FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Friday 27th August, 2021) **TIME : 9 : 00 AM to 12 : 00 NOON**

PHYSICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. A uniformly charged disc of radius R having surface charge density σ is placed in the xy plane with its center at the origin. Find the electric field intensity along the z-axis at a distance Z from origin :-

(1) $E = \dfrac{\sigma}{2\varepsilon_0}\left(1 - \dfrac{Z}{(Z^2 + R^2)^{1/2}}\right)$

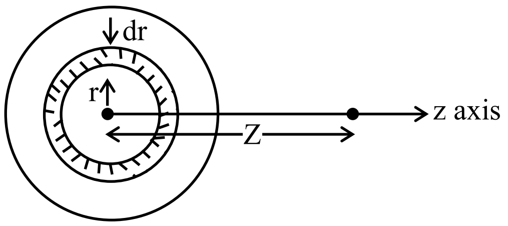
(2) $E = \dfrac{\sigma}{2\varepsilon_0}\left(1 + \dfrac{Z}{(Z^2 + R^2)^{1/2}}\right)$

(3) $E = \dfrac{2\varepsilon_0}{\sigma}\left(\dfrac{1}{(Z^2 + R^2)^{1/2}} + Z\right)$

(4) $E = \dfrac{\sigma}{2\varepsilon_0}\left(\dfrac{1}{(Z^2 + R^2)} + \dfrac{1}{Z^2}\right)$

Official Ans. by NTA (1)

Sol. Consider a small ring of radius r and thickness dr on disc.



area of elemental ring on disc

$dA = 2\pi r dr$

charge on this ring $dq = \sigma dA$

$$dE_z = \frac{kdqz}{\left(z^2 + r^2\right)^{3/2}}$$

$$E = \int_0^R dE_z = \frac{\sigma}{2\in_0}\left[1 - \frac{z}{\sqrt{R^2 + z^2}}\right]$$

2. There are 10^{10} radioactive nuclei in a given radioactive element, Its half-life time is 1 minute. How many nuclei will remain after 30 seconds ? $\left(\sqrt{2} = 1.414\right)$

(1) 2×10^{10}

(2) 7×10^9

(3) 10^5

(4) 4×10^{10}

Official Ans. by NTA (2)

Sol. $\dfrac{N}{N_0} = \left(\dfrac{1}{2}\right)^{\frac{t}{t_{1/2}}}$

$$\frac{N}{10^{10}} = \left(\frac{1}{2}\right)^{\frac{30}{60}}$$

$$\Rightarrow \quad N = 10^{10} \times \left(\frac{1}{2}\right)^{\frac{1}{2}} = \frac{10^{10}}{\sqrt{2}} \approx 7 \times 10^9$$

3. Which of the following is not a dimensionless quantity ?

(1) Relative magnetic permeability (μ_r)

(2) Power factor

(3) Permeability of free space (μ_0)

(4) Quality factor

Official Ans. by NTA (3)

Sol. $[\mu_r] = 1$ as $\mu_r = \dfrac{\mu}{\mu_m}$

[power factor ($\cos\phi$)] = 1

$\mu_0 = \dfrac{B_0}{H}$ (unit = NA^{-2}) : Not dimensionless

$[\mu_0] = [MLT^{-2}A^{-2}]$

quality factor (Q) = $\dfrac{\text{Energy stored}}{\text{Energy dissipated per cycle}}$

So Q is unitless & dimensionless.

4. If E and H represents the intensity of electric field and magnetising field respectively, then the unit of E/H will be :

(1) ohm (2) mho

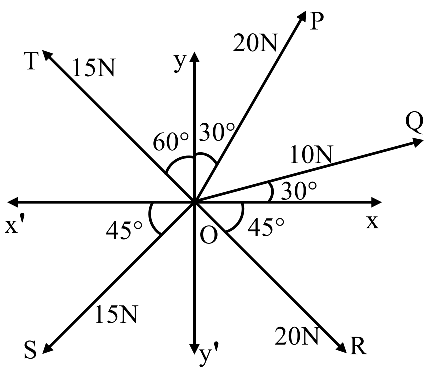
(3) joule (4) newton

Official Ans. by NTA (1)

Sol. Unit of $\frac{E}{H}$ is $\frac{\text{volt / metre}}{\text{Ampere / metre}} = \frac{\text{volt}}{\text{Ampere}} = \text{ohm}$

5. The resultant of these forces $\overrightarrow{OP}, \overrightarrow{OQ}, \overrightarrow{OR}, \overrightarrow{OS}$ and $\overrightarrow{OT}$ is approximately ...... N.

[Take $\sqrt{3} = 1.7, \sqrt{2} = 1.4$ Given $\hat{i}$ and $\hat{j}$ unit vectors along x, y axis]

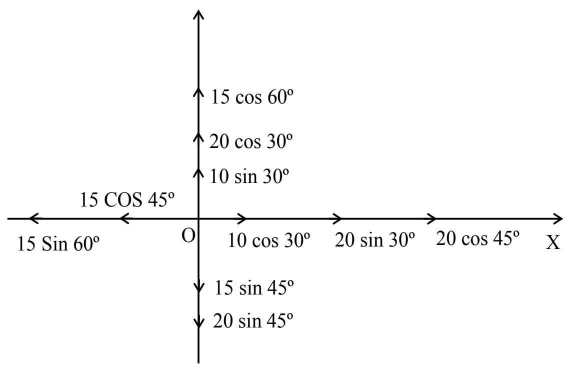


(1) $9.25\hat{i} + 5\hat{j}$ (2) $3\hat{i} + 15\hat{j}$

(3) $2.5\hat{i} - 14.5\hat{j}$ (4) $-1.5\hat{i} - 15.5\hat{j}$

Official Ans. by NTA (1)

Sol.



$$\vec{F}_x = \left(10 \times \frac{\sqrt{3}}{2} + 20\left(\frac{1}{2}\right) + 20\left(\frac{1}{\sqrt{2}}\right) - 15\left(\frac{1}{\sqrt{2}}\right) - 15\left(\frac{\sqrt{3}}{2}\right)\right)\hat{i}$$

$= 9.25\ \hat{i}$

$$\vec{F}_y = \left(15\left(\frac{1}{2}\right) + 20\left(\frac{\sqrt{3}}{2}\right) + 10\left(\frac{1}{2}\right) - 15\left(\frac{1}{\sqrt{2}}\right) - 20\left(\frac{1}{\sqrt{2}}\right)\right)\hat{j}$$

$= 5\ \hat{j}$

6. A balloon carries a total load of 185 kg at normal pressure and temperature of 27°C. What load will the balloon carry on rising to a height at which the barometric pressure is 45 cm of Hg and the temperature is –7°C. Assuming the volume constant ?

(1) 181.46 kg (2) 214.15 kg.

(3) 219.07 kg (4) 123.54 kg

Official Ans. by NTA (4)

Sol. $P_m = \rho RT$

$$\therefore \frac{P_1}{P_2} = \frac{\rho_1 T_1}{\rho_2 T_2}$$

$$\frac{\rho_1}{\rho_2} \Rightarrow \frac{P_1 T_2}{P_2 T_1} = \left(\frac{76}{45}\right) \times \frac{266}{300}$$

$$\frac{\rho_1}{\rho_2} \Rightarrow \frac{M_1}{M_2} = \frac{76 \times 266}{45 \times 300}$$

$$\therefore M_2 \Rightarrow \frac{45 \times 300 \times 185}{76 \times 266} = 123.54 \text{ kg}$$

7. An object is placed beyond the centre of curvature C of the given concave mirror. If the distance of the object is d_1 from C and the distance of the image formed is d_2 from C, the radius of curvature of this mirror is :

(1) $\frac{2d_1 d_2}{d_1 - d_2}$

(2) $\frac{2d_1 d_2}{d_1 + d_2}$

(3) $\frac{d_1 d_2}{d_1 + d_2}$

(4) $\frac{d_1 d_2}{d_1 - d_2}$

Official Ans. by NTA (1)

Sol. Using Newton"s formula

$$(f + d_1)(f - d_2) = f^2$$

$$f^2 + fd_1 - fd_2 - d_1 d_2 = f^2$$

$$f = \frac{d_1 d_2}{d_1 - d_2}$$

$$\therefore R = \frac{2d_1 d_2}{d_1 - d_2}$$

8. A huge circular arc of length 4.4 ly subtends an angle '4s' at the centre of the circle. How long it would take for a body to complete 4 revolution if its speed is 8 AU per second ?
Given : 1 ly = 9.46×10^{15} m
1 AU = 1.5×10^{11} m

(1) 4.1×10^{8} s (2) 4.5×10^{10} s
(3) 3.5×10^{6} s (4) 7.2×10^{8} s

Official Ans. by NTA (2)

Sol. $R = \frac{\ell}{\theta}$

$$\text{Time} = \frac{4\times 2\pi R}{v} = \frac{4\times 2\pi}{v}\left(\frac{\ell}{\theta}\right)$$

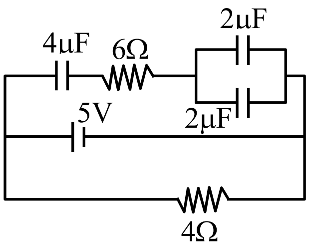
put $\ell = 4.4 \times 9.46 \times 10^{15}$

$v = 8 \times 1.5 \times 10^{11}$

$$\theta = \frac{4}{3600} \times \frac{\pi}{180} \text{rad.}$$

we get time = 4.5×10^{10} sec

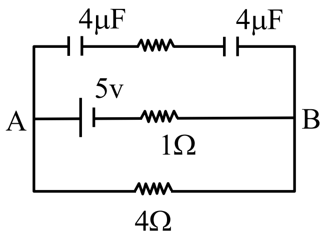
9. Calculate the amount of charge on capacitor of 4 μF. The internal resistance of battery is 1Ω :



(1) 8 μC (2) zero
(3) 16 μC (4) 4 μC

Official Ans. by NTA (1)

Sol. On simplifying circuit we get



No current in upper wire.

$$\therefore\ V_{AB} = \frac{5}{4+1} \times 4 = 4 \text{ v.}$$

$\therefore\ \theta = (C_{eq})v$

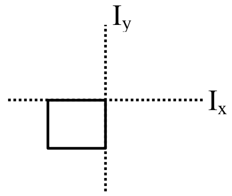
$\Rightarrow 2 \times 4 = 8\mu C$

10. Moment of inertia of a square plate of side l about the axis passing through one of the corner and perpendicular to the plane of square plate is given by :

(1) $\frac{Ml^2}{6}$ (2) Ml^2 (3) $\frac{Ml^2}{12}$ (4) $\frac{2}{3}Ml^2$

Official Ans. by NTA (4)

Sol. According to perpendicular Axis theorem.



