

# JEE-Main-25-07-2021-Shift-1 (Memory Based)

## PHYSICS

**Question:** A nucleus of mass  $4m$  disintegrates into two daughter nuclei of mass  $m$  and  $3m$ . Find the ratio of their respective De-Broglie wavelength?

**Options:**

- (a)  $\lambda_{e^-} > \lambda_{p^+} > \lambda_{\alpha}$
- (b)  $\lambda_{p^+} > \lambda_{\alpha} > \lambda_{e^-}$
- (c)  $\lambda_{\alpha} > \lambda_{p^+} > \lambda_{e^-}$
- (d)  $\lambda_{e^-} > \lambda_{\alpha} > \lambda_{p^+}$

**Answer:** (a)

**Solution:**  $\lambda_e = \frac{1.227}{\sqrt{v}} \text{ \AA}$

$$\lambda_p = \frac{0.286}{\sqrt{v}} \text{ \AA}$$

$$\lambda_{\alpha} = \frac{0.101}{\sqrt{v}} \text{ \AA}$$

$$\therefore \lambda_{e^-} > \lambda_{p^+} > \lambda_{\alpha}$$

**Question:** Earth moves around sun in elliptical orbit. If minimum and maximum distance between earth and sun are  $r_{\min}$  and  $r_{\max}$  respectively and minimum and maximum speed of earth are  $V_{\min}$  and  $V_{\max}$  respectively. Express  $V_{\max}$  in terms of others parameters.

**Options:**

(a)  $v_{\max} = \frac{r_{\min} v_{\min}}{r_{\max}}$

(b)  $v_{\max} = \frac{r_{\max} v_{\min}}{r_{\min} + r_{\max}}$

(c)  $v_{\max} = \frac{r_{\max} v_{\min}}{r_{\min}}$

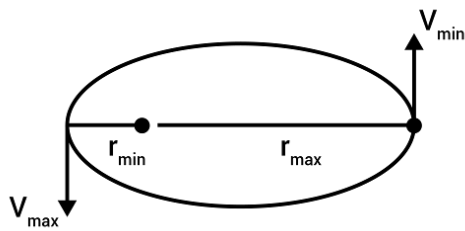
(d)  $v_{\max} = v_{\min}$

**Answer:** (c)

**Solution:** As at minimum distance  $v$  will be maximum.

By conservation of angular momentum.

$$mv_{\max}r_{\min} = mv_{\min}r_{\max}$$



$$v_{\max} = \frac{v_{\min} r_{\max}}{r_{\min}}$$

**Question:** In an amplitude modulator circuit, the carrier wave is given by,  
 $C(t) = 4 \sin(2\pi \times 10^7 t)$  while modulating signal is given by,  $m(t) = 2 \sin(2\pi \times 10^5 t)$ . Then  
the bandwidth of the broadcast signal will be

**Options:**

- (a) 0.2 MHz
- (b) 2 MHz
- (c) 20 MHz
- (d) 40 MHz

**Answer:** (c)

**Solution:**

$$c(t) = c \sin(2\pi w_c t)$$

$$m(t) = M \sin(2\pi w_m t)$$

$$\text{Lower side band (LSB)} = w_c - w_m$$

$$\text{Lower side band (USB)} = w_c + w_m$$

$$\text{Band width} = \text{USB} - \text{LSB}$$

$$= (w_c + w_m) - (w_c - w_m)$$

$$= 2w_c$$

$$= 2 \times 10^7 \text{ Hz}$$

$$= 20 \text{ MHz}$$

**Question:** Find the ratio of the impulse transferred to the wall by a ball incident normally and then at  $45^\circ$  with normal?

**Options:**

- (a) 1:2
- (b)  $\sqrt{2} : 1$
- (c)  $1 : \sqrt{2}$
- (d) 2:1

**Answer:** (b)

**Solution:**

For collision along normal

$$\text{Impulse } (\Delta p)_1 = 2mv$$

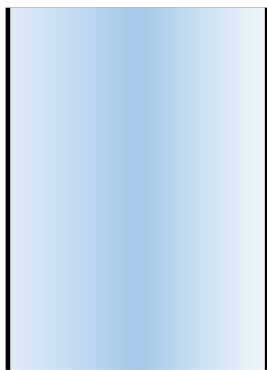
For collision at  $45^\circ$

$$\text{Impulse } (\Delta p)_2 = 2mv(\sin 45^\circ)$$

$$\text{Ratio} = \frac{2mv}{2mv \sin 45^\circ} = \frac{\sqrt{2}}{1}$$

**Question:** The electrical permittivity of dielectric varies with x as:

$$\begin{aligned} \epsilon &= \epsilon_0 \left[ 1 + \frac{kx}{d} \right]; 0 \leq x \leq \frac{d}{2} \\ &= \epsilon_0 \left[ 1 + \frac{k(d-x)}{d} \right]; \frac{d}{2} \leq x \leq d \end{aligned}$$



If the area of the plates is A then find capacitance of capacitor.

**Options:**

- (a)  $\frac{k\epsilon_0 A}{2d}$
- (b)  $\frac{k\epsilon_0 A}{2d} \ln \left[ 1 + \frac{k}{2} \right]$

$$(c) \frac{k\epsilon_0 A}{2d} \ln \left[ 2 + \frac{k}{2} \right]$$

$$(d) \left( 1 + \frac{k}{2} \right) \frac{\epsilon_0 A}{2}$$

**Answer:** (b)

**Solution:**

$$C_{eq} = \frac{\epsilon_0 A}{\frac{dx}{k_1} + \frac{dx}{k_2} + \frac{dx}{k_3} \dots}$$

$$\text{Let } m = \frac{dx}{k_1} + \frac{dx}{k_2} + \frac{dx}{k_3} \dots$$

$$m = \int_0^{d/2} \frac{dx}{\left[ 1 + \frac{kx}{d} \right]} + \int_{d/2}^d \frac{dx}{1 + \frac{k(d-x)}{d}}$$

$$= \int_0^{d/2} \frac{d \, dx}{d + kx} + \int_{d/2}^d \frac{d \, dx}{d + kd - kx}$$

$$= \frac{d}{k} \ln(d + kx) \Big|_0^{d/2} + -\frac{d}{k} \ln(d + kd - kx) \Big|_{d/2}^d$$

$$= \frac{2d}{k} \ln \left( 1 + \frac{k}{2} \right)$$

$$C_{eq} = \frac{k\epsilon_0 A}{2d} \ln \left[ 1 + \frac{k}{2} \right]$$

**Question:** A particle tied to string of length 0.5 m is given a velocity  $3\text{ms}^{-1}$  at its bottom point while undergoing vertical circular motion. What will be its speed when it makes an angle of  $60^\circ$  with the lower vertical.

**Options:**

(a) 1.5 m/s

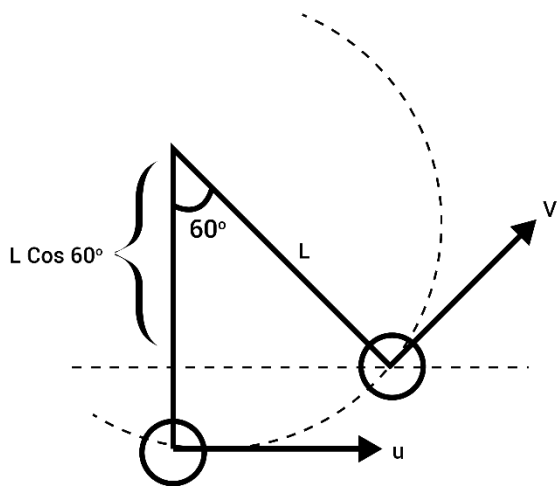
(b) 2.5 m/s

(c)  $2\sqrt{2}$  m/s

(d) 2 m/s

**Answer:** (d)

**Solution:**



Applying energy conservation

$$\frac{1}{2}mv^2 + mg(L - L \cos 60^\circ) = \frac{1}{2}mu^2$$

$$\Rightarrow v^2 = u^2 - 2gL\left(1 - \frac{1}{2}\right) = u^2 - gL$$

$$\Rightarrow v = \sqrt{3^2 - 10 \times 0.5} = \sqrt{4} = 2 \text{ m/s}$$

**Question:** Half life time of Gold is 3 days. Find the activity of a sample of 2 mg of Gold.

**Options:**

- (a) 85 curie
- (b) 594 curie
- (c) 441 curie
- (d) 121 curie

**Answer:** (c)

**Solution:**

$$T_{1/2} = 3 \text{ days}$$

$$= 3 \times 24 \times 60 \times 60 \text{ sec}$$

$$m = 2 \text{ mg} = 2 \times 10^{-3} \text{ g}$$

$$M = 196.96$$

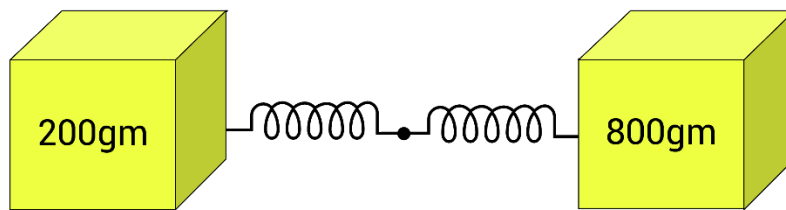
$$N = \frac{m}{M} \times \text{Avogadro's number}$$

$$= 2 \times 10^{-3} \times 6.0233 \times 10^{23} \text{ atoms}$$

Activity of the sample,

$$\begin{aligned}
 R &= \lambda N = \frac{0.693}{T_{1/2}} N \\
 &= \frac{0.693 \times 2 \times 10^{-3} \times 6.023 \times 10^{23}}{3 \times 24 \times 60 \times 60 \times 196.96} \\
 &= 163.36 \times 10^{11} \text{ Bq.} \\
 &= \frac{163.36 \times 10^{11}}{3.7 \times 10^{10}} \text{ curie} = 441 \text{ curie}
 \end{aligned}$$

