26^{th} Feb. 2021 | Shift - 1
PHYSICS

SECTION - A

1. If λ_1 and λ_2 are the wavelengths of the third member of Lyman and first member of the Paschen series respectively, then the value of $\lambda_1 : \lambda_2$ is :

(1) 1 : 3 (2) 1 : 9 (3) 7 : 135 (4) 7 : 108

Sol. **(3)**

For Lyman series

$n_1 = 1, \quad n_2 = 4$

$$\frac{1}{\lambda_1} = Rz^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda_1} = Rz^2\left(\frac{1}{1_1^2} - \frac{1}{4^2}\right)$$

$$\frac{1}{\lambda_1} = \frac{15Rz^2}{16}$$

$$\lambda_1 = \frac{16}{15Rz^2}$$

For paschen series

$n_1 = 3, \quad n_2 = 4$

$$\frac{1}{\lambda_2} = Rz^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$\frac{1}{\lambda_2} = Rz^2\left(\frac{16-9}{9\times16}\right)$$

$$\frac{1}{\lambda_2} = Rz^2\left(\frac{7}{9\times16}\right)$$

$$\lambda_2 = \frac{9\times16}{7Rz^2}$$

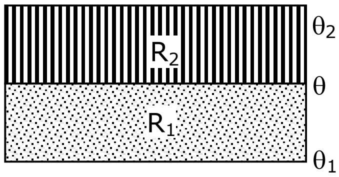
So,

$$\frac{\lambda_1}{\lambda_2} = \frac{\frac{16}{15Rz^2}}{\frac{9\times16}{7Rz^2}}$$

$$= \frac{16\times7}{15\times9\times16}$$

$$= \frac{7}{135}$$

2. The temperature θ at the junction of two insulating sheets, having thermal resistances R_1 and R_2 as well as top and bottom temperatures θ_1 and θ_2 (as shown in figure) is given by :



(1) $\frac{\theta_1 R_2 + \theta_2 R_1}{R_1 + R_2}$ (2) $\frac{\theta_1 R_2 - \theta_2 R_1}{R_2 - R_1}$ (3) $\frac{\theta_2 R_2 - \theta_1 R_1}{R_2 - R_1}$ (4) $\frac{\theta_1 R_1 + \theta_2 R_2}{R_1 + R_2}$

Sol. **(1)**

Temperature at the junction is θ.

so using the formula

$$\frac{T_2 - T}{R_1} = \frac{T - T_1}{R_2}$$

$$\frac{\theta_2 - \theta}{R_2} = \frac{\theta - \theta_1}{R_1}$$

$R_1(\theta_2 - \theta) = R_2(\theta - \theta_1)$

$R_1\theta_2 - R_1\theta = R_2\theta - R_2\theta_1$

$R_1\theta + R_2\theta = R_1\theta_2 + R_2\theta_1$

$$\theta = \frac{R_1\theta_2 + R_2\theta_1}{R_1 + R_2}$$

3. In a Young's double slit experiment two slits are separated by 2 mm and the screen is placed one meter away. When a light of wavelength 500 nm is used, the fringe separation will be :

(1) 0.75 mm (2) 0.50 mm (3) 1 mm (4) 0.25 mm

Sol. **(4)**

Fringe width $(\beta) = \frac{\lambda D}{d}$

$d = 2 \times 10^{-3}$m

$\lambda = 500 \times 10^{-9}$m

$D = 1$m

Now

$$\beta = \frac{500 \times 10^{-9} \times 1}{2 \times 10^{-3}}$$

$$\beta = \frac{5}{2} \times 10^{-4}$$

$\beta = 2.5 \times 10^{-4}$

$\beta = 0.25$ mm

4. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : An electron microscope can achieve better resolving power than an optical microscope.

Reason R : The de Broglie's wavelength of the electrons emitted from an electron gun is much less than wavelength of visible light.

In the light of the above statements, choose the correct answer from the options given below:

(1) A is true but R is false.

(2) Both A and R are true but R is NOT the correct explanation of A.

(3) Both A and R are true and R is the correct explanation of A.

(4) A is false but R is true.

Sol. **(3)**

Resolution limit $(\Delta\theta) = \dfrac{1.22\lambda}{d}$

Resolution power $= \dfrac{1}{\text{Resolution limit}}$

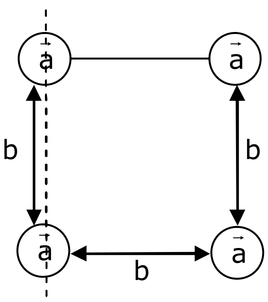
$\lambda \downarrow \Delta\theta \downarrow$

$\Delta\theta \downarrow$ Power $\uparrow$

5. Four identical solid spheres each of mass 'm' and radius 'a' are placed with their centres on the four corners of a square of side 'b'. The moment of inertia of the system about one side of square where the axis of rotation is parallel to the plane of the square is :

(1) $\dfrac{4}{5}ma^2$ (2) $\dfrac{8}{5}ma^2 + mb^2$ (3) $\dfrac{4}{5}ma^2 + 2mb^2$ (4) $\dfrac{8}{5}ma^2 + 2mb^2$

Sol. **(4)**



$$I = \frac{2}{5}ma^2 + \frac{2}{5}ma^2 + \left[\frac{2}{5}ma^2 + mb^2\right] + \frac{2}{5}ma^2 + mb^2$$

$$I = 4 \times \frac{2}{5}ma^2 + 2mb^2$$

$$= \frac{8}{5}ma^2 + 2mb^2$$

6. The normal density of a material is ρ and its bulk modulus of elasticity is K. The magnitude of increase in density of material, when a pressure P is applied uniformly on all sides, will be :

(1) $\frac{\rho K}{P}$ (2) $\frac{K}{\rho P}$ (3) $\frac{PK}{\rho}$ (4) $\frac{\rho P}{K}$

Sol. **(4)**

Bulk modulus $K = \frac{-\Delta P}{\frac{\Delta v}{v}} = \frac{-\Delta pv}{\Delta v}$

We know, $\rho = \frac{M}{V}$

So, $\frac{-\Delta\rho}{\rho} = \frac{\Delta v}{v}$

$$K = \frac{-\Delta P}{\left(-\frac{\Delta\rho}{\rho}\right)} = \frac{\rho\Delta P}{\Delta\rho}$$

$$\Delta\rho = \frac{\rho\Delta P}{K}$$

$$\Delta\rho = \frac{\rho P}{K}$$

7. LED is constructed from Ga-As-P semiconducting material. The energy gap of this LED is 1.9 eV. Calculate the wavelength of light emitted and its colour.

[$h=6.63\times10^{-34}$ Js and $c=3\times10^{8}$ ms^{-1}]

(1) 654 nm and red colour (2) 1046 nm and blue colour

(3) 1046 nm and red colour (4) 654 nm and orange colour

Sol. **(1)**

We know that $E = \frac{hc}{\lambda}$

$$\lambda = \frac{hc}{E} \Rightarrow \frac{1240(\text{in eV})}{E\ (\text{in eV})}$$

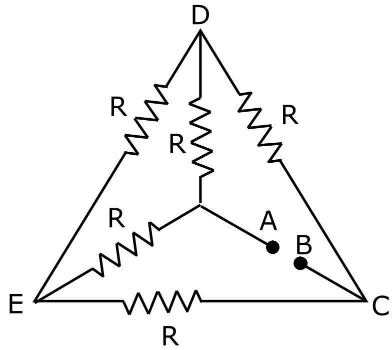
$$\lambda = \frac{1240}{1.9}$$

$= 652.63$ nm ≈ 654 nm

Wavelength of red light is 620 nm to 750 nm

So, answer is 1.

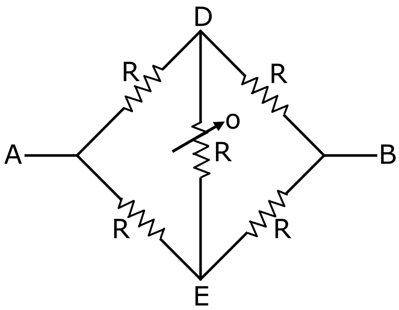
8. Five equal resistances are connected in a network as shown in figure. The net resistance between the points A and B is :



(1) $\frac{3R}{2}$ (2) $\frac{R}{2}$ (3) R (4) 2R

Sol. **(3)**

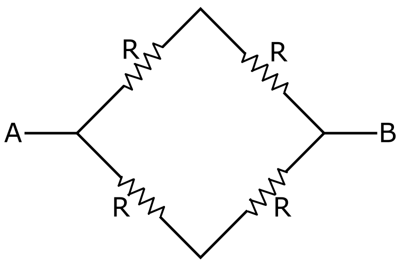
It is balanced wheat stone bridge

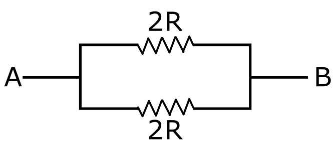


So, we know that

$R_1R_4 = R_2R_3$

$\frac{R_1}{R_2} = \frac{R_3}{R_4}$

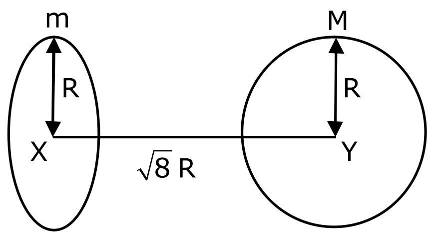




$$Req = \frac{2R \times 2R}{2R + 2R}$$

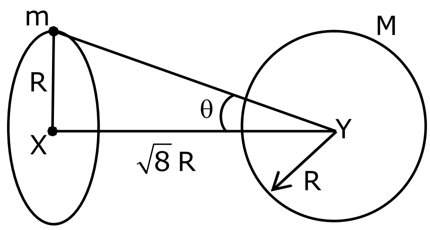
$$= \frac{4R^2}{4R} \Rightarrow R$$

9. Find the gravitational force of attraction between the ring and sphere as shown in the diagram, where the plane of the ring is perpendicular to the line joining the centres. If $\sqrt{8}R$ is the distance between the centres of a ring (of mass 'm') and a sphere (mass 'M') where both have equal radius 'R'.