$I_x + I_y = I_z$

$$I_z \Rightarrow \frac{m\ell^2}{3} + \frac{m\ell^2}{3}$$

$$= \frac{2m\ell^2}{3}$$

11. For a transistor in CE mode to be used as an amplifier, it must be operated in :
(1) Both cut-off and Saturation
(2) Saturation region only
(3) Cut-off region only
(4) The active region only

Official Ans. by NTA (4)

Sol. Active region of the CE transistor is linear region and is best suited for its use as an amplifier

12. An ideal gas is expanding such that PT^3 = constant. The coefficient of volume expansion of the gas is :

(1) $\frac{1}{T}$ (2) $\frac{2}{T}$ (3) $\frac{4}{T}$ (4) $\frac{3}{T}$

Official Ans. by NTA (3)

Sol. PT^3 = constant

$$\left(\frac{nRT}{v}\right)T^3 = \text{constant}$$

$T^4 V^{-1}$ = constant

$T^4 = kV$

$$\Rightarrow 4\frac{\Delta T}{T} = \frac{\Delta V}{V} \text{.........(1)}$$

$\Delta V = V\gamma\Delta T$............(2)

comparing (1) and (2)

we get

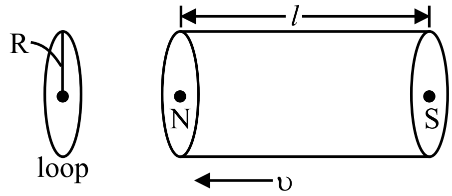
$$\gamma = \frac{4}{T}$$

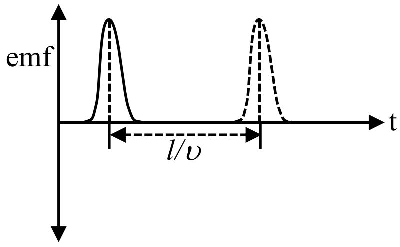
13. In a photoelectric experiment, increasing the intensity of incident light :
(1) increases the number of photons incident and also increases the K.E. of the ejected electrons
(2) increases the frequency of photons incident and increases the K.E. of the ejected electrons.
(3) increases the frequency of photons incident and the K.E. of the ejected electrons remains unchanged
(4) increases the number of photons incident and the K.E. of the ejected electrons remains unchanged

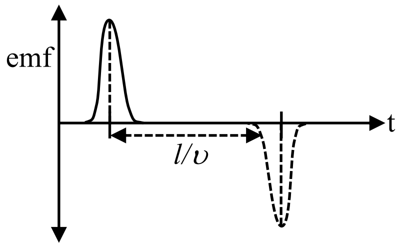
Official Ans. by NTA (4)

Sol. → Increasing intensity means number of incident photons are increased.
→Kinetic energy of ejected electrons depend on the frequency of incident photons, not the intensity.

14. A bar magnet is passing through a conducting loop of radius R with velocity υ. The radius of the bar magnet is such that it just passes through the loop. The induced e.m.f. in the loop can be represented by the approximate curve :



(1)


(2)


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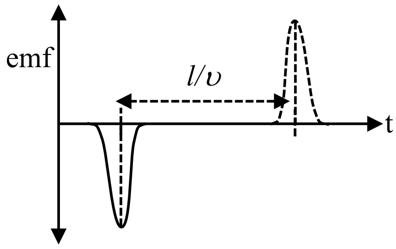

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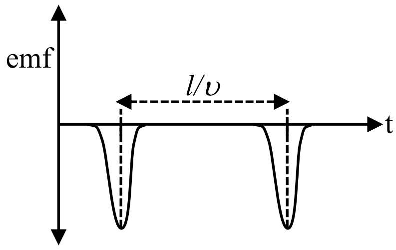
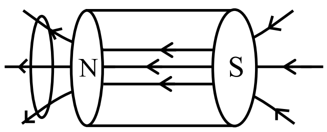


Official Ans. by NTA (3)

Sol.


$\rightarrow$ When magnet passes through centre region of solenoid , no current / Emf is induced in loop.

$\rightarrow$ While entering flux increases so negative induced emf

$\rightarrow$ While leaving flux decreases so positive induced emf.

15. Two ions of masses 4 amu and 16 amu have charges +2e and +3e respectively. These ions pass through the region of constant perpendicular magnetic field. The kinetic energy of both ions is same. Then :

(1) lighter ion will be deflected less than heavier ion
(2) lighter ion will be deflected more than heavier ion
(3) both ions will be deflected equally
(4) no ion will be deflected.

Official Ans. by NTA (2)

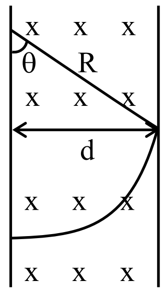
Sol. $r = \dfrac{P}{qB} = \dfrac{\sqrt{2mk}}{qB}$

Given they have same kinetic energy

$r \propto \dfrac{\sqrt{m}}{q}$

$\dfrac{r_1}{r_2} = \dfrac{\sqrt{4}}{2} \times \dfrac{3}{\sqrt{16}} = \dfrac{3}{4}$

$\boxed{r_2 = \dfrac{4r_1}{3}}$ (r_2 is for hearier ion and r_1 is for lighter ion)



$\sin\theta = \dfrac{d}{R}$

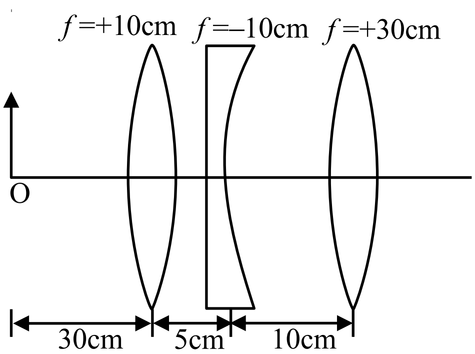
$\theta \rightarrow$ Deflection

$\theta \propto \dfrac{1}{R}$

($R \rightarrow$ Radius of path)

$\because R_2 > R_1 \Rightarrow \theta_2 < \theta_1$

16. Find the distance of the image from object O, formed by the combination of lenses in the figure :



(1) 75 cm (2) 10 cm
(3) 20 cm (4) infinity

Official Ans. by NTA (1)

Sol. $\frac{1}{V_1} + \frac{1}{30} = \frac{1}{10}$

$\frac{1}{V_1} = \frac{2}{30} \Rightarrow V_1 = 15$ cm

$\frac{1}{V_2} - \frac{1}{10} = -\frac{1}{10}$

$\frac{1}{V_2} = 0$ $\quad V_2 = \infty$

$V_3 = 30$ cm

$OV_3 = 75$ cm

17. In Millikan's oil drop experiment, what is viscous force acting on an uncharged drop of radius 2.0×10^{-5} m and density 1.2×10^3 kgm^{-3} ? Take viscosity of liquid = 1.8×10^{-5} Nsm^{-2}. (Neglect buoyancy due to air).

(1) 3.8×10^{-11} N
(2) 3.9×10^{-10} N
(3) 1.8×10^{-10} N
(4) 5.8×10^{-10} N

Official Ans. by NTA (2)

Sol. Viscous force = Weight

$= \rho \times \left(\frac{4}{3}\pi r^3\right) g$

$= 3.9 \times 10^{-10}$

18. Electric field in a plane electromagnetic wave is given by $E = 50 \sin(500x - 10 \times 10^{10}t)$ V/m

The velocity of electromagnetic wave in this medium is :

(Given C = speed of light in vacuum)

(1) $\frac{3}{2}C$ (2) C (3) $\frac{2}{3}C$ (4) $\frac{C}{2}$

Official Ans. by NTA (3)

Sol. $V = \frac{\omega}{K} = \frac{10 \times 10^{10}}{500} = 2 \times 10^8$

$\mathbf{V} = \frac{2C}{3}$.

19. Five identical cells each of internal resistance 1Ω and emf 5V are connected in series and in parallel with an external resistance 'R'. For what value of 'R', current in series and parallel combination will remain the same ?

(1) 1 Ω
(2) 25 Ω
(3) 5 Ω
(4) 10 Ω

Official Ans. by NTA (1)

Sol. $i_1 = \frac{25}{5+R}$

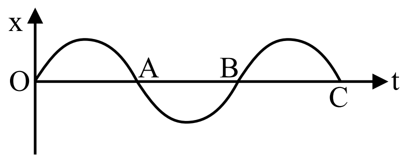
$i_2 = \frac{5}{R+\frac{1}{5}}$

$i_1 = i_2 \Rightarrow 5\left(R+\frac{1}{5}\right) = 5+R$

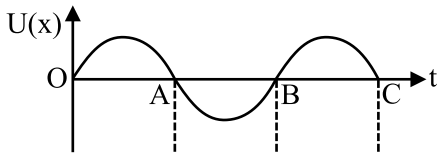
$4R = 4$

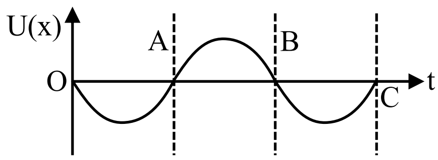
$R = 1\Omega$

20. The variation of displacement with time of a particle executing free simple harmonic motion is shown in the figure.



The potential energy U(x) versus time (t) plot of the particle is correctly shown in figure :

(1)


(2)


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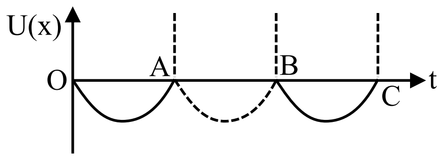

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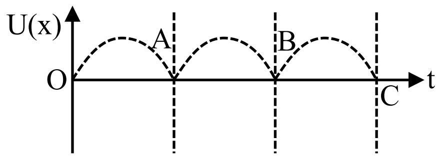
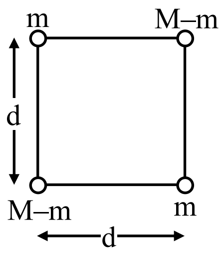


Official Ans. by NTA (4)

Sol. Potential energy is maximum at maximum distance from mean.

SECTION-B

1. A body of mass (2M) splits into four masses {m, M – m, m, M – m}, which are rearranged to form a square as shown in the figure. The ratio of $\frac{M}{m}$ for which, the gravitational potential energy of the system becomes maximum is x : 1. The value of x is ....



Official Ans. by NTA (2)

Sol. Energy is maximum when mass is split equally so

$$\frac{M}{m} = 2$$

2. The alternating current is given by

$$i = \left\{\sqrt{42}\sin\left(\frac{2\pi}{T}t\right) + 10\right\}A$$

The r.m.s. value of this current is .... A.

Official Ans. by NTA (11)

Sol. $f_{rms}^2 = f_{1rms}^2 + f_{2rms}^2$

$$= \left(\frac{\sqrt{42}}{\sqrt{2}}\right)^2 + 10^2$$

$= 121 \;\Rightarrow\; f_{rms} = 11$ A

3. A uniform conducting wire of length is 24a, and resistance R is wound up as a current carrying coil in the shape of an equilateral triangle of side 'a' and then in the form of a square of side 'a'. The coil is connected to a voltage source V_0. The ratio of magnetic moment of the coils in case of equilateral triangle to that for square is $1:\sqrt{y}$ where y is ....