**Question:** Two springs having constant,  $k$  and  $4k$  are connected in series and then connected to 2 masses, 200 gm and 800 gm at each end. Find angular frequency of their oscillation if  $k = 20 \text{ N/m}$ .



**Options:**

- (a) 10 rad/sec
- (b) 20 rad/sec
- (c) 5 rad/sec
- (d) 40 rad/sec

**Answer:** (a)

**Solution:**

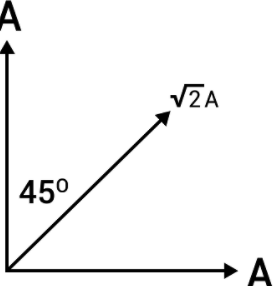
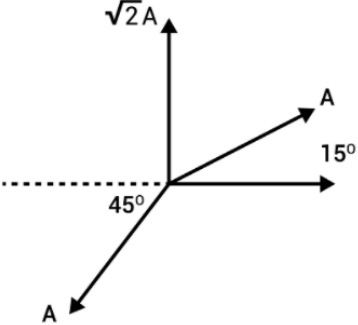
$$k_{eq} = \frac{k_1 k_2}{k_1 + k_2} = \frac{k \cdot 4k}{k + 4k} = 0.8k$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{200 \times 800}{200 + 800} = 160 \text{ g}$$

$$\omega = \sqrt{\frac{k_{eq}}{\mu}} = \sqrt{\frac{0.8 \times 20 \text{ N/m}}{0.160 \text{ kg}}} = 10 \text{ rad/s}$$

**Question:** In column (I) different orientation of 3 vectors are given and in column (II) magnitude of resultant of vectors is given

Column (I)	Column (I)
<p>(P)</p>	A
<p>(Q)</p>	Zero

 <p>(R)</p>	$2A$
 <p>(S)</p>	$2\sqrt{2}A$

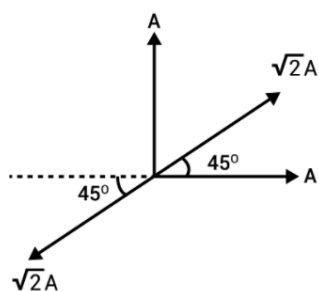
**Options:**

- (a)  $P \rightarrow 1, Q \rightarrow 2, R \rightarrow 3, S \rightarrow 4$
- (b)  $P \rightarrow 2, Q \rightarrow 3, R \rightarrow 4, S \rightarrow 1$
- (c)  $P \rightarrow 2, Q \rightarrow 3, R \rightarrow 1, S \rightarrow 4$
- (d)  $P \rightarrow 3, Q \rightarrow 2, R \rightarrow 4, S \rightarrow 1$

**Answer:** (b)

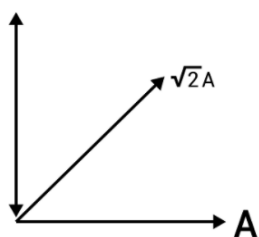
**Solution:**

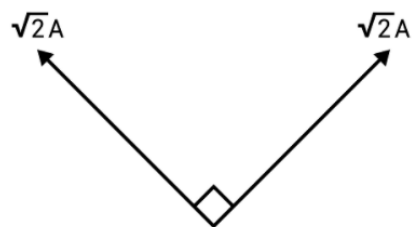
(P)



Resultant = 0

(Q)

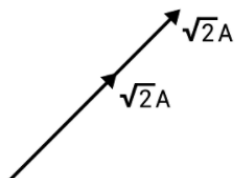
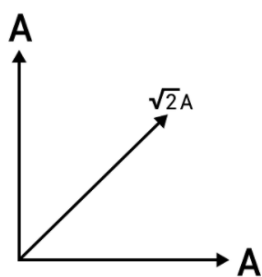




$$= \sqrt{2}(\sqrt{2}A)$$

$$\text{Resultant} = 2A$$

(r)



$$\text{Resultant} = 2\sqrt{2}$$

So,  $P \rightarrow 2, Q \rightarrow 3, R \rightarrow 4, S \rightarrow 1$

**Question:** If position vector of a particle is given by:  $\vec{r} = 10\alpha t^2 \hat{i} + [5\beta t - 5] \hat{j}$

Find time when its angular momentum about origin is 0.

**Options:**

(a)  $\beta$

(b)  $\frac{1}{\beta}$

(c)  $\frac{2}{\beta}$

(d)  $\frac{\beta}{2}$

**Answer:** (c)

**Solution:**

$$\vec{r} = 10\alpha t^2 \hat{i} + [5\beta t - 5] \hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = 20\alpha t \hat{i} + 5\beta \hat{j}$$

$$\vec{L} = \vec{r} \times \vec{p} = [\vec{r} \times m\vec{v}] = m[\vec{r} \times \vec{v}]$$

For constant m

When  $\vec{L} = 0$

$$\Rightarrow [10\alpha t^2 \hat{i} + [5\beta t - 5] \hat{j}] \times [20\alpha t \hat{i} + 5\beta \hat{j}] = 0$$

$$\Rightarrow (10\alpha t^2 \cdot 5\beta) \hat{k} + [20\alpha t (5\beta t - 5)] - \hat{k} = 0$$

$$\Rightarrow 10\alpha t^2 \cdot 5\beta = 20\alpha t [5\beta t - 5]$$

$$\Rightarrow 5\beta t = 2(5\beta t - 5)$$

$$\Rightarrow 10 = 5\beta t$$

$$\Rightarrow t = \frac{2}{\beta}$$

**Question:** Two wires of equal dimensions and Young's modulus  $Y_1$  and  $Y_2$  are connected end to end. What is the equivalent Young's modulus for the combination

**Options:**

(a)  $\frac{Y_1 + Y_2}{2}$

(b)  $\sqrt{Y_1 Y_2}$

(c)  $\frac{2Y_1 Y_2}{(Y_1 + Y_2)}$

(d)  $\sqrt{\frac{Y_1^2 Y_2^2}{2}}$

**Answer:** (c)

**Solution:**

$$Y = \frac{F}{A} \left( \frac{L}{\Delta L} \right)$$

$$\Rightarrow F = \left( \frac{YA}{L} \Delta L \right)$$

$$K_1 = \frac{Y_1 A}{L}$$

$$K_2 = \frac{Y_2 A}{l}$$

$$K_{net} = \frac{Y_{net} A}{2L} \quad [\because \text{total length} = 2L]$$

$$\text{In series } K_{net} = \frac{K_1 K_2}{K_1 + K_2}$$

$$\Rightarrow \frac{\left( \frac{Y_1 A}{L} \right) \left( \frac{Y_2 A}{L} \right)}{\left( \frac{Y_1 A}{L} \right) + \left( \frac{Y_2 A}{L} \right)} = \frac{(Y_{net}) A}{2L}$$

$$\Rightarrow Y_{net} = \frac{2Y_1 Y_2}{Y_1 + Y_2}$$

**Question:** For a radioactive sample, time taken for quarter of the sample to decay is  $T_1$  and time taken for half the sample to decay is  $T_2$ . Find the value of  $T_2 - T_1$

**Options:**

(a)  $\frac{1}{\lambda} \ln \left( \frac{3}{2} \right)$

(b)  $\lambda \left( \ln \left( \frac{3}{2} \right) \right)$

(c)  $\frac{1}{\lambda} \ln \left( \frac{2}{3} \right)$

(d)  $\lambda \ln \left( \frac{2}{3} \right)$

**Answer: (a)**

**Solution:**

$$N = N_0 e^{-\lambda t}$$

$$3 \frac{N_0}{4} = N_0 e^{-\lambda T_1}$$

$$\Rightarrow e^{-\lambda T_1} = \frac{3}{4} \dots (i)$$

$$\frac{N_0}{2} = N_0 e^{-\lambda T_2}$$

$$\Rightarrow e^{-\lambda T_2} = \frac{1}{2} \dots (ii)$$

Dividing equation (i) and (ii)

$$e^{-\lambda(T_1 - T_2)} = \frac{2 \times 3}{4}$$

Taking natural log on both sides

$$-\lambda(T_1 - T_2) = \ln\left(\frac{3}{2}\right)$$

$$T_2 - T_1 = \frac{1}{\lambda} \ln\left(\frac{3}{2}\right)$$

**Question:** A bulb rated 200W, 100V is connected to 200V supply. What external resistance should be connected in series to have same power as earlier?