(1) $\frac{\sqrt{8}}{9}\cdot\frac{GmM}{R}$ (2) $\frac{\sqrt{8}}{27}\cdot\frac{GmM}{R^2}$ (3) $\frac{2\sqrt{2}}{3}\cdot\frac{GMm}{R^2}$ (4) $\frac{1}{3\sqrt{8}}\cdot\frac{GMm}{R^2}$

Sol. **(2)**



We know that

$$F = ME = M\left(\frac{GM\sqrt{8}R}{\left(R^2+(\sqrt{8}R)^2\right)^{3/2}}\right)$$

$$F = \frac{GMm\sqrt{8}R}{\left(9R^2\right)^{3/2}} \Rightarrow \frac{2\sqrt{2}GmM}{\left(9R^2\right)^{3/2}}$$

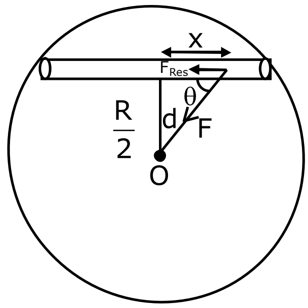
$$= \frac{2\sqrt{2}GmM}{27R^2}$$

$$F = \frac{\sqrt{8}GMm}{27R^2}$$

10. Assume that a tunnel is dug along a chord of the earth, at a perpendicular distance (R/2) from the earth's centre, where 'R' is the radius of the Earth. The wall of the tunnel is frictionless. If a particle is released in this tunnel, it will execute a simple harmonic motion with a time period :

(1) $2\pi\sqrt{\frac{R}{g}}$ (2) $\frac{1}{2\pi}\sqrt{\frac{g}{R}}$ (3) $\frac{2\pi R}{g}$ (4) $\frac{g}{2\pi R}$

Sol. **(1)**



$$\cos\theta = \frac{x}{d}$$

If displaced from equilibrium position,

$$F_{restoring} = \left(\frac{GMmd}{R^3}\right)\cos\theta$$

$$F_{Res.} = \frac{GMmd}{R^3}\cdot\frac{x}{d} = \frac{GMmx}{R^3}$$

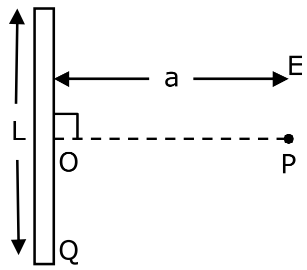
$$a_R = \frac{GMx}{R^3} \qquad GM_e = gR^2$$

$$T = 2\pi\sqrt{\left|\frac{x}{a}\right|}$$

$$T = 2\pi\sqrt{\frac{x}{\frac{GMx}{R^3}}}\sqrt{\frac{R^3}{gR^2}}$$

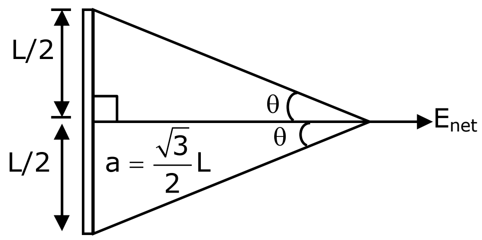
$$\boxed{T = 2\pi\sqrt{\frac{R}{g}}}$$

11. Find the electric field at point P (as shown in figure) on the perpendicular bisector of a uniformly charged thin wire of length L carrying a charge Q. The distance of the point P from the centre of the rod is $a = \frac{\sqrt{3}}{2}L$.



(1) $\frac{Q}{2\sqrt{3}\pi\varepsilon_0 L^2}$ (2) $\frac{\sqrt{3}Q}{4\pi\varepsilon_0 L^2}$ (3) $\frac{Q}{3\pi\varepsilon_0 L^2}$ (4) $\frac{Q}{4\pi\varepsilon_0 L^2}$

Sol. (1)



$$\tan\theta = \frac{L/2}{\frac{\sqrt{3}}{2}L} \Rightarrow \frac{1}{\sqrt{3}}$$

$$\theta = 30°$$

$$E_{net} = \frac{K\lambda}{\frac{\sqrt{3}}{2}L}(\sin 30° + \sin 30°) \Rightarrow \frac{2KQ}{\sqrt{3}L^2}\left(\frac{1}{2}+\frac{1}{2}\right)$$

$$E_{net} = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{\sqrt{3}L^2}$$

$$E_{net} = \frac{Q}{2\sqrt{3}\pi\varepsilon_0 L^2}$$

12. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

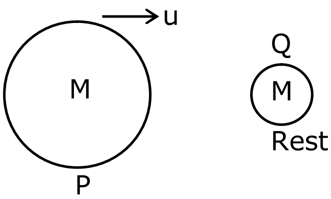
Assertion A : Body 'P' having mass M moving with speed 'u' has head-on collision elastically with another body 'Q' having mass 'm' initially at rest. If m<<M, body 'Q' will have a maximum speed equal to '2u' after collision.

Reason R : During elastic collision, the momentum and kinetic energy are both conserved.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) A is correct but R is not correct.
(2) Both A and R are correct but R is NOT the correct explanation of A.
(3) A is not correct but R is correct.
(4) Both A and R are correct and R is the correct explanation of A.

Sol. **(4)**



m << M

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

For elastic collision $\rightarrow$ e = 1

$$1 = \frac{v_2 - u}{u - 0}$$

$u = v_2 - u$

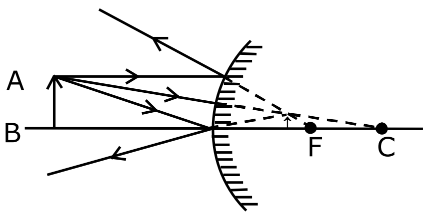
$\boxed{v_2 = 2u}$

In elastic collision kinetic energy & momentum are conserved.

13. A short straight object of height 100 cm lies before the central axis of a spherical mirror whose focal length has absolute value $|f|$ = 40 cm. The image of object produced by the mirror is of height 25 cm and has the same orientation of the object. One may conclude from the information :

(1) Image is real, same side of concave mirror.
(2) Image is virtual, opposite side of convex mirror.
(3) Image is virtual, opposite side of concave mirror.
(4) Image is real, same side of convex mirror.

Sol. **(2)**



Same orientation so image is virtual. It is combination of real object and virtual image using height it is possible only from convex mirror.

14. A particle is moving with uniform speed along the circumference of a circle of radius R under the action of a central fictitious force F which is inversely proportional to R^3. Its time period of revolution will be given by :

(1) $T \propto R^{\frac{5}{2}}$ (2) $T \propto R^2$ (3) $T \propto R^{\frac{4}{3}}$ (4) $T \propto R^{\frac{3}{2}}$

Sol. **(2)**

$$F \propto \frac{1}{R^3}$$

$$F = \frac{K}{R^3}$$

$$\frac{mv^2}{R} = \frac{K}{R^3}$$

$$m(\omega R)^2 = \frac{K}{R^2}$$

$$m\omega^2 R^2 = \frac{K}{R^2}$$

$$\omega^2 = \frac{K}{m}\left(\frac{1}{R^4}\right)$$

$$\left(\frac{2\pi}{T}\right)^2 \propto \frac{1}{R^4}$$

$$\frac{4\pi^2}{T^2} \propto \frac{1}{R^4}$$

$$\boxed{T \propto R^2}$$

15. A large number of water drops, each of radius r, combine to have a drop of radius R. If the surface tension is T and mechanical equivalent of heat is J, the rise in heat energy per unit volume will be :

(1) $\frac{2T}{rJ}$ (2) $\frac{3T}{rJ}$ (3) $\frac{2T}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$ (4) $\frac{3T}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$

Sol. **(4)**

R is the radius of bigger drop.
r is the radius of n water drops.
Water drops are combined to make bigger drop.
So,

Volume of n drops = volume of bigger drop

$$n\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$$

$$R = rn^{1/3} \Rightarrow n = \left(\frac{R}{r}\right)^3$$

$\Delta U = T$(Change in surface area)

$\Delta U = T(n4\pi r^2 - 4\pi R^2)$

$$\Delta U = 4\pi T\left[\left(\frac{R}{r}\right)^3 r^2 - R^2\right] \Rightarrow \frac{4\pi T\left(\frac{R^3}{r} - R^2\right)}{J}$$

$$\frac{\Delta U}{V} = \frac{4\pi T\left(\frac{R^3}{r} - R^2\right)}{J \times \frac{4}{3}\pi R^3} = \frac{3T}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$$

16. A planet revolving in elliptical orbit has :

A. a constant velocity of revolution.

B. has the least velocity when it is nearest to the sun.

C. its areal velocity is directly proportional to its velocity.

D. areal velocity is inversely proportional to its velocity.

E. to follow a trajectory such that the areal velocity is constant.

Choose the correct answer from the options given below :

(1) A only (2) E only (3) D only (4) C only

Sol. (2)

$$\boxed{\frac{d\vec{A}}{dt} = \frac{\vec{L}}{2m}}$$

17. An alternating current is given by the equation $i = i_1 \sin\omega t + i_2 \cos\omega t$. The rms current will be :

(1) $\frac{1}{2}\left(i_1^2 + i_2^2\right)^{\frac{1}{2}}$ (2) $\frac{1}{\sqrt{2}}\left(i_1^2 + i_2^2\right)^{\frac{1}{2}}$ (3) $\frac{1}{\sqrt{2}}\left(i_1 + i_2\right)^2$ (4) $\frac{1}{\sqrt{2}}\left(i_1 + i_2\right)$

Sol. (2)

$$I_0 = \sqrt{I_1^2 + I_2^2 + 2I_1I_2\cos\theta}$$

$$I_0 = \sqrt{I_1^2 + I_2^2 + 2I_1I_2\cos 90^\circ}$$

$$I_0 = \sqrt{I_1^2 + I_2^2 + 2I_1I_2(0)} \Rightarrow \sqrt{I_1^2 + I_2^2}$$

We , know that

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

So, $$I_{rms} = \frac{\sqrt{I_1^2 + I_2^2}}{\sqrt{2}}$$

18. Consider the combination of 2 capacitors C_1 and C_2, with $C_2 > C_1$, when connected in parallel, the equivalent capacitance is $\frac{15}{4}$ times the equivalent capacitance of the same connected in series. Calculate the ratio of capacitors, $\frac{C_2}{C_1}$.