Official Ans. by NTA (3)

Sol. In triangle shape $N_t = \frac{24a}{3a} = 8$

In square $N_s = \frac{24a}{4a} = 6$

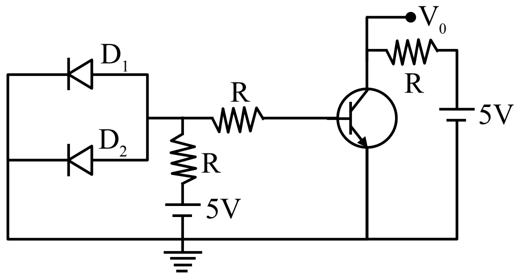
$\frac{M_t}{M_3} = \frac{N_t I A_t}{N_s I A_s}$ [I will be same in both]

$$= \frac{8 \times \frac{\sqrt{3}}{4} \times a^2}{6 \times a^2}$$

$$\frac{M_t}{M_s} = \frac{1}{\sqrt{3}}$$

$\boxed{y = 3}$

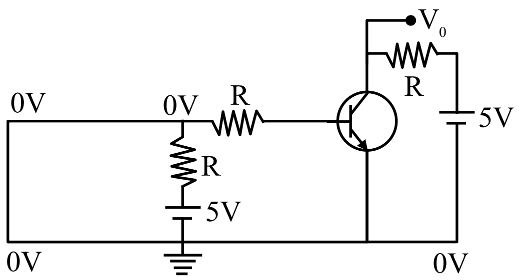
4. A circuit is arranged as shown in figure. The output voltage V_0 is equal to .... V.



Official Ans. by NTA (5)

Sol. As diodes D_1 and D_2 are in forward bias, so they acted as neligible resistances

$\Rightarrow$ Input voltage become zero



$\Rightarrow$ Input current is zero

$\Rightarrow$ Output current is zero

$\Rightarrow V_0 = 5$ volt

5. First, a set of n equal resistors of 10 Ω each are connected in series to a battery of emf 20V and internal resistance 10 Ω. A current I is observed to flow. Then, the n resistors are connected in parallel to the same battery. It is observed that the current is increased 20 times, then the value of n is ....

Official Ans. by NTA (20)

Sol. In series

$R_{eq} = nR = 10\ n$

$i_s = \dfrac{20}{10+10n} = \dfrac{2}{1+n}$

in parallel

$R_{eq} = \dfrac{10}{n}$

$i_p = \dfrac{20}{\dfrac{10}{n}+10} = \dfrac{2n}{1+n}$

$\dfrac{i_p}{i_s} = 20$

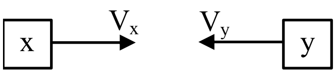
$\dfrac{\left(\dfrac{2n}{1+n}\right)}{\left(\dfrac{2}{1+n}\right)} = 20$

$n = 20$

6. Two cars X and Y are approaching each other with velocities 36 km/h and 72 km/h respectively. The frequency of a whistle sound as emitted by a passenger in car X, heard by the passenger in car Y is 1320 Hz. If the velocity of sound in air is 340 m/s, the actual frequency of the whistle sound produced is .... Hz.

Official Ans. by NTA (1210)

Sol.



$V_x = 36$ km/hr $= 10$ m/s

$V_y = 72$ km/hr $= 20$ m/s

by doppler's effect

$F' = F_0\left(\dfrac{V \pm V_0}{V \pm V_s}\right)$

$1320 = F_0\left(\dfrac{340+20}{340-10}\right) \Rightarrow F_0 = 1210$ Hz

7. If the velocity of a body related to displacement x is given by $\upsilon = \sqrt{5000 + 24x}$ m/s, then the acceleration of the body is ...... m/s^2.

Official Ans. by NTA (12)

Sol. $V = \sqrt{5000+24x}$

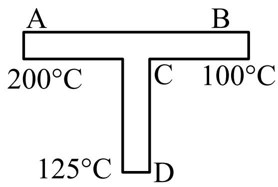
$\dfrac{dV}{dx} = \dfrac{1}{2\sqrt{5000+24x}} \times 24 = \dfrac{12}{\sqrt{5000+24x}}$

now $a = V\dfrac{dV}{dx}$

$= \sqrt{5000+24x} \times \dfrac{12}{\sqrt{5000+24x}}$

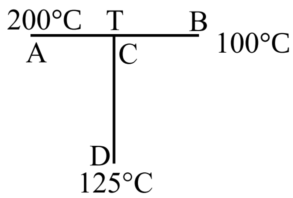
$\boxed{a = 12 m/s^2}$

8. A rod CD of thermal resistance 10.0 KW^{-1} is joined at the middle of an identical rod AB as shown in figure, The end A, B and D are maintained at 200°C, 100°C and 125°C respectively. The heat current in CD is P watt. The value of P is ....



Official Ans. by NTA (2)

Sol.



Rods are identical so

$R_{AB} = R_{CD} = 10\ Kw^{-1}$

C is mid-point of AB, so

$R_{AC} = R_{CB} = 5\ Kw^{-1}$

at point C

$\dfrac{200-T}{5} = \dfrac{T-125}{10} + \dfrac{T-100}{5}$

$2(200 - T) = T - 125 + 2(T - 100)$

$400 - 2T = T - 125 + 2T - 200$

$T = \dfrac{725}{5} = 145°C$

$I_h = \dfrac{145-125}{10} w = \dfrac{20}{10} w$

$\boxed{I_h = 2w}$

9. Two persons A and B perform same amount of work in moving a body through a certain distance d with application of forces acting at angle 45° and 60° with the direction of displacement respectively. The ratio of force applied by person A to the force applied by person B is $\frac{1}{\sqrt{x}}$. The value of x is ....

Official Ans. by NTA (2)

Sol. Given $W_A = W_B$

$F_A d\cos 45° = F_B d\cos 60°$

$$F_A \times \frac{1}{\sqrt{2}} = F_B \times \frac{1}{2}$$

$$\frac{F_A}{F_B} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$\boxed{x = 2}$$

10. A transmitting antenna has a height of 320 m and that of receiving antenna is 2000 m. The maximum distance between them for satisfactory communication in line of sight mode is 'd'. The value of 'd' is .... km.

Official Ans. by NTA (224)

Sol. $d_m = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$$d_m = \left(\sqrt{2 \times 6400 \times 10^3 \times 320} + \sqrt{2 \times 6400 \times 10^3 \times 2000}\right) m$$

$$\boxed{d_m = 224 km}$$

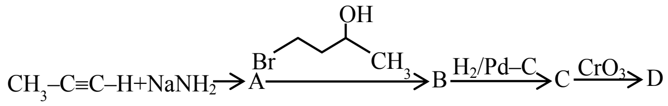
FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Friday 27th August, 2021) **TIME : 9 : 00 AM to 12 : 00 NOON**

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION-A

1. In the following sequence of reactions, the final product D is :



(1) $H_3C–CH_2–CH_2–CH_2–CH_2–\overset{\overset{O}{\|}}{C}–H$

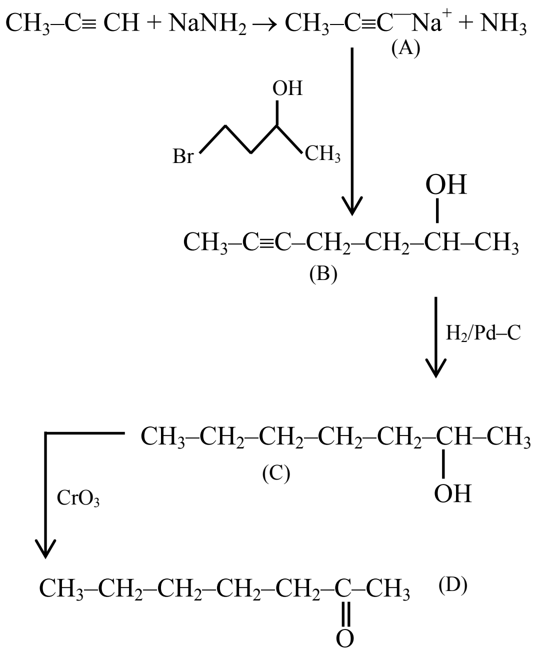
(2) $CH_3–CH=CH–CH_2–CH_2–CH_2–COOH$

(3) $H_3C–CH=CH–CH(OH)–CH_2–CH_2–CH_3$

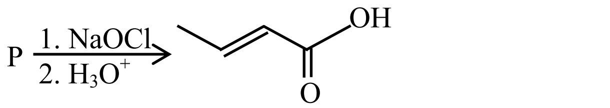
(4) $CH_3–CH_2–CH_2–CH_2–CH_2–\overset{\overset{O}{\|}}{C}–CH_3$

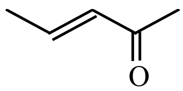
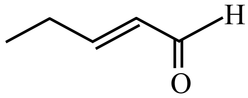
Official Ans. by NTA (4)

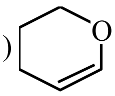
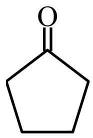
Sol.



2. The structure of the starting compound **P** used in the reaction given below is :

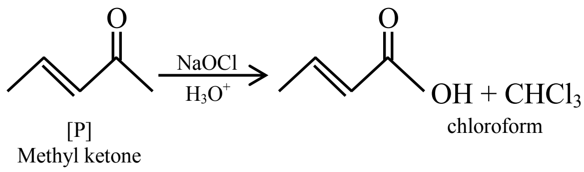


(1)  (2) 

(3)  (4) 

Official Ans. by NTA (1)

Sol.



NaOCl is used in haloform reaction as reagent.

3. Match **List–I** with **List–II** :

List–I (Species)	**List–II (Number of lone pairs of electrons on the central atom)**
(a) XeF_2	(i) 0
(b) XeO_2F_2	(ii) 1
(c) XeO_3F_2	(iii) 2
(d) XeF_4	(iv) 3

Choose the **most appropriate** answer from the options given below :

(1) (a)–(iv), (b)–(i), (c)–(ii), (d)–(iii)

(2) (a)–(iii), (b)–(iv), (c)–(ii), (d)–(i)

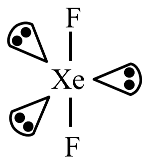
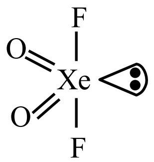
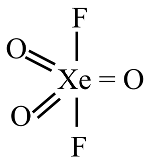
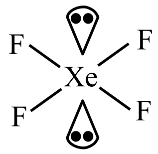
(3) (a)–(iii), (b)–(ii), (c)–(iv), (d)–(i)

(4) (a)–(iv), (b)–(ii), (c)–(i), (d)–(iii)

Official Ans. by NTA (4)

Sol. **Species** **(Number of lone pairs of electrons**

on the central atom)

XeF_2	3	
XeO_2F_2	1	
XeO_3F_2	0	
XeF_4	2	

4. In which one of the following molecules strongest back donation of an electron pair from halide to boron is expected?

(1) BCl_3 (2) BF_3

(3) BBr_3 (4) BI_3

Official Ans. by NTA (2)

Sol. Type of back bonding

BF_3	BCl	BBr_3	BI_3
$(2p\pi–2p\pi)$	$(2p\pi–3p\pi)$	$(2p\pi–4p\pi)$	$(2p\pi–5p\pi)$

Therefore back bonding strength is as follows

BF_3 > BCl > BBr_3 > BI_3

5. Deuterium resembles hydrogen in properties but :

(1) reacts slower than hydrogen

(2) reacts vigorously than hydrogen

(3) reacts just as hydrogen

(4) emits β^+ particles

Official Ans. by NTA (1)

Sol. The bond dissociation energy of D_2 is greater than H_2 and therefore D_2 reacts slower than H_2.

