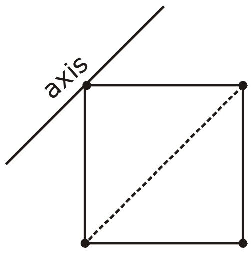
QUESTION PAPER WITH SOLUTION

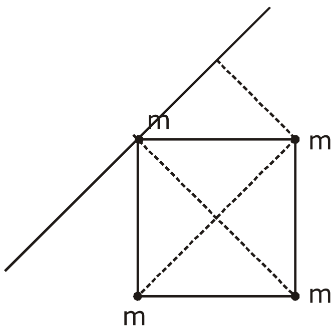
PHYSICS _ 6 Sep. _ SHIFT - 1

1. Four point masses, each of mass m, are fixed at the corners of a square of side l. The square is rotating with angular frequency ω, about an axis passing through one of the corners of the square and parallel to its diagonal, as shown in the figure. The angular momentum of the square about this axis is :



(1) $4\ ml^2\omega$ (2) $2\ ml^2\omega$ (3) $3\ ml^2\omega$ (4) $ml^2\omega$

Sol. **(3)**



$L = I\omega$

$$I = m\left(\frac{a}{\sqrt{2}}\right)^2 x\,2 + m\left(\sqrt{2}a\right)^2$$

$= ma^2 + 2ma^2$

$\therefore L = I\omega = 3ml^2\omega$ $(a = l)$

2. A screw gauge has 50 divisions on its circular scale. The circular scale is 4 units ahead of the pitch scale marking, prior to use. Upon one complete rotation of the circular scale, a displacement of 0.5mm is noticed on the pitch scale. The nature of zero error involved and the least count of the screw gauge, are respectively :

(1) Positive, 0.1 mm (2) Positive, 0.1 μm

(3) Positive, 10 μm (4) Negative, 2 μm

2. **(3)**

$$= L.C = \frac{0.5}{50} mm = 1 \times 10^{-5} m = 10 \mu m$$

3. An electron, a doubly ionized helium ion (He^{++}) and a proton are having the same kinetic energy. The relation between their respective de-Broglie wavelengths λ_e, $\lambda_{He^{++}}$ and λ_p is :

(1) $\lambda_e > \lambda_P > \lambda_{He^{++}}$ (2) $\lambda_e > \lambda_{He^{++}} > \lambda_P$

(3) $\lambda_e < \lambda_P < \lambda_{He^{++}}$ (4) $\lambda_e < \lambda_{He^{++}} = \lambda_P$

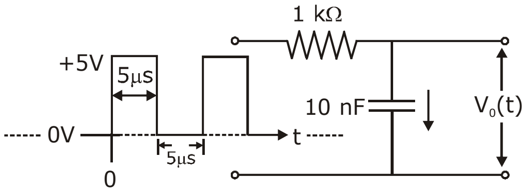
Sol. **(1)**

$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK \cdot E}}$ $\gamma = \frac{C_p}{C_d} = 1 + \frac{2}{f} = 1 + \frac{2}{5} = \frac{7}{5}$

$m_{He} > m_P > m_e$

$\lambda_{He} < \lambda_p < \lambda_e$

4. For the given input voltage waveform $V_{in}(t)$, the output voltage waveform $V_o(t)$, across the capacitor is correctly depicted by :



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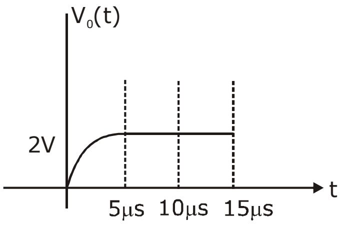

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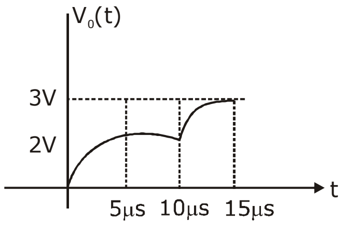

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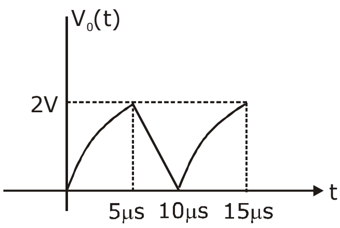

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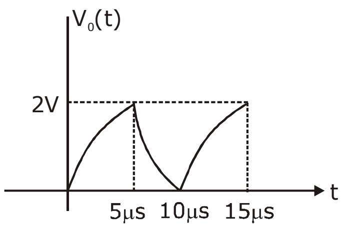
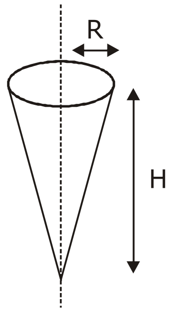


Sol. **(2)**

Answer is (2) because capacitor is charging then discharging then again charging.
But during discharging not possible to discharge 100%.