(1) $\frac{15}{11}$ (2) $\frac{29}{15}$ (3) $\frac{15}{4}$ (4) $\frac{111}{80}$

Sol. Bonus

$$C_1 + C_2 = \frac{15}{4}\left(\frac{C_1C_2}{C_1 + C_2}\right)$$

$$4(C_1 + C_2)^2 = 15C_1C_2$$

$$4C_1^2 + 4C_2^2 - 7C_1C_2 = 0$$

$$4 + 4\left(\frac{C_2}{C_1}\right)^2 - 7\frac{C_2}{C_1} = 0$$

$$4\left(\frac{C_2}{C_1}\right)^2 - 7\frac{C_2}{C_1} + 4 = 0$$

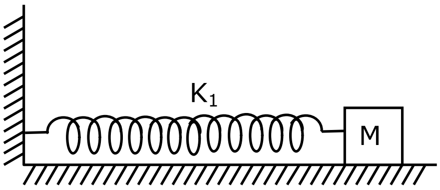
$\frac{C_2}{C_1}$ has not real value

$\frac{C_2}{C_1}$ = imaginary

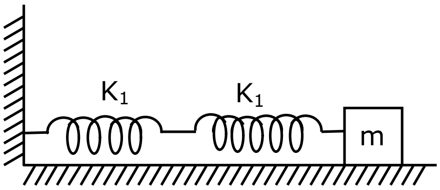
19. If two similar springs each of spring constant K_1 are joined in series, the new spring constant and time period would be changed by a factor :

(1) $\frac{1}{2}, \sqrt{2}$ (2) $\frac{1}{4}, 2\sqrt{2}$ (3) $\frac{1}{2}, 2\sqrt{2}$ (4) $\frac{1}{4}, \sqrt{2}$

Sol. **(1)**



$T = 2\pi\sqrt{\frac{m}{K_1}}$



$$K_{eq} = \frac{K_1 \times K_1}{K_1 + K_1} = \frac{K_1^2}{2K_1}$$

$$K_{eq} = \frac{K_1}{2}$$

$$T' = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2\pi\sqrt{\frac{m}{\frac{k_1}{2}}}$$

$$= \sqrt{2}T$$

20. In a typical combustion engine the workdone by a gas molecule is given by $W = \alpha^2\beta e^{\frac{-\beta x^2}{kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature. If α and β are constants, dimensions of α will be :

(1) $[M^0\ L\ T^0]$ (2) $[M^2\ L\ T^{-2}]$ (3) $[M\ L\ T^{-2}]$ (4) $[M\ L\ T^{-1}]$

Sol. **(1)**

$\frac{\beta x^2}{KT}$ is dimension less

so

$KT = \beta x^2 \quad \Rightarrow M^1L^2T^{-2}$

$\beta = \frac{M^1L^2T^{-2}}{L^2} \quad \Rightarrow M^1T^{-2}$

$M^1L^2T^{-2} = \alpha^2 M^1T^{-2}$

$\alpha^2 = L^2$

$\alpha = L$

$\alpha = M^0L^1T^0$

SECTION - B

1. The mass per unit length of a uniform wire is 0.135 g/cm. A transverse wave of the form $y=-0.21\sin(x+30t)$ is produced in it, where x is in meter and t is in second. Then, the expected value of tension in the wire is $x \times 10^{-2}$ N. Value of x is __________. (Round-off to the nearest integer)

Sol. **1215**

$y = -0.21\sin(x+30t)$

$$v = \frac{\omega}{K} = \frac{30}{1} = 30\text{m/s}$$

$$v = \sqrt{\frac{T}{\mu}}$$

$T = v^2 \times \mu$

$T = (30)^2 \times 0.135 \times 10^{-1}$ $\qquad \mu = 0.135\text{gm/cm}$

$T = 900 \times 0.135 \times 10^{-1}$ $\qquad \mu = 0.135 \times \dfrac{10^{-3}}{10^{-2}}\dfrac{\text{kg}}{\text{m}}$

$T = 12.15\text{N}$

$T = 1215 \times 10^{-2}$ N

$\boxed{x = 1215}$

2. A radiation is emitted by 1000 W bulb and it generates an electric field and magnetic field at P, placed at a distance of 2m. The efficiency of the bulb is 1.25%. The value of peak electric field at P is $x \times 10^{-1}$ V/m. Value of x is __________. (Rounded-off to the nearest integer)

[Take $\varepsilon_0 = 8.85\times10^{-12}\ C^2\ N^{-1}\ m^{-2}$, $c=3\times10^{8}\ ms^{-1}$]

Sol. **137**

Intensity of electro magnetic wave is,

$$I = \frac{1}{2}c\varepsilon_0 E_0^2 = \frac{P}{4\pi r^2}$$

$$\frac{1}{2}4\pi\varepsilon_0 \times c \times E_0^2 = \frac{P}{r^2}$$

$$\frac{1}{2} \times \frac{3\times10^5 \times E_0^2}{9\times10^9} = \frac{1000\times1.25}{(2)^2} \times \frac{1}{100}$$

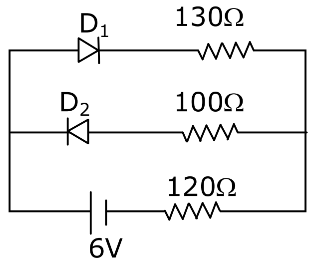
$$E_0^2 = \frac{60\times1000\times1.25}{4\times100} = \frac{125\times3}{2}$$

$$E_0^2 = \frac{375}{2} = 187.5$$

$\boxed{E_0 = 13.69}$

$E_0 \approx 137 \times 10^{-1}$ v/m

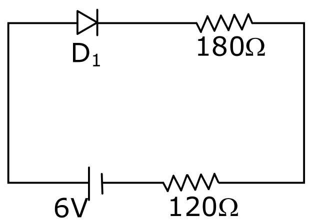
3. The circuit contains two diodes each with a forward resistance of 50Ω and with infinite reverse resistance. If the battery voltage is 6 V, the current through the 120 Ω resistance is __________ mA.



Sol. **20**

D_2 is reverse bias so current does not flow through D_2.

D_1 is forward bias.

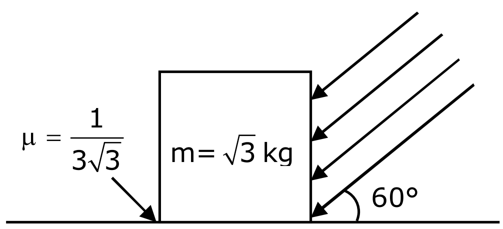


$I = \dfrac{6}{300} \qquad \Rightarrow 0.02\text{ A}$

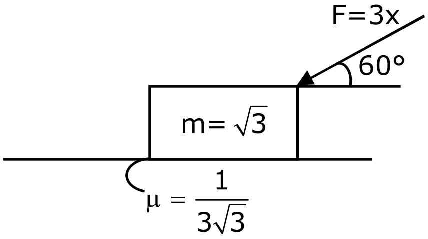
$= 20\text{ mA}$

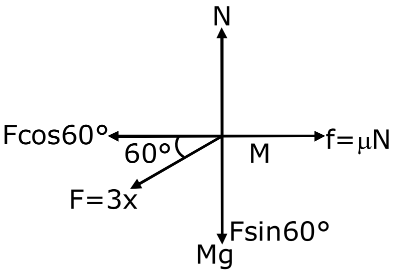
4. As shown in the figure, a block of mass $\sqrt{3}$ kg is kept on a horizontal rough surface of coefficient of friction $\dfrac{1}{3\sqrt{3}}$. The critical force to be applied on the vertical surface as shown at an angle 60° with horizontal such that it does not move, will be 3x. The value of x will be ________.

[$g=10\text{m/s}^2$; $\sin 60° = \dfrac{\sqrt{3}}{2}$; $\cos 60° = \dfrac{1}{2}$]



Sol. **3.33**





N = Mg + Fsin60°

$$N = \sqrt{3}g + \frac{F\sqrt{3}}{2}$$

For No slipping

Fcos60° = Friction

$$\frac{F}{2} = \mu N = \frac{1}{3\sqrt{3}}\left(\sqrt{3}g + \frac{F\sqrt{3}}{2}\right)$$

$$\frac{F}{2} = \frac{g}{3} + \frac{F}{6}$$

$$\frac{F}{2} - \frac{F}{6} = \frac{g}{3}$$

$$\frac{6F - 2F}{12} = \frac{g}{3}$$

4F = 4g

F = 10

F = 3x

$$x = \frac{F}{3} = \frac{10}{3} = 3.33$$

$$\boxed{x = 3.33}$$

5. In a series LCR resonant circuit, the quality factor is measured as 100. If the inductance is increased by two fold and resistance is decreased by two fold, then the quality factor after this change will be __________.

Sol. **282.84**

Quality factor = $\frac{X_L}{R} = \frac{\omega L}{R}$

$$Q = \frac{1}{\sqrt{LC}}\frac{L}{R}$$

$$Q = \left(\frac{1}{\sqrt{C}}\right)\frac{\sqrt{L}}{R}$$

$$Q = \frac{XL}{R} = \frac{\omega L}{R} = \frac{1}{\sqrt{LC}}\frac{L}{R} = \frac{1}{R}\frac{\sqrt{L}}{\sqrt{C}}$$

$$Q' = \frac{\sqrt{2L}}{\left(\frac{R}{2}\right)\sqrt{C}} = 2\sqrt{2}Q$$

$$\boxed{Q' = 282.84}$$

6. In an electrical circuit, a battery is connected to pass 20 C of charge through it in a certain given time. The potential difference between two plates of the battery is maintained at 15 V. The work done by the battery is __________ J.

Sol. **300**

Charge flown (Q) = 20 C

Potential difference (V) = 15V

Work done (w) = Q.V

= 20 × 15 = 300 J

$\boxed{w = 300\text{ J}}$

7. A container is divided into two chambers by a partition. The volume of first chamber is 4.5 litre and second chamber is 5.5 litre. The first chamber contain 3.0 moles of gas at pressure 2.0 atm and second chamber contain 4.0 moles of gas at pressure 3.0 atm. After the partition is removed and the mixture attains equilibrium, then, the common equilibrium pressure existing in the mixture is x × 10^{-1} atm. Value of x is _______.

Sol. **25**

By energy Conservation

$$\frac{3}{2}n_1RT_1 + \frac{3}{2}n_2RT_2 = \frac{3}{2}(n_1 + n_2)RT$$

Using PV = nRT

$P_1V_1 + P_2V_2 = P(V_1 + V_2)$

$$P = \frac{P_1V_1 + P_2V_2}{V_1 + V_2} = \frac{2 \times 4.5 + 3 \times 5.5}{4.5 + 5.5}$$

$$P = \frac{9 + 16.5}{10} = \frac{25.5}{10}$$

$\approx 25 \times 10^{-1}$ atm

8. A boy pushes a box of mass 2kg with a force $\vec{F} = \left(20\hat{i} + 10\hat{j}\right)$ N on a frictionless surface. If the box was initially at rest, then __________m is displacement along the x-axis after 10 s.