**Options:**

(a)  $100\Omega$

(b)  $50\Omega$

(c)  $10\Omega$

(d)  $5\Omega$

**Answer: (b)**

**Solution:**

Rated power of the bulb is 200 W and operating voltage is 100 V.

$$I = \frac{P}{V} = \frac{200}{100} = 2A$$

$$R = \frac{V}{I} = \frac{100}{2} = 50\Omega$$

When this bulb is connected in 200 V supply, we need to ensure the current passing through this bulb is 2 A by connecting additional resistance

Total resistance required to get 2 A from 200 V supply =  $200/2 = 100\Omega$

Since resistance of bulb is  $50\Omega$ , we need to add additional  $50\Omega$  resistance in series.

**Question:** Water drops are coming out of a tap at constant rate. A falling drop is observed at 4<sup>th</sup> second after fall. Distance between that drop and next drop is 34.3 m. Find the rate of water drops coming out of tap (drops/sec) [ $g = 9.8 \text{ m/s}^2$ ]

**Options:**

- (a) 2
- (b) 1
- (c) 1.5
- (d) 3

**Answer:** (b)**Solution:**

Distance moved by 1<sup>st</sup> drop in 4 second

$$S_1 = \frac{1}{2} \times 9.8 \times 4^2$$

$$S_1 = 9.8 \times 8 = 78.4m$$

Distance moved by 2<sup>nd</sup> drop

$$S_2 = 78.4 - 34.3$$

$$S_2 = 44.1m$$

$$44.1 = \frac{1}{2} \times 9.8 \times t^2$$

$$\frac{88.2}{9.8} = t^2$$

$$t = 3 \text{ sec}$$

That means 2<sup>nd</sup> drop fall 1 second after the first drop.

So, water drops coming out of tap at 1 drop/sec.

**Question:** In a potentiometer setup, EMF of primary cell is 6 V and internal resistance is  $20\Omega$ . The potentiometer wire is of length 10 m having  $0.1\Omega/cm$ . This setup is used to find EMF of a cell which is balanced at 5.5 m mark. Find EMF of the cell.

**Options:**

- (a) 2.75 V
- (b) 2.25 V
- (c) 2.5 V
- (d) 3 V

**Answer:** (a)**Solution:**

$$I = \frac{V}{R} = \frac{6}{(20 + 0.1 \times 100 \times 10)} = \frac{6}{120} A$$

$$EMF = IR' = \frac{1}{120} \times (0.1 \times 100 \times 5.5)$$

$$EMF = \frac{6}{120} \times 55 = 2.75V$$

**Question:** A nucleus of mass 4 m disintegrates into two daughter nuclei of mass m and 3 m. Find the ratio of their respective De Broglie wavelength?

**Options:**

(a) 1 : 3

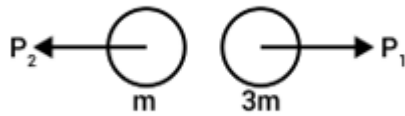
(b) 3 : 1

(c)  $1:\sqrt{3}$

(d) 1 : 1

**Answer:** (d)

**Solution:**



$$P_i = P_f = 0$$

$$0 = P_1 - P_2$$

$$P_1 = P_2$$

$$\lambda = \frac{h}{P} \Rightarrow \lambda_1 = \frac{h}{P_1}$$

$$\lambda_2 = \frac{h}{P_2}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{1}{1} = 1:1$$

**Question:** In a YDSE setup, the distance between the slits varies as  $d = d_0 + A \sin \omega t$ . What is the difference between the maximum and minimum fringe width?

**Options:**

(a)  $\frac{2\lambda AD}{d_0^2}$

(b)  $\frac{2\lambda AD}{d_0^2 - A^2}$

(c)  $\frac{2\lambda AD}{d_0^2 + A^2}$

(d)  $\frac{2\lambda d_0 D}{d_0^2 - A^2}$

**Answer:** (b)

**Solution:**

$$\text{Fringe width } (\beta) = \frac{\lambda D}{d}$$

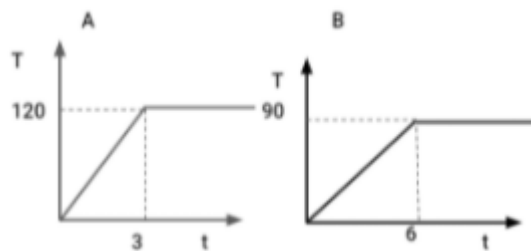
$$\beta = \frac{\lambda D}{d_0 + A \sin \omega t}$$

$$\beta_{\max} = \frac{\lambda D}{d_0 - A}$$

$$\beta_{\min} = \frac{\lambda D}{d_0 + A}$$

$$\beta_{\max} - \beta_{\min} = \frac{2\lambda AD}{d_0^2 - A^2}$$

**Question:** Temperature vs time graphs are given below for 2 substances. Compare specific heat capacity.



**Options:**

- (a)  $S_A > S_B$
- (b)  $S_B > S_A$
- (c)  $S_A = S_B$
- (d) Can't be determined

**Answer:** (b)

**Solution:**

$$C = \frac{Q}{m\Delta T}$$

Where, C = specific heat capacity

Now if mass and energy given are same

$$\text{Then } C \propto \frac{1}{\Delta T}$$

$$\Delta T \text{ in A in 3 sec} = 120^\circ$$

$$\Delta T \text{ in B in 6 sec} = 90^\circ$$

As we can see temperature of A is rising faster as compared to B for the same amount of energy given. We can say

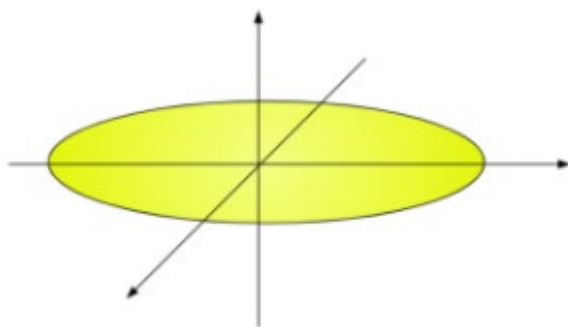
$$C_B > C_A$$

$$\text{Or } S_B > S_A$$

**Question:** x and y are the axes along the diameter of a disk of mass m and radius R. z-axis is perpendicular to plane of the disk.

Assertion: Radius of gyration is same about all three axes.

Reason: All axes are symmetry axes.



**Options:**

- (a) Assertion and Reason both are correct and Reason is correct explanation for assertion
- (b) Assertion and Reason both are correct but Reason doesn't explain assertion
- (c) Assertion is right and Reason is wrong
- (d) Assertion is wrong and Reason is right

**Answer:** (d)

**Solution:**

Assertion: Radius of gyration is same about all three axes.

$$K_x = \frac{R}{2} \quad K_y = \frac{R}{2} \quad K_z = \frac{R}{\sqrt{2}}$$

$$\text{As } mK^2 = I$$

$$K = \sqrt{\frac{I}{m}}$$

So, assertion is wrong.

Reason: All axes are symmetric axes.

Reason is true as

About x, y and z axes object is symmetry.

**Question:** A ball of mass 2 kg moving with 4 m/s collides elastically with a stationary ball. If it continues to move in original direction with  $\frac{1}{4}^{\text{th}}$  of its original velocity. Find velocity of centre of Mass of system.

**Options:**

- (a) 3 m/s
- (b)  $\frac{2}{5} m / s$
- (c)  $\frac{5}{2} m / s$
- (d) 2 m/s

**Answer:** (c)

**Solution:**

$$m_1 = 2\text{kg}$$

$$u_1 = 4\text{m} / \text{s}$$

$$e = 1$$

$$v_1 = 1m / s$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$2 \times 4 + 0 = 2 \times 1 + m_2 v_2$$

$$m_2 v_2 = 6kgm / s \dots(i)$$

$$\frac{v_2 - v_1}{u_1 - u_2} = 1$$

$$\frac{v_2 - 1}{4} = 1$$

$$v_2 = 5m / s \dots(ii)$$

From eq (i) and (ii)

$$m_2 = \frac{6}{5} kg$$

$$v_{com} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$= \frac{2 \times 1 + \frac{6}{5} \times 5}{2 + \frac{6}{5}}$$

$$= \frac{8 \times 5}{16} = \frac{40}{16} = \frac{5}{2} m / s$$

**Question:** A gas has  $C_p - C_v = R$  at temperature  $T = P$  and  $C_p - C_v = 1.1R$  at temperature  $T = Q$ . Then

**Options:**

- (a)  $P = Q$
- (b)  $P > Q$
- (c)  $P < Q$
- (d) None of these

**Answer:** (b)

**Solution:**

Since we know that,  $C_p - C_v = nR$

Therefore, at temperature P has lesser number of moles of gas than at temperature Q.

