I_1 + I_2)}(\omega_1 - \omega_2)^2$

(2) $\frac{(I_1 - I_2)^2 \omega_1 \omega_2}{2(I_1 + I_2)}$

(3) $\frac{I_1 I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2$

(4) $\frac{(\omega_1 - \omega_2)^2}{2(I_1 + I_2)}$

Official Ans. by NTA (3)

Sol. From conservation of angular momentum we get

$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

$$k_i = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$$

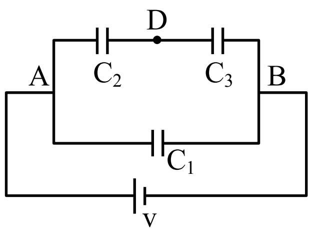
$$k_f = \frac{1}{2}(I_1 + I_2)\omega^2$$

$$k_i - k_f = \frac{1}{2}\left[I_1\omega_1^2 + I_2\omega_2^2 - \frac{(I_1\omega_1 + I_2\omega_2)^2}{I_1 + I_2}\right]$$

Solving above we get

$$k_i - k_f = \frac{1}{2}\left(\frac{I_1 I_2}{I_1 + I_2}\right)(\omega_1 - \omega_2)^2$$

6. Three capacitors $C_1 = 2\mu F$, $C_2 = 6\ \mu F$ and $C_3 = 12\ \mu F$ are connected as shown in figure. Find the ratio of the charges on capacitors C_1, C_2 and C_3 respectively :

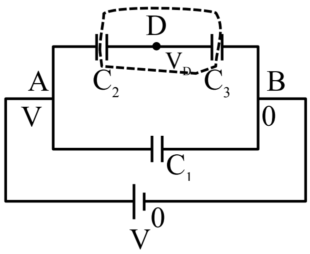


(1) 2 : 1 : 1 (2) 2 : 3 : 3

(3) 1 : 2 : 2 (4) 3 : 4 : 4

Official Ans. by NTA (3)

Sol.



$(V_D - V)\ C_2 + (V_D - 0)\ C_3 = 0$

$(V_D - V)\ 6 + (V_D - 0)\ 12 = 0$

$V_D - V + 2V_D = 0$

$$V_D = \frac{V}{3}$$

$$q_2 = (V - V_D)\ C_2 = \left(V - \frac{V}{3}\right)(6\ \mu F)$$

$q_2 = (4V)\ \mu F$

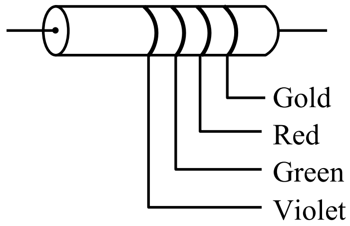
$$q_3 = (V_D - 0)\ C_3 = \frac{V}{3} \times 12\mu F = 4V\mu F$$

$q_1 = (V - 0)\ C_1 = V(2\mu F)$

$q_1 : q_2 : q_3 = 2 : 4 : 4$

$q_1 : q_2 : q_3 = 1 : 2 : 2$

7. The colour coding on a carbon resistor is shown in the given figure. The resistance value of the given resistor is :



(1) $(5700 \pm 285)\ \Omega$

(2) $(7500 \pm 750)\ \Omega$

(3) $(5700 \pm 375)\ \Omega$

(4) $(7500 \pm 375)\ \Omega$

Official Ans. by NTA (4)

Sol. $R = 75 \times 10^2 \pm 5\%$ of 7500

$R = (7500 \pm 375)\Omega$

8. An antenna is mounted on a 400 m tall building. What will be the wavelength of signal of signal that can be radiated effectively by the transmission tower upto a range of 44 km?

(1) 37.8 m

(2) 605 m

(3) 75.6 m

(4) 302 m

Official Ans. by NTA (2)

Sol. h : height of antenna

λ : wavelength of signal

$h < \lambda$

$\lambda > h$

$\lambda > 400$ m

9. If the rms speed of oxygen molecules at 0°C is 160 m/s, find the rms speed of hydrogen molecules at 0°C.

(1) 640 m/s (2) 40 m/s

(3) 80 m/s (4) 332 m/s

Official Ans. by NTA (1)

Sol. $V_{rms} = \sqrt{\frac{3KT}{M}}$

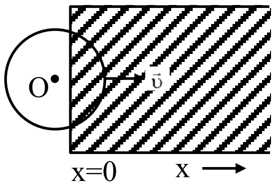
$$\frac{(V_{rms})_{O_2}}{(V_{rms})_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{O_2}}} = \sqrt{\frac{2}{32}}$$

$(V_{rms})_{H_2} = 4 \times (V_{rms})_{O_2}$

$= 4 \times 160$

$= 640$ m/s

10. A constant magnetic field of 1 T is applied in the x > 0 region. A metallic circular ring of radius 1m is moving with a constant velocity of 1 m/s along the x-axis. At t = 0s, the centre of O of the ring is at x = – 1m. What will be the value of the induced emf in the ring at t = 1s? (Assume the velocity of the ring does not change.)



(1) 1 V (2) 2π V

(3) 2 V (4) 0 V

Official Ans. by NTA (3)

Sol. emf = BLV

= 1.(2R) .1

= 2 V

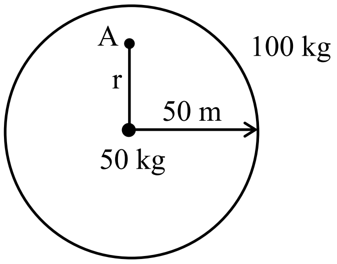
11. A mass of 50 kg is placed at the centre of a uniform spherical shell of mass 100 kg and radius 50 m. If the gravitational potential at a point, 25 m from the centre is V kg/m. The value of V is :

(1) – 60 G (2) + 2 G

(3) – 20 G (4) – 4 G

Official Ans. by NTA (4)

Sol.



$$V_A = \left[-\frac{GM_1}{r} - \frac{GM_2}{R}\right]$$

$$= \left[-\frac{50}{25}G - \frac{100}{50}G\right]$$

$= -4G$

12. For full scale deflection of total 50 divisions, 50 mV voltage is required in galvanometer. The resistance of galvanometer if its current sensitivity is 2 div/mA will be :

(1) 1 Ω (2) 5 Ω (3) 4 Ω (4) 2 Ω

Official Ans. by NTA (4)

Sol. $I_{max} = \frac{50}{2} = 25mA$

R $= \frac{V}{I} = \frac{50mV}{25mA} = 2\Omega$

13. A monochromatic neon lamp with wavelength of 670.5 nm illuminates a photo-sensitive material which has a stopping voltage of 0.48 V. What will be the stopping voltage if the source light is changed with another source of wavelength of 474.6 nm?

(1) 0.96 V (2) 1.25 V (3) 0.24 V (4) 1.5 V

Official Ans. by NTA (2)

Sol. $kE_{max} = \frac{hc}{\lambda_i} + \phi$

or $eV_o = \frac{hc}{\lambda_i} + \phi$

when $\lambda_i = 670.5$ nm ; $V_o = 0.48$

when $\lambda_i = 474.6$ nm ; $V_o = ?$

So, $e(0.48) = \frac{1240}{670.5} + \phi$ ...(1)

$e(V_o) = \frac{1240}{474.6} + \phi$ ...(2)

(2) – (1)

$$e(V_o - 0.48) = 1240\left(\frac{1}{474.6} - \frac{1}{670.5}\right)eV$$

$$V_o = 0.48 + 1240\left(\frac{670.5 - 474.6}{474.6 \times 670.5}\right) \text{ Volts}$$

$V_o = 0.48 + 0.76$

$V_o = 1.24\text{ V} \simeq 1.25\text{ V}$

14. Match List-I with List-II.

List-I	List-II
(a) R_H (Rydberg constant)	(i) $kg\ m^{-1}\ s^{-1}$
(b) h(Planck's constant)	(ii) $kg\ m^2\ s^{-1}$
(c) μ_B (Magnetic field energy density)	(iii) m^{-1}
(d) η(coefficient of viscocity)	(iv) $kg\ m^{-1}\ s^{-2}$

Choose the most appropriate answer from the options given below :

(1) (a)–(ii), (b)–(iii), (c)–(iv), (d)–(i)

(2) (a)–(iii), (b)–(ii), (c)–(iv), (d)–(i)

(3) (a)–(iv), (b)–(ii), (c)–(i), (d)–(iii)

(4) (a)–(iii), (b)–(ii), (c)–(i), (d)–(iv)

Official Ans. by NTA (2)

Sol. SI unit of Rydberg const. = m^{-1}

SI unit of Plank's const. = $kg\ m^2s^{-1}$

SI unit of Magnetic field energy density= $kg\ m^{-1}s^{-2}$

SI unit of coeff. of viscosity = $kg\ m^{-1}s^{-1}$

15. If force (F), length (L) and time (T) are taken as the fundamental quantities. Then what will be the dimension of density :

(1) $[FL^{-4}T^2]$

(2) $[FL^{-3}T^2]$

(3) $[FL^{-5}T^2]$

(4) $[FL^{-3}T^3]$

Official Ans. by NTA (1)

Sol. Density = $[F^aL^bT^c]$

$[ML^{-3}] = [M^aL^aT^{-2a}L^bT^c]$

$[M^1L^{-3}] = [M^aL^{a+b}T^{-2a+c}]$

$a = 1$; $a + b = -3$; $-2a + c = 0$

$1 + b = -3$ $\quad c = 2a$

$b = -4$ $\quad c = 2$

So, density = $[F^1L^{-4}T^2]$

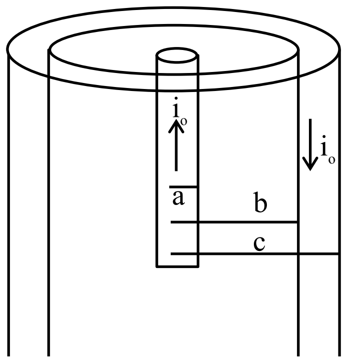
16. A coaxial cable consists of an inner wire of radius 'a' surrounded by an outer shell of inner and outer radii 'b' and 'c' respectively. The inner wire carries an electric current i_0, which is distributed uniformly across cross-sectional area. The outer shell carries an equal current in opposite direction and distributed uniformly. What will be the ratio of the magnetic field at a distance x from the axis when (i) x < a and (ii) a < x < b ?

(1) $\dfrac{x^2}{a^2}$ (2) $\dfrac{a^2}{x^2}$

(3) $\dfrac{x^2}{b^2-a^2}$ (4) $\dfrac{b^2-a^2}{x^2}$

Official Ans. by NTA (1)

Sol.



when x < a

$$B_1\ (2\pi x) = \mu_o\left(\frac{i_o}{\pi a^2}\right)\pi x^2$$

$$B(2\pi x) = \frac{\mu_o i_o x^2}{a^2}$$

$$B_1 = \frac{\mu_o i_o x}{2\pi a^2} \quad ...(1)$$

when a < x < b

$$B_2(2\pi x) = \mu_o i_o$$

$$B_2 = \frac{\mu_o i_o}{2\pi x} \quad ...(2)$$

$$\frac{B_1}{B_2} = \frac{\mu_o i_o \frac{x}{2\pi a^2}}{\frac{\mu_o i_o}{2\pi x}} = \frac{x^2}{a^2}$$

17. The height of victoria falls is 63 m. What is the difference in temperature of water at the top and at the bottom of fall ?

[Given 1 cal = 4.2 J and specific heat of water = 1 cal g^{-1} $^oC^{-1}$]

(1) 0.147° C

(2) 14.76° C

(3) 1.476°

(4) 0.014° C

Official Ans. by NTA (1)

Sol. Change in P.E. = Heat energy

$mgh = mS\Delta T$

$\Delta T = \frac{gh}{S}$

$= \frac{10 \times 63}{4200J / kgC}$

$= 0.147°C$

18. A player kicks a football with an initial speed of 25 ms^{-1} at an angle of 45° from the ground. What are the maximum height and the time taken by the football to reach at the highest point during motion ? (Take g = 10 ms^{-2})

(1) h_{max} = 10 m T = 2.5 s

(2) h_{max} = 15.625 m T = 3.54 s

(3) h_{max} = 15.625 m T = 1.77 s

(4) h_{max} = 3.54 m T = 0.125 s

Official Ans. by NTA (3)

Sol. $H = \frac{U^2 \sin^2 \theta}{2g}$

$= \frac{(25)^2.(\sin 45)^2}{2 \times 10}$

= 15.625 m

$T = \frac{U\sin\theta}{g}$

$= \frac{25 \times \sin 45°}{10}$

= 2.5 × 0.7

= 1.77 s

19. The light waves from two coherent sources have same intensity $I_1 = I_2 = I_0$. In interference pattern the intensity of light at minima is zero. What will be the intensity of light at maxima ?

