JEE Main 2020 Paper

Date of Exam: **8th January (Shift I)**
Time: **9:30 am – 12:30 pm**
Subject: **Physics**

1. A block of mass m is connected at one end of natural length l_o and spring constant k. The spring is fixed at its other end. The block is rotated with constant angular speed (ω) in gravity free space. The elongation in spring is

a. $\frac{l_o m\omega^2}{k-m\omega^2}$
b. $\frac{l_o m\omega^2}{k+m\omega^2}$
c. $\frac{l_o m\omega^2}{k-m\omega}$
d. $\frac{l_o m\omega^2}{k+m\omega}$

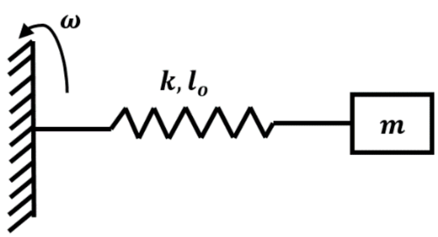
Solution: (a)

The centripetal force is provided by the spring force.

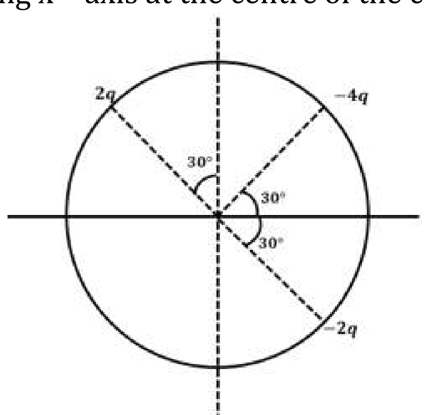
$$m\omega^2(l_o + x) = kx$$

$$\left(\frac{l_o}{x} + 1\right) = \frac{k}{m\omega^2}$$

$$x = \frac{l_o m\omega^2}{k-m\omega^2}$$



2. Three charges are placed on the circumference of a circle of radius d as shown in the figure. The electric field along x −axis at the centre of the circle is



a. $\frac{q}{4\pi\varepsilon_o d^2}$

b. $\frac{q\sqrt{3}}{4\pi\varepsilon_o d^2}$

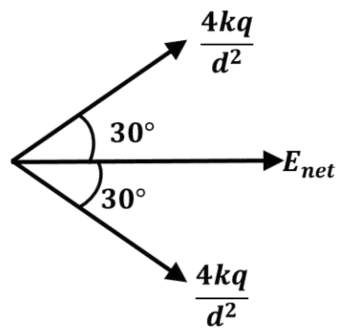
c. $\frac{q\sqrt{3}}{\pi\varepsilon_o d^2}$

d. $\frac{q\sqrt{3}}{2\pi\varepsilon_o d^2}$

Solution: (c)

Applying superposition principle,

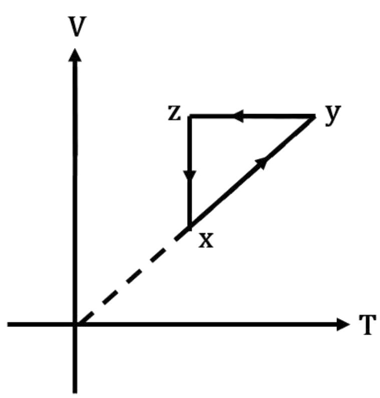
$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$



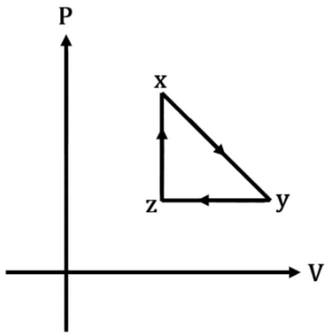
By symmetry, net electric field along the x-axis.

$$\left|\vec{E}_{net}\right| = \frac{4kq}{d^2} \times 2\cos 30^\circ = \frac{q\sqrt{3}}{\pi\varepsilon_o d^2}$$

3. Choose the correct $P-V$ graph of an ideal gas for the given $V-T$ graph.



a.