So, temperature in state P will be greater than the temperature in state Q

Since,  $T \propto \frac{1}{n}$

So,  $P > Q$

# JEE-Main-25-07-2021-Shift-1 (Memory Based)

## CHEMISTRY

**Question:** Which of the following gives  $\text{CO}_2$  on reaction with  $\text{NaHCO}_3$ ?

**Options:**

- (a)  $(\text{CH}_3)_4\text{N}^+\text{OH}^-$
- (b)  $\text{CH}_3\text{NH}_2$
- (c) Benzene Diazonium +  $\text{HCl}$
- (d) Acetic acid

**Answer: (d)**

**Solution:** Carboxylic acids and enols react with sodium bicarbonate to liberate carbon dioxide.

**Question:** Which of the following do not exist?

**Options:**

- (a)  $\text{SiF}_6^{2-}$
- (b)  $\text{GeCl}_6^{2-}$
- (c)  $\text{Sn}(\text{OH})_6^{2-}$
- (d)  $\text{SiCl}_6^{2-}$

**Answer: (d)**

**Solution:**  $\text{SiF}_6^{2-}$  is known to be existed because of small size of F.

As we go down the group in halides their existence is less stable because of large size of halogen that is the reason why  $\text{SiCl}_6^{2-}$  do not exist.

**Question:** Which of the following give para Bromination on phenol as a major product.

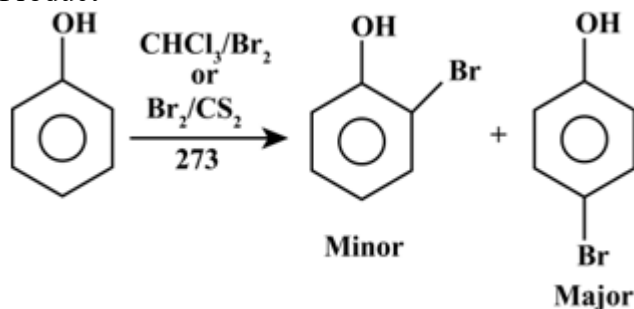
**Options:**

- (a)  $\text{Br}_2/\text{FeBr}_3$
- (b)  $\text{CS}_2/\text{Br}_2$
- (c)  $\text{CHCl}_3/\text{Br}_2$
- (d) Bromine water

**Answer: (b), (c)**

**Solution:**

Product



**Question:** Which of the following is water soluble protein?

**Options:**

- (a) Albumin
- (b) Collagen
- (c) Myosin
- (d) Fibrin

**Answer: (a)**

**Solution:** Albumin is water soluble protein.

Albumin is a protein that is produced in the liver. Albumin enters the bloodstream where it helps carry vitamins, enzymes, and other important substances.

**Question:** Which of the following is leached out from the extraction of Al from Bauxite?

**Options:**

- (a)  $\text{Al}_2\text{O}_3$
- (b) Al
- (c) AlN
- (d)  $\text{Al}_2\text{S}_3$

**Answer: (a)**

**Solution:** in the leaching process the bauxite ore is made to digest with a concentrated solution of NaOH that produces a complex named sodium aluminate. When this complex is made to bear  $\text{CO}_2$ , a hydrated compound that is hydrated alumina is precipitated.

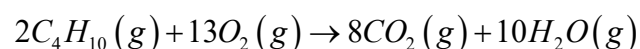
**Question:** In the complete combustion of Butane, 72g of water is given out. How much Butane was there?

**Options:**

- (a) 50 g
- (b) 54 g
- (c) 46.4 g
- (d) 44.6 g

**Answer: (c)**

**Solution:**



116 g butane gives 180 g water

72 g water will get release by = 46.4g

**Question:** Assertion (A): Primary aromatic amine cannot be Gabriel phthalimide reaction.

Reason (R): Primary aromatic amine cannot be produced by nucleophilic substitution reaction.

**Options:**

- (a) A – Both A and R are true and R is the correct explanation of A
- (b) B – Both A and R are true but R is not the correct explanation of A
- (c) C – A is true but R is false
- (d) D – A is false but R is true

**Answer: (a)**

**Solution:** During Gabriel phthalimide synthesis, the reaction between phthalimide and ethanolic potassium hydroxide gives potassium salt of phthalimide.

The salt on heating with alkyl halide followed by alkaline hydrolysis gives corresponding primary amine.

Aromatic primary amines cannot be prepared by Gabriel phthalimide synthesis as aryl halides do not undergo nucleophilic substitution with the salt formed by phthalimide.

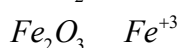
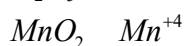
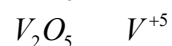
**Question:** Arrange the following in the increasing order of oxidation state:  $\text{CrO}_3$ ,  $\text{V}_2\text{O}_5$ ,  $\text{MnO}_2$ ,  $\text{Fe}_2\text{O}_3$

**Options:**

- (a)  $\text{Fe}_2\text{O}_3 < \text{MnO}_2 < \text{V}_2\text{O}_5 < \text{CrO}_3$
- (b)  $\text{MnO}_2 < \text{Fe}_2\text{O}_3 < \text{CrO}_3 < \text{V}_2\text{O}_5$
- (c)  $\text{Fe}_2\text{O}_3 < \text{MnO}_2 < \text{CrO}_3 < \text{MnO}_2$
- (d)  $\text{MnO}_2 < \text{Fe}_2\text{O}_3 < \text{CrO}_3 < \text{Fe}_2\text{O}_3$

**Answer: (a)**

**Solution:**



**Question:** Which of the following is not used in drycleaning?

Options:

- (a)  $\text{H}_2\text{O}_2$
- (b)  $\text{CCl}_4$
- (c)  $\text{CO}_2$
- (d)  $\text{C}_2\text{Cl}_4$

**Answer: (b)**

**Solution:**

Hydrogen peroxide ( $\text{H}_2\text{O}_2$ ) is an oxidizing agent that can be used as laundry.

Tetrachloroethylene is the most used chemical solvent in dry cleaning process.

Liquid carbon dioxide cleaning is a method that uses pressurized liquid  $\text{CO}_2$  in place of perc, in combination with other cleaning agents.

**Question:** S1 : None of the alkaline earth metal hydroxide are soluble in alkaline solution

S2: The solubility of alkaline earth metal hydroxide decreases down the group.

True or False type

**Options:**

- (a) Both S1 and S2 are true
- (b) Both S1 and S2 are false
- (c) S1 is true but S2 is false
- (d) S1 is false but S2 is true

**Answer: (b)**

**Solution:** Beryllium hydroxide dissolves in excess  $\text{NaOH}$ , So this is due to the formation of a complex,  $\text{Be}(\text{OH})_2 + 2\text{NaOH} \rightarrow \text{Na}_2[\text{Be}(\text{OH})_4]$

Solubility of alkaline earth metal hydroxide increase with increasing atomic number from  $\text{Mg}(\text{OH})_2$  To  $\text{Ba}(\text{OH})_2$

→ S<sub>1</sub> is wrong

→ S<sub>2</sub> is wrong

**Question:** Which of the following complexes show attraction in external magnetic field?

**Options:**

(a)  $\text{Co}(\text{CN})_6^{3-}$

(b)  $\text{Ni}(\text{CO})_4$

(c)  $\text{Ni}(\text{CN})_4^{2-}$

(d)  $\text{Fe}(\text{H}_2\text{O})_6^{3+}$

**Answer: (d)**

**Solution:**

$[\text{Co}(\text{CN})_6]^{3-} \Rightarrow d^6 \rightarrow$  Strong field all paired electrons.