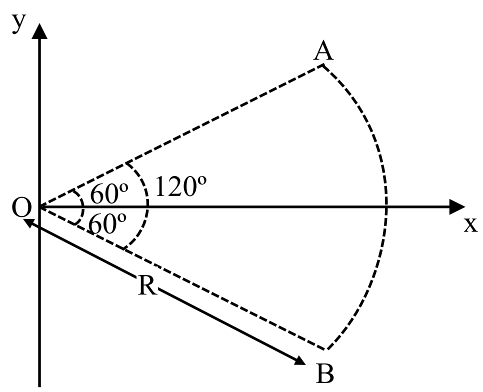
(1) I_0 (2) 2 I_0 (3) 5 I_0 (4) 4 I_0

Official Ans. by NTA (4)

Sol. $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

$= 4I_o$

20. Figure shows a rod AB, which is bent in a 120° circular arc of radius R. A charge (–Q) is uniformly distributed over rod AB. What is the electric field $\vec{E}$ at the centre of curvature O ?



(1) $\frac{3\sqrt{3}Q}{8\pi\varepsilon_0 R^2}(\hat{i})$ (2) $\frac{3\sqrt{3}Q}{8\pi^2\varepsilon_0 R^2}(\hat{i})$

(3) $\frac{3\sqrt{3}Q}{16\pi^2\varepsilon_0 R^2}(\hat{i})$ (4) $\frac{3\sqrt{3}Q}{8\pi^2\varepsilon_0 R^2}(-\hat{i})$

Official Ans. by NTA (2)

Sol. $\varepsilon = \frac{2k\lambda}{R}\sin\left(\frac{\theta}{2}\right)(-\hat{i})$

$\lambda = \left(\frac{-Q}{R\theta}\right) = \left(\frac{-Q}{R.\frac{2\pi}{3}}\right)$

$\lambda = \frac{-3Q}{2\pi R}$

$\varepsilon = \frac{2k}{R}.\frac{-3Q}{2\pi R}.\sin(60°)(-\hat{i})$

$\varepsilon = \frac{3\sqrt{3}Q}{8\pi^2 \epsilon_o R^2}(+\hat{i})$

SECTION-B

1. A heat engine operates between a cold reservoir at temperature T_2 = 400 K and a hot reservoir at temperature T_1. It takes 300 J of heat from the hot reservoir and delivers 240 J of heat to the cold reservoir in a cycle. The minimum temperature of the hot reservoir has to be _________K.

Official Ans. by NTA (500)

Sol. $Q_{in} = 300$ J ; $Q_{out} = 240$ J

Work done = $Q_{in} - Q_{out} = 300 - 240 = 60$ J

$$\text{Efficiency} = \frac{W}{Q_{in}} = \frac{60}{300} = \frac{1}{5}$$

$$\text{efficiency} = 1 - \frac{T_2}{T_1}$$

$$\frac{1}{5} = 1 - \frac{400}{T_1} \Rightarrow \frac{400}{T_1} = \frac{4}{5}$$

$T_1 = 500$ k

2. Two simple harmonic motion, are represented by the equations $y_1 = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$

$y_2 = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$

Ratio of amplitude of y_1 to y_2 = x : 1. The value of x is ________.

Official Ans. by NTA (1)

Sol. $y_1 = 10\sin\left(3\pi t + \frac{\pi}{3}\right) \Rightarrow$ Amplitude= 10

$y_2 = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$

$$y_2 = 10\left(\frac{1}{2}\sin 3\pi t + \frac{\sqrt{3}}{2}\cos 3\pi t\right)$$

$$y_2 = 10\left(\cos\frac{\pi}{3}\sin 3\pi t + \sin\frac{\pi}{3}\cos 3\pi t\right)$$

$$y_2 = 10\sin\left(3\pi t + \frac{\pi}{3}\right) \Rightarrow \text{Amplitude} = 10$$

$$\text{So ratio of amplitudes} = \frac{10}{10} = 1$$

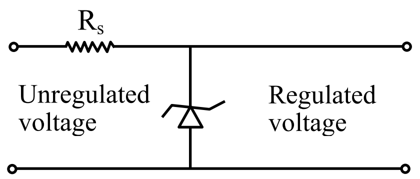
3. X different wavelengths may be observed in the spectrum from a hydrogen sample if the atoms are exited to states with principal quantum number n = 6 ? The value of X is _______.

Official Ans. by NTA (15)

Sol. No. of different wavelengths = $\frac{n(n-1)}{2}$

$$= \frac{6\times(6-1)}{2} = \frac{6\times 5}{2} = 15$$

4. A zener diode of power rating 2W is to be used as a voltage regulator. If the zener diode has a breakdown of 10 V and it has to regulate voltage fluctuated between 6 V and 14 V, the value of R_s for safe operation should be _______ Ω.



Official Ans. by NTA (20)

Sol. When unregulated voltage is 14 V voltage across zener diode must be 10 V So potential difference across resistor $\Delta V_{Rs} = 4V$

and $P_{zener} = 2W$

$VI = 2$

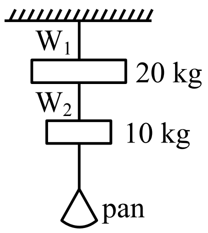
$$I = \frac{2}{10} = 0.2 \text{ A}$$

$\Delta V_{Rs} = I R_s$

$$4 \times 0.2 R_s \Rightarrow R_s = \frac{40}{2} = 20\Omega$$

5. Wires W_1 and W_2 are made of same material having the breaking stress of 1.25×10^9 N/m^2. W_1 and W_2 have cross-sectional area of 8×10^{-7} m^2 and 4×10^{-7} m^2, respectively. Masses of 20 kg and 10 kg hang from them as shown in the figure. The maximum mass that can be placed in the pan without breaking the wires is ____ kg.

(Use $g = 10$ m/s^2)



Official Ans. by NTA (40)

Sol. $B.S_1 = \dfrac{T_{1max}}{8 \times 10^{-7}} \Rightarrow T_{1\,max} = 8 \times 1.25 \times 100$

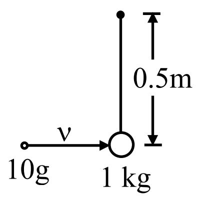
$= 1000$ N

$B.S_2 = \dfrac{T_{2max}}{4 \times 10^{-7}} \Rightarrow T_{2max} = 4 \times 1.25 \times 100$

$= 500$ N

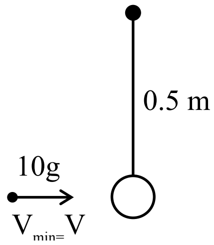
$m = \dfrac{500 - 100}{10} = 40$ kg

6. A bullet of 10 g, moving with velocity v, collides head-on with the stationary bob of a pendulum and recoils with velocity 100 m/s. The length of the pendulum is 0.5 m and mass of the bob is 1 kg. The minimum value of v = _____ m/s so that the pendulum describes a circle. (Assume the string to be inextensible and $g = 10$ m/s^2)



Official Ans. by NTA (400)

Sol.



$V' = \sqrt{5gR} = \sqrt{5 \times 10 \times 0.5}$

$V' = 5$ m/s

$m_1V = m_2 \times 5 - m_1 \times 100$

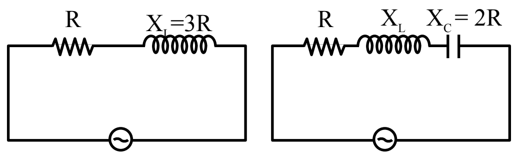
$\dfrac{10}{1000} \times V = 5 - \dfrac{10}{1000} \times 100$

$V = 400$ m/s

7. An ac circuit has an inductor and a resistor of resistance R in series, such that $X_L = 3R$. Now, a capacitor is added in series such that $X_C = 2R$. The ratio of new power factor with the old power factor of the circuit is $\sqrt{5} : x$. The value of x is _____.

Official Ans. by NTA (1)

Sol.

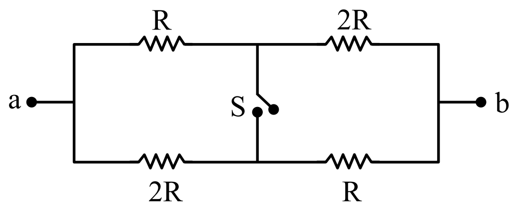


$\cos\phi = \dfrac{R}{\sqrt{R^2 + 3R^2}}$ $\qquad$ $\cos\phi' = \dfrac{R}{\sqrt{R^2 + R^2}}$

$= \dfrac{1}{\sqrt{10}}$ $\qquad$ $= \dfrac{1}{\sqrt{2}}$

$\dfrac{\cos\phi'}{\cos\phi} = \dfrac{\sqrt{10}}{\sqrt{2}} = \dfrac{\sqrt{5}}{1}$ $\qquad$ $\therefore x = 1$

8. The ratio of the equivalent resistance of the network (shown in figure) between the points a and b when switch is open and switch is closed is x : 8. The value of x is _______.



Official Ans. by NTA (9)

Sol. $R_{eq\ open} = \frac{3R}{2}$

$R_{eq\ closed} = 2 \times \frac{R \times 2R}{3R} = \frac{4R}{3}$

$\frac{R_{eq\ open}}{R_{eq\ closed}} = \frac{3R}{2} \times \frac{3}{4R} = \frac{9}{8}$

$\therefore x = 9$

9. A plane electromagnetic wave with frequency of 30 MHz travels in free space. At particular point in space and time, electric field is 6 V/m. The magnetic field at this point will be $x \times 10^{-8}$ T. The value of x is _____.

Official Ans. by NTA (2)

Sol. $|B| = \frac{|E|}{C} = \frac{6}{3 \times 10^8}$

$= 2 \times 10^{-8}$ T

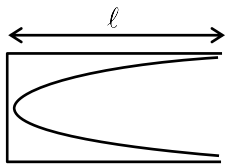
$\therefore x = 2$

10. A tuning fork is vibrating at 250 Hz. The length of the shortest closed organ pipe that will resonate with the tuning fork will be _____ cm.

(Take speed of sound in air as 340 ms^{-1})

Official Ans. by NTA (34)

Sol.



$\frac{\lambda}{4} = \ell \Rightarrow \lambda = 4\ell$

$f = \frac{V}{\lambda} = \frac{V}{4\ell}$

$\Rightarrow 250 = \frac{340}{4\ell}$

$\Rightarrow \ell = \frac{34}{4 \times 25} = 0.34m$

$\ell = 34cm$

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Friday 27th August, 2021) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION-A

1. Choose the **correct** statement from the following :

(1) The standard enthalpy of formation for alkali metal bromides becomes less negative on descending the group.

(2) The low solubility of CsI in water is due to its high lattice enthalpy.

(3) Among the alkali metal halides, LiF is least soluble in water.

(4) LiF has least negative standard enthalpy of formation among alkali metal fluorides.

Official Ans. by NTA (3)

Sol. 1. Standard enthalpy of formation for alkali metal bromides becomes more negative on desending down the group.

2. In case of CsI, lattice energy is less, but Cs^+ is having less hydration enthalpy due to which it is less soluble in water.

