JEE Main 2020 Paper

Date: **8th January 2020 (Shift 1)**
Time: **9:30 A.M. to 12:30 P.M.**
Subject: **Mathematics**

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1. The maximum values of ${}^{19}C_p, {}^{20}C_q, {}^{21}C_r$ are a, b, c respectively. Then, the relation between a, b, c is

a. $\frac{a}{22} = \frac{b}{42} = \frac{c}{11}$

b. $\frac{a}{11} = \frac{b}{22} = \frac{c}{42}$

c. $\frac{a}{22} = \frac{b}{11} = \frac{c}{42}$

d. $\frac{a}{21} = \frac{b}{11} = \frac{c}{22}$

Answer: (b)

Solution:

We know that, ${}^{n}C_r$ is maximum when $r = \begin{cases} \frac{n}{2}, & n \text{ is even} \\ \frac{n+1}{2} \text{ or } \frac{n-1}{2}, & n \text{ is odd} \end{cases}$

Therefore, $\max\left({}^{19}C_p\right) = {}^{19}C_9 = a$

$\max\left({}^{20}C_q\right) = {}^{20}C_{10} = b$

$\max\left({}^{21}C_r\right) = {}^{21}C_{11} = c$

$\therefore \frac{a}{{}^{19}C_9} = \frac{b}{\frac{20}{10} \times {}^{19}C_9} = \frac{c}{\frac{21}{11} \times \frac{20}{10} \times {}^{19}C_9}$

$\frac{a}{1} = \frac{b}{2} = \frac{c}{\frac{42}{11}}$

$\Rightarrow \frac{a}{11} = \frac{b}{22} = \frac{c}{42}$.

2. Let $P(A) = \frac{1}{3}, P(B) = \frac{1}{6}$ where A and B are independent events, then

a. $P\left(\frac{A}{B}\right) = \frac{2}{3}$

b. $P\left(\frac{A}{B'}\right) = \frac{5}{6}$

c. $P\left(\frac{A}{B'}\right) = \frac{1}{3}$

d. $P\left(\frac{A}{B}\right) = \frac{1}{6}$

Answer: (c)

Solution:

If X and Y are independent events, then

$$P\left(\frac{X}{Y}\right) = \frac{P(X \cap Y)}{P(Y)} = \frac{P(X)P(Y)}{P(Y)} = P(X)$$

Therefore, $P\left(\frac{A}{B}\right) = P(A) = \frac{1}{3} \Longrightarrow P\left(\frac{A}{B'}\right) = P(A) = \frac{1}{3}$.

3. If $f(x) = \frac{8^{2x}-8^{-2x}}{8^{2x}+8^{-2x}}$, then inverse of $f(x)$ is

a. $\frac{1}{2}\log_8\left(\frac{1+x}{1-x}\right)$

b. $\frac{1}{2}\log_8\left(\frac{1-x}{1+x}\right)$

c. $\frac{1}{4}\log_8\left(\frac{1-x}{1+x}\right)$

d. $\frac{1}{4}\log_8\left(\frac{1+x}{1-x}\right)$

Answer: (d)

Solution:

$$f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}} = \frac{8^{4x} - 1}{8^{4x} + 1}$$

Put $y = \frac{8^{4x}-1}{8^{4x}+1}$

Applying componendo-dividendo on both sides

$$\frac{y+1}{y-1} = \frac{2 \times 8^{4x}}{-2}$$

$\frac{y+1}{y-1} = -8^{4x} \Rightarrow 8^{4x} = \frac{1+y}{1-y}$

$\Rightarrow x = \frac{1}{4}\log_8\left(\frac{1+y}{1-y}\right)$

$f^{-1}(x) = \frac{1}{4}\log_8\left(\frac{1+x}{1-x}\right)$.