Sol. **500**

$F = 20\hat{i} + 10\hat{j}$

$F_x = 20N$

$F_y = 10N$

$$a_x = \frac{F_x}{M} = \frac{20}{2} = 10 \text{ m/s}^2$$

$$a_y = \frac{F_y}{M} = \frac{10}{2} = 5 \text{ m/s}^2$$

displacement on x axis is

$$S_x = u_x t + \frac{1}{2} a_x t^2$$

$$S = 0 \times 10 + \frac{1}{2} \times 10 \times (10)^2$$

$$\boxed{S = 500\,m}$$

9. The maximum and minimum amplitude of an amplitude modulated wave is 16 V and 8 V respectively. The modulation index for this amplitude modulated wave is x $\times 10^{-2}$. The value of x is ________.

Sol. **33**

$$A_m = \frac{A_{max} - A_{min}}{2}$$

$$A_C = \frac{A_{max} + A_{min}}{2} \qquad \begin{bmatrix} A_{max} = 16V \\ A_{min} = 8V \end{bmatrix}$$

$$\text{Modulation index (mi)} = \frac{A_m}{A_C} = \frac{\frac{A_{max} - A_{min}}{2}}{\frac{A_{max} + A_{min}}{2}} = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

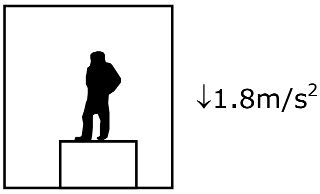
$$mi = \frac{16 - 8}{16 + 8} = \frac{8}{24} = \frac{1}{3} = 0.33$$

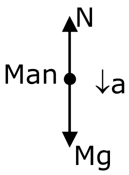
$mi = 33 \times 10^{-2}$

$$\boxed{x = 33}$$

10. A person standing on a spring balance inside a stationary lift measures 60 kg. The weight of that person if the lift descends with uniform downward acceleration of 1.8 m/s^2 will be ____________ N. [$g = 10 m/s^2$]

Sol. **492**





$Mg - N = Ma$

$N = M(g-a)$

$N = 60(10-1.8)$

$N = 60 \times 8.2 = 492N$

$\boxed{N = 492}$

26th Feb. 2021 | Shift - 1
CHEMISTRY

SECTION - A

1. $\underset{(C_4H_8Cl_2)}{A} \xrightarrow[373\text{ K}]{\text{Hydrolysis}} \underset{(C_4H_8O)}{B}$

B reacts with Hydroxyl amine but does not give Tollen's test. Identify A and B.

(1) 1, 1-Dichlorobutane and 2-Butanone

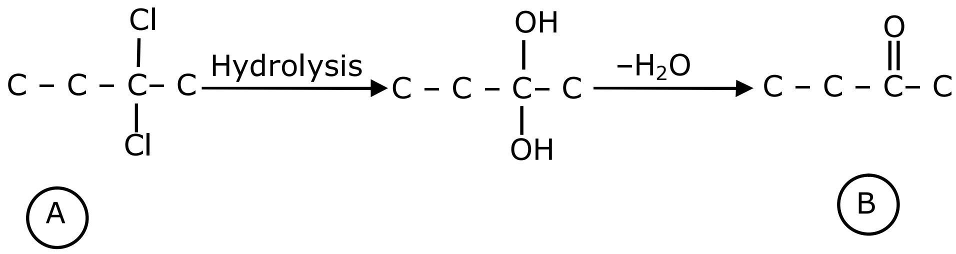
(2) 2, 2- Dichlorobutane and Butan-2-one

(3) 2, 2- Dichlorobutane and Butanal

(4) 1, 1- Dichlorobutane and Butanal

Ans. (2)

Sol.



Compound 'B' does not gives Tollen's test due to presence of kenotic group but react with hydroxyl amine

2. Match List-I with List-II.

List –I (Ore)	**List-II (Element Present)**
(a) Kernite	(i) Tin
(b) Cassiterite	(ii) Boron
(c) Calamine	(iii) Fluorine
(d) Cryolite	(iv) Zinc

Choose the most appropriate answer from the option given below :

(1) (a) – (ii), (b) – (iv), (c) – (i), (d) – (iii)

(2) (a) – (ii), (b) – (i), (c) – (iv), (d) – (iii)

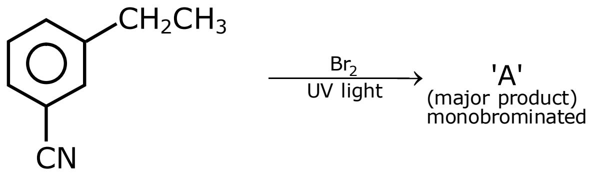
(3) (a) – (i), (b) – (iii), (c) – (iv), (d) – (ii)

(4) (a) – (iii), (b) – (i), (c) – (ii), (d) – (iv)

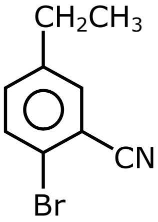
Ans. (2)

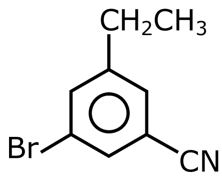
Sol. Fact

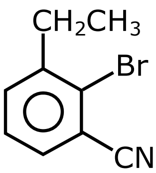
3. For the given reaction :

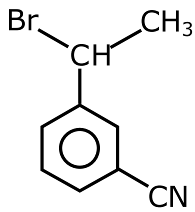


What is 'A' ?

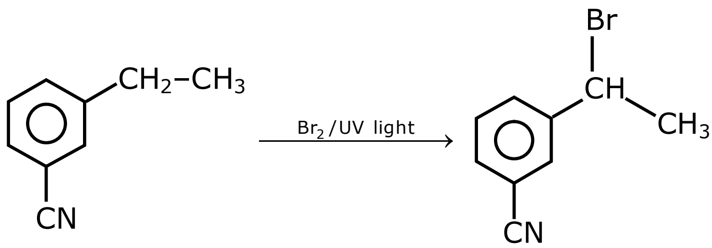
(1) 

(2) 

(3) 

(4) 

Ans. (4)

Sol. 

It is bezylic substitution reaction

4. The orbital having two radial as well as two angular nodes is

(1) 5d (2) 4f (3) 3p (4) 4d

Ans. (1)

Sol. A.N. = ℓ

R.N = n – ℓ – 1

Orbital	Angular Node	Radial Node
5d	2	2
4f	3	0
3p	1	1
4d	0	1

5. Given below are two statement :

Statement I : o-Nitrophenol is steam volatile due to intramolecular hydrogen bonding

Statement II : o-Nitrophenol has high melting point due to hydrogen bonding.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both Statement I and Statement II are false

(2) Statement I is false but Statement II is true

(3) Both Statement I and Statement II are true

(4) Statement I is true but Statement II is false

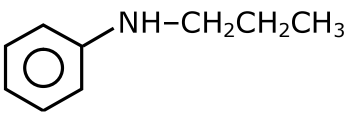
Ans. **(4)**

Sol. o-Nitrophenol is steam volatile due to intramolecular hydrogen H-bonding. but m-Nitrophenol has more melting point due to its symmetry.

6. An amine on reaction with benzenesulphonyl chloride produces a compound insoluble in alkaline solution. This amine can be prepared by ammonolysis of ethyl chloride. The correct structure of amine is :

(1) $CH_3CH_2CH_2\overset{H}{N}-CH_2CH_3$

(2) $CH_3CH_2CH_2NHCH_3$

(3) 

(4) $CH_3CH_2NH_2$

Ans. **(1)**

Sol.

$$\underset{1^\circ\text{-amine}}{R-NH_2} \xrightarrow{C_6H_5SO_2Cl} R-NH-\overset{O}{\overset{\|}{\underset{\underset{O}{\|}}{S}}}-C_6H_5$$

Soluble in alkalines

$$\underset{2^\circ\text{-amine}}{R-NH-R} \xrightarrow{C_6H_5SO_2Cl} R-\overset{R}{\overset{|}{N}}-\overset{O}{\overset{\|}{\underset{\underset{O}{\|}}{S}}}-C_6H_5$$

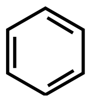
(In soluble in alkalines)

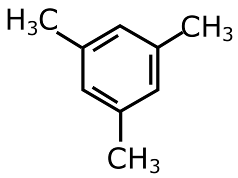
According to the question the amine should be 2°-amine, in which one of the alkyl group should be ethyl, because it can be formed by ammonolysis of ethyl chloride

7. For the given reaction :

$$\underset{\displaystyle CH_3}{\overset{|}{CH}}=CHBr \xrightarrow[\text{2. Red hot iron tube, 873 K}]{\text{1. } NaNH_2} \underset{\text{(major product)}}{(A)}$$

What is 'A'

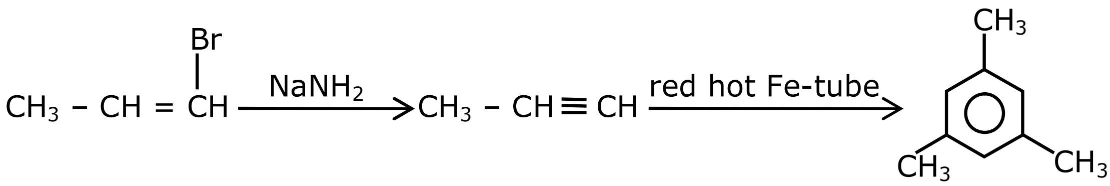
(1) $\underset{\displaystyle CH_3}{\overset{|}{CH}} = CH-NH_2$ (2) 

(3) 

(4) $CH_3CH_2CH_2NH_2$

Ans. (3)

Sol.