3. For alkali metal fluorides, the solubility in water increases from lithium to caesium. LiF is least soluble in water.

4. Standard enthalpy of formation for LiF is most negative among alkali metal fluorides.

2. The addition of dilute NaOH to Cr^{3+} salt solution will give :

(1) a solution of $[Cr(OH)_4]^-$

(2) precipitate of $Cr_2O_3(H_2O)_n$

(3) precipitate of $[Cr(OH)_6]^{3-}$

(4) precipitate of $Cr(OH)_3$

Official Ans. by NTA (2)

Sol. $$Cr^{3+} + \underset{\text{dil.}}{NaOH} \longrightarrow \underset{\text{precipitate}}{Cr_2O_3.(H_2O)_n}$$

3. Given below are two statements :

Statement I : Ethyl pent–4–yn–oate on reaction with CH_3MgBr gives a 3°–alcohol.

Statement II : In this reaction one mole of ethyl pent–4–yn–oate utilizes two moles of CH_3MgBr.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) Both **Statement I** and **Statement II** are false.

(2) **Statement I** is false but **Statement II** is true.

(3) **Statement I** is true but **Statement II** is false.

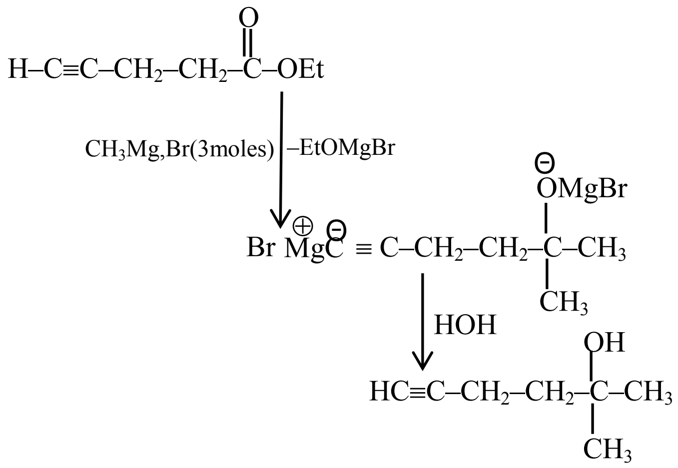
(4) Both **Statement I** and **Statement II** are true.

Official Ans. by NTA (3)

Sol. Statement 1 is true

But it consume 3 moles of G R

So statement 2 is false.



4. In stratosphere most of the ozone formation is assisted by :

(1) cosmic rays. (2) γ–rays.

(3) ultraviolet radiation. (4) visible radiations.

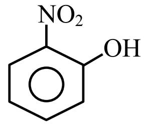
Official Ans. by NTA (3)

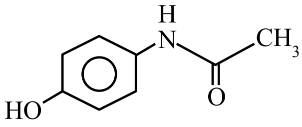
Sol. Ozone in the stratosphere is a product of UV radiations acting on dioxygen (O_2) molecules.

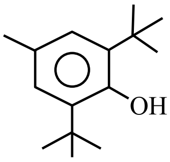
$$O_2(g) \xrightarrow{UV} O(g) + O(g)$$

$$O(g) + O_2(g) \xrightleftharpoons{UV} O_3(g)$$

5. The compound/s which will show significant intermolecular H–bonding is/are :


(a)


(b)


(c)

(1) (b) only (2) (c) only
(3) (a) and (b) only (4) (a), (b) and (c)

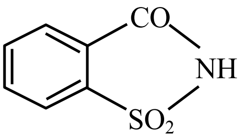
Official Ans. by NTA (1)

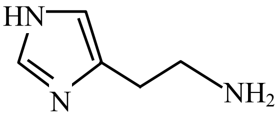
Sol. (a) Shows intra molecular H-bonding

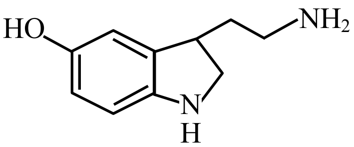
(b) Shows significant intermolecular H-bonding

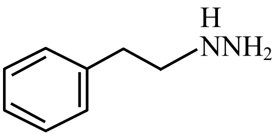
(c) It do not show intermolecular H-bonding due to steric hindrance.

6. Which one of the following chemicals is responsible for the production of HCl in the stomach leading to irritation and pain?

(1) 

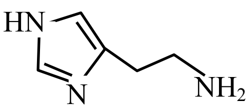
(2) 

(3) 

(4) 

Official Ans. by NTA (2)

Sol. Histamine stimulate the secretion of HCl


Histamine structure

7. The oxide that gives H_2O_2 most readily on treatment with H_2O is :

(1) PbO_2 (2) Na_2O_2
(3) SnO_2 (4) $BaO_2 \cdot 8H_2O$

Official Ans. by NTA (2)

Sol. 1. $PbO_2 + 2H_2O \rightarrow Pb(OH)_4$

2. $Na_2O_2 + 2H_2O \rightarrow 2NaOH + H_2O_2$

this reaction is possible at room temperature

3. $SnO_2 + 2H_2O \rightarrow Sn(OH)_4$

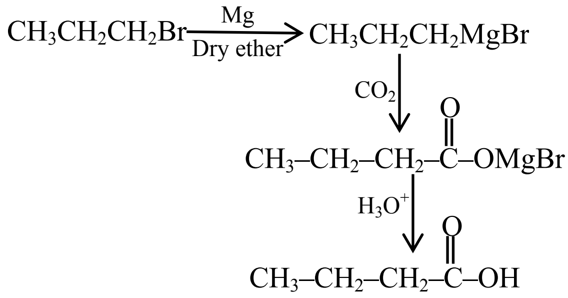
4. Acidified $BaO_2.8H_2O$ gives H_2O_2 after evaporation.

8. Which one of the following reactions will **not** yield propionic acid?

(1) $CH_3CH_2COCH_3 + OI^-/H_3O^+$
(2) $CH_3CH_2CH_3 + KMnO_4$(Heat)$,OH^-/H_3O^+$
(3) $CH_3CH_2CCl_3 + OH^-/H_3O^+$
(4) $CH_3CH_2CH_2Br + Mg$, CO_2 dry ether$/H_3O^+$

Official Ans. by NTA (4)

Sol. All gives propanoic acid as product but option 4 gives butanoic as product


Butanoic acid

9. The correct order of ionic radii for the ions, P^{3-}, S^{2-}, Ca^{2+}, K^+, Cl^- is :

(1) $P^{3-} > S^{2-} > Cl^- > K^+ > Ca^{2+}$
(2) $Cl^- > S^{2-} > P^{3-} > Ca^{2+} > K^+$
(3) $P^{3-} > S^{2-} > Cl^- > Ca^{2+} > K^+$
(4) $K^+ > Ca^{2+} > P^{3-} > S^{2-} > Cl^-$

Official Ans. by NTA (1)

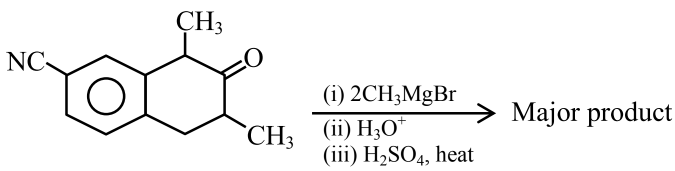
Sol. $P^{3-} > S^{2-} > Cl^- > K^+ > Ca^{2+}$

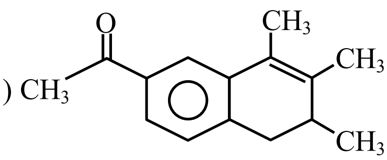
(Correct order of ionic radii)

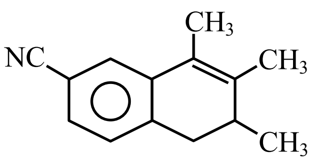
all the given species are isoelectronic species.

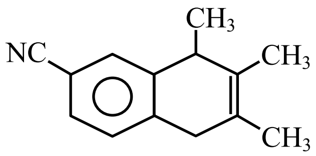
In isoelectronic species size increases with increase of negative charge and size decreases with increase in positive charge.

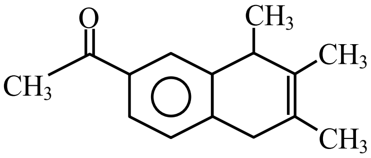
10. Which one of the following is the major product of the given reaction?



(1) 

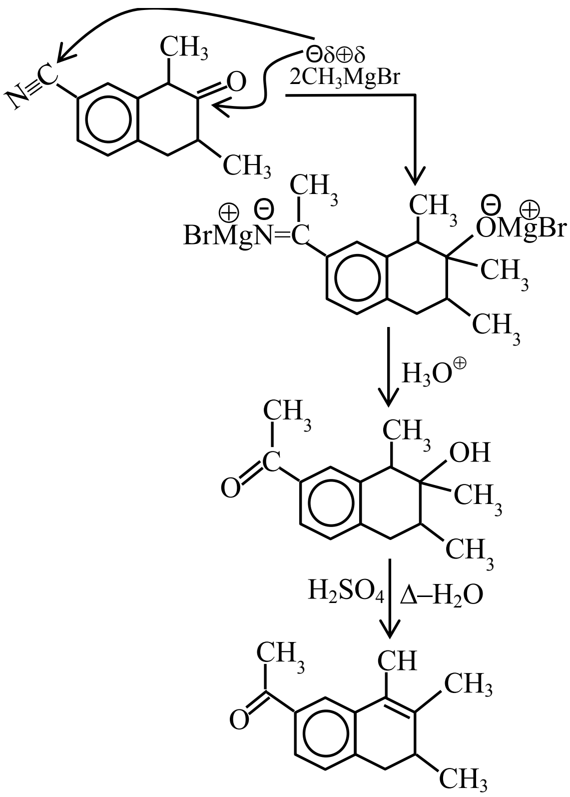
(2) 

(3) 

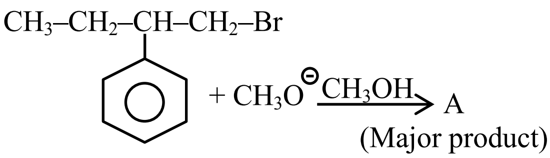
(4) 

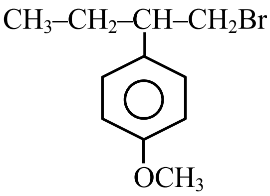
Official Ans. by NTA (1)

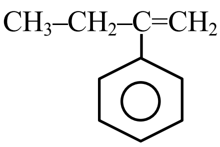
Sol.

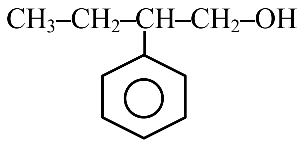


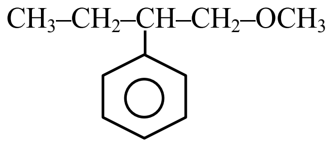
11. The major product (A) formed in the reaction given below is :



(1) 

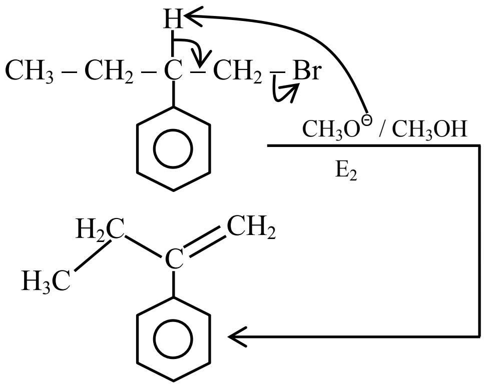
(2) 

(3) 

(4) 

Official Ans. by NTA (2)

Sol.



Major product (A)

12. Which one of the following is used to remove most of plutonium from spent nuclear fuel?

(1) ClF_3 (2) O_2F_2 (3) I_2O_5 (4) BrO_3

Official Ans. by NTA (2)

Sol. O_2F_2 oxidises plutonium to PuF_6 and the reaction is used in removing plutonium as PuF_6 from spent nuclear fuel.

13. Lyophilic sols are more stable than lyophobic sols because :

(1) there is a strong electrostatic repulsion between the negatively charged colloidal particles.

(2) the colloidal particles have positive charge.

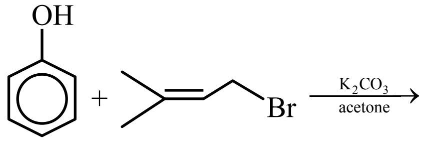
(3) the colloidal particles have no charge.