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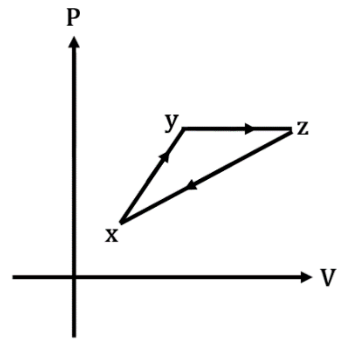

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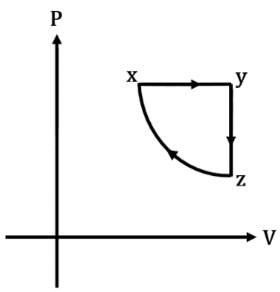
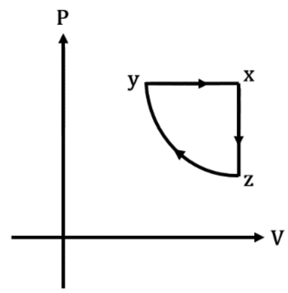


d.



Solution: (a)

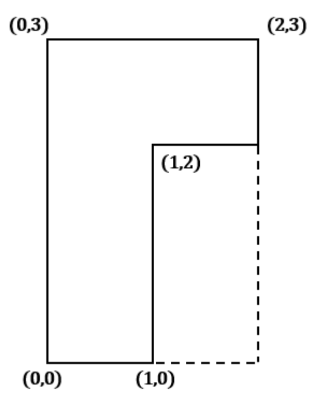
For the given $V - T$ graph

For the process $x \rightarrow y$; $V \propto T$; P =constant

For the process $y \rightarrow z$; V =constant

Only $'a'$ satisfies these two conditions.

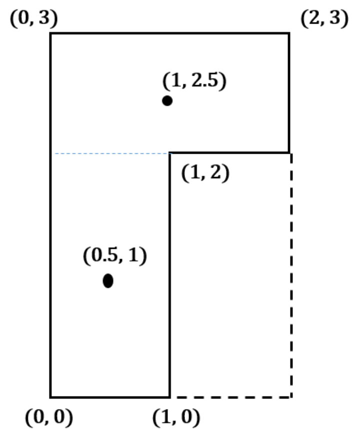
4. Find the co-ordinates of center of mass of the lamina shown in the figure below.



a. (0.75, 1.75)
b. (0.75, 1.5)
c. (0.5, 1.75)
d. (0.5, 1.5)

Solution: (a)

The Lamina can be divided into two parts having equal mass m each.

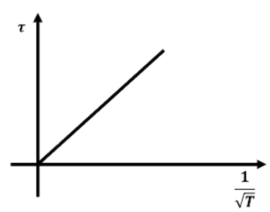


$$\vec{r}_{cm} = \frac{m \times \left(\frac{\hat{\imath}}{2} + \hat{\jmath}\right) + m \times \left(\hat{\imath} + \frac{5\hat{\jmath}}{2}\right)}{2m}$$

$$\vec{r}_{cm} = \frac{3}{4}\hat{\imath} + \frac{7}{4}\hat{\jmath}$$

5. Which graph correctly represents the variation between relaxation time (τ) of gas molecules with absolute temperature (T) of the gas?

a.



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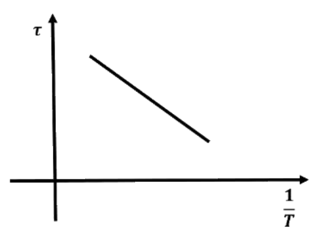
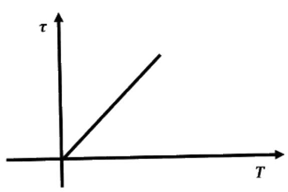
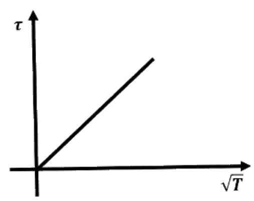


c.



d.