4. Roots of the equation $x^2 + bx + 45 = 0, b \in \boldsymbol{R}$ lies on the curve $|z+1| = 2\sqrt{10}$, where z is a complex number, then

a. $b^2 + b = 12$

b. $b^2 - b = 36$

c. $b^2 - b = 30$

d. $b^2 + b = 30$

Answer: (c)

Solution:

Given $x^2 + bx + 45 = 0, b \in \boldsymbol{R}$, let roots of the equation be $p \pm iq$

Then, sum of roots $= 2p = -b$

Product of roots $= p^2 + q^2 = 45$

As $p \pm iq$ lies on $|z+1| = 2\sqrt{10}$, we get

$(p+1)^2 + q^2 = 40$

$\Rightarrow p^2 + q^2 + 2p + 1 = 40$

$\Rightarrow 45 - b + 1 = 40$

$\Rightarrow b = 6$

$\Rightarrow b^2 - b = 30.$

5. Rolle's theorem is applicable on $f(x) = \ln\left(\frac{x^2+\alpha}{7x}\right)$ in $[3, 4]$. The value of $f''(c)$ is equal to

a. $\frac{1}{12}$
b. $\frac{-1}{12}$
c. $\frac{-1}{6}$
d. $\frac{1}{6}$

Answer: (a)

Solution:

Rolle's theorem is applicable on $f(x)$ in $[3, 4]$

$\Rightarrow f(3) = f(4)$

$$\Rightarrow \ln\left(\frac{9+\alpha}{21}\right) = \ln\left(\frac{16+\alpha}{28}\right)$$

$$\Rightarrow \frac{9+\alpha}{21} = \frac{16+\alpha}{28}$$

$\Rightarrow 36 + 4\alpha = 48 + 3\alpha$

$\Rightarrow \alpha = 12$

Now,

$$f(x) = \ln\left(\frac{x^2+12}{7x}\right) \Rightarrow f'(x) = \frac{7x}{x^2+12} \times \frac{7x \times 2x - (x^2+12) \times 7}{(7x)^2}$$

$$f'(x) = \frac{x^2 - 12}{x(x^2+12)}$$

$f'(c) = 0 \Rightarrow c = 2\sqrt{3}$

$$f''(x) = \frac{-x^4 + 48x^2 + 144}{x^2(x^2+12)^2}$$

$f''(c) = \frac{1}{12}.$

6. Let $f(x) = x\cos^{-1}(\sin(-|x|))$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then

a. $f'(0) = -\frac{\pi}{2}$
b. $f'(x)$ is not defined at $x = 0$
c. $f'(x)$ is decreasing in $\left(-\frac{\pi}{2}, 0\right)$ and $f'(x)$ is decreasing in $\left(0, \frac{\pi}{2}\right)$
d. $f'(x)$ is increasing in $\left(-\frac{\pi}{2}, 0\right)$ and $f'(x)$ is increasing in $\left(0, \frac{\pi}{2}\right)$

Answer: (c)

Solution:

$f(x) = x\cos^{-1}(\sin(-|x|))$

$\Rightarrow f(x) = x\cos^{-1}(-\sin|x|)$

$\Rightarrow f(x) = x[\pi - \cos^{-1}(\sin|x|)]$

$\Rightarrow f(x) = x\left[\pi - \left(\frac{\pi}{2} - \sin^{-1}(\sin|x|)\right)\right]$

$\Rightarrow f(x) = x\left(\frac{\pi}{2} + |x|\right)$

$$\Rightarrow f(x) = \begin{cases} x\left(\frac{\pi}{2} + x\right), & x \geq 0 \\ x\left(\frac{\pi}{2} - x\right), & x < 0 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} \left(\frac{\pi}{2} + 2x\right), & x \geq 0 \\ \left(\frac{\pi}{2} - 2x\right), & x < 0 \end{cases}$$

Therefore, $f'(x)$ is decreasing $\left(-\frac{\pi}{2}, 0\right)$ and increasing in $\left(0, \frac{\pi}{2}\right)$.

