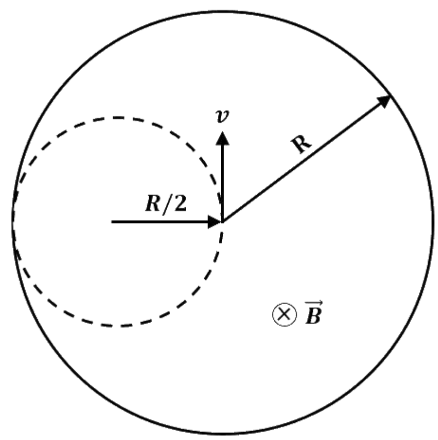
JEE Main 2020 Paper

Date of Exam: **9th January (Shift II)**
Time: **2:30 pm – 5:30 pm**
Subject: **Physics**

1. There is a long solenoid of radius R having n turns per unit length with current flowing in it. A particle having charge q and mass m is projected with speed v in the perpendicular direction of axis from a point on its axis. Find maximum value of v so that it will not collide with the solenoid.

   a. $\frac{Rq\mu_0 in}{5m}$ b. $\frac{Rq\mu_0 in}{2m}$
   c. $\frac{3Rq\mu_0 in}{m}$ d. $\frac{Rq\mu_0 in}{4m}$

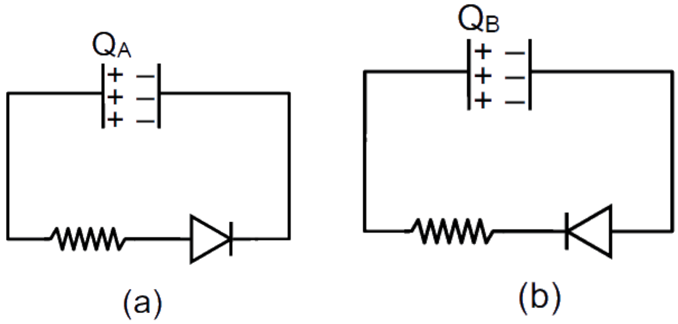
   Solution: (b)
   Looking at the cross-section of the solenoid, R_{max} of the particle's motion has to be $\frac{R}{2}$ for it not to strike the solenoid.



$$qvB = \frac{mv^2}{\frac{R}{2}}$$
$$R_{max} = \frac{R}{2} = \frac{mv_{max}}{q\mu_0 in}$$
$$V_{max} = \frac{Rq\mu_0 in}{2m}$$

2. A capacitor C and resistor R are connected to a battery of 5 V in series. Now the battery is disconnected and a diode is connected as shown in the figures (a) and (b) respectively.

The charge on the capacitor after time RC in (a) and (b) respectively is Q_A and Q_B. Their values are



a. $\frac{5CV}{e}, 5CV$

b. $\frac{5CV}{e}, \frac{5CV}{2e}$

c. $5CV, 5CV$

d. $5CV, \frac{5CV}{e}$

Solution:(a)

Maximum charge on capacitor = $5CV$

is forward biased and (b) is reverse biased

For case (a)

$q = q_{max}(1 - e^{\frac{-t}{RC}}) = 5CV$

$$Q_A = 5CVe^{-1}$$

For case (b)

$$Q_B = 5CV$$

3. Different values of a, b and c are given and their sum is d. Arrange the value of d in increasing order

	a	b	c
1	220.1	20.4567	40.118
2	218.2	22.3625	40.372
3	221.2	20.2435	39.432
4	221.4	18.3625	40.281

e. $d_1 = d_2 = d_3 = d_4$

f. $d_1 < d_2 < d_3 < d_4$

g. $d_1 > d_2 > d_3 > d_4$

h. $d_4 < d_1 < d_3 = d_2$

Solution:(d)

	a	b	c	$a+b+c=d$	Round off
1.	220.1	20.4567	40.118	$d_1 = 280.6747$	280.7
2.	218.2	22.3625	40.372	$d_2 = 280.9345$	280.9
3.	221.2	20.2435	39.432	$d_3 = 280.8755$	280.9
4.	221.4	18.3625	40.281	$d_4 = 280.0435$	280.0