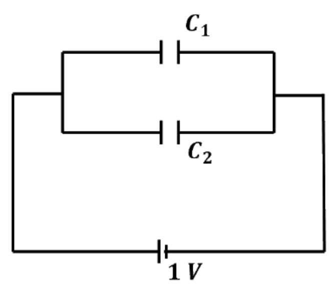
Solution: (a)

$\tau \propto \frac{1}{\sqrt{T}}$

6. If two capacitors C_1 and C_2 are connected in a parallel combination then the equivalent capacitance is 10 μF. If both the capacitors are connected across a 1 V battery, then energy stored in C_2 is 4 times of that in C_1. The equivalent capacitance if they are connected in series is

a. 1.6 μF
b. 16 μF
c. 4 μF
d. $\frac{1}{4}$ μF

Solution: (a)



Given that,

$C_1 + C_2 = 10\ \mu F$ .....(i)

$4\left(\frac{1}{2}C_1V^2\right) = \frac{1}{2}C_2V^2$

$\Longrightarrow 4C_1 = C_2$ ...(ii)

From equations (i) and (ii)

$C_1 = 2\ \mu F$

$C_2 = 8\ \mu F$

If they are in series

$$C_{eq} = \frac{C_1C_2}{C_1 + C_2} = 1.6\ \mu F$$

7. A rod of mass $4m$ and length L is hinged at its midpoint. A ball of mass m is moving in the plane of the rod with a speed v. It strikes the end at an angle of 45^0 and sticks to it. The angular velocity of system after collision is

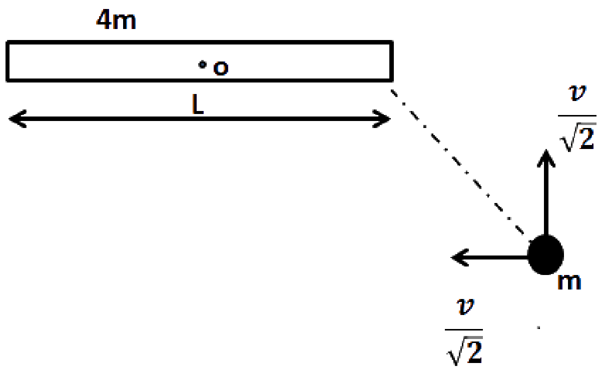
a. $\frac{3\sqrt{2}v}{7L}$

b. $\frac{\sqrt{2}v}{7L}$

c. $\frac{\sqrt{2}v}{3L}$

d. $\frac{3\mathrm{v}}{7L}$

Solution: (a)



There is no external torque on the system about the hinge point.

So,

$$\overrightarrow{L}_{\mathrm{i}} = \overrightarrow{L}_{\mathrm{f}}$$

$$\frac{mv}{\sqrt{2}} \times \frac{1}{2} = \left[\frac{4mL^{2}}{12} + \frac{mL^{2}}{4}\right] \times \omega$$

$$\omega = \frac{6\,\mathrm{V}}{7\sqrt{2}L} = \frac{3\sqrt{2}\mathrm{V}}{7L}$$

8. Two photons of energy 4 eV and 4.5 eV are incident on two metals A and B respectively. The maximum kinetic energy for an ejected electron is T_A for A, and $T_B = T_A - 1.5\ eV$ for the metal B. The relation between the de-Broglie wavelengths of the ejected electron of A and B are $\lambda_B = 2\lambda_B$. The work function of the metal B is

a. 3 eV

b. 1.5 eV

c. 4.5 eV

d. 4 eV

Solution: (d)

$$\lambda = \frac{\mathrm{h}}{\sqrt{2(\mathrm{KE})\mathrm{m_B}}} = \lambda \propto \frac{1}{\sqrt{\mathrm{KE}}}$$