7. Ellipse $2x^2 + y^2 = 1$ and $y = mx$ meet at a point P in the first quadrant. Normal to the ellipse at P meets x –axis at $\left(-\frac{1}{3\sqrt{2}}, 0\right)$ and y –axis at $(0, \beta)$, then $|\beta|$ is

a. $\frac{2}{3}$

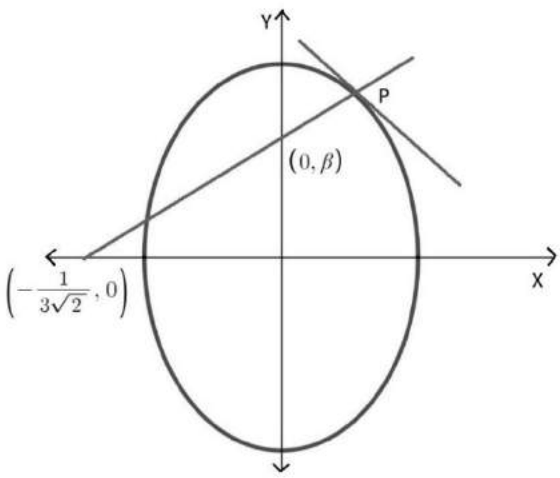
b. $\frac{2\sqrt{2}}{3}$

c. $\frac{\sqrt{2}}{3}$

d. $\frac{2}{\sqrt{3}}$

Answer: (c)

Solution:



Let $P \equiv (x_1, y_1)$

$2x^2 + y^2 = 1$ is given equation of ellipse.

$\Rightarrow 4x + 2yy' = 0$

$\Rightarrow y'|_{(x_1,y_1)} = -\frac{2x_1}{y_1}$

Therefore, slope of normal at $P(x_1, y_1)$ is $\frac{y_1}{2x_1}$

Equation of normal at $P(x_1, y_1)$ is

$(y - y_1) = \frac{y_1}{2x_1}(x - x_1)$

It passes through $\left(-\frac{1}{3\sqrt{2}}, 0\right)$

$\Rightarrow -y_1 = \frac{y_1}{2x_1}\left(-\frac{1}{3\sqrt{2}} - x_1\right)$

$\Rightarrow x_1 = \frac{1}{3\sqrt{2}}$

$\Rightarrow y_1 = \frac{2\sqrt{2}}{3}$ as P lies in first quadrant

Since $(0, \beta)$ lies on the normal of the ellipse at point P, hence we get

$\beta = \frac{y_1}{2} = \frac{\sqrt{2}}{3}$

8. If ABC is a triangle whose vertices are $A(1, -1)$, $B(0, 2)$, $C(x', y')$ and area of ΔABC is 5, and $C(x', y')$ lies on $3x + y - 4\lambda = 0$, then

a. $\lambda = 3$
b. $\lambda = 4$
c. $\lambda = -3$
d. $\lambda = 2$

Answer: (a)

Solution:

Area of triangle is

$$A = \frac{1}{2}\begin{vmatrix} 0 & 2 & 1 \\ 1 & -1 & 1 \\ x' & y' & 1 \end{vmatrix} = \pm 5$$

$-2(1 - x') + (y' + x') = \pm 10$

$-2 + 2x' + y' + x' = \pm 10$

$3x' + y' = 12$ or $3x' + y' = -8$

$\Rightarrow \lambda = 3$ or -2

9. Shortest distance between the lines $\frac{x-3}{1}=\frac{y-8}{4}=\frac{z-3}{22}$, $\frac{x+3}{1}=\frac{y+7}{1}=\frac{z-6}{7}$ is

a. $3\sqrt{30}$

b. $\sqrt{30}$

c. $2\sqrt{30}$

d. $4\sqrt{30}$

Answer: (a)

Solution:

$\overrightarrow{AB}=-3\hat{\imath}-7\hat{\jmath}+6\hat{k}-\left(3\hat{\imath}+8\hat{\jmath}+3\hat{k}\right)=-6\hat{\imath}-15\hat{\jmath}+3\hat{k}$