4. A particle starts moving with a velocity $\vec{u} = 3\hat{\imath}$ from origin and an acceleration $\vec{a} = 6\hat{\imath} + 4\hat{\jmath}$. Here, if the y-coordinate of the particle is 32 m, then its x-coordinate at that instant will be

a. 60
b. 48
c. 45
d. 22

Solution:(a)

$S_y = u_y t + \frac{1}{2} a_y t^2$

$32 = 0 + \frac{1}{2} \times 4t^2 \quad \rightarrow \quad t = 4\, sec$

$S_x = u_x t + \frac{1}{2} a_x t^2$

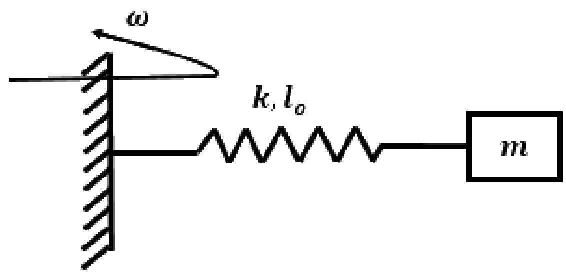
$= 3 \times 4 + \frac{1}{2} \times 6 \times 16$

$= 60\, m$

5. A mass m is attached to a spring of natural length l_o and spring constant k. One end of the spring is attached to the centre of a disc in the horizontal plane which is being rotated by a constant angular speed, ω. Find the extension per unit length in the spring (given $k >>> m\omega^2$)

a. $\frac{3m\omega^2}{2k}$
b. $\frac{\sqrt{2}}{3} \frac{m\omega^2}{k}$
c. $\frac{m\omega^2}{k}$
d. $\frac{3m\omega^2}{k}$

Solution: (c)



Using Newton's second law of dynamics,

$$m\omega^2(l_o + x) = kx$$

$$\left(\frac{l_o}{x} + 1\right) = \frac{k}{m\omega^2}$$

$$x = \frac{l_o m\omega^2}{k - m\omega^2}$$

$$k >> m\omega^2$$

So, $\frac{x}{l_o}$ is equal to $\frac{m\omega^2}{k}$

6. A loop of radius R and mass m is placed in a uniform magnetic field B with its plane perpendicular to the field. A current i is flowing in it. Now the loop is slightly rotated about its diameter and released. Find the time period of oscillations.

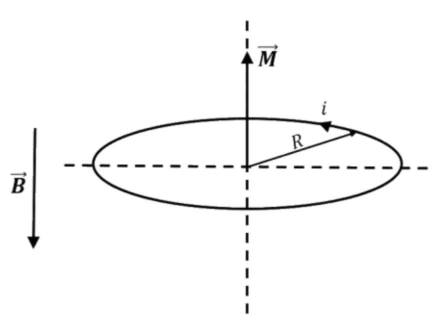
a. $2\sqrt{\frac{\pi M}{iB}}$

b. $\sqrt{\frac{2\pi M}{iB}}$

c. $2\sqrt{\frac{M}{\pi iB}}$

d. $\sqrt{\frac{M}{\pi iB}}$

Solution: (b)



Considering the torque situation on the loop,

$$\tau = MBsin\theta = -i\alpha$$

$$\pi R^2 iB\theta = -\frac{mR^2}{2}\alpha$$

The above equation is analogous to $\theta = -C\alpha$, where $C = \omega^2 = \frac{2\pi iB}{M}$

$$\omega = \sqrt{\frac{2\pi iB}{M}} = \frac{2\pi}{T}$$

$$T = \sqrt{\frac{2\pi M}{iB}}$$

7. A sphere of density ρ is half submerged in a liquid of density σ and surface tension T. The sphere remains in equilibrium. Find the radius of this sphere. (Assume the force due to surface tension acts tangentially to surface of sphere.)