(mesitylene)

8. Statement about heavy water are given below

A. Heavy water is used in exchange reactions for the study of reaction mechanisms

B. Heavy water is prepared by exhaustive electrolysis of water

C. Heavy water has higher boiling point than ordinary water

D. Viscosity of H_2O is greater than D_2O

(1) A and B only (2) A and D only

(3) A, B and C only (4) A and C only

Ans. (3)

Sol. Fact

9. Which of the following is 'a' FALSE statement ?

(1) Carius tube used in the estimation of sulphur in an organic compound

(2) Kjedahl's method is used for the estimation of nitrogen in an organic compound

(3) Phosphoric acid produced on oxidation of phosphorus present in an organic compound is precipitated as $Mg_2P_2O_7$ by adding magnesia mixture

(4) Carius method is used for the estimation of nitrogen in an organic compound

Ans. (4)

Sol. Fact

10. Given below are two statements :

Statement I : A mixture of chloroform and aniline can be separated by simple distillation

Statement II : When separating aniline from a mixture of aniline and water by steam distillation aniline boils below its boiling point

In the light of the above statements, choose the most appropriate answer from the options given below

(1) Statement I is true, statement II is false

(2) Both Statement I and Statement II are true

(3) Both Statement I and Statement II are false

(4) Statement I is false, Statement II is true

Ans. (2)

Sol. A suitable method for separating a mixture of aniline and chloro form would be steam distillation. Steam distillation is the process used to separate aromatic compound from a mixture because of their temperature sensitivity. Therefore, steam distillation is an ideal method for their separation

11. Which of the following vitamin is helpful in delaying the blood clotting ?

(1) Vitamin B (2) Vitamin C (3) Vitamin K (4) Vitamin E

Ans. (3)

Sol. Vitamin K is used by the body to help blood clot.

12. The presence of ozone in troposphere :

(1) generates photochemical smog (2) Protects us from the UV radiation

(3) Protects us from the X-ray radiation (4) Protects us from greenhouse effect

Ans. (2)

Sol. The presence of ozone in troposphere protect earth from ultra violet rays

13. On treating a compound with warm dil. H_2SO_4, gas X is evolved which turns $K_2Cr_2O_7$ paper acidified with dil. H_2SO_4 to a green compound Y. X and Y respectively are :

(1) X = SO_2, Y = $Cr_2(SO_4)_3$ (2) X = SO_2, Y = Cr_2O_3

(3) X = SO_3, Y = Cr_2O_3 (4) X = SO_3, Y = $Cr_2(SO_4)_3$

Ans. (1)

Sol. $\underset{(X)}{SO_2} + K_2Cr_2O_7 + H_2SO_4 \longrightarrow \underset{(Y)}{Cr_2(SO_4)_3} + K_2SO_4 + H_2O$

14. Find A, B and C in the following reaction :

$NH_3 + A + CO_2 \rightarrow (NH_4)_2CO_3$

$(NH_4)_2CO_3 + H_2O + B \rightarrow NH_4HCO_3$

$NH_4HCO_3 + NaCl \rightarrow NH_4Cl + C$

(1) A – H_2O ; B – CO_2 ; C – $NaHCO_3$
(2) A – H_2O ; B – O_2 ; C – Na_2CO_3
(3) A – O_2 ; B – CO_2 ; C – Na_2CO_3
(4) A – H_2O ; B – O_2 ; C – $NaHCO_3$

Ans. (1)

Sol. (1) $NH_3 + \underset{(A)}{H_2O} + CO_2 \rightarrow (NH_4)_2CO_3$

(2) $(NH_4)_2CO_3 + H_2O + \underset{(B)}{CO_2} \rightarrow NH_4HCO_3$

(3) $NH_4HCO_3 + NaCl \rightarrow NH_4Cl + \underset{(C)}{NaHCO_3}$

15. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Dipole-dipole interactions are the only non-covalent interactions, resulting in hydrogen bond formation

Reason R : Fluorine is the most electronegative element and hydrogen bonds in HF are symmetrical

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) A is false but R is true
(2) Both A and R are true and R is the correct explanation of A
(3) A is true but R is false
(4) Both A and R are true and R is not the correct explanation of A

Ans. (3)

Sol. Fact

16. Match List-I with List-II.

List –I Electronic configuration of elements	**List-II Δ_iH in kJ mol^{-1}**
(a) $1s^22s^2$	(i) 801
(b) $1s^22s^22p^4$	(ii) 899
(c) $1s^22s^22p^3$	(iii) 1314
(d) $1s^22s^22p^1$	(iv) 1402

(1) (a) – (ii), (b) – (iii), (c) – (iv), (d) – (i)
(2) (a) – (iv), (b) – (i), (c) – (ii), (d) – (iii)
(3) (a) – (i), (b) – (iv), (c) – (iii), (d) – (ii)
(4) (a) – (i), (b) – (iii), (c) – (iv), (d) – (ii)

Ans. (1)

Sol. Order of I.E. in second period

Li < B < Be < C < O < N < F < Ne

$2p^1$ $2s^2$ $2p^2$ $2p^4$ $2p^3$

17. Which one of the following lanthanoids does not form MO_2 ?

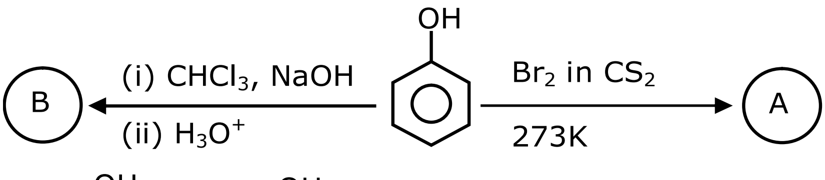
[M is lanthanoid metal]

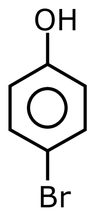
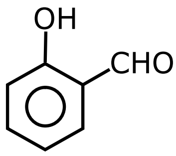
(1) Nd (2) Yb (3) Dy (4) Pr

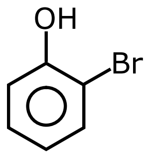
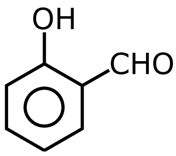
Ans. (2)

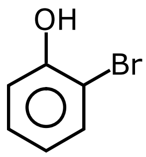
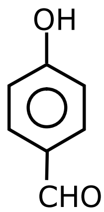
Sol. Fact

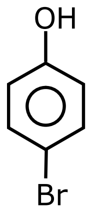
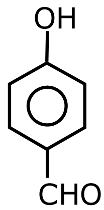
18. Identify the major products A and B respectively in the following reaction of phenol :



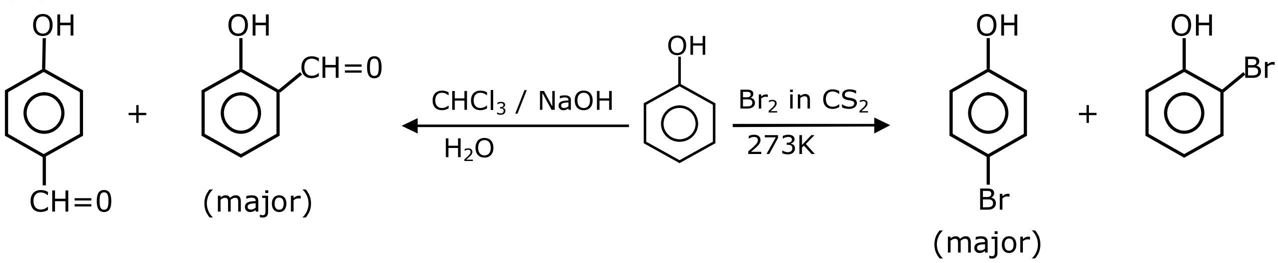
(1)  and 

(2)  and 

(3)  and 

(4)  and 

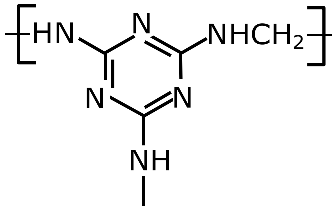
Ans. (1)

Sol. 

19. The structure of Neoprene is :

(1) $\left[CH_2-\underset{\displaystyle CN}{\underset{|}{CH}}\right]_n$

(2) $\left[CH_2-\overset{\displaystyle Cl}{\overset{|}{C}}=CH-CH_2\right]_n$

(3) 

(4) $\left[CH_2CH=CH-CH_2-CH_2-\underset{\displaystyle CN}{\underset{|}{CH}}\right]_n$

Ans. **(2)**

Sol. $\left[CH_2-\overset{\displaystyle Cl}{\overset{|}{C}}=CH-CH_2\right]_n$

neoprene

20. Compound A used as a strong oxidizing agent is amphoteric in nature. It is the part of lead storage batteries. Compound A is :

(1) Pb_3O_4 (2) PbO_2 (3) $PbSO_4$ (4) PbO

Ans. **(2)**

Sol. lead storage batteries PbO_2 is used. In this O.S. of Pb is +4 so it is always reduced and behaves as oxidizing agent

SECTION - B

1. 224 mL of $SO_{2(g)}$ at 298 K and 1 atm is passed through 100 mL of 0.1 M NaOH solution. The non-volatile solute produced is dissolved in 36 g of water. The lowering of vapour pressure of solution (assuming the solution is dilute) ($P^*_{(H_2O)}$ = 24 mm of Hg) is x × 10^{-2} mm of Hg, the value of x is _____ .

Ans. **(0.18)**

Sol. The balanced equation is

$SO_2 + 2NaOH \longrightarrow Na_2SO_3 + H_2O$

moles of NaOH = molarity × volume (in litre)

= 0.1 × 0.1

= 0.01 moles

Here NaOH is limiting Reagent

2 mole NaOH $\longrightarrow$ 1 mole Na_2SO_3

0.01 mole NaOH $\longrightarrow \frac{1}{2} \times 0.01$ mole Na_2SO_3

Moles of $Na_2SO_3 \longrightarrow 0.005$ mole

$Na_2SO_3 \longrightarrow 2Na^+ + SO_3^{2-}$

i = 3

Moles of H_2O = $\frac{36}{18}$ = 2 moles

Accoding to RLVP -

$$\frac{P_A^o - P_A}{P_A^o} = iX_B$$

$$\frac{P_A^o - P_A}{P_A^o} = \frac{in_A}{in_A + n_B} \quad (in_A \simeq 0)$$

$n_B << n_A$

$\{n_A + n_B \simeq n_A\}$

$$\frac{P_A^o - P_A}{P_A^o} = i \times \frac{n_B}{n_A}$$

$$\frac{2H - P_A}{2H} = 3 \times \frac{0.005}{2}$$

$\Rightarrow$ $2H - P_A = 0.18$

Lowering in pressure = 0.18 mm of Hg

lowering in pressure = 18×10^{-12} mm of Hg

$\boxed{x = 18}$

2. Consider the following reaction

$MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{+2} + 4H_2O$, $E° = 1.51$ V.

The quantity of electricity required in Faraday to reduce five moles of MnO_4^- is_____.