6. Which refining process is generally used in the purification of low melting metals ?

(1) Chromatographic method

(2) Liquation

(3) Electrolysis

(4) Zone refining

Official Ans. by NTA (2)

Sol. Liquation method is used to purify those impure metals which has lower melting point than the melting point of impurities associated.

$\therefore$ This method is used for metal having low melting point.

7. Match items of **List–I** with those of **List–II** :

List–I (Property)	**List–II (Example)**
(a) Diamagnetism	(i) MnO
(b) Ferrimagnetism	(ii) O_2
(c) Paramagnetism	(iii) NaCl
(d) Antiferromagnetism	(iv) Fe_3O_4

Choose the **most appropriate** answer from the options given below :

(1) (a)–(ii), (b)–(i), (c)–(iii), (d)–(iv)

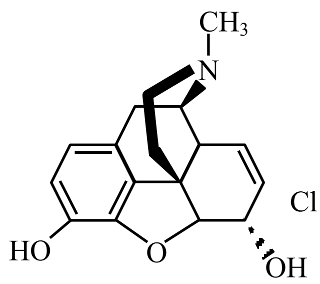
(2) (a)–(i), (b)–(iii), (c)–(iv), (d)–(ii)

(3) (a)–(iii), (b)–(iv), (c)–(ii), (d)–(i)

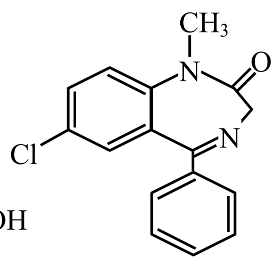
(4) (a)–(iv), (b)–(ii), (c)–(i), (d)–(iii)

Official Ans. by NTA (3)

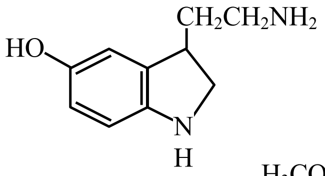
8.



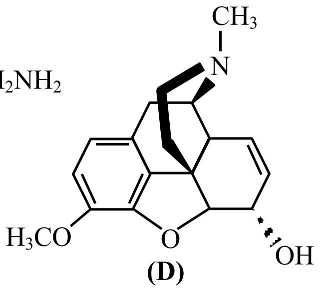
(A)



(B)



(C)



(D)

The correct statement about (A), (B), (C) and (D) is :

(1) (A), (B) and (C) are narcotic analgesics

(2) (B), (C) and (D) are tranquillizers

(3) (A) and (D) are tranquillizers

(4) (B) and (C) are tranquillizers

Official Ans. by NTA (4)

Sol. B and C are tranquilizers

9. The major product of the following reaction is :

$$CH_3-\underset{\underset{CH_3}{|}}{CH}-CH_2-CH_2-\overset{\overset{O}{||}}{C}-Cl \xrightarrow[\text{(iii) } NaNO_2,HCl \text{ (iv) } H_2O]{\text{(i) alcoholic } NH_3 \text{ (ii) } NaOH,\ Br_2} \text{Major product}$$

(1) $CH_3-\underset{\underset{CH_3}{|}}{CH}-\overset{\overset{Br}{|}}{CH}-CH_2OH$

(2) $CH_3-\underset{\underset{CH_3}{|}}{CH}-CH_2-CH_2-CH_2OH$

(3) $CH_3-\underset{\underset{CH_3}{|}}{CH}-CH_2-CH_2OH$

(4) $CH_3-\underset{\underset{CH_3}{|}}{CH}-CH_2-CH_2-Cl$

Official Ans. by NTA (3)

Sol.

$$CH_3-\underset{\underset{CH_3}{|}}{CH}-CH_2-CH_2-\overset{\overset{O}{||}}{C}-Cl \xrightarrow{\text{alc. } NH_3} CH_3-\underset{\underset{CH_3}{|}}{CH}-CH_2-CH_2-\overset{\overset{O}{||}}{C}-NH_2$$

$$\xrightarrow{NaOH,\ Br_2} CH_3-\underset{\underset{CH_3}{|}}{CH}-CH_2-CH_2-NH_2 \xrightarrow{NaNO_2,HCl} CH_3-\underset{\underset{CH_3}{|}}{CH}-CH_2-CH_2-N^+Cl^-$$

$$\xrightarrow{H_2O} CH_3-\underset{\underset{CH_3}{|}}{CH}-CH_2-CH_2-OH$$

10. Which of the following is **not** a correct statement for primary aliphatic amines?

(1) The intermolecular association in primary amines is less than the intermolecular association in secondary amines.

(2) Primary amines on treating with nitrous acid solution form corresponding alcohols except methyl amine.

(3) Primary amines are less basic than the secondary amines.

(4) Primary amines can be prepared by the Gabriel phthalimide synthesis.

Official Ans. by NTA (1)

Sol. The intermolecular association is more prominent in case of primary amines as compared to secondary, due to the availability of two hydrogen atom.

11. Acidic ferric chloride solution on treatment with excess of potassium ferrocyanide gives a Prussian blue coloured colloidal species. It is :

(1) $Fe_4[Fe(CN)_6]_3$ (2) $K_5Fe[Fe(CN)_6]_2$

(3) $HFe[Fe(CN)_6]$ (4) $KFe[Fe(CN)_6]$

Official Ans. by NTA (4)

Sol. $FeCl_3 + K_4[Fe(CN)_6]$ (excess)

$\downarrow$

$K\,Fe[Fe(CN)_6]$

Colloidal species

12. The gas 'A' is having very low reactivity reaches to stratosphere. It is non–toxic and non–flammable but dissociated by UV—radiations in stratosphere. The intermediates formed initially from the gas 'A' are :

(1) $Cl\dot{O}+\dot{C}F_2Cl$ (2) $Cl\dot{O}+\dot{C}H_3$

(3) $\dot{C}H_3+\dot{C}F_2Cl$ (4) $\dot{C}l+\dot{C}F_2Cl$

Official Ans. by NTA (4)

Sol. In stratosphere CFCs get broken down by powerful UV radiations releasing $Cl^{\bullet}$

$$CF_2Cl_2(g)\xrightarrow{\text{U.V.}}Cl^{\bullet}(g)+{}^{\bullet}CF_2Cl(g)$$

13. The number of water molecules in gypsum, dead burnt plaster and plaster of paris, respectively are:

(1) 2, 0 and 1 (2) 0.5, 0 and 2

(3) 5, 0 and 0.5 (4) 2, 0 and 0.5

Official Ans. by NTA (4)

Sol.

Gypsum	$CaSO_4.2H_2O$
Plaster of Paris	$CaSO_4.\frac{1}{2}H_2O$
Dead burnt plaster	$CaSO_4$

14. The nature of oxides V_2O_3 and CrO is indexed as 'X' and 'Y' type respectively. The correct set of X and Y is:

(1) X = basic Y = amphoteric

(2) X = amphoteric Y = basic

(3) X = acidic Y = acidic

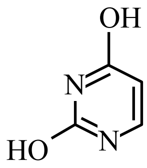
(4) X = basic Y = basic

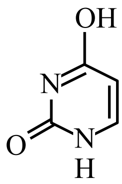
Official Ans. by NTA (4)

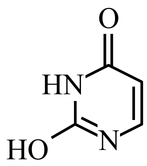
Sol. V_2O_3 basic

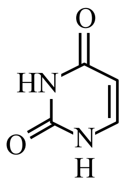
CrO basic

15. Out of following isomeric forms of uracil, which one is present in RNA ?

(1) 

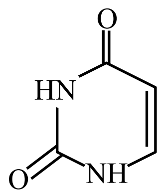
(2) 

(3) 

(4) 

Official Ans. by NTA (4)

Sol. Isomeric form of uracil present in RNA



16. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A): Synthesis of ethyl phenyl ether may be achieved by Williamson synthesis.

Reason (R): Reaction of bromobenzene with sodium ethoxide yields ethyl phenyl ether.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

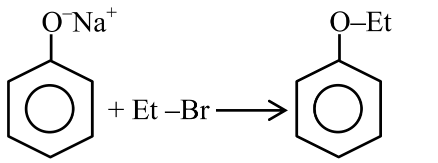
(2) **(A)** is correct but **(R)** is not correct

(3) **(A)** is not correct but **(R)** is correct

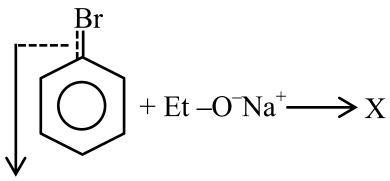
(4) Both **(A)** and **(R)** are correct but **(R)** is NOT the correct explanation of **(A)**

Official Ans. by NTA (2)

Sol.

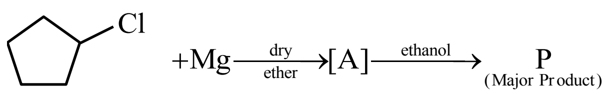


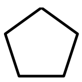
Sodium Phenoxide   Ethyl Phenyl ether

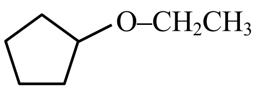


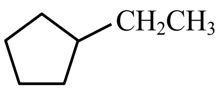
Partial double bond character

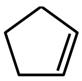
17. In the following sequence of reactions the P is :



(1) 

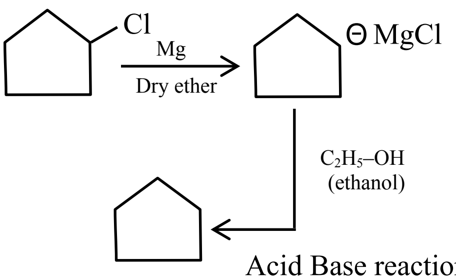
(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol.



Acid Base reaction

18. The unit of the van der Waals gas equation parameter 'a' in $\left(P+\frac{an^2}{V^2}\right)(V-nb)=nRT$ is :

(1) $kg\ m\ s^{-2}$ (2) $dm^3\ mol^{-1}$

(3) $kg\ m\ s^{-1}$ (4) $atm\ dm^6\ mol^{-2}$

Official Ans. by NTA (4)

Sol. $\frac{an^2}{V^2}=atm \Rightarrow a = atm \times \frac{dm^6}{mol^2}$

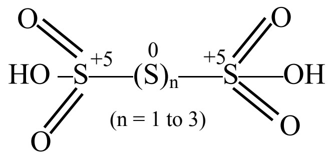
19. In polythionic acid, $H_2S_xO_6$(x = 3 to 5) the oxidation state(s) of sulphur is/are :

(1) + 5 only (2) + 6 only

(3) + 3 and + 5 only (4) 0 and + 5 only

Official Ans. by NTA (4)

Sol.