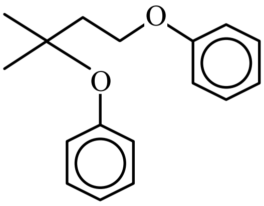
(4) the colloidal particles are solvated.

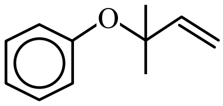
Official Ans. by NTA (4)

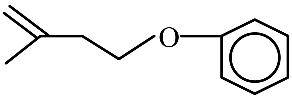
Sol. In the lyophilic colloids, the colloidal particles are extensively solvated.

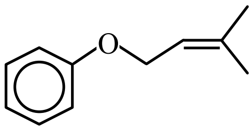
14. The major product of the following reaction, if it occurs by S_N2 mechanism is :



(1) 

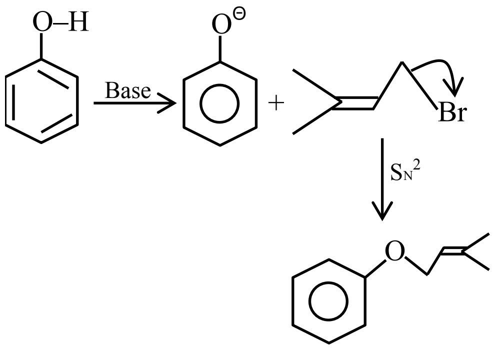
(2) 

(3) 

(4) 

Official Ans. by NTA (4)

Sol.



15. Potassium permanganate on heating at 513 K gives a product which is :

(1) paramagnetic and colourless

(2) diamagnetic and green

(3) diamagnetic and colourless

(4) paramagnetic and green

Official Ans. by NTA (4)

Sol. $$2KMnO_4 \xrightarrow[200°C]{\Delta} \underset{\text{Green}}{K_2MnO_4} + \underset{\text{Black}}{MnO_2} + O_2$$

In K_2MnO_4, manganese oxidation state is +6 and hence it has one unpaired e^-.

16. Which one of the following tests used for the identification of functional groups in organic compounds does not use copper reagent ?

(1) Barfoed's test

(2) Seliwanoff's test

(3) Benedict's test

(4) Biuret test for peptide bond

Official Ans. by NTA (2)

Sol. In Seliwanoff's reagent, Cu is not present.

In Barfoed, Biuret and in Benediet reagent Cu is present.

17. Hydrolysis of sucrose gives :

(1) α-D-(–)-Glucose and β-D-(–)-Fructose

(2) α-D-(+)-Glucose and α-D-(–)-Fructose

(3) α-D-(–)-Glucose and α-D-(+)-Fructose

(4) α-D-(+)-Glucose and β-D-(–)-Fructose

Official Ans. by NTA (4)

Sol. Sucrose is formed by α–D(+) . Glucose + β–D (–) Fructose.

we obtain these monomers on hydrolysis.

18. Match **List-I** with **List – II** :

List-I (Name of ore/mineral)	**List-II** (Chemical formula)
(a) Calamine	(i) Zns
(b) Malachite	(ii) $FeCO_3$
(c) Siderite	(iii) $ZnCO_3$
(d) Sphalerite	(iv) $CuCO_3 \cdot Cu(OH)_2$

Choose the **most appropriate** answer from the options given below :

(1) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

(2) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

(3) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)

(4) (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)

Official Ans. by NTA (1)

Sol. (Name of ore/mineral)

(a) Calamine $ZnCO_3$

(b) Malachite $CuCO_3.Cu(OH)_2$

(c) Siderite $FeCO_3$

(d) Sphalerite ZnS

19. Which one of the following is formed (mainly) when red phosphorus is heated in a sealed tube at 803 K ?

(1) White phosphorus

(2) Yellow phosphorus

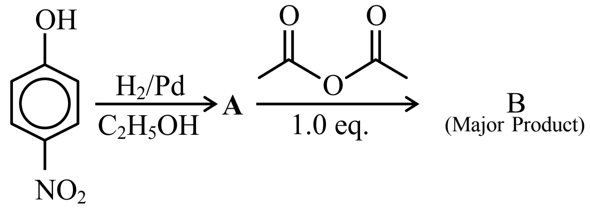
(3) β-Black phosphorus

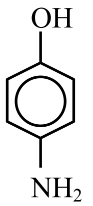
(4) α-Black phosphorus

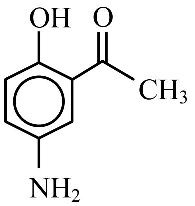
Official Ans. by NTA (4)

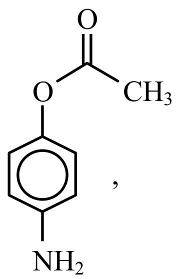
Sol. When red phosphorus is heated in a sealed tube at 803 K, α-black phosphorus is formed.

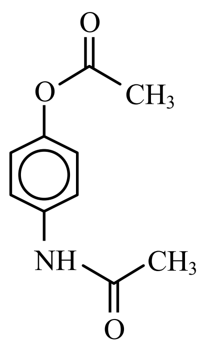
20. The correct structures of **A** and **B** formed in the following reactions are :

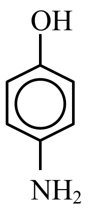


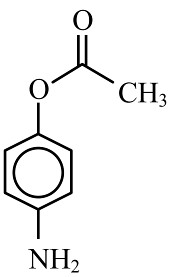
(1) **A** : 

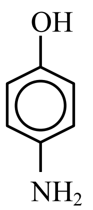
, **B** : 

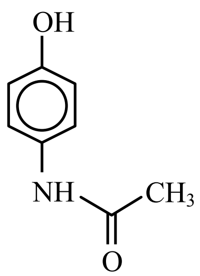
(2) **A** : 

, **B** : 

(3) **A** : 

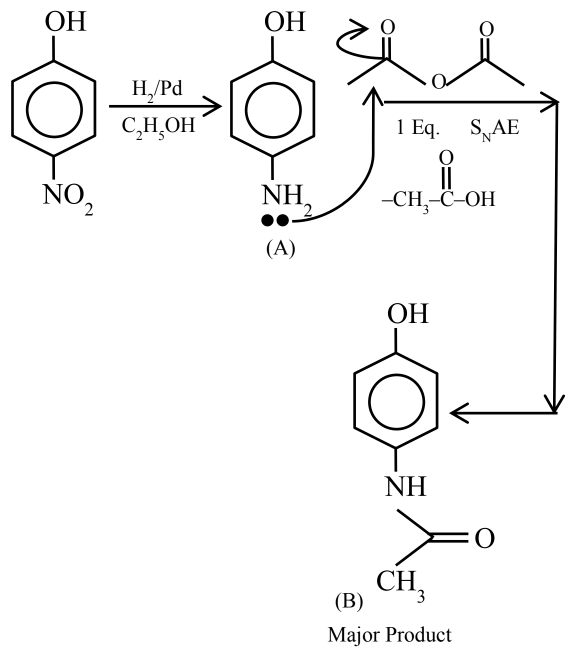
, **B** : 

(4) **A** : 

, **B** : 

Official Ans. by NTA (4)

Sol.



SECTION-B

1. The first order rate constant for the decomposition of $CaCO_3$ at 700 K is $6.36 \times 10^{-3} s^{-1}$ and activation energy is 209 kJ mol^{-1}. Its rate constant (in s^{-1}) at 600 K is $x \times 10^{-6}$. The value of x is _______. (Nearest integer)

[Given R = 8.31 J K^{-1} mol^{-1}; log $6.36 \times 10^{-3} = -2.19$, $10^{-4.79} = 1.62 \times 10^{-5}$]

Official Ans. by NTA (16)

Sol. $K_{700} = 6.36 \times 10^{-3} s^{-1}$;

$K_{600} = x \times 10^{-6} s^{-1}$

$E_a = 209$ kJ/mol

Applying ;

$$\log\left(\frac{K_{T_2}}{K_{T_1}}\right) = \frac{-E_a}{2.303R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

$$\log\left(\frac{K_{700}}{K_{600}}\right) = \frac{-E_a}{2.303R}\left(\frac{1}{700} - \frac{1}{600}\right)$$

$$\log\left(\frac{6.36 \times 10^{-3}}{K_{600}}\right) = \frac{+209 \times 1000}{2.303 \times 8.31}\left(\frac{100}{700 \times 600}\right)$$

$\log(6.36 \times 10^{-3}) - \log K_{600} = 2.6$

$\Rightarrow \log K_{600} = -2.19 - 2.6 = -4.79$

$\Rightarrow K_{600} = 10^{-4.79} = 1.62 \times 10^{-5}$

$= 16.2 \times 10^{-6}$

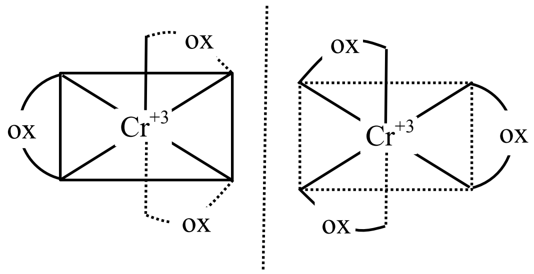
$= x \times 10^{-6}$

$\Rightarrow x = 16$

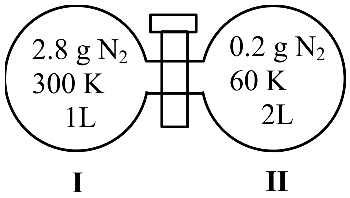
2. The number of optical isomers possible for $[Cr(C_2O_4)_3]^{3-}$ is ______.

Official Ans. by NTA (2)

Sol. The number of optical isomers for $[Cr(C_2O_4)_3]^{3-}$ is two.



3. Two flasks I and II shown below are connected by a valve of negligible volume.



When the valve is opened, the final pressure of the system in bar is $x \times 10^{-2}$. The value of x is ______.

(Integer answer)

[Assume–Ideal gas; 1 bar = 10^5 Pa; Molar mass of $N_2 = 28.0$ g mol^{-1} ; R = 8.31 J $mol^{-1}K^{-1}$]

Official Ans. by NTA (84)

Sol. Applying ; $(n_I + n_{II})_{initial} = (n_I + n_{II})_{final}$

$\Rightarrow$ Assuming the system attains a final temperature of T (such that 300 < T < 60)

$$\Rightarrow \begin{pmatrix}\text{Heat lost by} \\ N_2 \text{ of container} \\ \text{I}\end{pmatrix} = \begin{pmatrix}\text{Heat gained by} \\ N_2 \text{ of container} \\ \text{II}\end{pmatrix}$$

$\Rightarrow n_I C_m(300–T) = n_{II} C_m(T–60)$

$$\Rightarrow \left(\frac{2.8}{28}\right)(300 - T) = \frac{0.2}{28}(T - 60)$$

$\Rightarrow 14(300–T) = T–60$

$$\Rightarrow \frac{(14 \times 300 + 60)}{15} = T$$

$\Rightarrow$ T = 284 K (final temperature)

$\Rightarrow$ If the final pressure = P

$$\Rightarrow (n_I + n_{II})_{final} = \left(\frac{3.0}{28}\right)$$

$$\Rightarrow \frac{P}{RT}(V_I + V_{II}) = \frac{3.0gm}{28gm/mol}$$

$$P = \left(\frac{3}{28}mol\right) \times 8.31\frac{J}{mol - K} \times \frac{284K}{3 \times 10^{-3}m^3} \times 10^{-5}\frac{bar}{Pa}$$

$\Rightarrow$ 0.84287 bar

$\Rightarrow 84.28 \times 10^{-2}$ bar

$\Rightarrow$ 84

4. 100 g of propane is completely reacted with 1000 g of oxygen. The mole fraction of carbon dioxide in the resulting mixture is $x \times 10^{-2}$. The value of x is _______. (Nearest integer)

[Atomic weight : H = 1.008; C = 12.00; O = 16.00]

Official Ans. by NTA (19)