$$\frac{\lambda_A}{\lambda_B} = \frac{\sqrt{KE_B}}{\sqrt{KE_A}}$$

$$\frac{1}{2} = \sqrt{\frac{T_A - 1.5}{T_A}}$$

$$T_A = 2\ eV$$

$$KE_B = 2 - 1.5 = 0.5\ eV$$

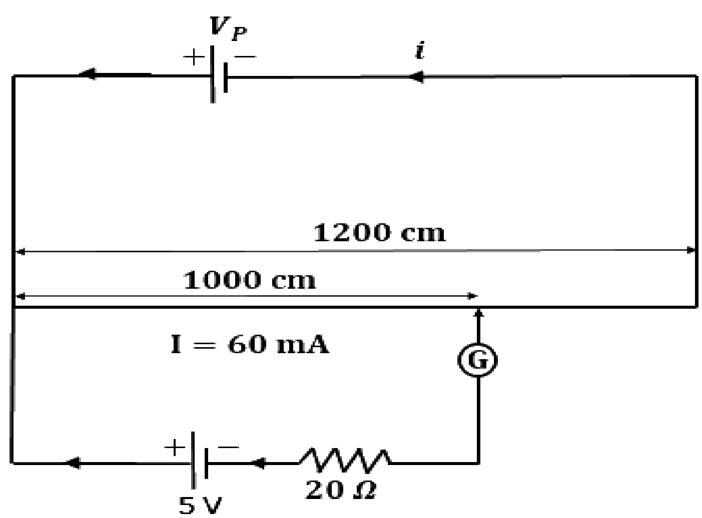
$$\emptyset_B = 4.5 - 0.5 = 4\ eV$$

9. There is a potentiometer wire of length 1200 cm and a 60 mA current is flowing in it. A battery of emf $5\ V$ and internal resistance of 20 Ω is balanced on this potentiometer wire with a balancing length1000 cm. The resistance of the potentiometer wire is

   a. 80 Ω b. 100 Ω

   c. 120 Ω d. 60 Ω

Solution: (b)



Assume the terminal voltage of the primary battery as V_p. As long as this potentiometer is operating on balanced length, V_p will remain constant.

As we know, potential gradient = $\frac{5}{1000} = \frac{V_p}{1200}$

$V_p = 6$ V

And $R_p = \frac{V_p}{i} = \frac{6}{60\times10^{-3}} = 100\ \Omega$

10. A telescope has a magnification equal to 5 and the length of its tube is 60 cm. The focal length of its eye piece is

a. 10 cm
b. 20 cm
c. 30 cm
d. 40 cm

Solution: (a)

$$m = \frac{f_0}{f_e} = 5$$

$$\Rightarrow f_0 = 5f_e$$

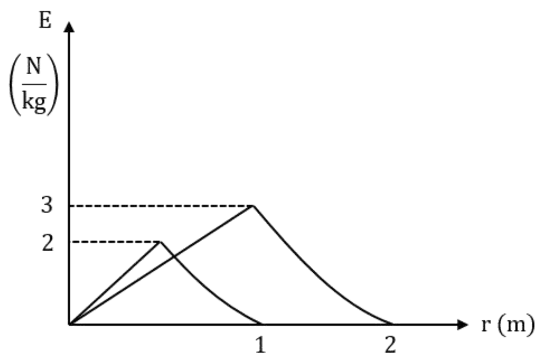
$$f_0 + f_e = 5f_e + f_e = 6f_e = \text{length of the tube}$$

$$\Rightarrow 6f_e = 60 \text{ cm}$$

$$\Rightarrow f_e = 10 \text{ cm}$$

11. Two spherical bodies of mass m_1 & m_2 have radii 1 m & 2 m respectively. The graph of the gravitational field of the two bodies with their radial distance is shown below. The value of $\frac{m_1}{m_2}$ is

a. $\frac{1}{6}$
b. $\frac{1}{3}$
c. $\frac{1}{2}$
d. $\frac{1}{4}$



Solution: (a)

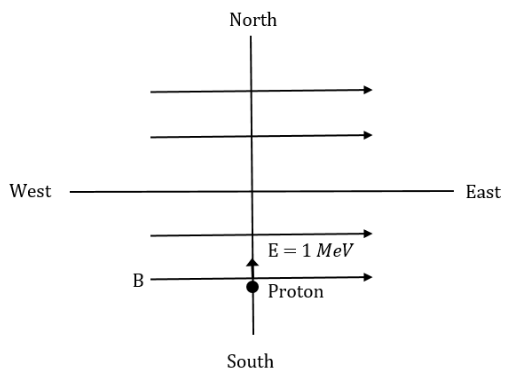
Gravitation field will be maximum at the surface of a sphere. Therefore,