20. Tyndall effect is more effectively shown by :

(1) true solution (2) lyophilic colloid

(3) lyophobic colloid (4) suspension

Official Ans. by NTA (3)

Sol. Tyndall effect is observed in lyophobic colloids

SECTION-B

1. In Carius method for estimation of halogens, 0.2 g of an organic compound gave 0.188 g of AgBr. The percentage of bromine in the compound is _______. (Nearest integer)

[Atomic mass : Ag = 108, Br = 80]

Official Ans. by NTA (40)

Sol. $n_{AgBr}=\frac{0.188g}{188g/mol}=10^{-3}mol$

$\Rightarrow n_{Br}=n_{AgBr}=0.001$ mol

$\Rightarrow mass_{Br}=(0.001\times 80)$ gm $=0.08$ gm

$\Rightarrow$ mass % $=\frac{0.08\times 100}{0.2}=40\%$

2. The reaction that occurs in a breath analyser, a device used to determine the alcohol level in a person's blood stream is

$2K_2Cr_2O_7 + 8H_2SO_4 + 3C_2H_6O \rightarrow 2Cr_2(SO_4)_3 + 3C_2H_4O_2 + 2K_2SO_4 + 11H_2O$

If the rate of appearance of $Cr_2(SO_4)_3$ is 2.67 mol min^{-1} at a particular time, the rate of disappearance of C_2H_6O at the same time is _______ mol min^{-1}. (Nearest integer)

Official Ans. by NTA (4)

Sol. $\left(\frac{\text{Rate of disappearance of } C_2H_6O}{3}\right)$

$= \left(\frac{\text{Rate of appearance of } Cr_2(SO_4)_3}{2}\right)$

$\Rightarrow \left(\frac{2.67 \text{mol}/\text{min} \times 3}{2}\right)$ = rate of disappearance of C_2H_6O.

$\Rightarrow$ Rate of disappearance of C_2H_6O = 4.005 mol/min

3. The kinetic energy of an electron in the second Bohr orbit of a hydrogen atom is equal to $\frac{h^2}{xma_0^2}$. The value of 10x is _______ . (a_0 is radius of Bohr's orbit) (Nearest integer) [Given : $\pi = 3.14$]

Official Ans. by NTA (3155)

Sol. $mvr = \frac{nh}{2\pi}$

$$K.E. = \frac{n^2h^2}{8\pi^2mr^2} = \frac{4h^2}{8\pi^2m(4a_0)^2}$$

$$= \left(\frac{4}{8\pi^2 \times 16}\right)\frac{h^2}{ma_0^2}$$

$\Rightarrow x = 315.507$

$\Rightarrow 10x = 3155$ (nearest integer)

4. 1 kg of 0.75 molal aqueous solution of sucrose can be cooled up to –4°C before freezing. The amount of ice (in g) that will be separated out is _______ . (Nearest integer)

[Given : $K_f(H_2O) = 1.86$ K kg mol^{-1}]

Official Ans. by NTA (518)

Sol. Let mass of water initially present = x gm

$\Rightarrow$ Mass of sucrose = (1000 - x) gm

$\Rightarrow$ moles of sucrose = $\left(\frac{1000-x}{342}\right)$

$$\Rightarrow 0.75 = \frac{\left(\frac{1000-x}{342}\right)}{\left(\frac{x}{1000}\right)} \Rightarrow \frac{x}{1000} = \frac{1000-x}{342 \times 0.75}$$

$\Rightarrow 256.5\ x = 10^6 - 1000x$

$\Rightarrow x = 795.86$ gm

$\Rightarrow$ moles of sucrose = 0.5969

New mass of H_2O = a kg

$\Rightarrow 4 = \frac{0.5969}{a} \times 1.86 \Rightarrow a = 0.2775$ kg

$\Rightarrow$ ice separated = (795.86 – 277.5) = 518.3 gm

5. 1 mol of an octahedral metal complex with formula $MCl_3 \cdot 2L$ on reaction with excess of $AgNO_3$ gives 1 mol of AgCl. The denticity of Ligand L is _______ . (Integer answer)

Official Ans. by NTA (2)

Sol. $MCl_3.2L$ octahedral

$\underset{1\text{mole}}{MCl_3.2L} \xrightarrow{\text{Ex.}AgNO_3} 1 \text{ mole of } AgCl$

Its means that one Cl^- ion present in ionization sphere.

$\therefore$ formula = $[MCl_2L_2]Cl$

For octahedral complex coordination no. is 6

$\therefore$ L act as bidentate ligand

6. The number of moles of CuO, that will be utilized in Dumas method for estimation nitrogen in a sample of 57.5g of N, N-dimethylaminopentane is _______ × 10^{-2}. (Nearest integer)

Official Ans. by NTA (1125)

Sol. Moles of N in N,N - dimethylaminopentane

$= \left(\frac{57.5}{115}\right) = 0.5 \text{mol}$

$\Rightarrow C_7H_{17}N + \frac{45}{2}CuO \rightarrow 7CO_2 + \frac{17}{2}H_2O + \frac{1}{2}N_2 + \frac{45}{2}Cu$

$$\frac{n_{CuO}\text{ reacted}}{\left(\frac{45}{2}\right)} = \frac{n_{C_7H_{17}N}\text{ reacted}}{1}$$

$$\Rightarrow n_{CuO}\text{ reacted} = \left(\frac{45}{2}\right)\times 0.5 = 11.25$$

7. The number of f electrons in the ground state electronic configuration of Np (Z = 93) is _______. (Nearest integer)

Official Ans. by NTA (4)

Allen Ans. (18)

Sol. Np = $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}\ 4p^6\ 5s^2\ 4d^{10}\ 5p^6\ 6s^2$ $\boxed{4f^{14}}\ 5d^{10}\ 6p^6\ 7s^2\ \boxed{5f^4}\ 6d^1$

Total no. of 'f' electron = $14\ e^- + 4e^- = 18$

8. 200 mL of 0.2 M HCl is mixed with 300 mL of 0.1 M NaOH. The molar heat of neutralization of this reaction is –57.1 kJ. The increase in temperature in °C of the system on mixing is x × 10^{-2}. The value of x is _______ . (Nearest integer)

[Given : Specific heat of water = 4.18 J g^{-1} K^{-1}

Density of water = 1.00 g cm^{-3}]

(Assume no volume change on mixing)

Official Ans. by NTA (82)

Sol. ⇒ Millimoles of HCl = 200 × 0.2 = 40

⇒ Millimoles of NaOH = 300 × 0.1 = 30

$$\Rightarrow \text{Heat released} = \left(\frac{30}{1000}\times 57.1\times 1000\right) = 1713\text{ J}$$

⇒ Mass of solution = 500 ml × 1 gm/ml = 500 gm

$$\Rightarrow \Delta T = \frac{q}{m\times C} = \frac{1713\text{J}}{500\text{g}\times 4.18\frac{\text{J}}{\text{g}-\text{K}}} = 0.8196\text{K}$$

= 81.96 × 10^{-2} K

9. The number of moles of NH_3, that must be added to 2 L of 0.80 M $AgNO_3$ in order to reduce the concentration of Ag^+ ions to 5.0 × 10^{-8} M ($K_{formation}$ for $[Ag(NH_3)_2]^+$ = 1.0 × 10^8) is _______ . (Nearest integer)

[Assume no volume change on adding NH_3]

Official Ans. by NTA (4)

Sol. Let moles added = a

$$Ag^+_{(aq.)} + 2NH_{3(aq.)} \rightleftharpoons Ag(NH_3)^+_{2(aq.)}$$

t = 0	0.8	$\left(\frac{a}{2}\right)$	
t = ∞	5 × 10^{-8}	$\left(\frac{a}{2}-1.6\right)$	0.8

$$\frac{0.8}{(5\times10^{-8})\left(\frac{a}{2}-1.6\right)^2} = 10^8$$

$$\Rightarrow \frac{a}{2}-1.6 = 0.4 \Rightarrow a = 4$$

10. When 10 mL of an aqueous solution of $KMnO_4$ was titrated in acidic medium, equal volume of 0.1 M of an aqueous solution of ferrous sulphate was required for complete discharge of colour. The strength of $KMnO_4$ in grams per litre is _______ × 10^{-2}. (Nearest integer)

[Atomic mass of K = 39, Mn = 55, O = 16]

Official Ans. by NTA (316)

Sol. Let molarity of $KMnO_4$ = x

$KMnO_4 + FeSO_4 \rightarrow Fe_2(SO_4)_3 + Mn^{2+}$

n = 5   n = 1

(Equivalents of $KMnO_4$ reacted) = (Equivalents of $FeSO_4$ reacted)

⇒ (5 × x × 10 ml) = 1 × 0.1 × 10 ml

⇒ x = 0.02 M

Molar mass of $KMnO_4$ = 158 gm/mol

⇒ Strength = (x × 158) = 3.16 g/ℓ

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Friday 27th August, 2021) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS | TEST PAPER WITH SOLUTION

SECTION-A

1. If $0 < x < 1$, then $\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + ...$, is equal to :

(1) $x\left(\frac{1+x}{1-x}\right) + \log_e(1-x)$

(2) $x\left(\frac{1-x}{1+x}\right) + \log_e(1-x)$

(3) $\frac{1-x}{1+x} + \log_e(1-x)$

(4) $\frac{1+x}{1-x} + \log_e(1-x)$

Official Ans. by NTA (1)

Sol. Let $t = \frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + ...\infty$

$$= \left(2 - \frac{1}{2}\right)x^2 + \left(2 - \frac{1}{3}\right)x^3 + \left(2 - \frac{1}{4}\right)x^4 + ...\infty$$

$$= 2\left(x^2 + x^3 + x^4 + ...\infty\right) - \left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + ...\infty\right)$$

$$= \frac{2x^2}{1-x} - \left(\ell n(1-x) - x\right)$$

$$\Rightarrow t = \frac{2x^2}{1-x} + x - \ell n(1-x)$$

$$\Rightarrow t = \frac{x(1+x)}{1-x} - \ell n(1-x)$$

2. If for $x, y \in \mathbf{R}$, $x > 0$,

$y = \log_{10}x + \log_{10}x^{1/3} + \log_{10}x^{1/9} + ...$ upto ∞ terms

and $\frac{2+4+6+....+2y}{3+6+9+....+3y} = \frac{4}{\log_{10} x}$, then the ordered pair (x, y) is equal to :

(1) $(10^6, 6)$ (2) $(10^4, 6)$

(3) $(10^2, 3)$ (4) $(10^6, 9)$

Official Ans. by NTA (4)