Sol. $C_3H_{8(g)} + 5O_{2(g)} \longrightarrow 3CO_{2(g)} + 4H_2O_{(\ell)}$

t = 0 2.27 mole 31.25 mol

t = ∞ 0 19.9 mol 6.81 mol 9.08 mol

mole fraction of CO_2 in the final reaction mixture (heterogenous)

$$X_{CO_2} = \frac{6.81}{19.9 + 6.81 + 9.08}$$

$= 0.1902 = 19.02 \times 10^{-2}$

$\Rightarrow$ 19

5. 40 g of glucose (Molar mass = 180) is mixed with 200 mL of water. The freezing point of solution is _____ K. (Nearest integer)

[Given : $K_f = 1.86$ K kg mol^{-1} ; Density of water = 1.00 g cm^{-3}; Freezing point of water = 273.15 K]

Official Ans. by NTA (271)

Sol. $$\text{molality} = \frac{\left(\frac{40}{180}\right)mol}{0.2Kg} = \left(\frac{10}{9}\right)molal$$

$$\Rightarrow \Delta T_f = T_f - T_f' = 1.86 \times \frac{10}{9}$$

$$\Rightarrow T_f' = 273.15 - 1.86 \times \frac{10}{9}$$

= 271.08 K

≃ 271 K (nearest-integer)

6. The resistance of a conductivity cell with cell constant 1.14 cm^{-1}, containing 0.001 M KCl at 298 K is 1500 Ω. The molar conductivity of 0.001 M KCl solution at 298 K in S cm^2 mol^{-1} is_______. (Integer answer)

Official Ans. by NTA (760)

Sol. $K = \frac{1}{R} \times \ell/A = \left(\left(\frac{1}{1500}\right) \times 1.14\right) S\ cm^{-1}$

$$\Rightarrow \Lambda_m = 1000 \times \frac{\left(\frac{1.14}{1500}\right)}{0.001} S\ cm^2 mol^{-1}$$

= 760 S cm^2 mol^{-1}

⇒ 760

7. The number of photons emitted by a monochromatic (single frequency) infrared range finder of power 1 mW and wavelength of 1000 nm, in 0.1 second is x × 10^{13}. The value of x is _____. (Nearest integer)

(h = 6.63 × 10^{-34} Js, c = 3.00 × 10^8 ms^{-1})

Official Ans. by NTA (50)

Sol. Energy emitted in 0.1 sec.

$$= 0.1\ sec. \times 10^{-3} \frac{J}{s}$$

= 10^{-4} J

If 'n' photons of λ = 1000 nm are emitted,

then ; $10^{-4} = n \times \frac{hc}{\lambda}$

$$\Rightarrow 10^{-4} = \frac{n \times 6.63 \times 10^{-34} \times 3 \times 10^8}{1000 \times 10^{-9}}$$

⇒ n = 5.02 × 10^{14} = 50.2 × 10^{13}

⇒ 50 (nearest integer)

8. When 5.1 g of solid NH_4HS is introduced into a two litre evacuated flask at 27°C, 20% of the solid decomposes into gaseous ammonia and hydrogen sulphide. The K_p for the reaction at 27°C is x × 10^{-2}. The value of x is _______. (Integer answer)

[Given R = 0.082 L atm K^{-1} mol^{-1}]

Official Ans. by NTA (6)

Sol. moles of NH_4HS initially taken = $\frac{5.1g}{51g/mol}$

= 0.1 mol

volume of vessel = 2ℓ

$$NH_4HS_{(s)} \rightleftharpoons NH_{3(g)} + H_2S_{(g)}$$

t = 0	0.1 mol		
t = ∞	0.1(1–0.2)	0.1×0.2	0.1×0.2

⇒ partial pressure of each component

$$P = \frac{nRT}{V} = \frac{0.1 \times 0.2 \times 0.082 \times 300}{2}$$

= 0.246 atm

$$\Rightarrow k_P = P_{NH_3} \times P_{H_2S} = (0.246)^2 = 0.060516$$

= 6.05 × 10^{-2}

⇒ 6

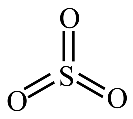
9. The number of species having non–pyramidal shape among the following is_______.

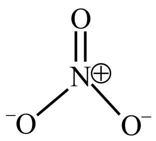
(A) SO_3 (B) NO_3^-

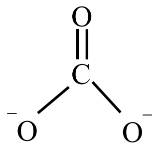
(C) PCl_3 (D) CO_3^{2-}

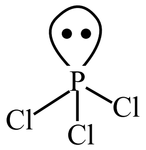
Official Ans. by NTA (3)

Sol.


Trigonal planar


Trigonal planar


Trigonal planar


Pyramidal

Hence non-pyramidal species are SO_3, NO_3^- and CO_3^{2-}.

10. Data given for the following reaction is as follows:

$FeO_{(s)} + C_{(graphite)} \longrightarrow Fe_{(s)} + CO_{(g)}$

Substance	$\Delta H°$ (kJ mol^{-1})	$\Delta S°$ (J mol^{-1}K^{-1})
$FeO_{(s)}$	–266.3	57.49
$C_{(graphite)}$	0	5.74
$Fe_{(s)}$	0	27.28
$CO_{(g)}$	–110.5	197.6

The minimum temperature in K at which the reaction becomes spontaneous is _______.

(Integer answer)

Official Ans. by NTA (964)

Sol. $T_{min} = \left(\dfrac{\Delta^0 H}{\Delta^0 S}\right)$

$$\Delta^0 H_{rxn} = \left[\Delta_f^0 H(Fe) + \Delta_f^0 H(CO)\right] -$$

$$= \left[\Delta_f^0 H(FeO) + \Delta_f^0 H(C_{(graphite)})\right]$$

= [0 – 110.5] – [–266.3+0]

= 155.8 kJ/mol

$$\Delta^0 S_{rxn} = \left[\Delta^0 S(Fe) + \Delta^0 S(CO)\right] -$$

$$\left[\Delta^0 S(FeO) + \Delta^0 S(C_{(graphite)})\right]$$

= [27.28 + 197.6] – [57.49 + 5.74]

= 161.65 J/mol-K

$$T_{min} = \frac{155.8 \times 10^3 J / mol}{161.65 J / mol - K} = 963.8K$$

≃ 964 k (nearest integer)

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Friday 27th August, 2021) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The angle between the straight lines, whose direction cosines are given by the equations $2l + 2m - n = 0$ and $mn + nl + lm = 0$, is :

(1) $\frac{\pi}{2}$ (2) $\pi - \cos^{-1}\left(\frac{4}{9}\right)$

(3) $\cos^{-1}\left(\frac{8}{9}\right)$ (4) $\frac{\pi}{3}$

Official Ans. by NTA (1)

Sol. $n = 2(\ell + m)$

$\ell m + n(\ell + m) = 0$

$\ell m + 2(\ell + m)^2 = 0$

$2\ell^2 + 2m^2 + 5m\ell = 0$

$2\left(\frac{\ell}{m}\right)^2 + 2 + 5\left(\frac{\ell}{m}\right) = 0.$

$2t^2 + 5t + 2 = 0$

$(t + 2)(2t + 1) = 0$

$\Rightarrow t = -2; -\frac{1}{2}$

(i) $\frac{\ell}{m} = -2$

$\frac{n}{m} = -2$

$(-2m, m, -2m)$

$(-2, 1, -2)$

(ii) $\frac{\ell}{m} = -\frac{1}{2}$

$n = -2\ell$

$(\ell, -2\ell, -2\ell)$

$(1, -2, -2)$

$\cos\theta = \frac{-2-2+4}{\sqrt{9}\ \sqrt{9}} = 0 \Rightarrow 0 = \frac{\pi}{2}$

2. Let $A = \begin{pmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{pmatrix}$, where $[t]$ denotes the greatest integer less than or equal to t. If det(A) = 192, then the set of values of x is the interval:

(1) [68, 69) (2) [62, 63)

(3) [65, 66) (4) [60, 61)

Official Ans. by NTA (2)

Sol. $\begin{vmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{vmatrix} = 192$

$R_1 \to R_1 - R_3$ & $R_2 \to R_2 - R_3$

$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ [x] & [x]+2 & [x]+4 \end{bmatrix} = 192$

$2[x] + 6 + [x] = 192 \Rightarrow [x] = 62$

3. Let M and m respectively be the maximum and minimum values of the function $f(x) = \tan^{-1}(\sin x + \cos x)$ in $\left[0, \frac{\pi}{2}\right]$, Then the value of tan(M – m) is equal to:

(1) $2+\sqrt{3}$ (2) $2-\sqrt{3}$

(3) $3+2\sqrt{2}$ (4) $3-2\sqrt{2}$

Official Ans. by NTA (4)

Sol. Let $g(x) = \sin x + \cos x = \sqrt{2}\ \sin\left(x + \frac{\pi}{4}\right)$

$g(x) \in [1, \sqrt{2}]$ for $x \in [0, \pi/2]$

$f(x) = \tan^{-1}(\sin x + \cos x) \in \left[\frac{\pi}{4}, \tan^{-1}\sqrt{2}\right]$

$\tan\left(\tan^{-1}\sqrt{2} - \frac{\pi}{4}\right) = \frac{\sqrt{2}-1}{1+\sqrt{2}} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = 3 - 2\sqrt{2}$

4. Each of the persons A and B independently tosses three fair coins. The probability that both of them get the same number of heads is :

(1) $\frac{1}{8}$ (2) $\frac{5}{8}$ (3) $\frac{5}{16}$ (4) 1

Official Ans. by NTA (3)

Sol. C – I '0' Head

T T T $\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3 = \frac{1}{64}$

C – II '1' head

H T T $\left(\frac{3}{8}\right)\left(\frac{3}{8}\right) = \frac{9}{64}$

C – III '2' Head

H H T $\left(\frac{3}{8}\right)\left(\frac{3}{8}\right) = \frac{9}{64}$

C–IV '3' Heads

H H H $\left(\frac{1}{8}\right)\left(\frac{1}{8}\right) = \frac{1}{64}$

Total probability = $\frac{5}{16}$.

5. A differential equation representing the family of parabolas with axis parallel to y-axis and whose length of latus rectum is the distance of the point (2, –3) form the line 3x + 4y = 5, is given by :

(1) $10\frac{d^2y}{dx^2} = 11$ (2) $11\frac{d^2x}{dy^2} = 10$

(3) $10\frac{d^2x}{dy^2} = 11$ (4) $11\frac{d^2y}{dx^2} = 10$

Official Ans. by NTA (4)

Sol. α. R = $\frac{|3(2)+4(-3)-5|}{5} = \frac{11}{5}$

$(x-h)^2 = \frac{11}{5}(y-k)$

differentiate w.r.t 'x' : –

$2(x-h) = \frac{11}{5}\frac{dy}{dx}$

again differentiate

$2 = \frac{11}{5}\frac{d^2y}{dx^2}$

$\frac{11d^2y}{dx^2} = 10$.

6. If two tangents drawn from a point P to the parabola $y^2 = 16(x – 3)$ are at right angles, then the locus of point P is :

(1) x + 3 = 0 (2) x + 1 = 0

(3) x + 2 = 0 (4) x + 4 = 0

Official Ans. by NTA (2)

Sol. Locus is directrix of parabola

x – 3 + 4 = 0 ⇒ x + 1 = 0.

7. The equation of the plane passing through the line of intersection of the planes $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=1$ and $\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0$ and parallel to the x-axis is:

(1) $\vec{r}.\left(\hat{j}-3\hat{k}\right)+6=0$ (2) $\vec{r}.\left(\hat{i}+3\hat{k}\right)+6=0$

(3) $\vec{r}.\left(\hat{i}-3\hat{k}\right)+6=0$ (4) $\vec{r}.\left(\hat{j}-3\hat{k}\right)-6=0$

Official Ans. by NTA (1)

Sol. Equation of planes are

$\vec{r}\,.\left(\hat{i}+\hat{j}+\hat{k}\right)-1=0 \Rightarrow x + y + z – 1 = 0$

and $\vec{r}\,.\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0 \Rightarrow 2x + 3y – z + 4 = 0$

equation of planes through line of intersection of these planes is :–

(x + y + z – 1) + λ (2x + 3y – z + 4) = 0

⇒ (1 + 2 λ) x + (1 + 3 λ) y + (1– λ) z – 1 + 4 λ = 0

But this plane is parallel to x–axis whose direction are (1, 0, 0)

∴ (1 + 2 λ)1 + (1 + 3 λ) 0 + (1 – λ) 0 = 0

$\lambda = -\frac{1}{2}$

∴ Required plane is

$0\,x + \left(1-\frac{3}{2}\right)y + \left(1+\frac{1}{2}\right)z - 1 + 4\left(\frac{-1}{2}\right) = 0$

$\Rightarrow \frac{-y}{2}+\frac{3}{2}z-3=0$

⇒ y – 3z + 6 = 0

$\Rightarrow \boxed{\vec{r}.\left(\hat{j}-3\hat{k}\right)+6=0}$ Ans.