$\vec{p}=\hat{\imath}+4\hat{\jmath}+22\hat{k}$

$\vec{q}=\hat{\imath}+\hat{\jmath}+7\hat{k}$

$$\vec{p}\times\vec{q}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & 4 & 22\\ 1 & 1 & 7\end{vmatrix}=6\hat{\imath}+15\hat{\jmath}-3\hat{k}$$

Shortest distance $=\frac{|\overrightarrow{AB}.(\vec{p}\times\vec{q})|}{|\vec{p}\times\vec{q}|}=\frac{|36+225+9|}{\sqrt{36+225+9}}=3\sqrt{30}.$

10. Let $\int\frac{\cos x}{\sin^3 x(1+\sin^6 x)^{\frac{2}{3}}}\,dx=f(x)(1+\sin^6 x)^{\frac{1}{\lambda}}+c$, then the value of $\lambda f\left(\frac{\pi}{3}\right)$ is

a. -4

b. -2

c. 8

d. 4

Answer: (b)

Solution:

Let $\sin x=t\Rightarrow\cos x\,dx=dt$

$$\therefore\int\frac{dt}{t^3\,(1+t^6)^{\frac{2}{3}}}=\int\frac{dt}{t^7\left(1+\frac{1}{t^6}\right)^{\frac{2}{3}}}$$

Let $1+\frac{1}{t^6}=u\Rightarrow-6t^{-7}dt=du$

$$\Rightarrow\int\frac{dt}{t^7\left(1+\frac{1}{t^6}\right)^{\frac{2}{3}}}=-\frac{1}{6}\int\frac{du}{u^{\frac{2}{3}}}=-\frac{3}{6}u^{\frac{1}{3}}+c=-\frac{1}{2}\left(1+\frac{1}{t^6}\right)^{\frac{1}{3}}+c$$

$$=-\frac{(1+\sin^6 x)^{\frac{1}{3}}}{2\sin^2 x}+c=f(x)(1+\sin^6 x)^{\frac{1}{\lambda}}$$

$\therefore\lambda=3$ and $f(x)=-\frac{1}{2\sin^2 x}$

$\Rightarrow\lambda f\left(\frac{\pi}{3}\right)=-2.$

11. If $y(x)$ is a solution of the differential equation $\sqrt{1-x^2}\frac{dy}{dx}+\sqrt{1-y^2}=0$, such that $y\left(\frac{1}{2}\right)=\frac{\sqrt{3}}{2}$, then

a. $y\left(\frac{1}{\sqrt{2}}\right)=-\frac{1}{\sqrt{2}}$

b. $y\left(\frac{1}{\sqrt{2}}\right)=\frac{1}{2}$

c. $y\left(\frac{1}{\sqrt{2}}\right)=\frac{1}{\sqrt{2}}$

d. $y\left(\frac{1}{\sqrt{2}}\right)=\frac{\sqrt{3}}{2}$

Answer: (c)

Solution:

$\sqrt{1-x^2}\frac{dy}{dx}+\sqrt{1-y^2}=0$

$\Rightarrow \frac{dy}{\sqrt{1-y^2}}+\frac{dx}{\sqrt{1-x^2}}=0$

$\Rightarrow \sin^{-1}y+\sin^{-1}x=c$

If $x=\frac{1}{2}, y=\frac{\sqrt{3}}{2}$ then,

$\sin^{-1}\frac{\sqrt{3}}{2}+\sin^{-1}\frac{1}{2}=c$

$\therefore \frac{\pi}{3}+\frac{\pi}{6}=c \Rightarrow c=\frac{\pi}{2}$

$\Rightarrow \sin^{-1}y=\frac{\pi}{2}-\sin^{-1}x=\cos^{-1}x$

$\therefore \sin^{-1}y=\cos^{-1}\frac{1}{\sqrt{2}}$

$\Longrightarrow \sin^{-1}y=\frac{\pi}{4}$

$\Longrightarrow y\left(\frac{1}{\sqrt{2}}\right)=\frac{1}{\sqrt{2}}$

12. $\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}}$ is equal to

a. e^{-2}

b. e^2

c. $e^{\frac{3}{7}}$

d. $e^{\frac{2}{7}}$

Answer: (a)

Solution:

Let $L=\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}}$ [Intermediate form 1^{∞}]

$\therefore L = e^{\lim\limits_{x\to 0} \frac{1}{x^2}\left(\frac{3x^2+2}{7x^2+2}-1\right)}$

$= e^{\lim\limits_{x\to 0} \frac{1}{x^2}\left(-\frac{4x^2}{7x^2+2}\right)}$

$= e^{-2}$.