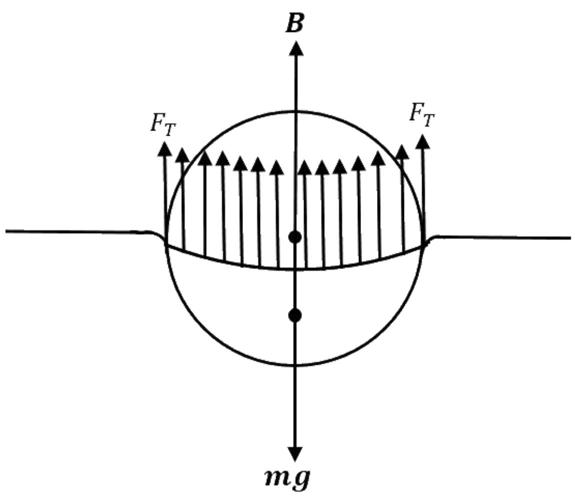
a. $\sqrt{\frac{3T}{2(\rho+\sigma)g}}$

b. $\sqrt{\frac{3T}{4(\rho-\sigma)g}}$

c. $\sqrt{\frac{3T}{2\left(\rho-\frac{\sigma}{2}\right)g}}$

d. $\sqrt{\frac{T}{(\rho+\sigma)g}}$

Solution:(c)



In equilibrium, net external force acting on the sphere is zero.

$mg = F_T + B$

$$\rho V g = \sigma\left(\frac{V}{2}\right) g + T 2\pi R$$

$$\rho \frac{4}{3}\pi R^3 g = \sigma \frac{2}{3}\pi R^3 g + T 2\pi R$$

$$R = \sqrt{\frac{3T}{2\left(\rho - \frac{\sigma}{2}\right) g}}$$

8. A string of mass per unit length $\mu = 6 \times 10^{-3}\ kg/m$ is fixed at both ends under the tension $540\ N$. If the string is in resonance with consecutive frequencies $420\ Hz$ and $490\ Hz$. Then what would be the length of the string?
   a. 2.8 m
   b. 2.1 m
   c. 4.5 m
   d. 4.2 m

Solution: (b)
Key Idea: The difference of two consecutive resonant frequencies is the fundamental resonant frequency.
Fundamental frequency= $490 - 420 = 70\ Hz$

$$70 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

$$\Rightarrow 70 = \frac{1}{2l}\sqrt{\frac{540}{6 \times 10^{-3}}}$$

$$\Rightarrow l = \frac{1}{2 \times 70}\sqrt{90 \times 10^{-3}} = \frac{300}{140}$$

$$\Rightarrow l \approx 2.14\ m$$

9. An EM wave is travelling in $\frac{\hat{\imath}+\hat{\jmath}}{2}$ direction. Axis of polarization of EM wave is found to be $\hat{k}$. Then equation of magnetic field will be

   a. $\frac{\hat{\imath}-\hat{\jmath}}{\sqrt{2}} cos\left(\omega t - k\left(\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}\right)\right)$
   b. $\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}} cos\left(\omega t + k\left(\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}\right)\right)$
   c. $\frac{\hat{\imath}-\hat{\jmath}}{\sqrt{2}} cos\left(\omega t + k\left(\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}\right)\right)$
   d. $\hat{k} cos\left(\omega t - k\left(\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}\right)\right)$

Solution:(a)

EM wave is in direction $\frac{\hat{\imath}+\hat{\jmath}}{2}$

Electric field is in direction $\hat{k}$

Direction of propagation of EM wave is given by $\vec{E} \times \vec{B}$

10. Two gases Ar (40) and Xe (131) at equal temperature have the same number density. Their diameters are 0.07 nm and 0.10 nm respectively. Find the ratio of their mean free time

a. 1.03 b. 2.04
c. 2.09 d. 2.49

Solution:(a)