$[\text{Ni}(\text{CO})_4] \Rightarrow d^{10} \rightarrow$  Strong field all paired electrons

$[\text{Ni}(\text{CN})_4]^{2-} \Rightarrow d^8 \rightarrow$  Strong field all paired electrons

$[\text{Fe}(\text{H}_2\text{O})_6]^{3+} \Rightarrow d^5 \rightarrow$  weak field 5 unpaired electrons

**Question:** Decreasing order of ionic radii :  $\text{Al}^{3+}$  ;  $\text{K}^+$  ;  $\text{Na}^+$  ;  $\text{Mg}^{2+}$

**Options:**

(a)  $\text{K}^+ > \text{Na}^+ > \text{Mg}^{2+} > \text{Al}^{3+}$

(b)  $\text{Na}^+ > \text{K}^+ > \text{Mg}^{2+} > \text{Al}^{3+}$

(c)  $\text{Mg}^{2+} > \text{Na}^+ > \text{Al}^{3+} > \text{Mg}^{2+}$

(d)  $\text{Al}^{3+} > \text{K}^+ > \text{Mg}^{2+} > \text{Na}^+$

**Answer: (a)**

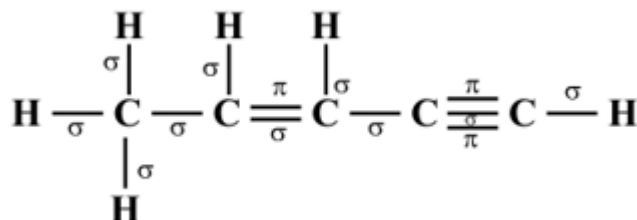
**Solution:**

$$\begin{cases} \text{Al}^{+3} < \text{Mg}^{+2} < \text{Na}^+ < \text{K}^+ \\ 53.5 \text{ pm} & 72 \text{ pm} & 102 \text{ pm} & 138 \text{ pm} \end{cases}$$

**Question:** Number of sigma bonds in  $\text{CH}_3\text{CH}=\text{CH}-\text{C}\equiv\text{CH}$

**Answer: 10.00**

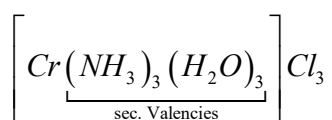
**Solution:**



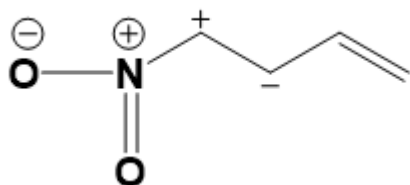
**Question:** Empirical formula of an octahedral complex is  $\text{CrCl}_3.3\text{NH}_3.3\text{H}_2\text{O}$ . It precipitates 3 moles of  $\text{AgCl}$ . Find the secondary valency of the central atom.

**Answer: 6.00**

**Solution:**

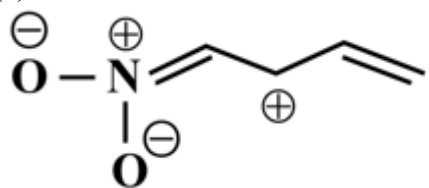


**Question:** Correct resonating structure of the given structure

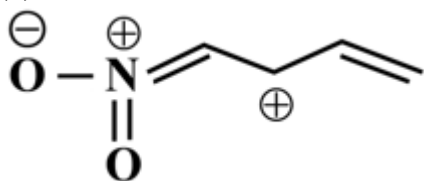


**Options:**

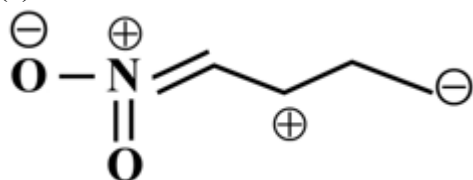
(a)



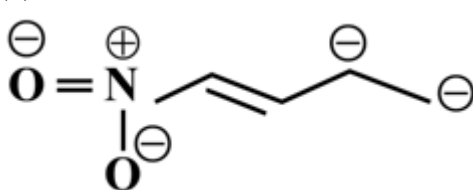
(b)



(c)

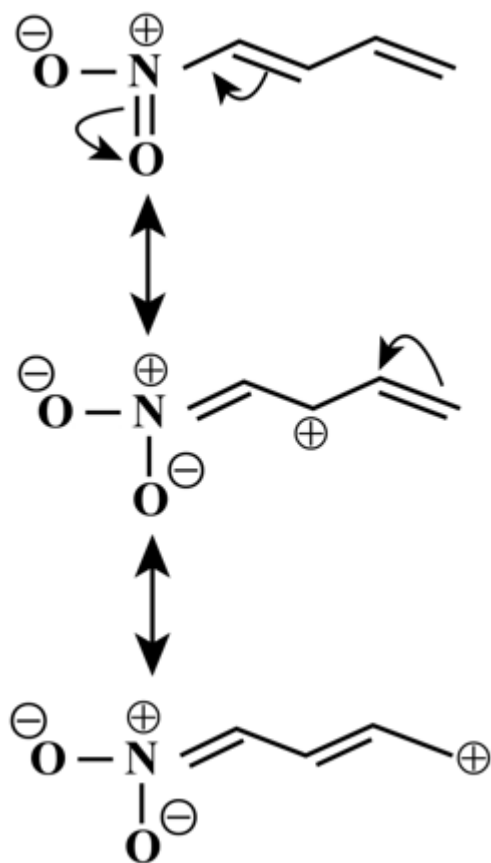


(d)

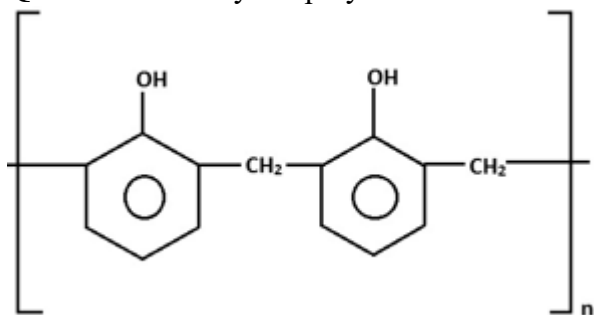


**Answer: (a)**

**Solution:**



**Question:** Identify the polymer.

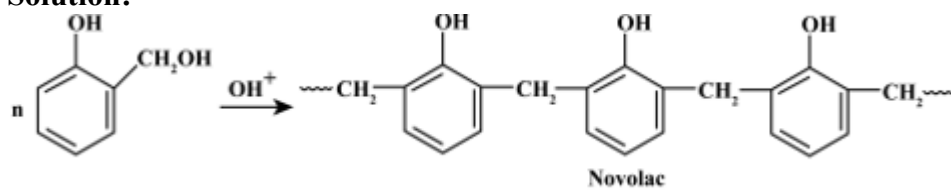


**Options:**

- (a) Novalac
- (b) Neoprene
- (c) Buna N
- (d) Nylon

**Answer:** (a)

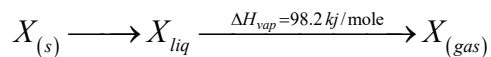
**Solution:**



**Question:** X Solid gets converts into X liquid , X liquid gets converts into X gas . Enthalpy of fusion of solid X = 2.8 kJ/mol , and enthalpy of vaporization of X = 98.2 kJ/mol , The enthalpy of sublimation is:

**Answer:**

**Solution:**



$$\Delta H_{Fusion} = 2.8 \text{ kJ / mole}$$

$$\Delta H_{fusion} + \Delta H_{vap} = 2.8 + 98.2 = 101 \text{ kJ / mol}$$

**Question:** Bond dissociation enthalpy of H ( $E_H$ ) and bond dissociation enthalpy of deuterium ( $E_D$ ) are related as:

**Options:**

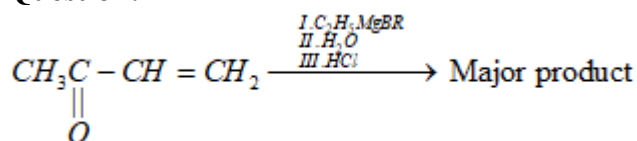
- (a)  $E_H = E_D$
- (b)  $E_H = \frac{1}{2} E_D$
- (c)  $E_H = 2E_D$
- (d)  $E_H \sim E_D - 7.5$

**Answer: (d)**

**Solution:** Bond dissociation enthalpy

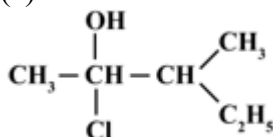
$$\begin{array}{ccc} D_2 & > & H_2 \\ 443.35 \text{ kJ/mol} & & 435.88 \text{ kJ/mol} \end{array}$$

**Question:**

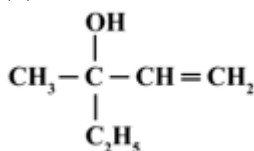


**Options:**

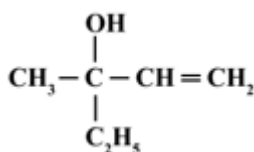
(a)



(b)