8. If the solution curve of the differential equation $(2x - 10y^3)\,dy + y\,dx = 0$, passes through the points $(0, 1)$ and $(2, \beta)$, then β is a root of the equation:

(1) $y^5 - 2y - 2 = 0$ (2) $2y^5 - 2y - 1 = 0$

(3) $2y^5 - y^2 - 2 = 0$ (4) $y^5 - y^2 - 1 = 0$

Official Ans. by NTA (4)

Sol. $(2x - 10y^3)\,dy + y\,dx = 0$

$$\Rightarrow \frac{dx}{dy} + \left(\frac{2}{y}\right)x = 10y^2$$

I. F. $= e^{\int \frac{2}{y}dy} = e^{2\ell n(y)} = y^2$

Solution of D.E. is

$$\therefore \quad x.y = \int (10y^2)\,y^2.dy$$

$$xy^2 = \frac{10y^5}{5} + C \Rightarrow xy^2 = 2y^5 + C$$

It passes through $(0, 1) \rightarrow 0 = 2 + C \Rightarrow C = -2$

$\therefore$ Curve is $\boxed{xy^2 = 2y^5 - 2}$

Now, it passes through $(2,\beta)$

$2\beta^2 = 2\beta^5 - 2 \Rightarrow \beta^5 - \beta^2 - 1 = 0$

$\therefore$ β is root of an equation $\boxed{y^5 - y^2 - 1 = 0}$ Ans.

9. Let $A(a, 0)$, $B(b, 2b + 1)$ and $C(0, b)$, $b \neq 0$, $|b| \neq 1$, be points such that the area of triangle ABC is 1 sq. unit, then the sum of all possible values of a is :

(1) $\frac{-2b}{b+1}$ (2) $\frac{2b}{b+1}$

(3) $\frac{2b^2}{b+1}$ (4) $\frac{-2b^2}{b+1}$

Official Ans. by NTA (4)

Sol. $$\left|\frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ b & 2b+1 & 1 \\ 0 & b & 1 \end{vmatrix}\right| = 1$$

$$\Rightarrow \begin{vmatrix} a & 0 & 1 \\ b & 2b+1 & 1 \\ 0 & b & 1 \end{vmatrix} = \pm 2$$

$\Rightarrow a(2b + 1 - b) - 0 + 1(b^2 - 0) = \pm 2$

$$\Rightarrow a = \frac{\pm 2 - b^2}{b+1}$$

$$\therefore \quad a = \frac{2 - b^2}{b+1} \text{ and } a = \frac{-2 - b^2}{b+1}$$

sum of possible values of 'a' is

$$= \frac{-2b^2}{a+1} \text{ Ans.}$$

10. Let $[\lambda]$ be the greatest integer less than or equal to λ. The set of all values of λ for which the system of linear equations $x + y + z = 4$, $3x + 2y + 5z = 3$, $9x + 4y + (28 + [\lambda])z = [\lambda]$ has a solution is:

(1) **R**

(2) $(-\infty, -9) \cup (-9, \infty)$

(3) $[-9, -8)$

(4) $(-\infty, -9) \cup [-8, \infty)$

Official Ans. by NTA (1)

Sol. $$D = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & 5 \\ 9 & 4 & 28 + [\lambda] \end{vmatrix} = -24 - [\lambda] + 15 = -[\lambda] - 9$$

if $[\lambda] + 9 \neq 0$ then unique solution

if $[\lambda] + 9 = 0$ then $D_1 = D_2 = D_3 = 0$

so infinite solutions

Hence λ can be any red number.

11. The set of all values of $k > -1$, for which the equation $(3x^2 + 4x + 3)^2 - (k + 1)(3x^2 + 4x + 3)(3x^2 + 4x + 2) + k(3x^2 + 4x + 2)^2 = 0$ has real roots, is :

(1) $\left(1, \frac{5}{2}\right]$ (2) $[2, 3)$

(3) $\left[-\frac{1}{2}, 1\right)$ (4) $\left(\frac{1}{2}, \frac{3}{2}\right] - \{1\}$

Official Ans. by NTA (1)

Sol. $(3x^2 + 4x + 3)^2 - (k + 1)(3x^2 + 4x + 3)(3x^2 + 4x + 2) + k(3x^2 + 4x + 2)^2 = 0$

Let $3x^2 + 4x + 3 = a$

and $3x^2 + 4x + 2 = b \Rightarrow b = a - 1$

Given equation becomes

$\Rightarrow a^2 - (k + 1)ab + kb^2 = 0$

$\Rightarrow a(a - kb) - b(a - kb) = 0$

$\Rightarrow (a - kb)(a - b) = 0 \Rightarrow a = kb$ or $a = b$ (reject)

$\because a = kb$

$\Rightarrow 3x^2 + 4x + 3 = k(3x^2 + 4x + 2)$

$\Rightarrow 3(k - 1)x^2 + 4(k - 1)x + (2k - 3) = 0$

for real roots

$D \geq 0$

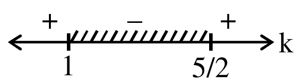
$\Rightarrow 16(k - 1)^2 - 4(3(k - 1))(2k - 3) \geq 0$

$\Rightarrow 4(k - 1)\{4(k - 1) - 3(2k - 3)\} \geq 0$

$\Rightarrow 4(k - 1)\{-2k + 5\} \geq 0$

$\Rightarrow -4(k - 1)\{2k - 5\} \geq 0$

$\Rightarrow (k - 1)(2k - 5) \leq 0$



$\therefore \quad k \in \left[1, \frac{5}{2}\right]$

$\because \quad k \neq 1$

$\therefore \quad k \in \left(1, \frac{5}{2}\right]$ Ans.

12. A box open from top is made from a rectangular sheet of dimension a × b by cutting squares each of side x from each of the four corners and folding up the flaps. If the volume of the box is maximum, then x is equal to :

(1) $\frac{a+b-\sqrt{a^2+b^2-ab}}{12}$

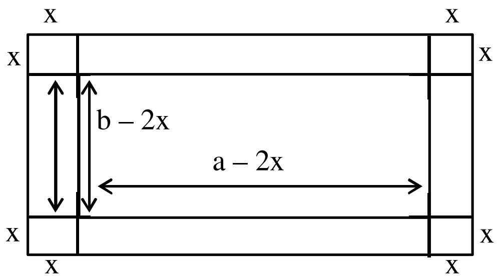
(2) $\frac{a+b-\sqrt{a^2+b^2+ab}}{6}$

(3) $\frac{a+b-\sqrt{a^2+b^2-ab}}{6}$

(4) $\frac{a+b+\sqrt{a^2+b^2-ab}}{6}$

Official Ans. by NTA (3)

Sol.



$V = \ell . b. h \quad = (a-2x)(b-2x)\, x$

$\Rightarrow V(x) = (2x-a)(2x-b)\,x$

$\Rightarrow V(x) = 4x^3 - 2(a+b)x^2 + abx$

$\Rightarrow \frac{d}{dx} v(x) = 12x^2 - 4(a+b)x + ab$

$\frac{d}{dx}(v(x)) = 0 \quad \Rightarrow 12x^2 - 4(a+b)x + ab = 0 <^{\alpha}_{\beta}$

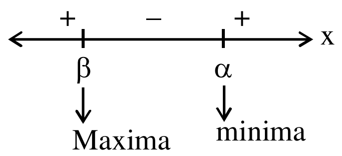
$\Rightarrow x = \frac{4(a+b) \pm \sqrt{16(a+b)^2 - 48ab}}{2(12)}$

$= \frac{(a+b) \pm \sqrt{a^2+b^2-ab}}{6}$

Let $x = \alpha = \frac{(a+b)+\sqrt{a^2+b^2-ab}}{6}$

$\beta = \frac{(a+b)-\sqrt{a^2+b^2-ab}}{6}$

Now, $12(x-\alpha)(x-\beta) = 0$



$\therefore \; x = \beta$

$= \frac{a+b-\sqrt{a^2+b^2-ab}}{b}$

13. The Boolean expression $(p \wedge q) \Rightarrow ((r \wedge q) \wedge p)$ is equivalent to :

(1) $(p \wedge q) \Rightarrow (r \wedge q)$ (2) $(q \wedge r) \Rightarrow (p \wedge q)$

(3) $(p \wedge q) \Rightarrow (r \vee q)$ (4) $(p \wedge r) \Rightarrow (p \wedge q)$

Official Ans. by NTA (1)

Sol. $(p \wedge q) \Rightarrow ((r \wedge q) \wedge p)$

$\sim (p \wedge q) \vee ((r \wedge q) \wedge p)$

$\sim (p \wedge q) \vee ((r \wedge p) \wedge (p \wedge q)$

$\Rightarrow [\sim (p \wedge q) \vee (p \wedge q)] \wedge (\sim (p \wedge q) \vee (r \wedge p))$

$\Rightarrow t \wedge [\sim (p \wedge q) \vee (r \wedge p)]$

$\Rightarrow \sim (p \wedge q) \vee (r \wedge p)$

$\Rightarrow (p \wedge q) \Rightarrow (r \wedge p)$

Aliter :

given statement says

" if p and q both happen then

p and q and r will happen"

it Simply implies

" If p and q both happen then

'r' too will happen "

i.e.

" if p and q both happen then r and p too will happen

i.e.

$(p \wedge q) \Rightarrow (r \wedge p)$

14. Let $\mathbb{Z}$ be the set of all integers,

$A = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : (x-2)^2 + y^2 \le 4\}$,

$B = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x^2 + y^2 \le 4\}$ and

$C = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : (x-2)^2 + (y-2)^2 \le 4\}$

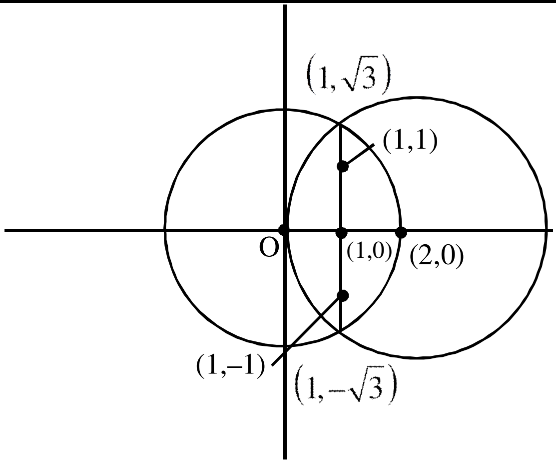
If the total number of relation from $A \cap B$ to $A \cap C$ is 2^p, then the value of p is :

(1) 16 (2) 25

(3) 49 (4) 9

Official Ans. by NTA (2)

Sol.



$(x-2)^2 + y^2 \le 4$

$x^2 + y^2 \le 4$

No. of points common in C_1 & C_2 is 5.

(0, 0) , (1, 0) , (2, 0) , (1, 1), (1, –1)

Similarly in C_2 & C_3 is 5.

No. of relations = $2^{5\times5} = 2^{25}$.