Sol. $\frac{2(1+2+3+...+y)}{3(1+2+3+...+y)} = \frac{4}{\log_{10} x}$

$\Rightarrow \log_{10}x = 6 \Rightarrow x = 10^6$

Now,

$$y = (\log_{10} x) + \left(\log_{10} x^{\frac{1}{3}}\right) + \left(\log_{10} x^{\frac{1}{9}}\right) + ..\infty$$

$$= \left(1 + \frac{1}{3} + \frac{1}{9} + ...\infty\right)\log_{10} x$$

$$= \left(\frac{1}{1-\frac{1}{3}}\right)\log_{10} x = 9$$

So, $(x,y) = (10^6, 9)$

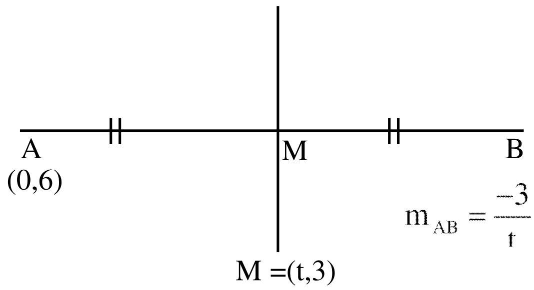
3. Let A be a fixed point (0, 6) and B be a moving point (2t, 0). Let M be the mid-point of AB and the perpendicular bisector of AB meets the y-axis at C. The locus of the mid-point P of MC is :

(1) $3x^2 - 2y - 6 = 0$ (2) $3x^2 + 2y - 6 = 0$

(3) $2x^2 + 3y - 9 = 0$ (4) $2x^2 - 3y + 9 = 0$

Official Ans. by NTA (3)

Sol. A(0,6) and B(2t,0)



Perpendicular bisector of AB is

$$(y-3) = \frac{t}{3}(x-t)$$

So, $C = \left(0, 3 - \frac{t^2}{3}\right)$

Let P be (h,k)

$$h = \frac{t}{2}; k = \left(3 - \frac{t^2}{6}\right)$$

$$\Rightarrow k = 3 - \frac{4h^2}{6} \Rightarrow 2x^2 + 3y - 9 = 0 \text{ option (3)}$$

4. If $(\sin^{-1}x)^2 - (\cos^{-1}x)^2 = a$; $0 < x < 1$, $a \neq 0$, then the value of $2x^2 - 1$ is :

(1) $\cos\left(\frac{4a}{\pi}\right)$ (2) $\sin\left(\frac{2a}{\pi}\right)$

(3) $\cos\left(\frac{2a}{\pi}\right)$ (4) $\sin\left(\frac{4a}{\pi}\right)$

Official Ans. by NTA (2)

Sol. Given $a = (\sin^{-1}x)^2 - (\cos^{-1}x)^2$

$= (\sin^{-1}x + \cos^{-1}x)(\sin^{-1}x - \cos^{-1}x)$

$= \frac{\pi}{2}\left(\frac{\pi}{2} - 2\cos^{-1}x\right)$

$\Rightarrow 2\cos^{-1}x = \frac{\pi}{2} - \frac{2a}{\pi}$

$\Rightarrow \cos^{-1}(2x^2 - 1) = \frac{\pi}{2} - \frac{2a}{\pi}$

$\Rightarrow 2x^2 - 1 = \cos\left(\frac{\pi}{2} - \frac{2a}{\pi}\right)$ option (2)

5. If the matrix $A = \begin{pmatrix} 0 & 2 \\ K & -1 \end{pmatrix}$ satisfies $A(A^3 + 3I) = 2I$, then the value of K is :

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) -1 (4) 1

Official Ans. by NTA (1)

Sol. Given matrix $A = \begin{bmatrix} 0 & 2 \\ k & -1 \end{bmatrix}$

$A^4 + 3IA = 2I$

$\Rightarrow A^4 = 2I - 3A$

Also characteristic equation of A is

$|A - \lambda I| = 0$

$\Rightarrow \begin{vmatrix} 0-\lambda & 2 \\ k & -1-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda + \lambda^2 - 2k = 0$

$\Rightarrow A + A^2 = 2K.I$

$\Rightarrow A^2 = 2KI - A$

$\Rightarrow A^4 = 4K^2I + A^2 - 4AK$

Put $A^2 = 2KI - A$

and $A^4 = 2I - 3A$

$2I - 3A = 4K^2I + 2KI - A - 4AK$

$\Rightarrow I(2 - 2K - 4K^2) = A(2 - 4K)$

$\Rightarrow -2I(2K^2 + K - 1) = 2A(1 - 2K)$

$\Rightarrow -2I(2K - 1)(K + 1) = 2A(1 - 2K)$

$\Rightarrow (2K - 1)(2A) - 2I(2K - 1)(K + 1) = 0$

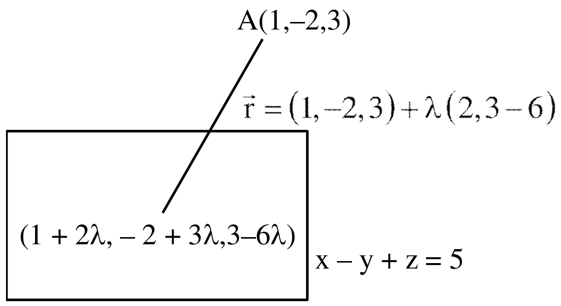
$\Rightarrow (2K - 1)[2A - 2I(K + 1)] = 0$

$\Rightarrow K = \frac{1}{2}$

6. The distance of the point (1, –2, 3) from the plane $x - y + z = 5$ measured parallel to a line, whose direction ratios are 2, 3, –6 is :

(1) 3 (2) 5 (3) 2 (4) 1

Official Ans. by NTA (4)



Sol.

$(1 + 2\lambda) + 2 - 3\lambda + 3 - 6\lambda = 5$

$\Rightarrow 6 - 7\lambda = 5 \Rightarrow \lambda = \frac{1}{7}$

so, $P = \left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7}\right)$

$AP = \sqrt{\left(1 - \frac{9}{7}\right)^2 + \left(-2 + \frac{11}{7}\right)^2 + \left(3 - \frac{15}{7}\right)^2}$

$AP = \sqrt{\left(\frac{4}{49}\right) + \frac{9}{49} + \frac{36}{49}} = 1$

7. If $S = \left\{z \in \mathbb{C} : \frac{z - i}{z + 2i} \in \mathbb{R}\right\}$, then :

(1) S contains exactly two elements

(2) S contains only one element

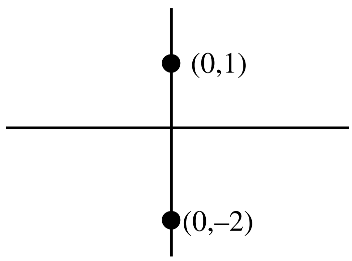
(3) S is a circle in the complex plane

(4) S is a straight line in the complex plane

Official Ans. by NTA (4)

Sol. Given $\frac{z - i}{z + 2i} \in R$

Then $\arg\left(\frac{z - i}{z + 2i}\right)$ is 0 or Π



$\Rightarrow$ S is straight line in complex

8. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx}$ = 2 (y + 2 sin x – 5) x – 2 cosx such that y(0) = 7. Then y(π) is equal to :

(1) $2e^{\pi^2}+5$ (2) $e^{\pi^2}+5$

(3) $3e^{\pi^2}+5$ (4) $7e^{\pi^2}+5$

Official Ans. by NTA (1)

Sol. $\frac{dy}{dx}-2xy=2(2\sin x-5)x-2\cos x$

$IF=e^{-x^2}$

so, $y.e^{-x^2}=\int e^{-x^2}(2x(2\sin x-5)-2\cos x)dx$

$\Rightarrow y.e^{-x^2}=e^{-x^2}(5-2\sin x)+c$

$\Rightarrow y=5-2\sin x+c.e^{x^2}$

Given at x = 0,y = 7

$\Rightarrow 7=5+c\Rightarrow c=2$

So, $y=5-2\sin x+2e^{x^2}$

Now at x = π,

$y=5+2e^{\pi^2}$

9. Equation of a plane at a distance $\sqrt{\frac{2}{21}}$ from the origin, which contains the line of intersection of the planes x – y – z – 1 = 0 and 2x + y – 3z + 4 = 0, is :

(1) 3x – y – 5z + 2 = 0 (2) 3x – 4z + 3 = 0

(3) –x + 2y + 2z – 3 = 0 (4) 4x – y – 5z + 2 = 0

Official Ans. by NTA (4)

Sol. Required equation of plane

$P_1+\lambda P_2=0$

(x – y – z – 1) + λ(2x + y – 3z + 4) = 0

Given that its dist. From origin is $\frac{2}{\sqrt{21}}$

Thus $\frac{|4\lambda-1|}{\sqrt{(2\lambda+1)^2+(\lambda-1)^2+(-3\lambda-1)^2}}=\frac{\sqrt{2}}{\sqrt{21}}$

$\Rightarrow 21(4\lambda-1)^2=2(14\lambda^2+8\lambda+3)$

$\Rightarrow 336\lambda^2-168\lambda+21=28\lambda^2+16\lambda+6$

$\Rightarrow 308\lambda^2-184\lambda+15=0$

$\Rightarrow 308\lambda^2-154\lambda-30\lambda+15=0$

$\Rightarrow (2\lambda-1)(154\lambda-15)=0$

$\Rightarrow \lambda=\frac{1}{2}$ or $\frac{15}{154}$

for $\lambda=\frac{1}{2}$ reqd. plane is

4x – y – 5z + 2 = 0

10. If $U_n=\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)^2\ldots\left(1+\frac{n^2}{n^2}\right)^n$, then $\lim_{n\to\infty}(U_n)^{\frac{-4}{n^2}}$ is equal to :

(1) $\frac{e^2}{16}$ (2) $\frac{4}{e}$ (3) $\frac{16}{e^2}$ (4) $\frac{4}{e^2}$

Official Ans. by NTA (1)

Sol. $U_n=\prod_{r=1}^{n}\left(1+\frac{r^2}{n^2}\right)^r$

$L=\lim_{n\to\infty}(U_n)^{-4/n^2}$

$\log L=\lim_{n\to\infty}\frac{-4}{n^2}\sum_{r=1}^{n}\log\left(1+\frac{r^2}{n^2}\right)^r$

$\Rightarrow \log L=\lim_{n\to\infty}\sum_{r=1}^{n}-\frac{4r}{n}.\frac{1}{n}\log\left(1+\frac{r^2}{n^2}\right)$

$\Rightarrow \log L\Rightarrow -4\int_0^1 x\log(1+x^2)dx$

put $1+x^2=t$

Now, 2xdx = dt

$=-2\int_1^2\log(t)dt=-2[t\log t-t]_1^2$

$\Rightarrow \log L=-2(2\log 2-1)$

$\therefore L=e^{-2(2\log 2-1)}$

$=e^{-2\left(\log\left(\frac{4}{e}\right)\right)}$

$=e^{\log\left(\frac{4}{e}\right)^{-2}}$

$=\left(\frac{e}{4}\right)^2=\frac{e^2}{16}$

11. The statement $(p\wedge(p\to q)\wedge(q\to r))\to r$ is :

(1) a tautology

(2) equivalent to $p\to\sim r$

(3) a fallacy

(4) equivalent to $q\to\sim r$

Official Ans. by NTA (1)