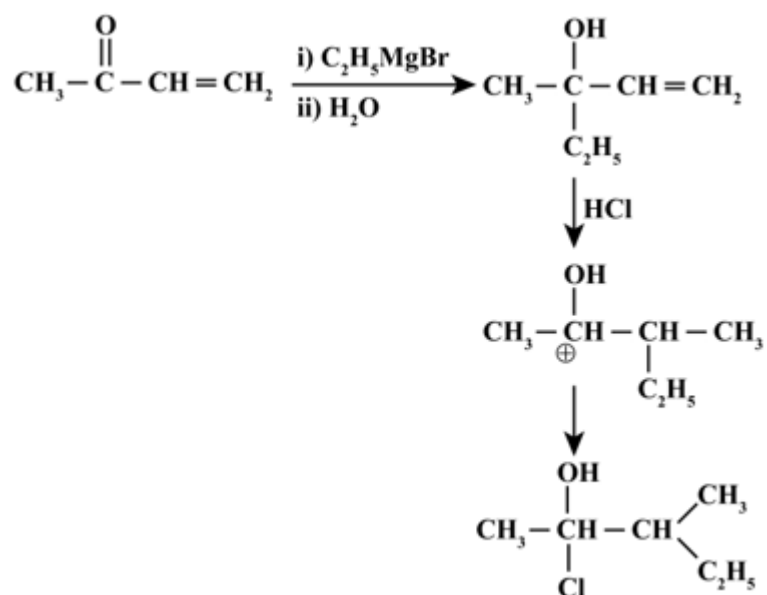
(c)



(d) None of these

**Answer: (a)**

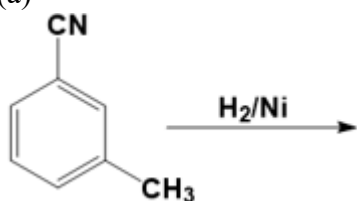
**Solution:**



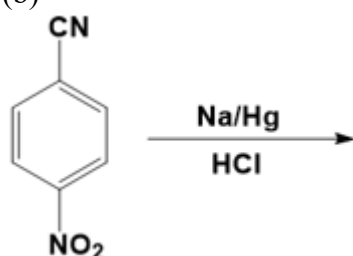
**Question:** Product of which of the following reaction will not give Hinsberg reagent ?

**Options:**

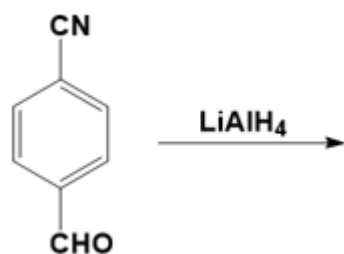
(a)



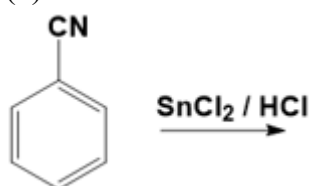
(b)



(c)

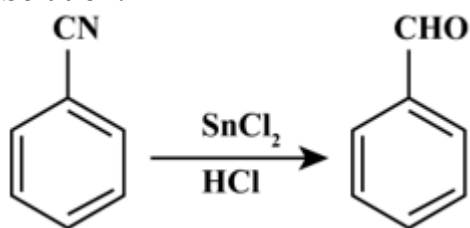


(d)



**Answer: (d)**

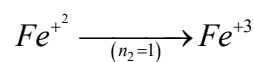
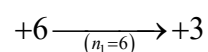
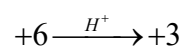
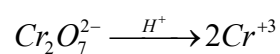
**Solution:**



**Question:**  $\text{Fe}^{2+}$  reacts with  $\text{K}_2\text{Cr}_2\text{O}_7$  in acidic medium. Volume of  $\text{Fe}^{2+} = 10 \text{ ml}$ ,  
Volume of  $\text{K}_2\text{Cr}_2\text{O}_7 = 15 \text{ ml}$ . molarity of  $\text{K}_2\text{Cr}_2\text{O}_7 = 0.1 \text{ M}$   
Find the molarity of  $\text{Fe}^{2+}$

**Answer: 0.9M**

**Solution:**



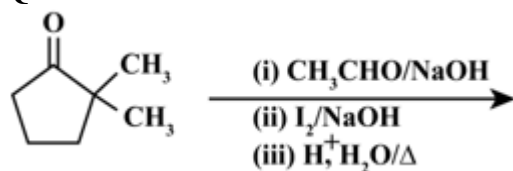
eq. of  $\text{K}_2\text{Cr}_2\text{O}_7 = \text{eq. of } \text{Fe}^{+2}$

$$N_1V_1 = N_2V_2$$

$$n_1m_1v_1 = n_2m_2v_2$$

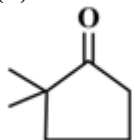
$$6 \times 0.1 \times 15 \times 10^{-3} = 1 \times m_2 \times 10 \times 10^{-3}$$

**Question:**

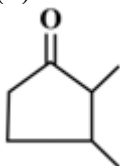


**Options:**

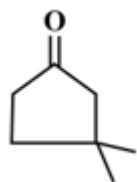
(a)



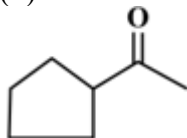
(b)



(c)

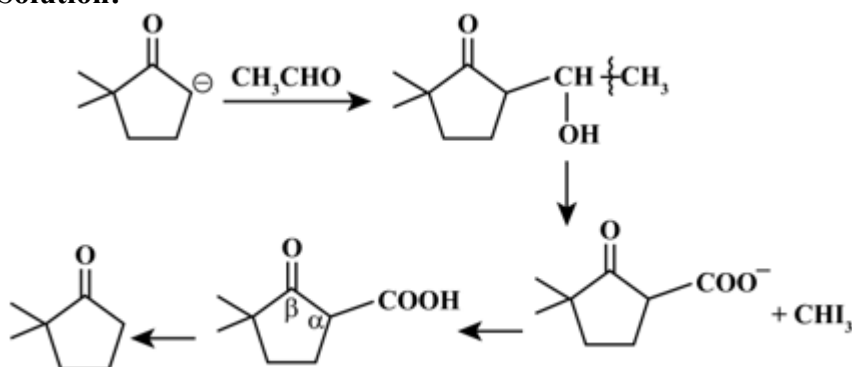


(d)



**Answer: (a)**

**Solution:**

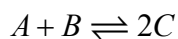


**Question:**  $A + B \rightleftharpoons 2C$

Concentration of  $A = B = C$  (initially) . If  $K_c = 100$  , find concentration  $C$  at equilibrium?

**Answer:**

**Solution:**

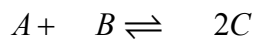


at

$a \quad a \quad a$

$$a = \frac{(C)^2}{(A)(B)} = 1$$

Reaction will go forward ( $a < k_c$ ) ( $k_c = 100$ )



Initially  $a \quad a \quad a$

At equilibrium  $a - x \quad a - x \quad a + 2x$

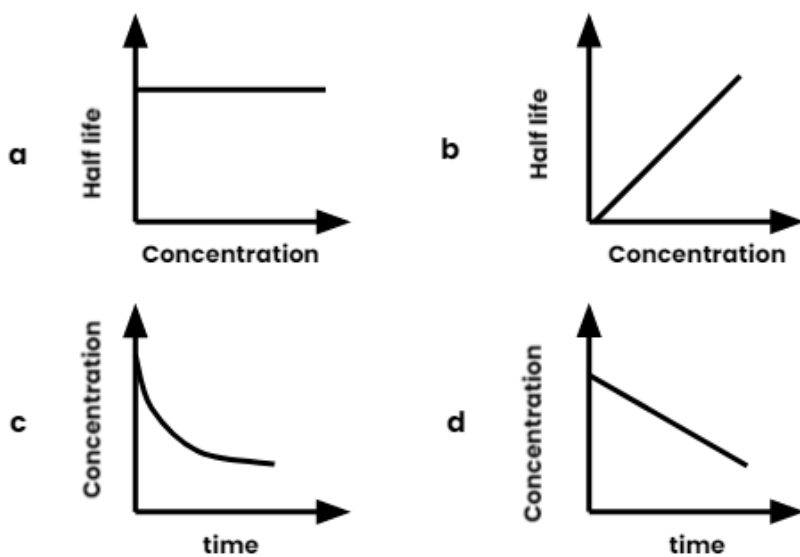
$$\text{Therefore, } \frac{(a + 2x)^2}{(a - x)^2} = 100$$

$$\text{Or, } \frac{(a + 2x)}{(a - x)} = 10$$

$$\text{On solving, } a = \frac{12x}{9}$$

$$[C]_{eq} = \frac{12x}{9} + 2x = \frac{30x}{9}$$

**Question:** Select the correct option for the given graph:



**Options:**

- (a) a , c = first order & b , d = 0 order
- (b) b , d = first order & a , c = 0 order
- (c) All are first order
- (d) All are zero order

**Answer: (a)**

**Solution:** For 1<sup>st</sup> order reaction half life does not depend on concentration but for zero order reaction half life  $\propto$  concentration

# JEE-Main-25-07-2021-Shift-1 (Memory Based)

## MATHEMATICS

**Question:**  $I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{dn}{1 + \sqrt[3]{\tan 2x}}$

**Options:**

- (a)  $\frac{\pi}{4}$
- (b)  $\frac{\pi}{2}$
- (c)  $\frac{\pi}{6}$
- (d)  $\frac{\pi}{12}$

**Answer:** (b)

**Solution:**

$$I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{\sqrt[3]{\sin 2x} dx}{\sqrt[3]{\sin 2x} + \sqrt[3]{\cos 2x}} = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{\sqrt[3]{\cos 2x} dx}{\sqrt[3]{\sin 2x} + \sqrt[3]{\cos 2x}}$$

$$\therefore 2I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} dx = \frac{\pi}{6} \Rightarrow I = \frac{\pi}{12}$$

**Question:** For a parabola, it's vertex is at a distance of 2 units from origin, focus is at distances of 4 units from origin. A pair of tangents are drawn from origin to the parabola which meet it at P and Q. Find area of  $\triangle OPQ$  (O : origin).