13. In a bag there are 5 red balls, 3 white balls and 4 black balls. Four balls are drawn from the bag. Find the number of ways in which at most 3 red balls are selected.

a. 360
b. 450
c. 490
d. 510

Answer: (c)

Solution:

Number of ways to select at most 3 red balls $= P(0 \text{ red balls}) + P(1 \text{ red ball}) + P(2 \text{ red balls }) + P(3 \text{ red balls})$

$= {}^7C_4 + {}^5C_1 \times {}^7C_3 + {}^5C_2 \times {}^7C_2 + {}^5C_3 \times {}^7C_1$

$= 35 + 175 + 210 + 70 = 490.$

14. Let $f(x) = [\sin(\tan^{-1} x) + \sin(\cot^{-1} x)]^2 - 1$ where $|x| > 1$ and $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}(\sin^{-1} f(x))$. If $y(\sqrt{3}) = \frac{\pi}{6}$ then $y(-\sqrt{3})$ is equal to

a. $\frac{5\pi}{6}$
b. $-\frac{\pi}{6}$
c. $\frac{2\pi}{3}$
d. $\frac{\pi}{3}$

Answer: (b)

Solution:

$f(x) = [\sin(\tan^{-1} x) + \sin(\cot^{-1} x)]^2 - 1$

$\Rightarrow f(x) = [\sin(\tan^{-1} x) + \cos(\tan^{-1} x)]^2 - 1$

$\Rightarrow f(x) = \sin(2\tan^{-1} x)$

Now, $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}(\sin^{-1}(f(x)))$

$\Rightarrow 2y = \sin^{-1}\big(f(x)\big) + c$

If $x = \sqrt{3}, y = \frac{\pi}{6}$

$\therefore \frac{\pi}{3} = \sin^{-1}\big(\sin\big(2\tan^{-1}\sqrt{3}\big)\big) + c$

$\Rightarrow \frac{\pi}{3} = \sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) + c$

$\Rightarrow \frac{\pi}{3} = \frac{\pi}{3} + c \Rightarrow c = 0$

$\Rightarrow 2y = \sin^{-1}\sin(2\tan^{-1}x)$

When $x = -\sqrt{3}$

$2y = \sin^{-1}\left(\sin\left(2\tan^{-1}(-\sqrt{3})\right)\right) = \sin^{-1}\left(\sin\left(-\frac{2\pi}{3}\right)\right) = -\frac{\pi}{3}$

$\Rightarrow y = -\frac{\pi}{6}$.

15. The system of equation

$$3x + 4y + 5z = \mu$$
$$x + 2y + 3z = 1$$
$$4x + 4y + 4z = \delta$$

is inconsistent, then $(\mu,\ \delta)$ can be

a. $(4,\ 6)$
b. $(1,\ 0)$
c. $(4,\ 3)$
d. $(3,\ 4)$

Answer: (c)

Solution:

$$D = \begin{vmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 4 & 4 & 4 \end{vmatrix}$$

$R_3 \to R_3 - 2R_1 + 2R_2$

$$D = \begin{vmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

For inconsistent system, one of $D_x,\ D_y,\ D_z$ should not be equal to 0

$$D_x = \begin{vmatrix} \mu & 4 & 5 \\ 1 & 2 & 3 \\ \delta & 4 & 4 \end{vmatrix} \qquad D_y = \begin{vmatrix} 3 & \mu & 5 \\ 1 & 1 & 3 \\ 4 & \delta & 4 \end{vmatrix}$$

$$D_z = \begin{vmatrix} 3 & 4 & \mu \\ 1 & 2 & 1 \\ 4 & 4 & \delta \end{vmatrix}$$

For inconsistent system, $2\mu \neq \delta + 2$

$\therefore$ The system will be inconsistent for $\mu = 4,\ \delta = 3$.