15. The area of the region bounded by the parabola $(y-2)^2 = (x-1)$, the tangent to it at the point whose ordinate is 3 and the x-axis is :

(1) 9 (2) 10 (3) 4 (4) 6

Official Ans. by NTA (1)

Sol. $y = 3 \Rightarrow x = 2$

Point is (2,3)

Diff. w.r.t x

$2(y-2)y' = 1$

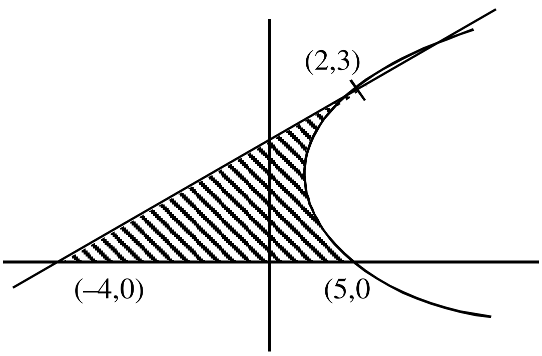
$\Rightarrow y' = \dfrac{1}{2(y-2)}$

$\Rightarrow y'_{(2,3)} = \dfrac{1}{2}$

$\Rightarrow \dfrac{y-3}{x-2} = \dfrac{1}{2} \Rightarrow x - 2y + 4 = 0$

Area $= \int_0^3 \left((y-2)^2 + 1 - (2y-4)\right) dy$

= 9 sq. units



16. If $y(x) = \cot^{-1}\left(\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$, $x \in \left(\dfrac{\pi}{2}, \pi\right)$, then $\dfrac{dy}{dx}$ at $x = \dfrac{5\pi}{6}$ is:

(1) $-\dfrac{1}{2}$ (2) –1 (3) $\dfrac{1}{2}$ (4) 0

Official Ans. by NTA (1)

Sol. $y(x) = \cot^{-1}\left[\dfrac{\cos\frac{x}{2}+\sin\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}-\sin\frac{x}{2}+\cos\frac{x}{2}}\right]$

$y(x) = \cot^{-1}\left(\tan\dfrac{x}{2}\right) = \dfrac{\pi}{2} - \dfrac{x}{2}$

$y'(x) = \dfrac{-1}{2}$

17. Two poles, AB of length a metres and CD of length a + b (b ≠ a) metres are erected at the same horizontal level with bases at B and D. If BD = x and $\tan\underline{|ACB} = \dfrac{1}{2}$, then:

(1) $x^2 + 2(a + 2b)x - b(a + b) = 0$

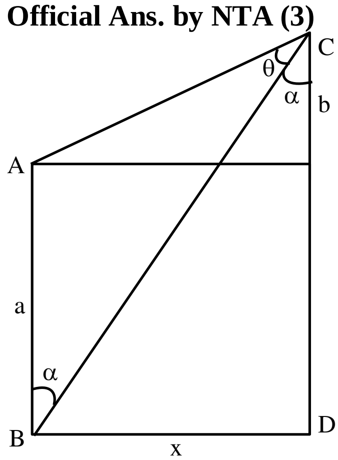
(2) $x^2 + 2(a + 2b)x + a(a + b) = 0$

(3) $x^2 - 2ax + b(a + b) = 0$

(4) $x^2 - 2ax + a(a + b) = 0$

Official Ans. by NTA (3)

Sol.



$\tan\theta = \dfrac{1}{2}$

$\tan(\theta + \alpha) = \dfrac{x}{b}$, $\tan\alpha = \dfrac{x}{a+b}$

$\Rightarrow \dfrac{1}{2} + \dfrac{x}{a+b}$

$\Rightarrow \dfrac{\frac{1}{2}+\frac{x}{a+b}}{1-\frac{1}{2}\times\frac{x}{a+b}} = \dfrac{x}{b}$.

$\Rightarrow x^2 - 2ax + ab + b^2 = 0$

18. If $0 < x < 1$ and $y = \frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \ldots$, then the value of e^{1+y} at $x = \frac{1}{2}$ is:

(1) $\frac{1}{2}e^2$ (2) $2e$

(3) $\frac{1}{2}\sqrt{e}$ (4) $2e^2$

Official Ans. by NTA (1)

Sol. $y = \left(1-\frac{1}{2}\right)x^2 + \left(1-\frac{1}{3}\right)x^3 + \ldots..$

$= (x^2 + x^3 + x^4 + \ldots\ldots.) - \left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots.\right)$

$= \frac{x^2}{1-x} + x - \left(x + \frac{x^2}{2} + \frac{x^2}{3} + \ldots\right)$

$= \frac{x}{1-x} + \ell n(1-x)$

$x = \frac{1}{2} \Rightarrow y = 1 - \ell n2$

$e^{1+y} = e^{1+1-\ell n2}$

$= e^{2-\ell n2} = \frac{e^2}{2}$

19. The value of the integral $\int_0^1 \frac{\sqrt{x}\,dx}{(1+x)(1+3x)(3+x)}$ is:

(1) $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{2}\right)$ (2) $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{6}\right)$

(3) $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{6}\right)$ (4) $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{2}\right)$

Official Ans. by NTA (1)

Sol. $I = \int_0^1 \frac{\sqrt{x}}{(1+x)(1+3x)(3+x)}dx$

Let $x = t^2 \Rightarrow dx = 2t.dt$

$I = \int_0^1 \frac{t(2t)}{(t^2+1)(1+3t^2)(3+t^2)}dt$

$I = \int_0^1 \frac{(3t^2+1)-(t^2+1)}{(3t^2+1)(t^2+1)(3+t^2)}dt$

$I = \int_0^1 \frac{dt}{(t^2+1)(3+t^2)} - \int_0^1 \frac{dt}{(1+3t^2)(3+t^2)}$

$= \frac{1}{2}\int_0^1 \frac{(3+t^2)-(t^2+1)}{(t^2+1)(3+t^2)}dt + \frac{1}{8}\int_0^1 \frac{(1+3t^2)-3(3+t^2)}{(1+3t^2)(3+t^2)}dt$

$= \frac{1}{2}\int_0^1 \frac{dt}{1+t^2} - \frac{1}{2}\int_0^1 \frac{dt}{t^2+3} + \frac{1}{8}\int_0^1 \frac{dt}{t^2+3} - \frac{3}{8}\int_0^1 \frac{dt}{(1+3t^2)}$

$= \frac{1}{2}\int_0^1 \frac{dt}{t^2+1} - \frac{3}{8}\int_0^1 \frac{dt}{t^2+3} - \frac{3}{8}\int_0^1 \frac{dt}{1+3t^2}$

$= \frac{1}{2}\left(\tan^{-1}(t)\right)_0^1 - \frac{3}{8\sqrt{3}}\left(\tan^{-1}\left(\frac{t}{\sqrt{3}}\right)\right)_0^1$

$-\frac{3}{8\sqrt{3}}\left(\tan^{-1}(\sqrt{3}t)\right)_0^1$

$= \frac{1}{2}\left(\frac{\pi}{4}\right) - \frac{\sqrt{3}}{8}\left(\frac{\pi}{6}\right) - \frac{\sqrt{3}}{8}\left(\frac{\pi}{3}\right)$

$= \frac{\pi}{8} - \frac{\sqrt{3}}{16}\pi$

$= \frac{\pi}{8}\left(1-\frac{\sqrt{3}}{2}\right)$

20. If $\lim_{x\to\infty}\left(\sqrt{x^2-x+1}-ax\right) = b$, then the ordered pair (a, b) is:

(1) $\left(1, \frac{1}{2}\right)$ (2) $\left(1, -\frac{1}{2}\right)$

(3) $\left(-1, \frac{1}{2}\right)$ (4) $\left(-1, -\frac{1}{2}\right)$

Official Ans. by NTA (2)]

Sol. (2)

$\lim_{x\to\infty}\left(\sqrt{x^2-x+1}\right)-ax=b \qquad (\infty-\infty)$

$\Rightarrow a>0$

Now, $\lim_{x\to\infty}\frac{(x^2-x+1-a^2x^2)}{\sqrt{x^2-x+1}+ax}=b$

$\Rightarrow \lim_{x\to\infty}\frac{(1-a^2)x^2-x+1}{\sqrt{x^2-x+1}+ax}=b$

$\Rightarrow \lim_{x\to\infty}\frac{(1-a^2)x^2-x+1}{x\left(\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a\right)}=b$

$\Rightarrow 1-a^2=0 \Rightarrow a=1$

Now, $\lim_{x\to\infty}\frac{-x+1}{x\left(\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a\right)}=b$

$\Rightarrow \frac{-1}{1+a}=b \Rightarrow b=-\frac{1}{2}$

$(a,b)=\left(1,-\frac{1}{2}\right)$

SECTION-B

1. Let S be the sum of all solutions (in radians) of the equation $\sin^4\theta+\cos^4\theta-\sin\theta\cos\theta=0$ in $[0,4\pi]$. Then $\frac{8S}{\pi}$ is equal to ________.

Official Ans. by NTA (56)

Sol. Given equation

$\sin^4\theta+\cos^4\theta-\sin\theta\cos\theta=0$

$\Rightarrow 1-\sin^2\theta\cos^2\theta-\sin\theta\cos\theta=0$

$\Rightarrow 2-(\sin 2\theta)^2-\sin 2\theta=0$

$\Rightarrow (\sin 2\theta)^2+(\sin 2\theta)-2=0$

$\Rightarrow (\sin 2\theta+2)(\sin 2\theta-1)=0$

$\Rightarrow \sin 2\theta=1$ or $\underbrace{\boxed{\sin 2\theta=-2}}_{\text{(not possible)}}$

$\Rightarrow 2\theta=\frac{\pi}{2},\frac{5\pi}{2},\frac{9\pi}{2},\frac{13\pi}{2}$

$\Rightarrow \theta=\frac{\pi}{4},\frac{5\pi}{4},\frac{9\pi}{4},\frac{13\pi}{4}$

$\Rightarrow S=\frac{\pi}{4}+\frac{5\pi}{4}+\frac{9\pi}{4}+\frac{13\pi}{4}=7\pi$

$\Rightarrow \frac{8S}{\pi}=\frac{8\times 7\pi}{\pi}=56.00$

2. Let S be the mirror image of the point Q(1, 3, 4) with respect to the plane $2x-y+z+3=0$ and let R (3, 5, γ) be a point of this plane. Then the square of the length of the line segment SR is ________.

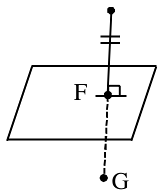
Official Ans. by NTA (72)

Sol. Since R $(3,5,\gamma)$ lies on the plane $2x-y+z+3=0$.

Therefore, $6-5+\gamma+3=0$

$\Rightarrow \gamma=-4$

Now,



dr's of line QS are 2, –1,1

equation of line QS is

$\frac{x-1}{2}=\frac{y-3}{-1}=\frac{z-4}{1}=\lambda$ (say)

$\Rightarrow F(2\lambda+1,-\lambda+3,\lambda+4)$

F lies in the plane

$\Rightarrow 2(2\lambda+1)-(-\lambda+3)+(\lambda+4)+3=0$

$\Rightarrow 4\lambda+2+\lambda-3+\lambda+7=0$

$\Rightarrow 6\lambda+6=0 \Rightarrow \lambda=-1.$

$\Rightarrow F(-1,4,3)$

Since, F is mid-point of QS.

Therefore, co-ordinated of S are (–3,5,2).

So, $SR=\sqrt{36+0+36}=\sqrt{72}$

$SR^2=72.$

3. The probability distribution of random variable X is given by:

X	1	2	3	4	5
P(X)	K	2K	2K	3K	K

Let $p=P(1<X<4 \mid X<3)$. If $5p=\lambda K$, then λ equal to ________.

Official Ans. by NTA (30)

Sol. $\sum P(X) = 1 \Rightarrow k + 2k + 2k + 3k + k = 1$

$\Rightarrow k = \frac{1}{9}$

Now, $p = P\left(\frac{kX < 4}{X < 3}\right) = \frac{P(X=2)}{P(X<3)} = \frac{\frac{2k}{9k}}{\frac{k}{9k} + \frac{2k}{9k}} = \frac{2}{3}$

$\Rightarrow p = \frac{2}{3}$

Now, $5p = \lambda k$

$\Rightarrow (5)\left(\frac{2}{3}\right) = \lambda(1/9)$

$\Rightarrow \lambda = 30$

4. Let z_1 and z_2 be two complex numbers such that $\arg(z_1 - z_2) = \frac{\pi}{4}$ and z_1, z_2 satisfy the equation $|z - 3| = \text{Re}(z)$. Then the imaginary part of $z_1 + z_2$ is equal to __________.