Ans. **(25)**

Sol. $MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{+2} + 4H_2O$

1 mole of MnO_4^- require 5 faraday charge

5 moles of MnO_4^- will require 25 faraday charge.

3. 3.12 g of oxygen is adsorbed on 1.2 g of platinum metal. The value of oxygen adsorbed per gram of the adsorbent at 1 atm and 300 K in L is ________.

[R = 0.0821 L atm K^{-1} mol^{-1}]

Ans. **(2)**

Sol. Moles of O_2 = $\frac{3.12}{32}$ = 0.0975

volume of O_2 = $\frac{nRT}{p}$ = $\frac{0.0975 \times 0.082 \times 300}{1}$

= 2.3985L $\simeq$ 2.4 L

volume of O_2 absorbed per gm of pt = $\frac{2.4}{1.2}$ = 2

4. The number of significant figures in 50000.020×10^{-3} is ________.

Ans. (7)

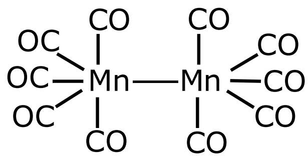
Sol. $\underline{50000.02}0 \times 10^{-3}$

Number of significant figure = 7

5. Number of bridging CO ligands in $[Mn_2(CO)_{10}]$ is ______.

Ans. (0)

Sol. Fact



6. For a chemical reaction $A + B \rightleftharpoons C + D$

($\Delta_r H^\ominus$ = 80 kJ mol^{-1}) the entropy change $\Delta_r S^\ominus$ depends on the temperature T (in K) as

$\Delta_r S^\ominus = 2T$ (J K^{-1} mol^{-1}).

Minimum temperature at which it will become spontaneous is _________K.

Ans. (200)

Sol. $\Delta G° = \Delta H° - T\Delta S°$

To make the process spontaneous

$\Delta G° < 0$

$\Delta H° - T\Delta S° < 0$

$T > \dfrac{\Delta H°}{\Delta S°}$

$T > \dfrac{80000}{2T}$

$2T^2 > 80000$

$T^2 > 40000$

$T > 200$

The minimum temperature to make it spontaneous is 200 K.

7. An exothermic reaction $X \rightarrow Y$ has an activation energy 30 kJ mol^{-1}. If energy change ΔE during the reaction is –20 kJ, then the activation energy for the reverse reaction in kJ is _______.

Ans. (50)

Sol. $\Delta H = E_{a,f} - E_{a,b}$

$-20 = 30 - E_{a,b}$

$E_{a,b}$ = 50 kJ/mole

8. A certain gas obeys $P(V_m - b) = RT$. The value of $\left(\frac{\partial Z}{\partial P}\right)_T$ is $\frac{xb}{RT}$. The value of x is ______.

Ans. **(1)**

Sol. $P(v - b) = RT$

$PV - Pb = RT$

$$\frac{PV}{RT} - \frac{Pb}{RT} = 1$$

$$Z = 1 + \frac{PV}{RT}$$

$$\frac{dz}{dp} = 0 + \frac{b}{RT}$$

$$\Rightarrow \frac{b}{RT} = \frac{xb}{RT}$$

$x = 1$

9. A homogeneous ideal gaseous reaction $AB_{2(g)} \rightleftharpoons A_{(g)} + 2B_{(g)}$ is carried out in a 25 litre flask at 27°C. The initial amount of AB_2 was 1 mole and the equilibrium pressure was 1.9 atm.
The value of K_p is $x \times 10^{-2}$. The value of x is ______.
$[R = 0.08206\ dm^3\ atm\ K^{-1}\ mol^{-1}]$

Ans. **(74)**

Sol. $AB_{2(g)} \rightleftharpoons A_{(g)} + 2B_{(g)}$

initial $1-x$ $\quad x \quad 2x$

at eq. $\frac{1}{1+2x} \quad \frac{1}{1.9}$

By ratio of pressure & mole

$$\frac{1}{1+2x} = \frac{0.985}{1.9}$$

$1.9 = 0.985 + 1.9\ x$

$0.915 = 1.9\ x$

$$\frac{0.915}{1.9} = x \ ; \ K_p = \frac{4x^2.x}{(1-x)} \left[\frac{P_{total}}{n_{total}}\right]^2$$

$$\Rightarrow \frac{4x^3}{1-x}\left(\frac{RT}{V}\right)^2$$

On substituting the values

$K_p = 74 \times 10^{-2}$

10. Dichromate ion is treated with base, the oxidation number of Cr in the product formed is :

Ans. **(6)**

Sol. $Cr_2O_7^{2-} + 2OH^- \rightleftharpoons 2CrO_4^{2-} + H_2O$

$2CrO_7^{2-}$

$x + (-2 \times 4) = -2$

$x = 6$

26^{th} Feb. 2021 | Shift - 1
MATHEMATICS

1. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1,2 and 3 only is

(1) 77

(2) 42

(3) 35

(4) 82

Ans. (1)

Sol. CASE-I: 1, 1, 1, 1, 1, 2, 3

WAYS = $\frac{7!}{5!}$ = 42

CASE-II: 1, 1, 1, 1, 2, 2, 2

WAYS = $\frac{7!}{4!\cdot 3!} = 35$

TOTAL WAYS = 42 + 35 = 77

2. The maximum value of the term independent of 't' in the expansion of $\left(tx^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t}\right)^{10}$ where $x \in (0,1)$ is:

(1) $\frac{10!}{\sqrt{3}(5!)^2}$

(2) $\frac{2.10!}{3(5!)^2}$

(3) $\frac{10!}{3(5!)^2}$

(4) $\frac{2.10!}{3\sqrt{3}(5!)^2}$

Ans. **(4)**

Sol. $T_{r+1} = {}^{10}C_r (tx^{1/5})^{10-r}\left[\dfrac{(1-x)^{1/10}}{t}\right]^r$

$= {}^{10}C_r t^{(10-2r)} \times x^{\frac{10-r}{5}} \times (1-x)^{\frac{r}{10}}$

$\Rightarrow$ 10 −2r = 0 $\Rightarrow$ r = 5

$T_6 = {}^{10}C_5 x\sqrt{1-x}$

$\dfrac{dT_6}{dx} = {}^{10}C_5\left[\sqrt{1-x} - \dfrac{x}{2\sqrt{1-x}}\right] = 0$

= 1 − x = x/2 $\Rightarrow$ 3x = 2

$\Rightarrow$ x = 2/3

$T_6|_{max} = \dfrac{10!}{5!5!} \times \dfrac{2}{3\sqrt{3}}$

3. The value of $\sum\limits_{n=1}^{100}\int\limits_{n-1}^{n} e^{x-[x]}dx$, where [x] is the greatest integer $\le x$, is:

(1) 100 (e–1)
(2) 100e
(3) 100(1-e)
(4) 100 (1 + e)

Ans. **(1)**

Sol. $\sum\limits_{n=1}^{100}\int\limits_{n-1}^{n} e^{x-[x]}dx$

$= \int\limits_0^1 e^{\{x\}}dx + \int\limits_1^2 e^{\{x\}}dx + \int\limits_2^3 e^{\{x\}}dx + \ldots\ldots. \int\limits_{99}^{100} e^{\{x\}}dx$ $\quad (\because \{x\} = x - [x])$

$= e^x\,|_0^1 + e^{(x-1)}\,|_1^2 + e^{(x-2)}\,|_2^3 + \ldots\ldots. + e^{(x-99)}\,|_{99}^{100}$

$= (e-1) + (e-1) + (e-1) + \ldots..(e-1)$

$= 100(e-1)$

4. The rate of growth of bacteria in a culture is proportional to the number of bacteria present and the bacteria count is 1000 at initial time t = 0. The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after $\dfrac{k}{\log_e\left(\dfrac{6}{5}\right)}$ hours,

then $\left(\dfrac{k}{\log_e 2}\right)^2$ is equal to

(1) 4
(2) 2
(3) 16
(4) 8

Ans. **(1)**

Sol. $\frac{dx}{dt} \propto x$

$\frac{dx}{dt} = \lambda x$

$\int_{1000}^{x} \frac{dx}{x} = \int_{0}^{t} \lambda dt$

$\ln x - \ln 1000 = \lambda t$

$\ln\left(\frac{x}{1000}\right) = \lambda t$

Put t = 2, x = 1200

$\ln\left(\frac{12}{10}\right) = 2\lambda \quad \Rightarrow \lambda = \frac{1}{2}\ln\frac{6}{5}$

Now $\ln\left(\frac{x}{1000}\right) = \frac{t}{2}\ln\left(\frac{6}{5}\right)$

$x = 1000e^{\frac{t}{2}\ln\left(\frac{6}{5}\right)}$

x = 2000 at t = $\frac{k}{\ln\left(\frac{6}{5}\right)}$

$\Rightarrow\ 2000 = 1000\ e^{\frac{k}{2\ln(6/5)} \times \ln(6/5)}$

$\Rightarrow\ 2 = e^{k/2}$

$\Rightarrow\ \ln 2 = \frac{k}{2}$

$\Rightarrow\ \frac{k}{\ln 2} = 2$

$\Rightarrow\ \left(\frac{k}{\ln 2}\right)^2 = 4$

5. If $\vec{a}$ & $\vec{b}$ are perpendicular vactors, then

$\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \vec{b}\right)\right)\right)$ is equal to

(1) $\frac{1}{2}|\vec{a}|^4\ \vec{b}$

(2) $\vec{a} \times \vec{b}$

(3) $|\vec{a}|^4\ \vec{b}$

(4) $\vec{0}$

Ans. **(3)**

Sol. $\vec{a} \times \left(\vec{a} \times \left(\left(\vec{a}.\vec{b}\right)\vec{a} - |\vec{a}|^2 \vec{b}\right)\right)$

$\vec{a} \times (-|\vec{a}|^2 (\vec{a} \times \vec{b})) = -|\vec{a}|^2 ((\vec{a}.\vec{b})\vec{a} - |\vec{a}|^2 \vec{b})$

$= -(\vec{a}.\vec{b})\vec{a}|\vec{a}|^2 + |\vec{a}|^4 \vec{b}$

$= |\vec{a}|^4 \vec{b} \quad \left(\because \vec{a} \cdot \vec{b} = 0\right)$

6. In an increasing, geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of 4^{th}, 6^{th} and 8^{th} terms is equal to :

(1) 35

(2)30

(3) 26

(4) 32

Ans **(1)**

Sol. $ar + ar^5 = \frac{25}{2}$

$ar^2 \times ar^4 = 25$

$a^2r^6 = 25$

$ar^3 = 5$

$\boxed{a = \frac{5}{r^3}}$ ....(1)

$\frac{5r}{r^3} + \frac{5r^5}{r^3} = \frac{25}{2}$

$\frac{1}{r^2} + r^2 = \frac{5}{2}$

Put $r^2 = t$

$\frac{t^2 + 1}{t} = \frac{5}{2}$

$2t^2 - 5t + 2 = 0$

$2t^2 - 4t - t + 2 = 0$

$(2t - 1)(t - 2) = 0$

$t = \frac{1}{2}, 2 \quad \Rightarrow \boxed{r^2 = \frac{1}{2}, 2}$

$\boxed{r = \sqrt{2}}$
$= ar^3 + ar^5 + ar^7$
$= ar^3 (1 + r^2 + r^4)$
$= 5[1 + 2 + 4] = 35$

7. Consider the three planes

$P_1 : 3x + 15y + 21z = 9,$

$P_2 : x - 3y - z = 5$, and

$P_3 : 2x + 10y + 14z = 5$

Then, which one of the following is true?