16. If volume of parallelepiped whose conterminous edges are $\vec{u} = \hat{\imath} + \hat{\jmath} + \lambda\,\hat{k}$, $\vec{v} = 2\hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{w} = \hat{\imath} + \hat{\jmath} + 3\hat{k}$ is 1 cubic units. Then, the cosine of angle between $\vec{u}$ and $\vec{v}$ is

a. $\frac{5}{3\sqrt{10}}$

b. $\frac{5}{7}$

c. $\frac{7}{6\sqrt{3}}$

d. $\frac{7}{3\sqrt{3}}$

Answer: (c)

Solution:

Volume of parallelepiped $= [\vec{u} \ \ \vec{v} \ \ \vec{w}]$

$$\Rightarrow \begin{vmatrix} 1 & 1 & \lambda \\ 1 & 1 & 3 \\ 2 & 1 & 1 \end{vmatrix} = \pm 1$$

$\Rightarrow \lambda = 2$ or 4

For $\lambda = 4$,

$\cos\theta = \frac{2+1+4}{\sqrt{6}\sqrt{18}} = \frac{7}{6\sqrt{3}}$.

17. If $2^{1-x} + 2^{1+x}, f(x), 3^x + 3^{-x}$ are in $A.P.$ then the minimum value of $f(x)$ is

a. 4

b. 2

c. 3

d. 1

Answer: (c)

Solution:

$2^{1-x} + 2^{1+x}, f(x), 3^x + 3^{-x}$ are in A.P.

$$\therefore f(x) = \frac{3^x + 3^{-x} + 2^{1+x} + 2^{1-x}}{2} = \frac{(3^x + 3^{-x})}{2} + \frac{2^{1+x} + 2^{1-x}}{2}$$

Applying A.M. ≥ G.M. inequality, we get

$$\frac{(3^x + 3^{-x})}{2} \geq \sqrt{3^x . 3^{-x}}$$

$$\Longrightarrow \frac{(3^x+3^{-x})}{2} \geq 1 \qquad \dots (1)$$

Also, Applying A.M. ≥ G.M. inequality, we get

$$\frac{2^{1+x} + 2^{1-x}}{2} \geq \sqrt{2^{1+x} . 2^{1-x}}$$

$$\Longrightarrow \frac{2^{1+x} + 2^{1-x}}{2} \geq 2 \quad \dots (2)$$

Adding (1) and (2), we get

$f(x) \geq 1 + 2 = 3$

Thus, minimum value of $f(x)$ is 3.

18. Which of the following is tautology?

a. $(p \wedge (p \longrightarrow q)) \longrightarrow q$

b. $q \longrightarrow p \wedge (p \longrightarrow q)$

c. $(p \wedge (p \vee q))$

d. $(p \vee (p \wedge q)$

Answer: (a)

Solution:

$(p \wedge (p \longrightarrow q)) \longrightarrow q$

$= (p \wedge (\backsim p \vee q)) \longrightarrow q$

$= [(p \wedge \sim p) \vee (p \wedge q)] \longrightarrow q$

$= (p \wedge q) \longrightarrow q$

$= \sim (p \wedge q) \vee q$

$= \sim p \vee \sim q \vee q$

$= T$

19. A is a 3×3 matrix whose elements are from the set $\{-1, 0, 1\}$. Find the number of matrices A such that $tr(AA^T) = 3$ where $tr(A)$ is sum of diagonal elements of matrix A

a. 612

b. 572

c. 672

d. 682

Answer: (c)

Solution:

$tr(AA^T) = 3$

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$

$tr(AA^T) = a_{11}^2 + a_{12}^2 + a_{13}^2 + a_{21}^2 + a_{22}^2 + a_{23}^2 + a_{31}^2 + a_{32}^2 + a_{33}^2 = 3$

So out of 9 elements (a_{ij})'s, 3 elements must be equal to 1 or -1 and rest elements must be 0.