Sol. $(p\wedge(p\to q)\wedge(q\to r))\to r$

$\equiv(p\wedge(\sim p\vee q)\vee(\sim q\vee r))\to r$

$\equiv((p\wedge q)\wedge(\sim p\vee r))\to r$

$\equiv(p\wedge q\wedge r)\to r$

$\equiv\sim(p\wedge q\wedge r)\vee r$

$\equiv(\sim p)\vee(\sim q)\vee(\sim r)\vee r$

$\Rightarrow$ tautology

12. Let us consider a curve, $y = f(x)$ passing through the point $(-2, 2)$ and the slope of the tangent to the curve at any point $(x, f(x))$ is given by $f(x) + xf'(x) = x^2$. Then :

(1) $x^2 + 2xf(x) - 12 = 0$

(2) $x^3 + xf(x) + 12 = 0$

(3) $x^3 - 3xf(x) - 4 = 0$

(4) $x^2 + 2xf(x) + 4 = 0$

Official Ans. by NTA (3)

Sol. $y + \frac{xdy}{dx} = x^2$ (given)

$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = x$

If $= e^{\int \frac{1}{x}dx} = x$

Solution of DE

$\Rightarrow y.x = \int x.x\ dx$

$\Rightarrow xy = \frac{x^3}{3} + \frac{c}{3}$

Passes through $(-2,2)$, so

$-12 = -8 + c \Rightarrow c = -4$

$\therefore\ 3xy = x^3 - 4$

ie. $3x.f(x) = x^3 - 4$

13. $\sum_{k=0}^{20} \left({}^{20}C_k\right)^2$ is equal to :

(1) ${}^{40}C_{21}$ (2) ${}^{40}C_{19}$ (3) ${}^{40}C_{20}$ (4) ${}^{41}C_{20}$

Official Ans. by NTA (3)

Sol. $\sum_{k=0}^{20} {}^{20}C_k . {}^{20}C_{20-k}$

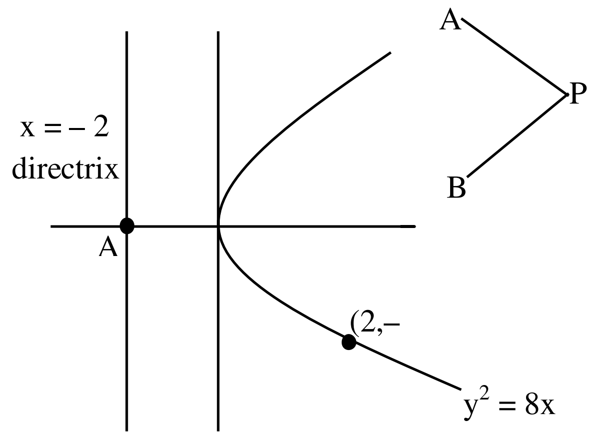
sum of suffix is const. so summation will be ${}^{40}C_{20}$

14. A tangent and a normal are drawn at the point $P(2, -4)$ on the parabola $y^2 = 8x$, which meet the directrix of the parabola at the points A and B respectively. If $Q(a, b)$ is a point such that AQBP is a square, then $2a + b$ is equal to :

(1) –16 (2) –18 (3) –12 (4) –20

Official Ans. by NTA (1)

Sol.



Equation of tangent at $(2,-4)$ $(T = 0)$

$-4y = 4(x + 2)$

$x + y + 2 = 0 \quad \ldots(1)$

equation of normal

$x - y + \lambda = 0$

$\downarrow(2,-4)$

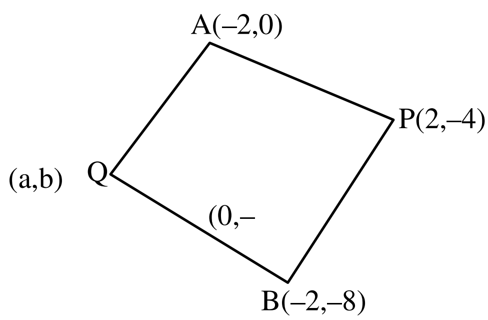
$\lambda = -6$

thus $x - y = 6$ …(2) equation of normal

POI of (1) & $x = -2$ is $A(-2,0)$

POI of (2) & $x = -2$ is $A(-2,8)$

Given AQBP is a sq.



$\Rightarrow m_{AQ}.m_{AP} = -1$

$\Rightarrow \left(\frac{b}{a+2}\right)\left(\frac{4}{-4}\right) = -1 \Rightarrow a + 2 = b \quad \ldots(1)$

Also PQ must be parallel to x-axis thus

$\Rightarrow b = -4$

$\therefore\ a = -6$

Thus $2a + b = -16$

15. Let $\frac{\sin A}{\sin B}=\frac{\sin(A-C)}{\sin(C-B)}$, where A, B, C are angles of a triangle ABC. If the lengths of the sides opposite these angles are a, b, c respectively, then :

(1) $b^2 - a^2 = a^2 + c^2$

(2) b^2, c^2, a^2 are in A.P.

(3) c^2, a^2, b^2 are in A.P.

(4) a^2, b^2, c^2 are in A.P.

Official Ans. by NTA (2)

Sol. $\frac{\sin A}{\sin B}=\frac{\sin(A-C)}{\sin(C-B)}$

As A,B,C are angles of triangle

$A + B + C = \pi$

$A = \pi - (B + C)$

So, $\sin A = \sin(B + C)$ ...(1)

Similarly $\sin B = \sin(A + C)$ ...(2)

From (1) and (2)

$\frac{\sin(B+C)}{\sin(A+C)}=\frac{\sin(A-C)}{\sin(C-B)}$

$\sin(C + B).\ \sin(C - B) = \sin(A - C)\sin(A + C)$

$\sin^2 C-\sin^2 B=\sin^2 A-\sin^2 C$

$\{\because \sin(x+y)\sin(x-y)=\sin^2 x-\sin^2 y\}$

$2\sin^2 C=\sin^2 A+\sin^2 B$

By sine rule

$2c^2 = a^2 + b^2$

$\Rightarrow b^2, c^2$ and a^2 are in A.P.

16. If α, β are the distinct roots of $x^2 + bx + c = 0$, then $\lim\limits_{x\to\beta}\frac{e^{2(x^2+bx+c)}-1-2(x^2+bx+c)}{(x-\beta)^2}$ is equal to:

(1) $b^2 + 4c$ (2) $2(b^2 + 4c)$

(3) $2(b^2 - 4c)$ (4) $b^2 - 4c$

Official Ans. by NTA (3)

Sol. $\lim\limits_{x\to\beta}\frac{e^{2(x^2+bx+c)}-1-2(x^2+bx+c)}{(x-\beta)^2}$

$\Rightarrow \lim\limits_{x\to\beta}\frac{1\left(1+\frac{2(x^2+bx+c)}{1!}+\frac{2^2(x^2+bx+c)^2}{2!}+...\right)-1-2(x^2+bx+c)}{(x-\beta)^2}$

$\Rightarrow \lim\limits_{x\to\beta}\frac{2(x^2+bx+1)^2}{(x-\beta)^2}$

$\Rightarrow \lim\limits_{x\to\beta}\frac{2(x-\alpha)^2(x-\beta)^2}{(x-\beta)^2}$

$\Rightarrow 2(\beta-\alpha)^2=2(b^2-4c)$

17. When a certain biased die is rolled, a particular face occurs with probability $\frac{1}{6}-x$ and its opposite face occurs with probability $\frac{1}{6}+x$. All other faces occur with probability $\frac{1}{6}$. Note that opposite faces sum to 7 in any die. If $0 < x < \frac{1}{6}$, and the probability of obtaining total sum = 7, when such a die is rolled twice, is $\frac{13}{96}$, then the value of x is:

(1) $\frac{1}{16}$ (2) $\frac{1}{8}$ (3) $\frac{1}{9}$ (4) $\frac{1}{12}$

Official Ans. by NTA (2)

Sol. Probability of obtaining total sum 7 = probability of getting opposite faces.

Probability of getting opposite faces

$=2\left[\left(\frac{1}{6}-x\right)\left(\frac{1}{6}+x\right)+\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}\right]$

$\Rightarrow 2\left[\left(\frac{1}{6}-x\right)\left(\frac{1}{6}+x\right)+\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}\right]=\frac{13}{96}$

(given)

$x=\frac{1}{8}$

18. If $x^2 + 9y^2 - 4x + 3 = 0$, $x, y \in \mathbb{R}$, then x and y respectively lie in the intervals:

(1) $\left[-\frac{1}{3},\frac{1}{3}\right]$ and $\left[-\frac{1}{3},\frac{1}{3}\right]$

(2) $\left[-\frac{1}{3},\frac{1}{3}\right]$ and [1, 3]

(3) [1, 3] and [1, 3]

(4) [1, 3] and $\left[-\frac{1}{3},\frac{1}{3}\right]$

Official Ans. by NTA (4)

Sol. $x^2 + 9y^2 - 4x + 3 = 0$

$(x^2 - 4x) + (9y^2) + 3 = 0$

$(x^2 - 4x + 4) + (9y^2) + 3 - 4 = 0$

$(x - 2)^2 + (3y)^2 = 1$

$\frac{(x-2)^2}{(1)^2}+\frac{y^2}{\left(\frac{1}{3}\right)^2}=1$ (equation of an ellipse).

As it is equation of an ellipse, x & y can vary inside the ellipse.

So, $x-2\in[-1,1]$ and $y\in\left[-\frac{1}{3},\frac{1}{3}\right]$

$x\in[1,3]$ $y\in\left[-\frac{1}{3},\frac{1}{3}\right]$

19. $\int_{6}^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e (x^2 - 44x + 484)} dx$ is equal to:

(1) 6 (2) 8

(3) 5 (4) 10

Official Ans. by NTA (3)

Sol. Let $I = \int_{6}^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e (x^2 - 44x + 484)} dx$

$$I = \int_{6}^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e (x-22)^2} dx \quad \ldots(1)$$

We know

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx \quad \text{(king)}$$

So $I = \int_{6}^{16} \frac{\log_e (22-x)^2}{\log_e (22-x)^2 + \log_e (22-(22-x))^2}$

$$I = \int_{0}^{16} \frac{\log_e (22-x)^2}{\log_e x^2 + \log_e (22-x)^2} dx \quad \ldots(2)$$

(1) + (2)

$$2I = \int_{6}^{16} 1.dx = 10$$

$I = 5$

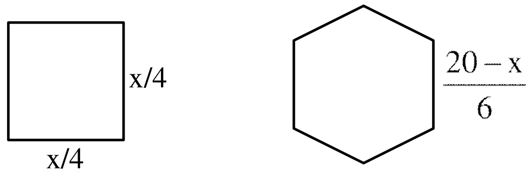
20. A wire of length 20 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a regular hexagon. Then the length of the side (in meters) of the hexagon, so that the combined area of the square and the hexagon is minimum, is:

(1) $\frac{5}{2+\sqrt{3}}$ (2) $\frac{10}{2+3\sqrt{3}}$

(3) $\frac{5}{3+\sqrt{3}}$ (4) $\frac{10}{3+2\sqrt{3}}$

Official Ans. by NTA (4)

Sol. Let the wire is cut into two pieces of length x and 20 – x.