Mean free time = $\frac{1}{\sqrt{2}n\pi d^2}$

$$\frac{t_{Ar}}{t_{Xe}} = \frac{d_{Xe}^2}{d_{Ar}^2} = \left(\frac{0.1}{0.07}\right)^2 = \left(\frac{10}{7}\right)^2 = 2.04$$

11. When the same mass is suspended from two steel rods, the ratio of their energy densities is 1: 4. If the lengths of both the rods are equal, then the ratio of their diameters will be

a. $\sqrt{2}:1$ b. $1:\sqrt{2}$
c. $1:2$ d. $2:1$

Solution: (a)

$$\frac{dU}{dV} = \frac{1}{2} \times stress \times \frac{stress}{Y}$$

$$= \frac{1}{2} \times \frac{F^2}{A^2Y}$$

$$\frac{dU}{dV} \propto \frac{1}{D^4}$$

$$\frac{\left(\frac{dU}{dV}\right)_1}{\left(\frac{dU}{dV}\right)_2} = \frac{{D_2}^4}{{D_1}^4} = \frac{1}{4}$$

$$\frac{D_1}{D_2} = (4)^{\frac{1}{4}}$$

$\therefore D_1 : D_2 = \sqrt{2} : 1$

12. Two planets of masses M and $\frac{M}{2}$ have radii R and $\frac{R}{2}$ respectively. If the ratio $\left(\frac{v_1}{v_2}\right)$ of the escape velocities from their surfaces is $\frac{n}{4}$, then n is

a. 8
b. 2
c. 4
d. 1

Solution: (c)

We know that the escape velocity is given by,

$$V_e = \sqrt{\frac{2GM}{R}}$$

Now

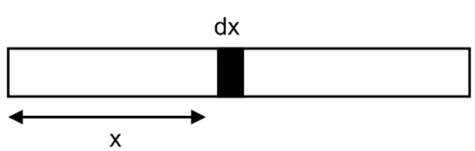
$$\frac{V_1}{V_2} = \frac{\sqrt{\frac{2GM}{R}}}{\sqrt{\frac{\frac{2GM}{2}}{\frac{R}{2}}}} = 1$$

We are given that $\frac{V_1}{V_2} = \frac{n}{4}$

$$\Rightarrow \frac{n}{4} = 1$$

$$\Rightarrow n = 4$$

13. Find the centre of mass of the given rod of linear mass density $\lambda = \left(a + b\left(\frac{x}{l}\right)^2\right)$. Here, x is the distance from one of its end. (Length of the rod is l)



a. $\frac{3l}{4}\left(\frac{2a+b}{2a+b}\right)$
b. $\frac{3l}{4}\left(\frac{2a+b}{3a+b}\right)$

c. $\frac{l}{4}\left(\frac{a+b}{3a+b}\right)$

d. $l\left(\frac{a+b}{3a+b}\right)$

Solution:(b)

Here we take a small element along the length as dx at a distance x from the left end as shown.

$$x_{cm} = \frac{1}{M}\int_0^l x.dm$$

$$\Rightarrow dM = \lambda.dx = \left(a + b\left(\frac{x}{l}\right)^2\right).dx$$

$$x_{cm} = \frac{\int x\lambda dx}{\int \lambda dx} = \frac{\int_0^l x\left(a + \frac{bx^2}{l^2}\right)dx}{\int_0^l \left(a + \frac{bx^2}{l^2}\right)dx}$$

$$= \frac{a\left(\frac{x^2}{2}\right)_0^l + \frac{b}{l^2}\left(\frac{x^4}{4}\right)_0^l}{a(x)_0^l + \frac{b}{l^2}\left(\frac{x^3}{3}\right)_0^l}$$

$$= \frac{\frac{al^2}{2} + \frac{bl^2}{4}}{al + \frac{bl}{3}}$$

$$= \frac{3l}{4}\left(\frac{2a+b}{3a+b}\right)$$

14. A particle is projected from the ground with a speed u at an angle of 60° from horizontal. It collides with a second particle of equal mass moving with a horizontal speed u in the same direction at the highest point of its trajectory. If the collision is perfectly inelastic then find the horizontal distance travelled by them after this collision when they reached the ground.