$\frac{Gm_2}{2^2} = 3$ & $\frac{Gm_1}{1^2} = 2$

$$\Rightarrow \frac{m_2}{m_1} \times \frac{1}{4} = \frac{3}{2}$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{1}{6}$$

12. When a proton of KE = 1.0 MeV moving towards North enters a magnetic field (directed along East), it accelerates with an acceleration, $a = 10^{12}\ m/s^2$. The magnitude of the magnetic field is



a. 0.71 mT
b. 7.1 mT
c. 71 mT
d. 710 mT

Solution: (a)

$K.E = 1 \times 10^6\ eV = 1.6 \times 10^{-13} J$

$$= \frac{1}{2} m_e v^2$$

Where m_e is the mass of the electron = 1.6×10^{-27}kg

$$\Rightarrow 1.6 \times 10^{-13} = \frac{1}{2} \times\ 1.6 \times 10^{-27} \times v^2$$

$$\therefore v = \sqrt{2} \times 10^7 m/s$$

$$Bqv = m_e a$$

$$\therefore B = \frac{1.6 \times 10^{-27} \times 10^{12}}{1.6 \times 10^{-19} \times \sqrt{2} \times 10^7}$$

$$= 0.71 \times 10^{-3} T = 0.71\ mT$$

13. If the electric field around a surface is given by $\left|\vec{E}\right| = \frac{Q_{in}}{\varepsilon_0 |A|}$ where A is the normal area of surface and Q_{in} is the charge enclosed by the surface. This relation of Gauss' law is valid when
   a. the surface is equipotential.
   b. the magnitude of the electric field is constant.
   c. the magnitude of the electric field is constant and the surface is equipotential.
   d. for all the Gaussian surfaces.

Solution: (c)

The magnitude of the electric field is constant and the electric field must be along the area vector i.e. the surface is equipotential.

14. The stopping potential depends on the Planks constant(h), the current (I), the universal gravitational constant (G) and the speed of light (C). Choose the correct option for the dimension of the stopping potential (V)
   a. $h^1 I^{-1} G^1 C^5$
   b. $h^{-1} I^1 G^6$
   c. $h^0 I^1 G^1 C^6$
   d. $h^0 I^{-1} G^{-1} C^5$

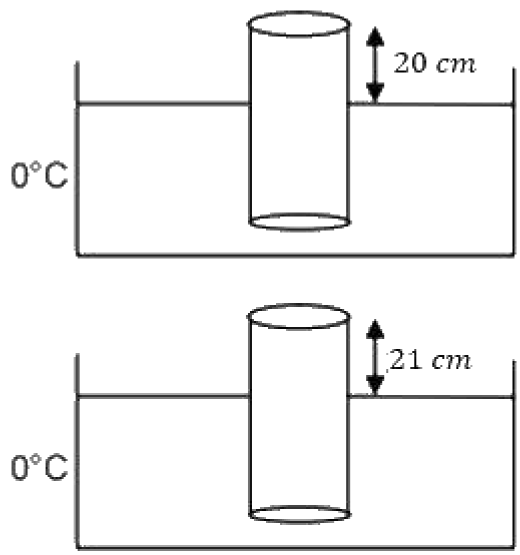
Solution: (d)

$V = K(h)^a (I)^b (G)^c (C)^d$ Unit of stopping potential is (V) Volt.
We know $[h] = ML^2T^{-1}$
$[I] = A$
$[G] = M^{-1}L^3T^{-2}$
$[C] = LT^{-1}$
$[V] = ML^2T^{-3}A^{-1}$
$ML^2T^{-3}A^{-1} = (ML^2T^{-1})^a (A)^b\ (M^{-1}L^3T^{-2})^c (LT^{-1})^d$
$ML^2T^{-3}A^{-1} = M^{a-c}L^{2a+3c+d}T^{-a-2c-d}A^b$
$a - c = 1$
$2a + 3c + d = 2$
$-a - 2c - d = -3$
$b = -1$
On solving,
$c = -1$
$a = 0$
$d = 5$
$b = -1$
$V = K(h)^0 (I)^{-1} (G)^{-1} (C)^5$

15. A cylinder of height 1 m is floating in water at 0^o C with 20 cm height in air. Now the temperature of water is raised to 4^o C, the height of the cylinder in air becomes 21 cm. The ratio of density of water at 4^o C to that at 0^o C is (Consider expansion of the cylinder is negligible)

a. 1.01
b. 1.03
c. 2.01
d. 1.04

Solution: (a)



Since the cylinder is in equilibrium, it's weight is balanced by the Buoyant force.