Official Ans. by NTA (6)

Sol. $|z - 3| = \text{Re}(z)$

let $Z = x = iy$

$\Rightarrow (x - 3)^2 + y^2 = x^2$

$\Rightarrow x^2 + 9 - 6x + y^2 = x^2$

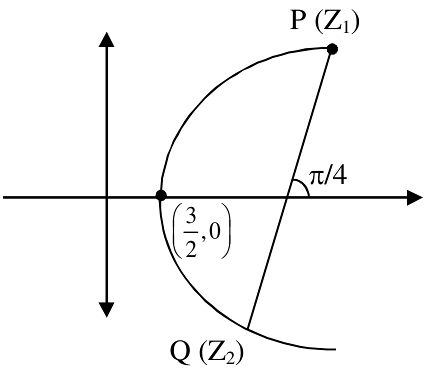
$\Rightarrow y^2 = 6x - 9$

$\Rightarrow y^2 = 6\left(x - \frac{3}{2}\right)$

$\Rightarrow z_1$ and z_2 lie on the parabola mentioned in eq.(1)

$\arg(z_1 - z_2) = \frac{\pi}{4}$

$\Rightarrow$ Slope of PQ = 1.



Let $P\left(\frac{3}{2} + \frac{3}{2}t_1^2, 3t_1\right)$ and $Q\left(\frac{3}{2} + \frac{3}{2}t_2^2, 3t_2\right)$

Slope of PQ = $\frac{3(t_2 - t_1)}{\frac{3}{2}(t_2^2 - t_1^2)} = 1$

$\Rightarrow \frac{2}{t_2 + t_2} = 1$

$\Rightarrow t_2 + t_1 = 2$

$\text{Im}(z_1 + z_2) = 3t_1 + 3t_2 = 3(t_1 + t_2) = 3\ (2)$

Ans. 6.00

Aliter :

Let $z_1 = x_1 + iy_1$; $z_2 = x_2 + iy_2$

$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$

$\therefore \arg(z_1 - z_2) = \frac{\pi}{4} \Rightarrow \tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \frac{\pi}{4}$

$y_1 - y_2 = x_1 - x_2$ ______(1)

$|z_1 - 3| = \text{Re}(z_1) \Rightarrow (x_1 - 3)^2 + y_1^2 = x_1^2$ __ (2)

$|z_2 - 3| = \text{Re}(z_2) \Rightarrow (x_2 - 3)^2 + y_2^2 = x_2^2$ __(2)

sub (2) & (3)

$(x_1 - 3)^2 - (x_2 - 3)^2 + y_1^2 - y_2^2 = x_1^2 - x_2^2$

$(x_1 - x_2)(x_1 + x_2 - 6) + (y_1 - y_2)(y_1 + y_2)$

$= (x_1 - x_2)(x_1 + x_2)$

$x_1 + x_2 - 6 + y_1 + y_2 = x_1 + x_2 \Rightarrow y_1 + y_2 = 6.$

5. Let S = {1, 2, 3, 4, 5, 6, 9}. Then the number of elements in the set $T = \{A \subseteq S : A \neq \phi$ and the sum of all the elements of A is not a multiple of 3} is __________.

Official Ans. by NTA (80)

Sol. 3n type $\rightarrow 3, 6, 9 = P$

$3n - 1$ type $\rightarrow 2, 5 = Q$

$3n - 2$ type $\rightarrow 1, 4 = R$

number of subset of S containing one element which are not divisible by 3 = ${}^2C_1 + {}^2C_1 = 4$

number of subset of S containing two numbers whose some is not divisible by 3

$= {}^3C_1 \times {}^2C_1 + {}^3C_1 \times {}^2C_1 + {}^2C_2 + {}^2C_2 = 14$

number of subsets containing 3 elements whose sum is not divisible by 3

$= {}^3C_2 \times {}^4C_1 + \left({}^2C_2 \times {}^2C_1\right)2 + {}^3C_1\left({}^2C_2 + {}^2C_2\right) = 22$

number of subsets containing 4 elements whose sum is not divisible by 3

$= {}^3C_3 \times {}^4C_1 + {}^3C_2\left({}^2C_2 + {}^2C_2\right) + \left({}^3C_1{}^2C_1 \times {}^2C_2\right)2$

$= 4 + 6 + 12 = 22.$

number of subsets of S containing 5 elements whose sum is not divisible by 3.

$= {}^3C_3\left({}^2C_2 + {}^2C_2\right) + \left({}^3C_2{}^2C_1 \times {}^2C_2\right) \times 2 = 2 + 12 = 14$

number of subsets of S containing 6 elements whose sum is not divisible by 3 = 4

$\Rightarrow$ Total subsets of Set A whose sum of digits is not divisible by 3 = 4 + 14 + 22 + 22 + 14 + 4 = 80.

6. Let A ($\sec\theta$, $2\tan\theta$) and B ($\sec\phi$, $2\tan\phi$), where $\theta + \phi = \pi/2$, be two points on the hyperbola $2x^2 - y^2 = 2$. If (α, β) is the point of the intersection of the normals to the hyperbola at A and B, then $(2\beta)^2$ is equal to ___________.

Official Ans. by NTA (36)

ALLEN Ans. (Bonus)

Sol. Since, point A ($\sec\theta$, $2\tan\theta$) lies on the hyperbola

$2x^2 - y^2 = 2$

Therefore, $2\sec^2\theta - 4\tan^2\theta = 2$

$\Rightarrow 2 + 2\tan^2\theta - 4\tan^2\theta = 2$

$\Rightarrow \tan\theta = 0 \Rightarrow \theta = 0$

Similarly, for point B, we will get $\phi = 0$.

but according to question $\theta + \phi = \frac{\pi}{2}$

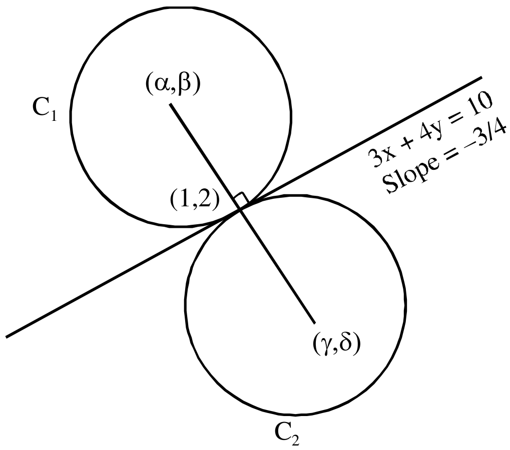
which is not possible.

Hence it must be a 'BONUS'.

7. Two circles each of radius 5 units touch each other at the point (1, 2). If the equation of their common tangent is $4x + 3y = 10$, and $C_1(\alpha, \beta)$ and $C_2(\gamma, \delta)$, $C_1 \neq C_2$ are their centres, then $|(\alpha + \beta)(\gamma + \delta)|$ is equal to ___________.

Official Ans. by NTA (40)

Sol. Slope of line joining centres of circles $= \frac{4}{3} = \tan\theta$



$\Rightarrow \cos\theta = \frac{3}{5}, \sin\theta = \frac{4}{5}$

Now using parametric form

$\frac{x-1}{\cos\theta} = \frac{y-2}{\sin\theta} = \pm 5$

$\oplus$ $(x,y) = (1 + 5\cos\theta, 2 + 5\sin\theta)$

$(\alpha,\beta) = (4,6)$

Θ $(x,y) = (\gamma,\delta) = (1 - 5\cos\theta, 2 - 5\sin\theta)$

$(\gamma, s) = (-2,-2)$

$\Rightarrow |(\alpha + \beta)(\gamma + \delta)| = |10x - 4| = 40$

8. $3 \times 7^{22} + 2 \times 10^{22} - 44$ when divided by 18 leaves the remainder __________.

Official Ans. by NTA (15)

Sol. $3(1 + 6)^{22} + 2\cdot(1 + 9)^{22} - 44 = (3 + 2 - 44) = 18\,.I$

$= -39 + 18.I$

$= (54 - 39) + 18(I - 3)$

$= 15 + 18\,I_1$

$\Rightarrow$ Remainder = 15.

9. An online exam is attempted by 50 candidates out of which 20 are boys. The average marks obtained by boys is 12 with a variance 2. The variance of marks obtained by 30 girls is also 2. The average marks of all 50 candidates is 15. If μ is the average marks of girls and σ^2 is the variance of marks of 50 candidates, then $\mu + \sigma^2$ is equal to _________.

Official Ans. by NTA (25)

Sol. $\sigma_b^2 = 2$ (variance of boys) $\quad n_1$ = no. of boys

$\bar{x}_b = 12 \quad n_2$ = no. of girls

$\sigma_g^2 = 2$

$$\bar{x}_g = \frac{50 \times 15 - 12 \times \sigma_b}{30} = \frac{750 - 12 \times 20}{30} = 17 = \mu$$

variance of combined series

$$\sigma^2 = \frac{n_1\sigma_b^2 + n_2\sigma_g^2}{n_1 + n_2} + \frac{n_1 \cdot n_2}{(n_1 + n_2)^2}(\bar{x}_b - \bar{x}_g)^2$$

$$\sigma^2 = \frac{20 \times 2 + 30 \times 2}{20 + 30} + \frac{20 \times 30}{(20 + 30)^2}(12 - 17)^2$$

$\sigma^2 = 8$.

$\Rightarrow \mu + \sigma^2 = 17 + 8 = 25$

10. If $\int \frac{2e^{x}+3e^{-x}}{4e^{x}+7e^{-x}}dx = \frac{1}{14}(ux + v\log_e(4e^x + 7e^{-x})) + C$, where C is a constant of integration, then u + v is equal to ________.

Official Ans. by NTA (7)

Sol. $\int \frac{2e^{x}}{4e^{x}+7e^{-x}}dx \;\; +3\int \frac{e^{-x}}{4e^{x}+7e^{-x}}dx$

$= \int \frac{2e^{2x}}{4e^{2x}+7}dx + 3\int \frac{e^{-2x}}{4+7e^{-2x}}dx$

Let $4e^{2x} + 7 = T$ Let $4 + 7e^{-2x} = t$

$8e^{2x}dx = dT$ $-14e^{-2x}dx = dt$

$2e^{2x}dx = \frac{dT}{4}$ $e^{-2x}dx = -\frac{dt}{14}$

$\int \frac{dT}{4T} - \frac{3}{14}\int \frac{dt}{t}$

$= \frac{1}{4}\log T - \frac{3}{14}\log t + C$

$= \frac{1}{4}\log(4e^{2x}+7) - \frac{3}{14}\log(4+7e^{-2x}) + C$

$= \frac{1}{14}\left[\frac{1}{2}\log(4e^{x}+7e^{-x}) + \frac{13}{2}x\right] + C$

$u = \frac{13}{2}, v = \frac{1}{2} \Rightarrow u + v = 7$

Aliter :

$2e^{x} + 3e^{-x} = A(4e^{x} + 7e^{-x}) + B(4e^{x} - 7e^{-x}) + \lambda$

$2 = 4A + 4B$; $3 = 7A - 7B$; $\lambda = 0$

$A + B = \frac{1}{2}$

$A - B = \frac{3}{7}$

$A = \frac{1}{2}\left(\frac{1}{2} + \frac{3}{7}\right) = \frac{7+6}{28} = \frac{13}{28}$

$B = A - \frac{3}{7} = \frac{13}{28} - \frac{3}{7} = \frac{13-12}{28} = \frac{1}{28}$

$\int \frac{13}{28}dx + \frac{1}{28}\int \frac{4e^{x}-7e^{-x}}{4e^{x}+7e^{-x}}dx$

$\frac{13}{28}x + \frac{1}{28}\ell n|4e^{x}+7e^{-x}| + C$

$u = \frac{13}{2}; v = \frac{1}{2}$

$\Rightarrow u + v = 7$