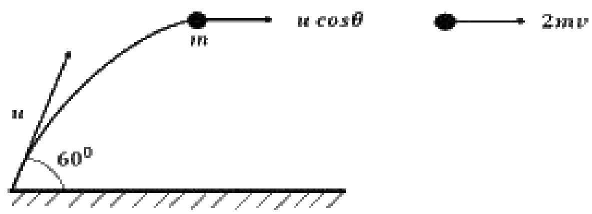
a. $\frac{3\sqrt{6}u^2}{8g}$

b. $\frac{3\sqrt{3}u^2}{8g}$

c. $\frac{u^2}{8g}$

d. $\frac{\sqrt{3}u^2}{g}$

Solution: (b)



The only external force acting on the colliding system during the collision is the gravitational force. Since gravitational force is non- impulsive, the linear momentum of the system is conserved just before and just after the collision.

$$p_i = p_f$$

$$mu + mu\,cos\theta = 2mv$$

$$\Longrightarrow \quad v = \frac{u(1 + cos60^0)}{2} = \frac{3}{4}u$$

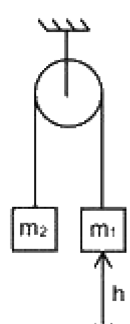
So the horizontal range after the collision $= vt$

$$= v\sqrt{\frac{2H_{\max}}{g}}$$

$$= \frac{3}{4}u\sqrt{\frac{2u^2 \sin^2 60^0}{2g^2}}$$

$$= \frac{3}{4}u^2\,\frac{\sqrt{\frac{3}{4}}}{g} = \frac{3\sqrt{3}u^2}{8g}$$

15. System is released from rest. Moment of inertia of pulley is I. Find angular speed of pulley when m_1 block falls by h. (Given $m_1 > m_2$ and assume no slipping between string and pulley)



a. $\frac{1}{R}\sqrt{\frac{2(m_1-m_2)gh}{m_1+m_2+\frac{I}{R^2}}}$

b. $\frac{1}{R}\sqrt{\frac{4(m_2+m_1)gh}{m_1+m_2+\frac{I}{R^4}}}$

c. $\frac{1}{R}\sqrt{\frac{(m_1-m_2)gh}{m_1+m_2+\frac{I}{R}}}$

d. $\frac{1}{R}\sqrt{\frac{2(m_2+m_1)gh}{m_1+m_2+\frac{I}{R^2}}}$

Solution: (a)

Assume initial potential energy of the blocks to be zero. Initial kinetic energy is also zero since the blocks are at rest.

When block m_1 falls by h, m_2 goes up by h (because of length constraint)

Final P.E $= m_2gh - m_1gh$

Let the final speed of the blocks be v and angular velocity of the pulley be ω

Final K.E $= \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}I\omega^2$

Total energy is conserved. Hence,

$$0 = m_2gh - m_1gh + \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}I\omega^2$$

$v = \omega r$ (due to no slip condition)

$$\Rightarrow \frac{1}{2}(m_1 + m_2)\omega^2R^2 + \frac{1}{2}I\omega^2 = (m_1 - m_2)gh$$

$$\Rightarrow \omega^2\left[\frac{1}{2}(m_1 + m_2)R^2 + \frac{1}{2}I\right] = (m_1 - m_2)gh$$

$$\Rightarrow \omega^2 = \frac{2(m_1 - m_2)gh}{R^2\left[(m_1 + m_2) + \frac{I}{R^2}\right]}$$

$$\Rightarrow \omega = \frac{1}{R}\sqrt{\frac{2(m_1 - m_2)gh}{\left[(m_1 + m_2) + \frac{I}{R^2}\right]}}$$

16. An H-like atom has its ionization energy equal to 9R. Find the wavelength of light emitted (in nm) when an electron jumps from the second excited state to the ground state. (R is Rydberg constant)

a. 12.40
b. 11.39
c. 5.80
d. 22.76

Solution:(b)

$$\frac{hc}{\lambda} = (13.6\ eV)Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$n_1 = 1$$

$$n_2 = 3$$

For an H-like atom, ionization energy is $(R)Z^2$.