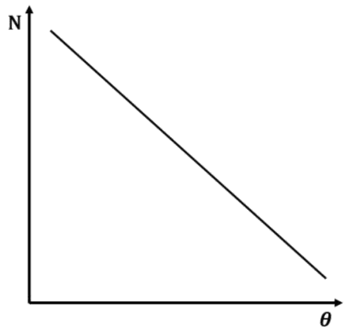
$$mg = A(80)(\rho_{0^oC})g$$

$$mg = A(79)(\rho_{4^oC})g$$

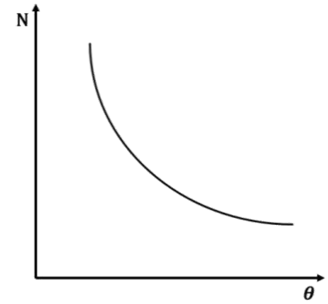
$$\frac{\rho_{4^oC}}{\rho_{0^oC}} = \frac{80}{79} = 1.01$$

16. Number of the α-particle deflected in Rutherford's α- particle scattering experiment varies with the angle of deflection. The graph between the two is best represented by

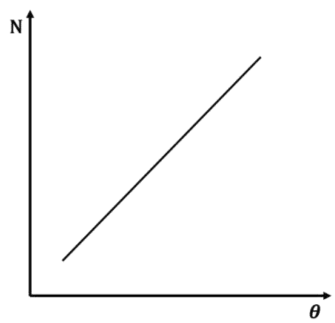
a.



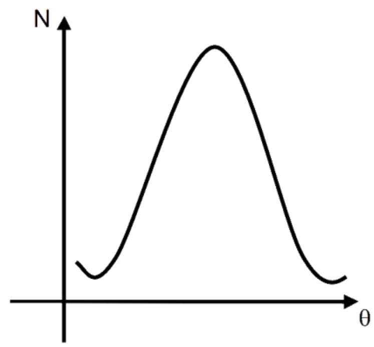
b.



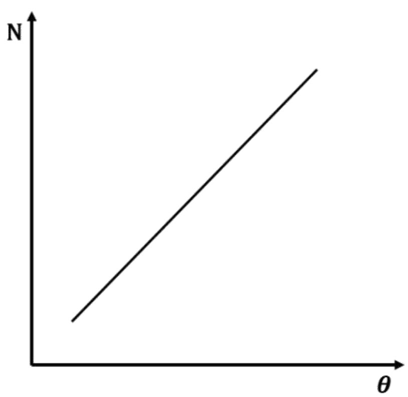
c.



d.



Solution: (c)



$N \propto \frac{1}{\sin^4(\theta/2)}$

17. If relative permittivity and relative permeability of a medium are 3 and $\frac{4}{3}$ respectively, the critical angle for this medium is

a. 45^o
b. 60^o
c. 30^0
d. 15^0

Solution: (c)

If the speed of light in the given medium is V then,

$$V = \frac{1}{\sqrt{\mu\epsilon}}$$

We know that, $\quad n = \frac{c}{v}$

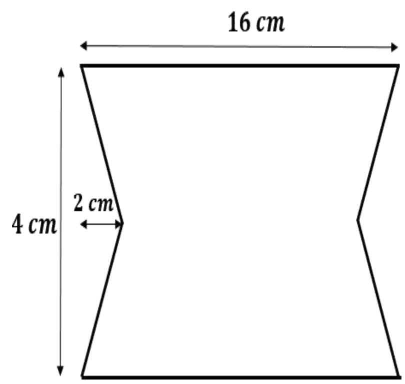
$$n = \sqrt{\mu_r \epsilon_r} = 2$$

$$\sin \theta_c = \frac{1}{2}$$

$$\theta_c = 30^o$$

18. The given loop is kept in a uniform magnetic field perpendicular to the plane of the loop. The field changes from 1000 *Gauss* to 500 *Gauss* in 5 seconds. The average induced emf in the loop is