So, the total possible cases will be

When there is $6(0's)$ and $3(1's)$ then the total possibilities is ${}^9C_6$

For $6(0's)$ and $3(-1's)$ total possibilities is ${}^9C_6$

For $6(0's), 2(1's)$ and $1(-1's)$ total possibilities is ${}^9C_6 \times 3$

For $6(0's), 1(1's)$ and $2(-1's)$ total possibilities is ${}^9C_6 \times 3$

$\therefore$ Total number of cases $= {}^9C_6 \times 8 = 672$.

20. Mean and standard deviation of 10 observations are 20 and 2 respectively. If $p\ (p \neq 0)$ is multiplied to each observation and then $q\ (q \neq 0)$ is subtracted from each of them, then new mean and standard deviation becomes half of it's original value. Then find q

a. 10

b. -20

c. -5

d. -10

Answer: (b)

Solution:

If mean $\bar{x}$ is multiplied by p and then q is subtracted from it,

then new mean $\bar{x}' = p\bar{x} - q$

$\therefore \bar{x}' = \frac{1}{2}\bar{x}$ and $\bar{x} = 10$

$\Rightarrow 10 = 20p - q \quad \dots(1)$

If standard deviation is multiplied by p, new standard deviation (σ') is p times of the initial standard deviation (σ).

$\sigma' = |p|\sigma$

$$\Rightarrow \frac{1}{2}\sigma = |p|\sigma \Rightarrow |p| = \frac{1}{2}$$

If $p = \frac{1}{2}$, $q = 0$

If $p = -\frac{1}{2}$, $q = -20$.

21. Let P be a point on $x^2 = 4y$. The segment joining $A(0, -1)$ and P is divided by a point Q in the ratio $1:2$, then locus of point Q is

a. $9x^2 = 3y + 2$

b. $9x^2 = 12y + 8$

c. $9y^2 = 3x + 2$

d. $9y^2 = 12x + 8$

Answer: (b)

Solution:

Let point P be $(2t, t^2)$ and Q be (h, k).

$h = \frac{2t}{3}, k = \frac{-2+t^2}{3}$

Now, eliminating t from the above equations we get:

$3k + 2 = \left(\frac{3h}{2}\right)^2$

Replacing h and k by x and y, we get the locus of the curve as $9x^2 = 12y + 8$.

22. If the curves $y^2 = ax$ and $x^2 = ay$ intersect each other at A and B such that the area bounded by the curves is bisected by the line $x = b$ (given a $> b > 0$) and the area of triangle formed by the lines $AB, x = b$ and the x-axis is $\frac{1}{2}$. Then

a. $a^6 + 12a^3 + 4 = 0$

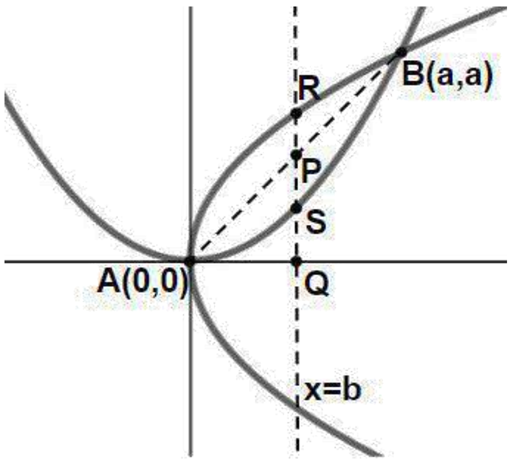
b. $a^6 + 12a^3 - 4 = 0$

c. $a^6 - 12a^3 + 4 = 0$

d. $a^6 - 12a^3 - 4 = 0$

Answer: (c)