**Options:**

- (a) 16
- (b) 32
- (c)  $16\sqrt{2}$
- (d)  $32\sqrt{2}$

**Answer:** (a)

**Solution:**

$$a = 2, y^2 = 8(x - 2)$$

$\therefore$  origin lies on directrix of parabola.

$$\therefore \text{Area of } \triangle OPQ = \frac{1}{2} \times 2a \times 4a = 4a^2 = 16$$

**Question:** If  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  ellipse passes through  $\left(\sqrt{\frac{3}{2}}, 1\right)$ ,  $e = \frac{1}{\sqrt{3}}$ . Circle centred at one of the focus and radius  $\frac{2}{\sqrt{3}}$ . These ellipse & circle intersect at two points. Find square of the distance between the two points.

**Options:**

- (a)  $\frac{4}{3}$
- (b)  $\frac{2}{3}$
- (c)  $\frac{16}{3}$
- (d)  $\frac{32}{3}$

**Answer:** (c)

**Solution:**

$$e^2 = 1 - \frac{b^2}{a^2} \Rightarrow \frac{b^2}{a^2} = \frac{2}{3} \quad \dots(1)$$

$$\frac{3}{2a^2} + \frac{3}{2a^2} = 1 \Rightarrow a^2 = 3, b^2 = 2 \Rightarrow \frac{x^2}{3} + \frac{y^2}{2} = 1$$

$$\therefore \text{ focus } = (1, 1, 0)$$

$$\therefore \text{ Equation of circle is } (x-1)^2 + y^2 = \frac{4}{3}$$

$$\Rightarrow 3(x-1)^2 + 6x^2 = 4$$

$$\Rightarrow x^2 - 6x + 5 = 0 \Rightarrow x = 1, 5$$

$$\therefore P\left(1, \frac{2}{\sqrt{3}}\right), Q = \left(1, \frac{-2}{\sqrt{3}}\right)$$

$$PQ = \frac{4}{\sqrt{3}} \Rightarrow PQ^2 = \frac{16}{3}$$

**Question:**  $(p \rightarrow q) \wedge (q \rightarrow \sim p)$  is equivalent to

**Options:**

- (a)  $\sim p$
- (b)  $p$
- (c)  $\sim q$

(d)  $q$

**Answer:** (a)

**Solution:**

$$\begin{aligned} & (p \rightarrow q) \wedge (q \rightarrow \sim p) \\ &= (\sim p \vee q) \wedge (\sim q \vee \sim p) \\ &= \sim p \vee (q \wedge \sim q) \\ &= \sim p \vee \text{false} \\ &= \sim p \end{aligned}$$

**Question:**  $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$ . Find sum of roots that lying in  $[0, 2\pi]$

**Options:**

- (a)  $8\pi$
- (b)  $9\pi$
- (c)  $11\pi$
- (d)  $12\pi$

**Answer:** (b)

**Solution:**

$$\begin{aligned} & (\sin x + \sin 3x) + (\sin 2x + \sin 4x) = 0 \\ & 2 \sin 2x \cos x + 2 \sin 3x \cos x = 0 \\ & 2 \cos x \cdot 2 \sin \frac{5x}{2} \cos \frac{5x}{2} = 0 \\ & \Rightarrow \cos x \cdot \cos \frac{x}{2} \cdot \sin \frac{5x}{2} = 0 \\ & \text{(i) } \cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2} \\ & \text{(ii) } \cos \frac{x}{2} = 0 \Rightarrow x = \pi \\ & \text{(iii) } \sin \frac{5x}{2} = 0 \Rightarrow x = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, 2\pi \\ & \therefore \text{sum of roots} = 9\pi \end{aligned}$$

**Question:** In An A.P.,  $S_{3n} = 3S_{2n}$ . Find  $\frac{S_{4n}}{S_{2n}}$ .

**Options:**

- (a) 2
- (b) 4
- (c) 6
- (d) 8

**Answer:** (c)

**Solution:**

$$S_{3n} = 3S_{2n}$$

$$\left(\frac{3n}{2}\right)[2a + (3n-1)d] = 3\left(\frac{2n}{2}\right)[2a + (2n-1)d]$$

$$2a + 3nd - d = 4a + 4nd - 2d$$

$$2a = d - nd \Rightarrow d = \frac{2a}{1-n}$$

$$\text{Now, } \frac{S_{4n}}{S_{2n}} = 2 \left[ \frac{2a + (4n-1)d}{2a + (2n-1)d} \right] = \frac{2 \times 3}{1} = 6$$

**Question:**  $\frac{1}{a-b} + \frac{1}{a-2b} + \frac{1}{a-3b} + \dots + \frac{1}{a-nb} = \alpha \cdot n + \beta \cdot n^2, \frac{b}{a}$  is small such that  $\left(\frac{b}{a}\right)^3$  & other

higher powers are neglected. Then find  $r$ .

**Options:**

(a)  $\frac{b^2}{3a^3}$

(b)  $\frac{b^2 + a}{3a^2}$

(c)

(d)

**Answer:** (a)

**Solution:**

$$\frac{1}{a} \left[ \left(1 - \left(\frac{b}{a}\right)\right)^{-1} + \left(1 - 2\left(\frac{b}{a}\right)\right)^{-1} + \left(1 - 3\left(\frac{b}{a}\right)\right)^{-1} + \dots + \left(1 - n\left(\frac{b}{a}\right)\right)^{-1} \right]$$

$$= \frac{1}{a} \left[ \left(1 + \frac{b}{a} + \frac{b^2}{a^2}\right) + \left(1 + \frac{2b}{a} + \frac{4b^2}{a^2}\right) + \left(1 + \frac{3b}{a} + \frac{9b^2}{a^2}\right) + \dots \right]$$

$$= \frac{1}{a} \left[ n + \frac{n(n+1)b}{2a} + \frac{n(n+1)(2n+1)}{6} \frac{b^2}{a^2} \right]$$

$$\therefore \text{coefficient of } n^3 = r = \frac{b^2}{3a^3}$$

**Question:** If the coefficient middle term of  $(1+x)^{20}$  is A and the coefficient of middle terms

of  $(1+x)^{19}$  are B and C then find  $\frac{A}{B+C}$

**Answer:** 1.00

**Solution:**

$$A = {}^{20}C_{10}, B = {}^{19}C_9, C = {}^{19}C_{10}$$

$$\therefore \frac{A}{B+C} = \frac{{}^{20}C_{10}}{2 \cdot {}^{19}C_9} = 1$$

**Question:** In class 12 there are 8 students, In class 11 there are 6 students, In class 10 there are 5 students. The probability of selecting 10 students, such that there are at least 2 students from each class and at most 5 students from 11 students of class 10 & 11 combined is  $100k$ . Find  $k$ ....

**Answer:** 238.00

**Solution:**

$$\begin{aligned} & {}^5C_2 \left[ {}^6C_3 \times {}^8C_5 + {}^6C_2 \times {}^8C_6 \right] + {}^5C_3 \left[ {}^6C_2 \times {}^8C_5 \right] \\ &= 10[1540] + 10[840] \\ &= 23800 \\ \therefore k &= 238 \end{aligned}$$

**Question:** Find the locus of centroid formed by any point P & foci of hyperbola.

$$16x^2 - 9y^2 + 32x + 36y - 164 = 0.$$

**Answer:** 1.00

**Solution:**

$$16x^2 + 32x - 9y^2 + 36y = 164$$

$$16(x^2 + 2x) - 9(y^2 - 4y) = 164$$

$$16(x+1)^2 - 9(y-2)^2 = 164 + 16 - 36$$

$$\therefore \frac{(x+1)^2}{9} - \frac{(y-2)^2}{16} = 1$$

$\therefore$  Any point on hyperbola is  $(-1, 3 + \sec \theta, 2 + 4 \tan \theta)$  and foci  $= (-1 \pm 5, 2)$ , i.e.,

$(-6, 2)$  &  $(4, 2)$

$$\therefore A(4, 2), B(-6, 2), C(-1 + 3 \sec \theta, 2 + 4 \tan \theta)$$

$$\therefore \text{Centroid} \Rightarrow \left( \frac{-3 + 3 \sec \theta}{3}, \frac{6 + 4 \tan \theta}{3} \right)$$

$$\therefore \text{locus is } (x+1)^2 - 9 \frac{(y-2)^2}{16} = 1$$

**Question:** Let  $S = \left\{ n \in N, \begin{bmatrix} 0 & i \end{bmatrix}^n \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \forall a, b, c, d \in R \right\}$ . Find number of 2-digit numbers in  $S$

**Answer:** 22.00

**Solution:**

$$\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}^n = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

If  $n$  is multiple of 4

Thus, 2 digit number which are multiple of 4 are

$$S = \{12, 16, 20, \dots, 96\}$$

So, thus, S contains 22, two digit numbers.