a. 56 μV
b. 28 μV
c. 30 μV
d. 48 μV



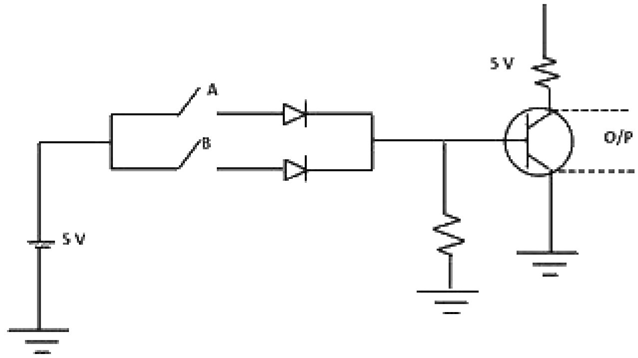
Solution: (a)

$$\epsilon = \left|-\frac{d\phi}{dt}\right| = \left|-\frac{AdB}{dt}\right|$$

$$= (16 \times 4 - 4 \times 2)\frac{(1000 - 500)}{5} \times 10^{-4} \times 10^{-4}$$

$$= 56 \times \frac{500}{5} \times 10^{-8} = 56 \times 10^{-6}\ V$$

19. Choose the correct Boolean expression for the given circuit diagram:



a. A.B
b. $\bar{A} + \bar{B}$
c. $A + B$
d. $\bar{A}.\bar{B}$

Solution: (d)
First part of figure shown OR gate and second part of figure shown NOT gate.
So, $Y = \overline{A+B} = \bar{A}.\bar{B}$

20. A Solid sphere of density $\rho = \rho_o\left(1 - \frac{r^2}{R^2}\right)$, $0 < r \leq R$ just floats in a liquid, then the density of the liquid is (r is the distance from the centre of the sphere)

a. $\frac{2}{5}\rho_o$
b. $\frac{5}{2}\rho_o$
c. $\frac{3}{5}\rho_o$
d. ρ_o

Solution: (a)

Let the mass of the sphere be m and the density of the liquid be ρ_L

$$\rho = \rho_o\left(1 - \frac{r^2}{R^2}\right), 0 < r \leq R$$

Since the sphere is floating in the liquid, buoyancy force (F_B) due to liquid will balance the weight of the sphere.

$$F_B = mg$$

$$\rho_L \frac{4}{3}\pi R^3 g = \int \rho(4\pi r^2 dr)g$$

$$\rho_L \frac{4}{3}\pi R^3 = \int \rho_o\left(1 - \frac{r^2}{R^2}\right)4\pi r^2 dr$$

$$\rho_L \frac{4}{3}\pi R^3 = \int_0^R \rho_o 4\pi\left(r^2 - \frac{r^4}{R^2}\right)dr = \rho_o 4\pi\left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right)_0^R$$

$$\rho_L = \frac{2}{5}\rho_o$$

21. Two masses each of mass 0.10 kg are moving with velocities 3 m/s along x −axis and 5 m/s along y −axis respectively. After an elastic collision one of the mass moves with velocity $4\hat{\imath} + 4\hat{\jmath}\ m/s$. If the energy of the other mass after the collision is $\frac{x}{10}$, then x is

Solution: (1)

Mass of each object, $m_1 = m_2 = 0.1\ kg$
Initial velocity of 1st object, $u_1 = 5\ m/s$
Initial velocity of 2nd object, $u_2 = 3\ m/s$
Final velocity of 1st object, $V_1 = 4\hat{\imath} + 4\hat{\jmath}\ m/s$ $=\sqrt{4^2+4^2} = 16\sqrt{2}\ m/s$