(1) P_1 and P_3 are parallel.

(2) P_2 and P_3 are parallel.

(3) P_1 and P_2 are parallel.

(4) P_1 , P_2 and P_3 all are parallel.

Ans. (1)

Sol. $P_1 = x + 5y + 7z = 3$

$P_2 = x - 3y - z = 5$

$P_3 = x + 5y + 7z = 5/2$

$\Rightarrow P_1 || P_3$

8. The sum of the infinite series $1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \ldots$ is equal to

(1) $\frac{9}{4}$

(2) $\frac{15}{4}$

(3) $\frac{13}{4}$

(4) $\frac{11}{4}$

Ans. **(3)**

Sol. $s = 1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + ...$

$\frac{s}{3} = \quad \frac{1}{3} + \frac{2}{3^2} + \frac{7}{3^3} + ...\infty$

$\frac{2s}{3} = 1 + \frac{1}{3} + \frac{5}{3^2} + \frac{5}{3^3} + ...\infty$

$\frac{2s}{3} = \frac{4}{3} + \frac{5}{3}\left\{\frac{1/3}{1-\frac{1}{3}}\right\} \quad = \frac{5}{6} + \frac{4}{3} = \frac{13}{6}$

$\boxed{s = \frac{13}{4}}$

9. The value of $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$ is

(1) -2

(2) (a+1) (a+2) (a+3)

(3) 0

(4) (a+2) (a+3) (a+4)

Ans. **(1)**

Sol. $c_1 \to c_1 - c_2$, $c_2 \to c_2 - c_3$

$= \begin{vmatrix} (a+2)a & a+1 & 1 \\ (a+3)(a+1) & a+2 & 1 \\ (a+4)(a+2) & a+3 & 1 \end{vmatrix}$

$R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$= \begin{vmatrix} a^2+2a & a+1 & 1 \\ 2a+3 & 1 & 0 \\ 4a+8 & 2 & 0 \end{vmatrix}$

$= 6 - 8 = -2$

10. If $\frac{\sin^{-1}x}{a}=\frac{\cos^{-1}x}{b}=\frac{\tan^{-1}y}{c}; 0<x<1,$ then the value of $\cos\left(\frac{\pi c}{a+b}\right)$ is:

(1) $\frac{1-y^2}{2y}$

(2) $\frac{1-y^2}{1+y^2}$

(3) $1-y^2$

(4) $\frac{1-y^2}{y\sqrt{y}}$

Ans. (2)

Sol. $\frac{\sin^{-1}x}{a}=\frac{\cos^{-1}x}{b}=\frac{\tan^{-1}y}{c}$

$$\frac{\sin^{-1}x}{a}=\frac{\cos^{-1}x}{b}=\frac{\sin^{-1}x+\cos^{-1}x}{a+b}=\frac{\pi}{2(a+b)}$$

Now, $\frac{\tan^{-1}y}{c}=\frac{\pi}{2(a+b)}$

$$2\tan^{-1}y=\frac{\pi c}{a+b}$$

$$\Rightarrow \quad \cos\left(\frac{\pi c}{a+b}\right)=\cos\left(2\tan^{-1}y\right)=\frac{1-y^2}{1+y^2}$$

11. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is:

(1) 6

(2) 1

(3) 4

(4)12

Ans. (3)

Sol. Let $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$$A^2 = \begin{bmatrix} a & b \\ b & c \end{bmatrix}\begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} a^2+b^2 & ab+bc \\ ab+bc & c^2+b^2 \end{bmatrix}$$

$= a^2 + 2b^2 + c^2 = 1$

$a = 1,\ b = 0,\ c = 0$

$a = 0,\ b = 0,\ c = 1$

$a = -1,\ b = 0,\ c = 0$

$c = -1,\ b = 0,\ a = 0$

12. The intersection of three lines $x-y = 0$, $x + 2y = 3$ and $2x + y = 6$ is a:

(1) Equilateral triangle

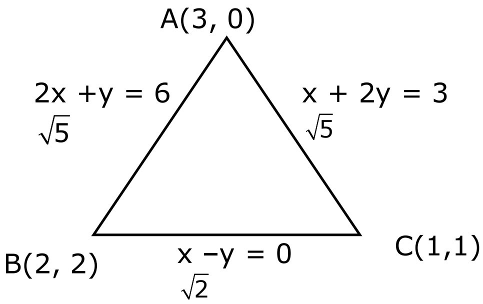
(2) None of the above

(3) Isosceles triangle

(4) Right angled triangle

Ans. (3)

Sol.



13. The maximum slope of the curve $y = \frac{1}{2}x^4 - 5x^3 + 18x^2 - 19x$ occurs at the point:

(1) (2, 9)

(2) (2,2)

(3) $\left(3, \frac{21}{2}\right)$

(4) (0, 0)

Ans. (2)

Sol. $\frac{dy}{dx} = 2x^3 - 15x^2 + 36x - 19$

Let $f(x) = 2x^3 - 15x^2 + 36x - 19$

$f'(x) = 6x^2 - 30x + 36 = 0$

$x^2 - 5x + 6 = 0$

$\boxed{x = 2, 3}$

$f''(x) = 12x - 30$

$f''(x) < 0$ for $\boxed{x = 2}$

At $\boxed{x = 2}$

$y = 8 - 40 + 72 - 38$

$y = 72 - 70 = 2$

$\Rightarrow (2,2)$

14. Let *f* be any function defined on R and let it satisfy the condition :

$(|f(x)-f(y)| \le |(x-y)^2|, \forall (x,y) \in R$

If f(0) = 1, then:

(1) f(x) < 0, $\forall x \in R$

(2) f(x) can take any value in R

(3) f(x) = 0, $\forall\ x \in R$

(4) f(x) >0, $\forall\ x \in R$

Ans. (4)

Sol. $|f(x)-f(y)| \le |(x-y)^2|, \forall (x,y) \in R$

$$\left|\frac{f(x)-f(y)}{x-y}\right| \le |x-y|$$

$$\lim_{x \to y}\left|\frac{f(x)-f(y)}{x-y}\right| \le 0$$

$|f'(y)| \le 0 \Rightarrow f'(y) = 0$

$f(y) = C$

$\Rightarrow \boxed{c = 1}$ $\qquad \because \quad f(0) = 1$

$\Rightarrow$ f(x) = 1

15. The value of $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx$ is:

(1) 2π
(2) 4π
(3) $\frac{\pi}{2}$
(4) $\frac{\pi}{4}$

Ans. **(4)**

Sol. Let $I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx$

$$I = \int_{-\pi/2}^{\pi/2} \frac{3^x \cos^2 x}{1+3^x} dx$$

$$2I = \int_{-\pi/2}^{\pi/2} \cos^2 x dx$$

$$I = \int_{0}^{\pi/2} \cos^2 x dx = \frac{\pi}{4}$$

16. The value of $\lim_{h \to 0} \left\{ \frac{\sqrt{3} \sin\left(\frac{\pi}{6} + h\right) - \cos\left(\frac{\pi}{6} + h\right)}{\sqrt{3}h\left(\sqrt{3}\cosh - \sinh\right)} \right\}$ is:

(1) $\frac{3}{4}$

(2) $\frac{2}{\sqrt{3}}$

(3) $\frac{4}{3}$

(4) $\frac{2}{3}$

Ans. (3)

Sol. $\lim_{h \to 0} 2 \times 2 \left\{ \frac{\sin\left(\frac{\pi}{6} + h - \frac{\pi}{6}\right)}{2\sqrt{3}h\left(\cos(h + \frac{\pi}{6}\right)} \right\}$

$$= \frac{2}{\sqrt{3}} \times \frac{2}{\sqrt{3}} = \frac{4}{3}$$

17. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is :

(1) $\frac{15}{2^{12}}$

(2) $\frac{15}{2^{13}}$

(3) $\frac{15}{2^{14}}$

(4) $\frac{15}{2^{8}}$

Ans. (2)

Sol. $p(x = 9) = p(x = 7)$

$$^{n}C_{9}\left(\frac{1}{2}\right)^{n-9} \times \left(\frac{1}{2}\right)^{9} = ^{n}C_{7}\left(\frac{1}{2}\right)^{n-7} \times \left(\frac{1}{2}\right)^{7}$$

$$^{n}C_{9} \times \left(\frac{1}{2}\right)^{2} = \left(\frac{1}{2}\right)^{2} \times {}^{n}C_{7}$$

$x + y = n \quad \Rightarrow \quad n = 16$

$$p(x = 2) = {}^{16}C_{2} \times \left(\frac{1}{2}\right)^{14} \times \left(\frac{1}{2}\right)^{2}$$

$$= {}^{16}C_{2} \times \left(\frac{1}{2}\right)^{16} = \frac{15}{2^{13}}$$

18. If (1,5,35), (7,5,5), (1, λ, 7) and (2λ, 1, 2) are coplanar, then the sum of all possible values of λ is:

(1) $-\frac{44}{5}$

(2) $\frac{39}{5}$

(3) $-\frac{39}{5}$

(4) $\frac{44}{5}$

Ans. **(4)**

Sol. Let P(1,5,35), Q(7,5,5), R(1, λ,7), S(2λ,1,2)

$$\left[\overrightarrow{PQ}\ \ \overrightarrow{PR}\ \ \overrightarrow{PS}\right] = 0$$

$$\begin{vmatrix} 6 & 0 & -30 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -33 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 0 & -5 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -33 \end{vmatrix} = 0$$

$$\{-33\lambda + 165 - 112\} + 5(\lambda-5)(2\lambda-1) = 0$$

$$53 - 33\lambda + 5\{2\lambda^2 - 11\lambda + 5\} = 0$$

$$16\lambda^2 - 88\lambda + 78 = 0$$

$$5\lambda^2 - 44\lambda + 39 = 0 <^{\lambda_1}_{\lambda_2}$$

$$\Rightarrow \lambda_1 + \lambda_2 = 44/5$$

19. Let R = {P,Q)|P and Q are at the same distance from the origin} be a relation, then the equivalence class of (1,–1) is the set:

(1) S = {(x,y)|$x^2+y^2=1$}

(2) S = {(x,y)|$x^2+y^2=4$}

(3) S = {(x,y)|$x^2+y^2=\sqrt{2}$}

(4) S = {(x,y)|$x^2+y^2=2$}

Ans. **(4)**

Sol. P(a, b) , Q(c, d), PO = QO

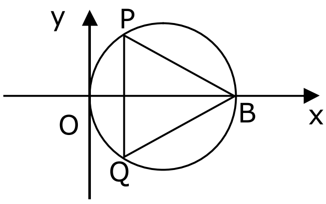
$\Rightarrow a^2 + b^2 = c^2 + d^2$

R(x,y) s = (1, –1) $\Rightarrow$ RO = SO

($\because$ equivalence class)

$x^2 + y^2 = 2$

20. In the circle given below, let OA = 1 unit, OB = 13 unit and $PQ \perp OB$. Then, the area of the triangle PQB (in square units) is:



(1) $26\sqrt{3}$

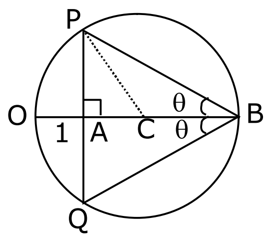
(2) $24\sqrt{2}$

(3) $24\sqrt{3}$

(4) $26\sqrt{2}$

Ans. (3)

Sol.



$OC = \frac{13}{2} = 6.5$

$AC = CO - AO$

$= 6.5 - 1$

$= 5.5$

In ΔPAC

$PA = \sqrt{6.5^2 - 5.5^2}$

$PA = \sqrt{12}$

$\Rightarrow PQ = 2PA = 2\sqrt{12}$

Now, area of $\Delta PQB = \frac{1}{2} \times PQ \times AB$

$= \frac{1}{2} \times 2\sqrt{12} \times 12$

$= 12\sqrt{12}$

$= 24\sqrt{3}$

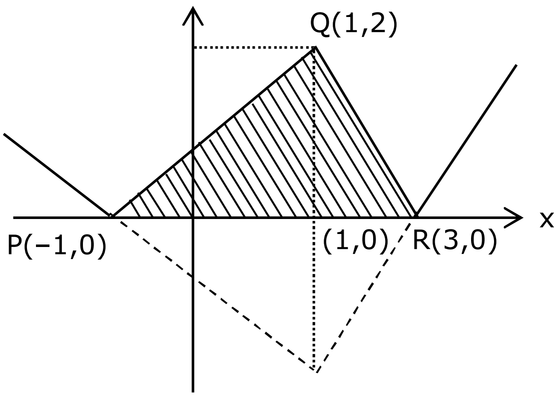
Section-B

1. The area bounded by the lines y = || x-1|-2| is.......

Ans. **Bonus**

NTA Ans. (8)

Sol.



2. The number of integral values of 'k' for which the equation 3sinx + 4cos x = k + 1 has a solution, $k \in R$ is ______.

Ans. **(11)**

Sol. 3 sinx + 4cosx = k+1

$-5 \le k + 1 \le 5$

$-6 \le k \le 4$

$\boxed{-6,-5,-4,-3,-2,-1,0,1,2,3,4}$ $\Rightarrow$ 11 integral values

3. Let $m, n \in N$ and gcd (2, n) = 1. If $30\binom{30}{0} + 29\binom{30}{1} + ... + 2\binom{30}{28} + 1\binom{30}{29} = n \cdot 2^m$,

then n + m is equal to_____.

Ans. **(45)**

Sol. Let $S = \sum_{r=0}^{30} (30 - r)\ {}^{30}C_r$

$= 30\sum_{r=0}^{30} {}^{30}C_r - \sum_{r=0}^{30} r\ {}^{30}C_r$

$= 20 \times 2^{30} - \sum_{r=1}^{30} r.\frac{30}{4}.{}^{29}C_{r-1}$

$= 30 \times 2^{30} - 30.2^{29}$

$= (30 \times 2 - 30).2^{29} = 30.2^{29} \Rightarrow 15.2^{30}$

$= n = 15, \ m = 30$

$n + m = 45$

4. If $y = y(x)$ is the solution of the equation $e^{\sin y} \cos y \frac{dy}{dx} + e^{\sin y} \cos x = \cos x, y(0) = 0$; then

$1 + y\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2} y\left(\frac{\pi}{3}\right) + \frac{1}{\sqrt{2}} y\left(\frac{\pi}{4}\right)$ is equal to ______.

Ans. **(1)**

Sol. $e^{\sin y} \cos y \frac{dy}{dx} + e^{\sin y} \cos x = \cos x$

Put $e^{\sin y} = t$

$e^{\sin y} \times \cos y \frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dt}{dx} + t \cos x = \cos x$

I.F. $= e^{\int \cos x dx} = e^{\sin x}$

Solution of diff equation:

$t.e^{\sin x} = \int e^{\sin x}.\cos x dx$

$e^{\sin y}.e^{\sin x} = e^{\sin x} + c$

at $x = 0, y = 0$

$1 = 1 + c \quad \Rightarrow c = 0$

$e^{\sin x + \sin y} = e^{\sin x}$

$\sin x + \sin y = \sin x$

$y = 0$

$\Rightarrow y\left(\frac{\pi}{6}\right) = 0, \quad y\left(\frac{\pi}{3}\right) = 0, \quad y\left(\frac{\pi}{4}\right) = 0$

$\Rightarrow 1 + 0 + 0 + 0 = 1$

5. The number of solutions of the equation $\log_4(x-1) = \log_2(x-3)$ is _______.

Ans. **(1)**

Sol. $\frac{1}{2}\log_2(x-1) = \log_2(x-3)$

$x-1 = (x-3)^2$

$x^2 - 6x + 9 = x - 1$

$x^2 - 7x + 10 = 0$

$x = 2, 5$

X = 2 Not possible as $\log_2(x-3)$ is not defined

$\Rightarrow$ No. of solution = 1

6. If $\sqrt{3}(\cos^2 x) = (\sqrt{3}-1)\cos x + 1,$ the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ is ________.

Ans. **(1)**

Sol. $\sqrt{3}t^2 - \left(\sqrt{3}-1\right)t - 1 = 0$ (where t = cosx)

Now, $t = \frac{\left(\sqrt{3}-1\right) \pm \sqrt{4+2\sqrt{3}}}{2\sqrt{3}}$

t = cos x = 1 or $-\frac{1}{\sqrt{3}} \rightarrow$ rejected as $x \in \left[0, \frac{\pi}{2}\right]$

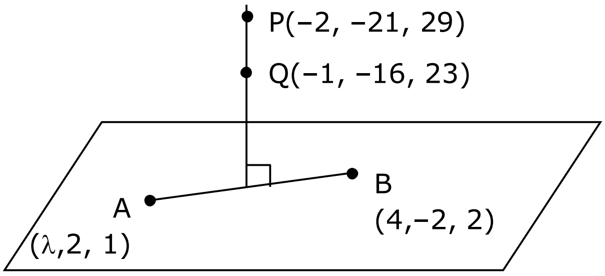
$\Rightarrow$ cos x = 1

$\Rightarrow$ No. of solution = 1

7. Let (λ, 2, 1) be a point on the plane which passes through the point (4, −2, 2). If the plane is perpendicular to the line joining the point (−2, −21, 29) and (−1, −16, 23), then $\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$ is equal to _______.

Ans. **(8)**

Sol.



$$\overrightarrow{AB} \perp \overrightarrow{PQ}$$

$$\left[(4-\lambda)\hat{i} - 4\hat{j} + \hat{k}\right].\left[+\hat{i} + 5\hat{j} - 6\hat{k}\right] = 0$$

$$4 - \lambda - 20 - 6 = 0$$

$$\boxed{\lambda = -22}$$

Now , $\frac{\lambda}{11} = -2$

$$\Rightarrow \left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$$

$$\Rightarrow \quad 4 + 8 - 4 = 8$$

8. The difference betweeen degree and order of differential equation that represents the family of curves given by $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right), a > 0$ is _______.

Ans. **(2)**

Sol. $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right)$

$2yy' = a$

$y^2 = 2yy'\left(x + \frac{\sqrt{2yy'}}{2}\right)$

$$y = 2y'\left(x + \frac{\sqrt{yy'}}{\sqrt{2}}\right)$$

$$y - 2xy' = \sqrt{2}y'\sqrt{yy'}$$

$$\left(y - 2x\frac{dy}{dx}\right)^2 = 2y\left(\frac{dy}{dx}\right)^3$$

D = 3 & O = 1

D– O = 3 – 1 = 2

9. The sum of 162^{th} power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is______.

Ans. (3)

Sol. Let roots of $x^3 - 2x^2 + 2x - 1 = 0$ are α, β, γ

$(x - 1)(x^2 - x + 1) = 0$

$$x = \underset{\alpha}{\underset{\downarrow}{1}},\ \underset{\beta}{\underset{\downarrow}{-\omega}},\ \underset{\gamma}{\underset{\downarrow}{-\omega^2}}$$

Now $\alpha^{162} + \beta^{162} + \gamma^{162}$

$= 1 + \omega^{162} + (\omega^2)^{162}$

$= 1 + (\omega^3)^{54} + (\omega^3)^{108}$

$= 3$

10. The value of the integeral $\int_0^{\pi} |\sin 2x|\, dx$ is ________

Ans. (2)

Sol. $I = \int_0^{\pi} |\sin 2x|\, dx$

$$I = 2\int_0^{\pi/2} |\sin 2x|\, dx = 2\int_0^{\pi/2} \sin 2x\, dx$$

$$I = 2\left[\frac{-\cos(2x)}{2}\right]_0^{\pi/2} = 2$$