This gives $Z = 3$

$$\frac{hc}{\lambda} = (13.6\ eV)(3^2)\left(\frac{1}{1^2} - \frac{1}{3^2}\right)$$

$$\Longrightarrow \frac{hc}{\lambda} = (13.6\ eV)(9) \times \frac{8}{9}$$

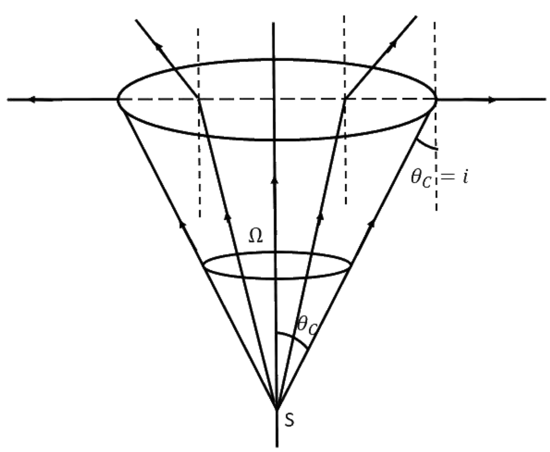
$$\text{wavelength} = \frac{1240}{8 \times 13.6}\ nm$$

$$\lambda = 11.39\ nm$$

17. A point source is placed at a depth h in a liquid of refractive index is $\frac{4}{3}$. Find the percentage of energy of light that escapes from the liquid. (Assuming 100 % transmission of emerging light)

a. 17 %
b. 25 %
c. 10 %
d. 8 %

Solution:(a)



The portion of light escaping into the air from the liquid will form a cone. As long as the angle of incidence on the liquid – air interface is less than the critical angle, i.e. $i < \theta_C$, the light rays will undergo refraction and emerge into the air.

For $i > \theta_C$, the light rays will suffer TIR. So, these rays will not emerge into the air.

The portion of light rays emerging into the air from the liquid will form a cone of half angle = θ_C

$$\sin\theta_C = \frac{1}{\mu_{Liq}} = \frac{3}{4}, \quad \cos\theta_C = \frac{\sqrt{7}}{4}$$

Solid angle contained in this cone is

$\Omega = 2\pi(1 - \cos\theta_C)$

Percentage of light that escapes from liquid $= \frac{\Omega}{4\pi} \times 100$

Putting values we get

Percentage $= \frac{4-\sqrt{7}}{8} \times 100 \approx 17\%$

18. An electron $(-|e|, m)$ is released in Electric field E from rest. Rate of change of de-Broglie wavelength with time will be.

a. $-\frac{h}{2|e|}$

b. $-\frac{h}{2|e|t}$

c. $-\frac{h}{|e|Et^2}$

d. $-\frac{2ht^2}{|e|E}$

Solution:(c)

$$\lambda_D = \frac{h}{mv}$$

$where, v = at$

$$v = \frac{eE}{M}t \quad \left(a = \frac{eE}{M}\right)$$

$$\lambda_D = \frac{h}{m\frac{eE}{M}t}$$

$$\lambda_D = \frac{h}{eEt}$$

$$\frac{d\lambda_D}{dt} = \frac{h}{|e|Et^2}$$

19. An AC source is connected to the LC series circuit with $V = 10\,sin\,(314t)$. Find the current in the circuit as function of time ? $(L = 40\,mH, C = 100\,\mu F)$

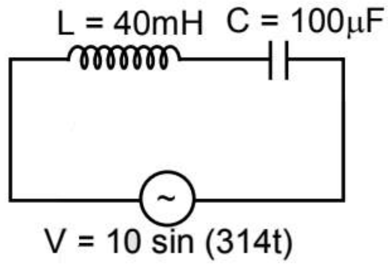
a. $10.4\,sin\,(314t)$

b. $0.52\,cos\,(314t)$

c. $0.52\,sin\,$(314t)

d. $5.2\,cos\,(314t)$

Solution:(b)