Area of square $= \left(\frac{x}{4}\right)^2$ Area of regular hexagon

$$= 6 \times \frac{\sqrt{3}}{4}\left(\frac{20-x}{6}\right)^2$$

Total area $= A(x) = \frac{x^2}{16} + \frac{3\sqrt{3}}{2}\frac{(20-x)^2}{36}$

$$A'(x) = \frac{2x}{16} + \frac{3\sqrt{3} \times 2}{2 \times 36}(20-x)(-1)$$

$A'(x) = 0$ at $x = \frac{40\sqrt{3}}{3+2\sqrt{3}}$

Length of side of regular Hexagon $= \frac{1}{6}(20-x)$

$$= \frac{1}{6}\left(20 - \frac{4.\sqrt{3}}{3+2\sqrt{3}}\right)$$

$$= \frac{10}{2+2\sqrt{3}}$$

SECTION-B

1. Let $\vec{a} = \hat{i} + 5\hat{j} + \alpha\hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} + \beta\hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{j} - 3\hat{k}$ be three vectors such that, $|\vec{b} \times \vec{c}| = 5\sqrt{3}$ and $\vec{a}$ is perpendicular to $\vec{b}$. Then the greatest amongst the values of $|\vec{a}|^2$ is _______.

Official Ans. by NTA (90)

Sol. since, $\vec{a}.\vec{b} = 0$

$1 + 15 + \alpha\beta = 0 \Rightarrow \alpha\beta = -16 \quad \ldots(1)$

Also,

$$|\vec{b} \times \vec{c}|^2 = 75 \Rightarrow (10+\beta^2)14 - (5-3\beta)^2 = 75$$

$$\Rightarrow 5\beta^2 + 30\beta + 40 = 0$$

$$\Rightarrow \beta = -4, -2$$

$$\Rightarrow \alpha = 4, 8$$

$$\Rightarrow |\vec{a}|^2_{max} = (26 + \alpha^2)_{max} = 90$$

2. The number of distinct real roots of the equation $3x^4 + 4x^3 - 12x^2 + 4 = 0$ is ________.

Official Ans. by NTA (4)

Sol. $3x^4 + 4x^3 - 12x^2 + 4 = 0$

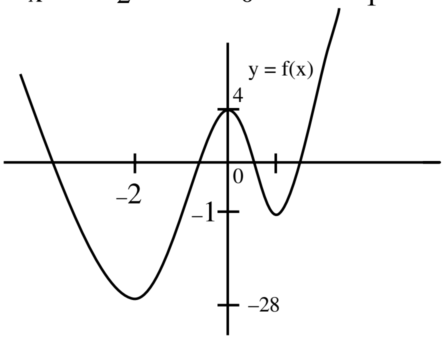
So, Let $f(x) = 3x^4 + 4x^3 - 12x^2 + 4$

$\therefore f'(x) = 12x(x^2 + x - 2)$

$= 12x\,(x + 2)\,(x - 1)$

f '(x) = –   +   –   +

x=   –2   0   1



3. Let the equation $x^2 + y^2 + px + (1 - p)y + 5 = 0$ represent circles of varying radius $r \in (0, 5]$. Then the number of elements in the set $S = \{q : q = p^2$ and q is an integer$\}$ is ________.

Official Ans. by NTA (61)

Sol. $r = \sqrt{\frac{p^2}{4} + \frac{(1-p)^2}{4} - 5} = \frac{\sqrt{2p^2 - 2p - 19}}{2}$

Since, $r \in (0, 5]$

So, $0 < 2p^2 - 2p - 19 \leq 100$

$\Rightarrow p \in \left[\frac{1-\sqrt{239}}{2}, \frac{1-\sqrt{39}}{2}\right) \cup \left(\frac{1+\sqrt{39}}{2}, \frac{1+\sqrt{239}}{2}\right]$ so, number of integral values of p^2 is 61

4. If $A = \{x \in \mathbf{R} : |x - 2| > 1\}$, $B = \{x \in \mathbf{R} : \sqrt{x^2 - 3} > 1\}$, $C = \{x \in \mathbf{R} : |x - 4| \geq 2\}$ and $\mathbf{Z}$ is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^c \cap \mathbf{Z}$ is ________.

Official Ans. by NTA (256)

Sol. $A = (-\infty, 1) \cup (3, \infty)$

$B = (-\infty, -2) \cup (2, \infty)$

$C = (-\infty, 2] \cup [6, \infty)$

So, $A \cap B \cap C = (-\infty, -2) \cup [6, \infty)$

$z \cap (A \cap B \cap C)' = \{-2, -1, 0, -1, 2, 3, 4, 5\}$

Hence no. of its subsets $= 2^8 = 256$.

5. If $\int \frac{dx}{(x^2 + x + 1)^2} = a \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + b\left(\frac{2x+1}{x^2+x+1}\right) + C$, $x > 0$ where C is the constant of integration, then the value of $9(\sqrt{3}a + b)$ is equal to ________.

Official Ans. by NTA (15)

Sol. $I = \int \frac{dx}{\left[\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}\right]^2}$

$\int \frac{dt}{\left(t^2 + \frac{3}{4}\right)^2} \quad \left(\text{Put } x + \frac{1}{2} = t\right)$

$= \frac{\sqrt{3}}{2} \int \frac{\sec^2\theta\, d\theta}{\frac{9}{16}\sec^4\theta} \quad \left(\text{Put } t = \frac{\sqrt{3}}{2}\tan\theta\right)$

$= \frac{4\sqrt{3}}{9} \int (1 + \cos 2\theta)\, d\theta$

$= \frac{4\sqrt{3}}{9}\left[\theta + \frac{\sin 2\theta}{2}\right] + c$

$= \frac{4\sqrt{3}}{9}\left[\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + \frac{\sqrt{3}(2x+1)}{3 + (2x+1)^2}\right] + c$

$= \frac{4\sqrt{3}}{9}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + \frac{1}{3}\left(\frac{2x+1}{x^2+x+1}\right) + c$

Hence, $9(\sqrt{3}a + b) = 15$

6. If the system of linear equations

$2x + y - z = 3$

$x - y - z = \alpha$

$3x + 3y + \beta z = 3$

has infinitely many solution, then $\alpha + \beta - \alpha\beta$ is equal to ________.

Official Ans. by NTA (5)

Sol. $2 \times (i) - (ii) - (iii)$ gives :

$-(1 + \beta)z = 3 - \alpha$

For infinitely many solution

$\beta + 1 = 0 = 3 - \alpha \Rightarrow (\alpha, \beta) = (3, -1)$

Hence, $\alpha + \beta - \alpha\beta = 5$

7. Let n be an odd natural number such that the variance of 1, 2, 3, 4, ..., n is 14. Then n is equal to ________.

Official Ans. by NTA (13)

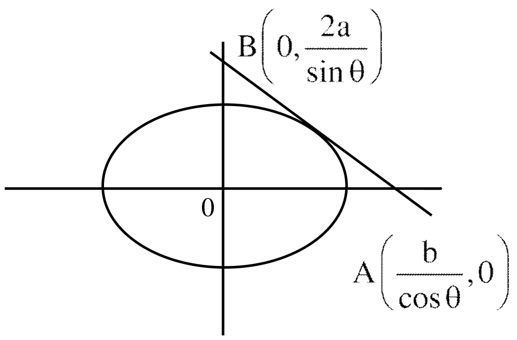
Sol. $\frac{n^2-1}{12} = 14 \Rightarrow n = 13$

8. If the minimum area of the triangle formed by a tangent to the ellipse $\frac{x^2}{b^2}+\frac{y^2}{4a^2}=1$ and the co-ordinate axis is kab, then k is equal to _______.

Official Ans. by NTA (2)

Sol. Tangent

$$\frac{x\cos\theta}{b}+\frac{y\sin\theta}{2a}=1$$



$$\text{So, area}(\Delta OAB) = \frac{1}{2}\times\frac{b}{\cos\theta}\times\frac{2a}{\sin\theta}$$

$$=\frac{2ab}{\sin 2\theta}\ge 2ab$$

$$\Rightarrow k = 2$$

9. A number is called a palindrome if it reads the same backward as well as forward. For example 285582 is a six digit palindrome. The number of six digit palindromes, which are divisible by 55, is _________.

Official Ans. by NTA (100)

Sol.

5	a	b	b	a	5

It is always divisible by 5 and 11.

So, required number = 10 × 10 = 100

10. If $y^{1/4}+y^{-1/4}=2x$, and $(x^2-1)\frac{d^2y}{dx^2}+\alpha x\frac{dy}{dx}+\beta y=0$, then $|\alpha-\beta|$ is equal to __________.

Official Ans. by NTA (17)

Sol. $y^{\frac{1}{4}}+\frac{1}{y^{\frac{1}{4}}}=2x$

$$\Rightarrow\left(y^{\frac{1}{4}}\right)^2-2xy^{\left(\frac{1}{4}\right)}+1=0$$

$$\Rightarrow y^{\frac{1}{4}}=x+\sqrt{x^2-1}\ \text{or}\ x-\sqrt{x^2-1}$$

$$\text{So, }\frac{1}{4}\frac{1}{y^{\frac{3}{4}}}\frac{dy}{dx}=1+\frac{x}{\sqrt{x^2-1}}$$

$$\Rightarrow\frac{1}{4}\frac{1}{y^{3/4}}\frac{dy}{dx}=\frac{y^{\frac{1}{4}}}{\sqrt{x^2-1}}$$

$$\Rightarrow\frac{dy}{dx}=\frac{4y}{\sqrt{x^2-1}}\quad\ldots(1)$$

$$\text{Hence, }\frac{d^2y}{dx^2}=4\frac{\left(\sqrt{x^2-1}\right)y'-\frac{yx}{\sqrt{x^2-1}}}{x^2-1}$$

$$\Rightarrow(x^2-1)y''=4\frac{(x^2-1)y'-xy}{\sqrt{x^2-1}}$$

$$\Rightarrow(x^2-1)y''=4\left(\sqrt{x^2-1}y'-\frac{xy}{\sqrt{x^2-1}}\right)$$

$$\Rightarrow(x^2-1)y''=4\left(4y-\frac{xy'}{4}\right)\ \text{(from I)}$$

$$\Rightarrow(x^2-1)y''+xy'-16y=0$$

So, $|\alpha-\beta| = 17$