**Question:** If  $\left(1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots\right)^{\log_{(0.25)}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots\right)} = l$  then find  $l^2$

**Answer:** 3.00

**Solution:**

$$\left(1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots\right)^{\log_{0.25}\left(1 + \frac{1}{3} + \frac{1}{3^2} + \dots\right)} = l$$

$$x = \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots = \frac{\frac{1}{3}}{1 - \frac{1}{3}} = \frac{1}{2} = 0.5$$

$$\begin{aligned} \text{Also, } y &= 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots \\ \frac{y}{3} &= \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \dots \end{aligned}$$

$$\frac{2y}{3} = \frac{4}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \dots = \frac{\frac{4}{3}}{1 - \frac{1}{3}} = 2$$

$$\therefore y = 3$$

$$\Rightarrow (3)^{\log_{0.25}(0.5)} = \sqrt{3}$$

$$\therefore l^2 = 3$$

**Question:** If  $\left[ \frac{x+1}{x^{\frac{2}{3}} + 1 - x^{\frac{1}{3}}} - \frac{x-1}{(x+\sqrt{x})} \right]^{10}$ . Find term independent of  $x$ .

**Answer:** 210.00

**Solution:**

$$\left[ \left( x^{\frac{1}{3}} + 1 \right) - \frac{\left( x^{\frac{1}{2}} + 1 \right)}{x^{\frac{1}{2}}} \right]^{10} = \left( x^{\frac{1}{3}} - x^{\frac{-1}{2}} \right)^{10}$$

$$\therefore T_{r+1} = {}^{10}C_r \left( x^{\frac{1}{3}} \right)^r \cdot \left( -x^{\frac{1}{2}} \right)^{10-r}$$

$$= {}^{10}C_r (-1)^{10-r} \cdot x^{\frac{r}{3} + \frac{r}{2} - 5}$$

$$\text{For term independent} \Rightarrow \frac{r}{3} + \frac{r}{2} = 5 \Rightarrow r = 6$$

$$\therefore \text{Term independent} = T_7 = {}^{10}C_6 = 210$$

**Question:**  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ ,  $a, b, c, d \in \{3, -3, -2, 2, 1, -1, 0\}$ ,  $f(A) = \det(A)$ . Find probability that  $f(A) = 15$ .

**Answer:**  $\frac{16}{7^4}$

**Solution:**

$$|A| = ad - bc$$

(I)  $a = \pm 3, d = \pm 3, b = \pm 2, c = \mp 3 \rightarrow 4$  cases

(II)  $a = \pm 3, d = \pm 3, b = \mp 3, c = \pm 2 \rightarrow 4$  cases

(III)  $a = \pm 2, d = \pm 3, b = \mp 3, c = \pm 3 \rightarrow 4$  cases

(IV)  $a = \pm 3, d = \pm 2, b = \mp 3, c = \pm 3 \rightarrow 4$  cases

$\therefore$  Total favourable cases = 16

$$\therefore \text{Probability} = \frac{16}{7^4}$$

**Question:**  $\vec{p} = (3\hat{i} + 2\hat{j} + \hat{k}), \vec{q} = (2\hat{i} + \hat{j} + \hat{k}), \vec{r}$  is perpendicular to both  $\vec{p} + \vec{q}$  and  $(\vec{p} - \vec{q}), |\vec{r}| = \sqrt{3}, \vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$ . Find  $|a| + |b| + |c|$ .

**Answer:** 3.00

**Solution:**

$$\vec{p} + \vec{q} = 5\hat{i} + 3\hat{j} + 2\hat{k}; \vec{p} - \vec{q} = \hat{i} + \hat{j}$$

$$\therefore \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 3 & 2 \\ 1 & 1 & 0 \end{vmatrix} = \lambda(-2\hat{i} + 2\hat{j} + 2\hat{k})$$

$$|\vec{r}| = 2\lambda\sqrt{3} = \sqrt{3} \Rightarrow \lambda = \frac{1}{2}$$

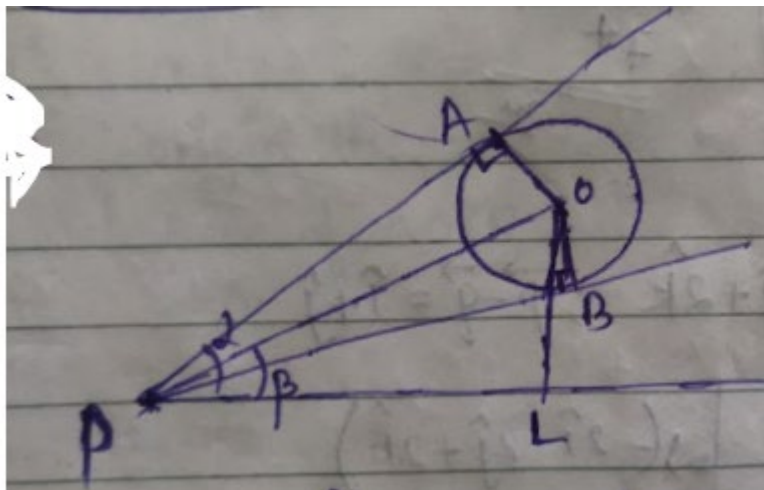
$$\therefore \vec{r} = -\hat{i} + \hat{j} + \hat{k}$$

$$\therefore |\vec{a}| + |\vec{b}| + |\vec{c}| = 3$$

**Question:** A spherical balloon of radius 16m subtends  $60^\circ$  at eye of an observer on the ground. The angle of elevation of centre from the same point of observation is  $75^\circ$ . Find the height of top most point of the balloon.

**Answer:**

**Solution:**



$$\alpha = 60^\circ, \beta = 75^\circ$$

$$\sin \frac{\alpha}{2} = \frac{r}{OP} = \frac{1}{2}$$

$$OP = 2r = 32$$

$$\therefore \sin \beta = \frac{OL}{OP} \Rightarrow OL = 32 \times \sin 75 = 8\sqrt{2}(\sqrt{3} + 1)$$

$$\therefore \text{Required Height} = 8\sqrt{2}(\sqrt{3} + 1) + 16$$

**Question:**  $f(x) = \begin{cases} \mu, & x = 2 \\ e^{\frac{\tan(x-2)}{x-[x]}}, & x < 2, \text{ } f(x) \text{ is continuous. Find } \mu + \lambda = \\ \frac{|x^2 - 5x + 6|\lambda}{(-x^2 + 5x - 6)\mu}, & x > 2 \end{cases}$

**Answer:** 2.00

**Solution:**

$$e^{\frac{\tan(x-2)}{(x-1)}} = \mu = \frac{\lambda}{\mu} = 1$$

$$\therefore \lambda = \mu = 1 \Rightarrow \lambda + \mu = 2$$

**Question:**  $x^2 + 5\sqrt{2}x + 10 = 0, P_n = \alpha^n - \beta^n, \frac{P_{17}P_{20} + 5\sqrt{2}}{P_{18}P_{19} + 5\sqrt{2}} \frac{P_{17}P_{19}}{P_{18}^2} = ?$

**Answer:** 1.00

**Solution:**

$$x^2 + 5\sqrt{2}x + 10 = 0; P_n = \alpha^n - \beta^n$$

$$\frac{P_{17} \left[ P_{20} + 5\sqrt{2}P_{19} \right]}{P_{18} \left[ P_{19} + 5\sqrt{2}P_{18} \right]} = \frac{P_{17} \left[ \left( \alpha^{20} - \beta^{20} \right) + 5\sqrt{2} \left( \alpha^{19} - \beta^{19} \right) \right]}{P_{18} \left[ \left( \alpha^{19} - \beta^{19} \right) + 5\sqrt{2} \left( \alpha^{18} - \beta^{18} \right) \right]}$$

$$\Rightarrow \frac{P_{17}}{P_{18}} \left[ \frac{-10\alpha^{18} + 10\beta^{18}}{-10\alpha^{17} + 10\beta^{17}} \right] = 1$$