Impedance $Z = \sqrt{R^2 + (X_C - X_L)^2}$

$= \sqrt{(X_C - X_L)^2}$

$= X_C - X_L$

$= \frac{1}{\omega C} - \omega L$

$= \frac{1}{314 \times 100 \times 10^{-6}} - 314 \times 40 \times 10^{-3}$

$= 31.84 - 12.56 = 19.28\ \Omega$

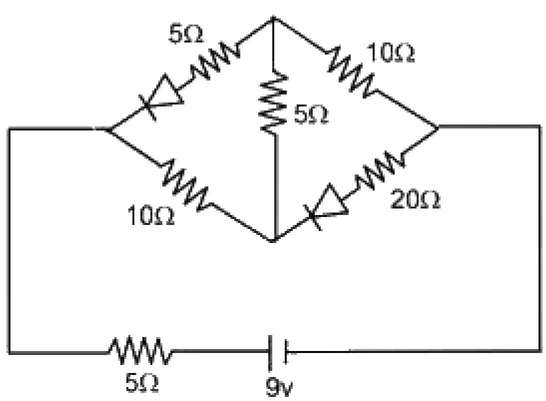
For $X_C > X_L$, current leads voltage by $\frac{\pi}{2}$

$\therefore i = \frac{V}{Z} = \frac{10 \sin(314t + \frac{\pi}{2})}{19.28}$

$= 0.52 \cos\ (314t)$

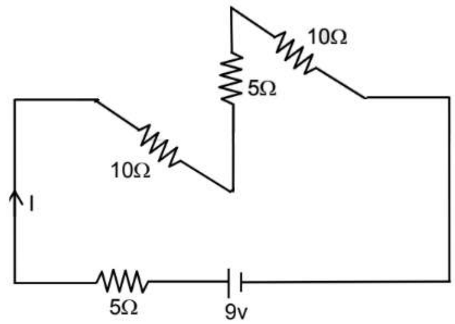
20. Find the current supplied by the battery.

a. $0.1\ A$
b. $0.3\ A$
c. $0.4\ A$
d. $0.5\ A$



Solution:(b)

Since the diodes are reverse biased, they will not conduct.
Hence, the circuit will look like

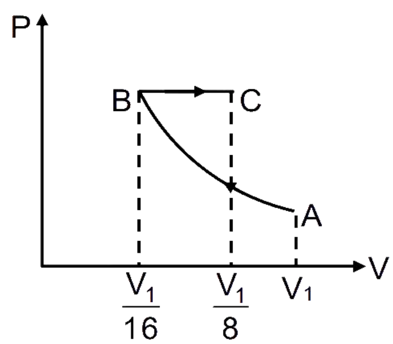


$$R_{eff} = 5 + 10 + 5 + 10 = 30\ \Omega$$

$$I = \frac{9}{30} = 0.3\ A$$

21. An ideal gas at an initial temperature 300 K is compressed adiabatically (γ = 1.4) to its initial volume. The gas is then expanded isobarically to double its volume. Then the final temperature of the gas rounded off to nearest integer is.

Solution:(1819 K)



PV^{γ} =Constant
$TV^{(\gamma-1)}$=constant

$$300(V_1)^{(1.4-1)} = T_B\left(\frac{V_1}{16}\right)^{\frac{2}{5}}$$

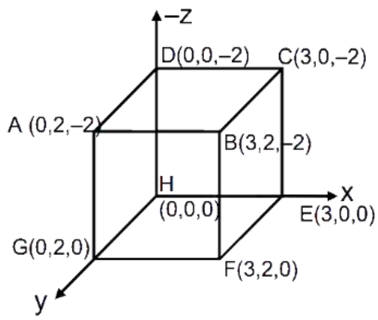
$$T_B = 300 \times 2^{\left(\frac{8}{5}\right)}$$

Now for BC process

$$\frac{V_B}{T_B} = \frac{V_C}{T_C}$$

$$T_C = \frac{V_C T_B}{T_B} = 2 \times 300 \times 2^{\left(\frac{8}{5}\right)}$$
$$T_C = 1819\ K$$

22. If electric field in the space is given by $\vec{E} = 4x\hat{\imath} - (y^2 + 1)\hat{\jmath}$, and electric flux through ABCD is ϕ_1 and electric flux through BCEF is ϕ_2, then find $(\phi_1 - \phi_2)$.