Solution:



Given, $ar(\Delta APQ) = \frac{1}{2}$

$\Rightarrow \frac{1}{2} \times b \times b = \frac{1}{2}$

$\Rightarrow b = 1$

As per the question

$\Rightarrow \int_0^1 \left(\sqrt{ax} - \frac{x^2}{a}\right) dx = \frac{1}{2} \int_0^a \left(\sqrt{ax} - \frac{x^2}{a}\right) dx$

$\Rightarrow \frac{2}{3}\sqrt{a} - \frac{1}{3a} = \frac{a^2}{6}$

$\Rightarrow 2a\sqrt{a} - 1 = \frac{a^3}{2}$

$\Rightarrow 4a\sqrt{a} = 2 + a^3$

$\Rightarrow 16a^3 = 4 + a^6 + 4a^3$

$\Rightarrow a^6 - 12a^3 + 4 = 0.$

23. The sum $\sum_{k=1}^{20}(1 + 2 + 3 + \ldots + k)$ is ___________.

Answer: (1540)

Solution:

$= \sum_{k=1}^{20} \frac{k(k+1)}{2}$

$= \frac{1}{2}\sum_{k=1}^{20} k^2 + k$

$= \frac{1}{2}\left[\frac{20(21)(41)}{6} + \frac{20(21)}{2}\right]$

$= \frac{1}{2}[2870 + 210]$

$= 1540.$

24. If $2x^2 + (a-10)x + \frac{33}{2} = 2a, a \in \mathbf{Z}^+$ has real roots, then minimum value of $'a'$ is equal to ___________.

Answer: (8)

Solution:

$\because 2x^2 + (a-10)x + \frac{33}{2} = 2a, a \in \mathbf{Z}^+$ has real roots

$\Rightarrow D \geq 0$

$\Rightarrow (a-10)^2 - 4 \times 2 \times \left(\frac{33}{2} - 2a\right) \geq 0$

$\Rightarrow (a-10)^2 - 4(33 - 4a) \geq 0$

$\Rightarrow a^2 - 4a - 32 \geq 0 \Rightarrow a \in (-\infty, 4] \cup [8, \infty)$

Thus, minimum value of $'a'$ $\forall\ a \in \mathbf{Z}^+$ is 8.

25. If normal at P on the curve $y^2 - 3x^2 + y + 10 = 0$ passes through the point $\left(0, \frac{3}{2}\right)$ and the slope of tangent at P is n. The value of $|n|$ is equal to__________.

Answer: (4)

Solution:

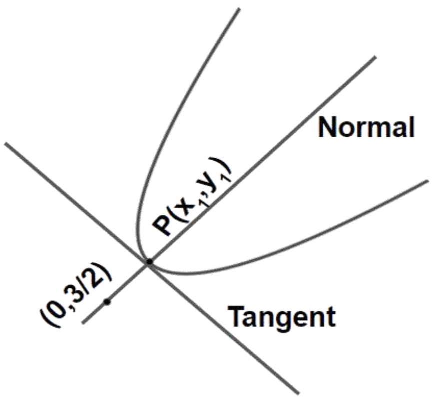
Let co-ordinate of P be (x_1, y_1)

Differentiating the curve w.r.t x

$2yy' - 6x + y' = 0$

Slope of tangent at P

$\Rightarrow \ y' = \dfrac{6x_1}{1 + 2y_1}$



$\because \ m_{\text{normal}} = \left(\dfrac{y_1 - \frac{3}{2}}{x_1 - 0}\right)$

$\because \ m_{\text{normal}} \times m_{\text{tangent}} = -1$

$\Rightarrow \dfrac{\frac{3}{2} - y_1}{-x_1} \times \dfrac{6x_1}{1 + 2y_1} = -1$

$\Rightarrow y_1 = 1$

$\Rightarrow x_1 = \pm 2$

Slope of tangent $= \pm \frac{12}{3} = \pm 4$

$\Rightarrow |n| = 4$