For elastic collision, kinetic energy remains conserved

$$Initial\ kinetic\ energy\ (K_i) = Final\ kinetic\ energy(K_f)$$

$$\frac{1}{2}mu_1^2 + \frac{1}{2}mu_2^2 = \frac{1}{2}mV_1^2 + \frac{1}{2}mV_2^2$$

$$\frac{1}{2}m(5)^2 + \frac{1}{2}m(3)^2 = \frac{1}{2}m(16\sqrt{2})^2 + \frac{1}{2}mV_2^2$$

$$V_2 = \sqrt{2}\ m/s$$

Kinetic energy of second object $= \frac{1}{2}mV_2^2 = \frac{1}{2} \times 0.1 \times \sqrt{2}^2 = \frac{1}{10}$

$$\Rightarrow x = 1$$

22. A plano-convex lens of radius of curvature 30 cm and refractive index 1.5 is kept in air. Find its focal length (in cm).

Solution: (60 cm)

Applying Lens makers' formula,

$$\frac{1}{\mathrm{f}} = (\mu - 1)\left(\frac{1}{\mathrm{R}_1} - \frac{1}{\mathrm{R}_2}\right)$$

$$\mathrm{R}_1 = \infty$$

$$\mathrm{R}_2 = -30\ \mathrm{cm}$$

$$\frac{1}{\mathrm{f}} = (1.5 - 1)\left(\frac{1}{\infty} - \frac{1}{-30}\right)$$

$$\frac{1}{f} = \frac{0.5}{30}$$

$$f = 60 \text{ cm}$$

23. The position of two particles *A* and *B* as a function of time are given by $X_A = -3t^2 + 8t + c$ and $Y_B = 10 - 8t^3$. The velocity of *B* with respect to *A* at t = 1 is $\sqrt{v}$. Find v.

Solution: (580 m/s)

$$X_A = -3t^2 + 8t + c$$

$$\overrightarrow{v_A} = (-6t + 8)\hat{\imath}$$

$$= 2\hat{\imath}$$

$$Y_B = 10 - 8t^3$$

$$\overrightarrow{v_B} = -24t^2\hat{\jmath}$$

$$\left|\overrightarrow{v_{B/A}}\right| = |\overrightarrow{v_B} - \overrightarrow{v_A}| = |-24\hat{\jmath} - 2\hat{\imath}|$$

$$v = \sqrt{24^2 + 2^2}$$

$$v = 580 \text{ m/s}$$

24. An open organ pipe of length 1 m contains a gas whose density is twice the density of the atmosphere at STP. Find the difference between its fundamental and second harmonic frequencies if the speed of sound in atmosphere is 300 m/s.

Solution: (105.75 Hz)

$$V = \sqrt{\frac{B}{\rho}}$$

$$\frac{V_{pipe}}{V_{air}} = \frac{\sqrt{\frac{B}{2\rho}}}{\sqrt{\frac{B}{\rho}}} = \frac{1}{\sqrt{2}}$$

$$V_{pipe} = \frac{V_{air}}{\sqrt{2}}$$

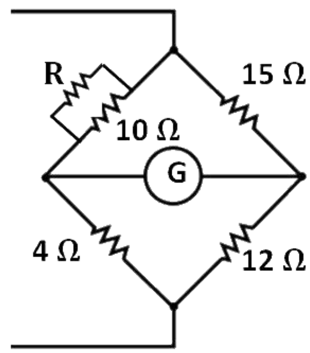
$$f_n = \frac{(n+1)}{2l} V_{pipe}$$

$$f_1 - f_0 = \frac{V_{pipe}}{2l} = \frac{300}{2\sqrt{2}} = 105.75 \text{ Hz (if } \sqrt{2} = 1.41)$$

$$= 106.05 \text{ Hz (if } \sqrt{2} = 1.414)$$

25. Four resistors of resistance 15 Ω, 12 Ω, 4 Ω and 10 Ω are connected in cyclic order to form a wheat stone bridge. The resistance (in Ω) that should be connected in parallel across the 10 Ω resistor to balance the wheat stone bridge is

Solution: (10 Ω)



$$\frac{10\text{ R}}{10+\text{R}} \times 12 = 15 \times 4 \Rightarrow \text{R} = 10\ \Omega$$