Solution: (-40)

Electric flux through a surface is $\emptyset = \int \vec{E} = d\vec{A}$

For surface $ABCD$,

$d\vec{A}$ is along $\left(-\hat{k}\right)$

So, at all the points of this surface,

$$\vec{E}.d\vec{A} = 0$$

Because, $\emptyset_{ABCD} = \emptyset_1 = 0$

For surface $BCEF$,

$d\vec{A}$ is along $(\hat{\imath})$

So,

$$\vec{E}.d\vec{A} = E_x dA$$
$$\emptyset_{BCEF} = \emptyset_2 = 4x(2 \times 2)$$

If $if\ x = 3$

$$\emptyset_2 = 48\ \frac{N - m^2}{C}$$

Hence, $\emptyset_1 - \emptyset_2 = -48\ \frac{N-m^2}{C}$

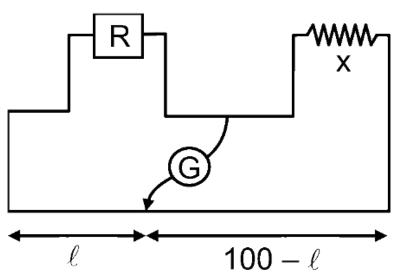
23. In a YDSE interference pattern obtained with light of wavelength $\lambda_1 = 500\ nm$, 15 fringes are obtained on a certain segment of screen. If number of fringes for light of wavelength λ_2 on same segment of screen is 10, then the value of λ_2 (in nm) is

Solution:(750)

If the length of the segment is y,
Then $y = n\ \beta$
$n =$ no. of fringes,
$\beta =$ fringe width
$15 \times 500 \times \frac{D}{d} = 10 \times \lambda_2 \times \frac{D}{d}$

$$\lambda_2 = 15 \times 50\ nm$$
$$\lambda_2 = 750\ nm$$

24. In a meter bridge experiment, the balancing length was 25 cm for the situation shown in the figure. If the length and diameter of the of wire of resistance R is halved, then the new balancing length in centimetre is

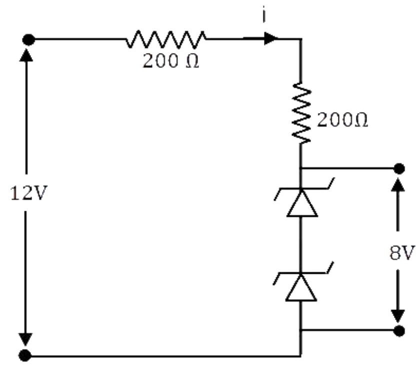


Solution:(40)

$$\frac{X}{R} = \frac{75}{25} = 3$$
$$R = \frac{\rho l}{A} = \frac{4\rho l}{\pi d^2}$$
$$R' = \frac{4\rho \frac{l}{2}}{\pi \left(\frac{d}{2}\right)^2} = 2R$$
$$\text{Then } \frac{X}{R'} = \left(\frac{100-l}{l}\right)$$
$$\frac{100-l}{l} = \frac{X}{2R} = \frac{3}{2}$$

$$l = 40.00cm$$

25. Find the power loss in each diode (in mW), if potential drop across the Zener diode is $8V$.



Solution:(40)

$i = \left(\frac{12-8}{200+200}\right) A = \frac{4}{400} = 10^{-2} A$

Power loss in each diode = $(4)(10^{-2})\ W = 40\ mW$