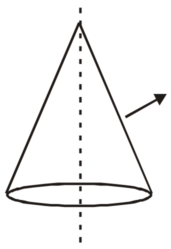
5. Shown in the figure is a hollow icecream cone (it is open at the top). If its mass is M, radius of its top, R and height, H, then its moment of inertia about its axis is :



(1) $\frac{MR^2}{2}$ (2) $\frac{MR^2}{3}$ (3) $\frac{M\left(R^2+H^2\right)}{4}$ (4) $\frac{MH^2}{3}$

Sol. **(1)**

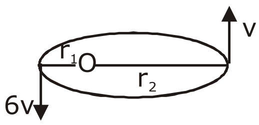
$I = \frac{mR^2}{2}$



6. A satellite is in an elliptical orbit around a planet P. It is observed that the velocity of the satellite when it is farthest from the planet is 6 times less than that when it is closest to the planet. The ratio of distances between the satellite and the planet at closest and farthest points is:

(1) 1 : 2 (2) 1 : 3 (3) 1 : 6 (4) 3 : 4

Sol. **(3)**



$L_i = L_f$

$m \cdot 6vr_1 = m \cdot vr_2$

$6r_1 = r_2$

$\Rightarrow \frac{r_1}{r_2} = \frac{1}{6}$

7. You are given that Mass of ${}_{3}^{7}Li = 7.0160\,u$,

Mass of ${}_{2}^{4}He = 4.0026\,u$

and Mass of ${}_{1}^{1}H = 1.0079\,u$.

When 20 g of ${}_{3}^{7}Li$ is converted into ${}_{2}^{4}He$ by proton capture, the energy liberated, (in kWh), is: [Mass of nucleon = 1 GeV/c²]

(1) 6.82×10^{5} (2) 4.5×10^{5} (3) 8×10^{6} (4) 1.33×10^{6}

Sol. **(4)**

${}_{3}^{7}Li + {}_{1}e^{+} \rightarrow 2\,{}_{2}^{4}He$

$\Delta m \Rightarrow [m_{Li} + m_{H}] - 2\,[M_{He}]$

$\rightarrow \Delta m = (7.0160 + 1.0079) - 2 \times 4.0003$

$= 0.0187$

Energy released in 1 reaction $\Rightarrow \Delta mc^2$

In use of 7.016 u Li energy is Δmc^2

In use of 1gm Li energy is $\dfrac{\Delta mc^2}{m_{Li}}$

In use of 20gm energy is $\Rightarrow \dfrac{\Delta mc^2}{m_{Li}} \times 20gm$

$$\frac{0.0187 \times 931.5 \times 10^{6} \times 1.6 \times 10^{-19} \times \frac{20}{7} \times 6.023 \times 10^{23}}{36 \times 10^{5}}$$

$= 1.33 \times 10^{6}$

8. If the potential energy between two molecules is given by $U = -\dfrac{A}{r^6} + \dfrac{B}{r^{12}}$, then at equilibrium, separation between molecules, and the potential energy are :

(1) $\left(\dfrac{2B}{A}\right)^{1/6}, -\dfrac{A^2}{4B}$ (2) $\left(\dfrac{2B}{A}\right)^{1/6}, -\dfrac{A^2}{2B}$ (3) $\left(\dfrac{B}{A}\right)^{1/6}, 0$ (4) $\left(\dfrac{B}{2A}\right)^{1/6}, -\dfrac{A^2}{2B}$

Sol. **(1)**

$F = \dfrac{-dU}{dr} = \dfrac{-d}{dr}(-Ar^{-6} + Br^{-12})$ for equation $F = 0$

$= \dfrac{A(-6)}{r^7} + \dfrac{B \cdot 12}{r^{13}} = 0$

$\dfrac{12B}{r^{13}} = \dfrac{6A}{r^7}$

$r = \left(\dfrac{2B}{A}\right)^{1/6}$

$$U = \frac{-A}{\frac{2B}{A}} + \frac{B}{\left(\frac{2B}{A}\right)^2}$$

$$= \frac{-A^2}{2B} + \frac{A^2}{4B} = \frac{-A^2}{4B}$$

$\therefore$ Answer (1)

9. A clock has a continuously moving second's hand of 0.1 m length. The average acceleration of the tip of the hand (in units of ms^{-2}) is of the order of:

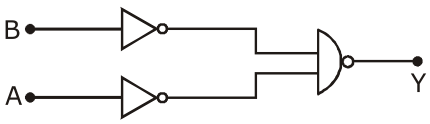
(1) 10^{-3} (2) 10^{-1} (3) 10^{-2} (4) 10^{-4}

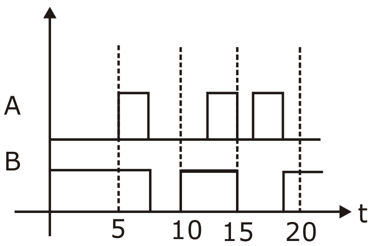
Sol. **(1)**

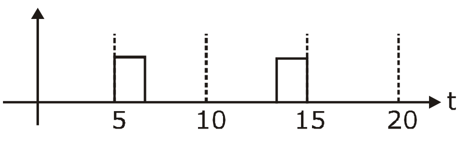
$$a = \frac{v^2}{R} \qquad v = \frac{2\pi R}{60}$$

$$= \frac{4\pi^2 \cdot R^2}{(60)^2 R} = \frac{4\pi^2 R}{(60)^2} = \frac{4}{(60)^2} \times 10 \times 0.1 \approx 10^{-3}$$

10. Identify the correct output signal Y in the given combination of gates (as shown) for the given inputs A and B.





(1)


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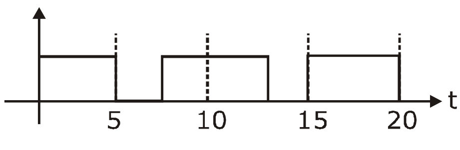

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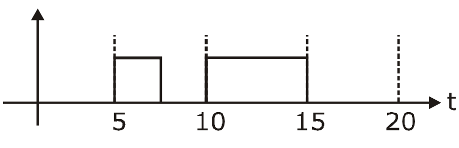

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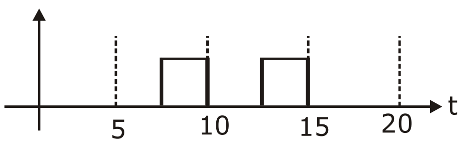
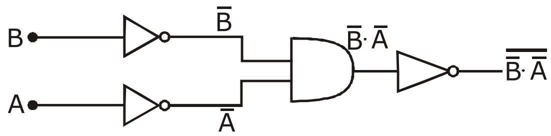


Sol. **None of the option is correct**

$$\overline{\overline{A}\cdot\overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$$

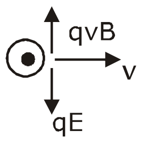


11. An electron is moving along +x direction with a velocity of 6×10^{6} ms^{-1}. It enters a region of uniform electric field of 300 V/cm pointing along +y direction. The magnitude and direction of the magnetic field set up in this region such that the electron keeps moving along the x direction will be:

(1) 3×10^{-4} T, along –z direction
(2) 5×10^{-3} T, along –z direction
(3) 5×10^{-3} T, along +z direction
(4) 3×10^{-4} T, along +z direction

Sol. **(3)**

$\vec{B}$ must be in +z axis.

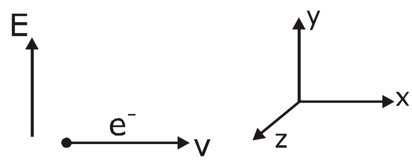


$qE = qVB$

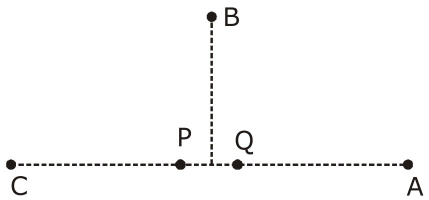
$E = 300\,\dfrac{v}{10^{-2}m}$

= 30000 v/m

$B = \dfrac{E}{V} = \dfrac{3\times10^{4}}{6\times10^{6}} = 5\times10^{-3}T$

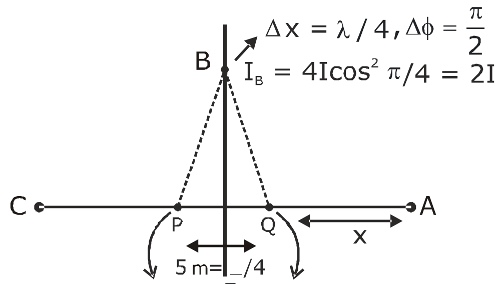


12. In the figure below, P and Q are two equally intense coherent sources emitting radiation of wavelength 20 m. The separation between P and Q is 5 m and the phase of P is ahead of that of Q by 90°. A, B and C are three distinct points of observation, each equidistant from the midpoint of PQ. The intensities of radiation at A, B, C will be in the ratio:



(1) 4 : 1 : 0
(2) 2 : 1 : 0
(3) 0 : 1 : 2
(4) 0 : 1 : 4

Sol. **(2)**



$A\sin(\omega t - k(x + \lambda/4) + \frac{\pi}{2})$ $\quad$ $A\sin(\omega t - kx)$

$A\sin(\omega t - kx - \frac{2\pi}{\lambda} \times \frac{\lambda}{4} + \frac{\pi}{2})$

$A\sin(\omega t - kx)$

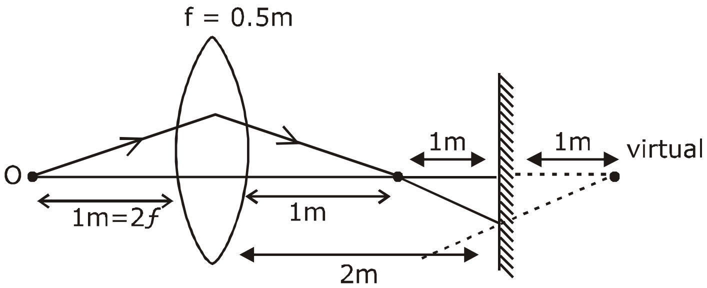
$\left| \begin{array}{l} \Delta x = \lambda/2 \\ \Delta\phi = \pi \\ I_C = O \end{array} \right|$

$\therefore$ at A $\Delta x_{effective} = 0$ or phase difference = 0
$\therefore$ $I_A = 4I$
{Same logic as A point but opposits}
$\therefore$ Answer is 2.

13. A point like object is placed at a distance of 1 m in front of a convex lens of focal length 0.5 m. A plane mirror is placed at a distance of 2 m behind the lens. The position and nature of the final image formed by the system is:

(1) 1 m from the mirror, virtual
(2) 2.6 m from the mirror, virtual
(3) 1 m from the mirror, real
(4) 2.6 m from the mirror, real

Sol. **(1, 2 Both are correct)**



for III[rd] Refraction, $u = -3$

$\frac{1}{V} + \frac{1}{3} = \frac{2}{1}$

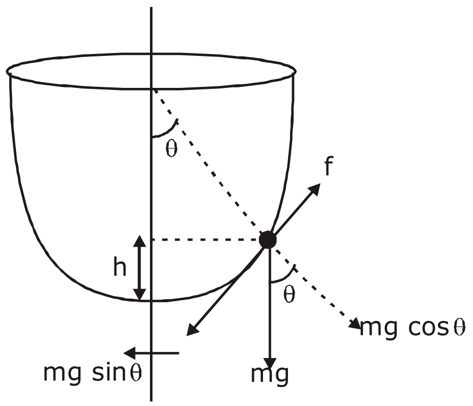
$V = \frac{3}{5} = 0.6$

from mirror = 2.6m

14. An insect is at the bottom of a hemispherical ditch of radius 1 m. It crawls up the ditch but starts slipping after it is at height h from the bottom. If the coefficient of friction between the ground and the insect is 0.75, then h is: (g = 10 ms^{-2})

(1) 0.45 m (2) 0.60 m (3) 0.20 m (4) 0.80 m

Sol. **(3)**



$f = mg \sin \theta$

$f = \mu mg \cos \theta$

$\mu mg \cos \theta = mg \sin \theta$

$\tan\theta = \mu$

$\tan \theta = \frac{3}{4}$

$\cos \theta = \frac{4}{\sqrt{16+9}} = \frac{4}{5}$

$h = 1(1 - \cos \theta) = 1 - \frac{4}{5} = \frac{1}{5}$

$h = \frac{1}{5} = 0.2m$

15. Molecules of an ideal gas are known to have three translational degrees of freedom and two rotational degrees of freedom.The gas is maintained at a temperature of T. The total internal energy, U of a mole of this gas, and the value of $\gamma\left(=\frac{C_p}{C_v}\right)$ are given, respectively by:

(1) $U = \frac{5}{2}RT$ and $\gamma = \frac{7}{5}$ (2) $U = 5RT$ and $\gamma = \frac{6}{5}$

(3) $U = 5RT$ and $\gamma = \frac{7}{5}$ (4) $U = \frac{5}{2}RT$ and $\gamma = \frac{6}{5}$

Sol. **(1)**

$$U = \frac{f}{2}nRT = \frac{5}{2}nRT\begin{pmatrix} C_p - C_v = R \\ C_v = \frac{f}{2}R \end{pmatrix}, \gamma = \frac{C_p}{C_d} \Rightarrow 1 + \frac{2}{f} = 1 + \frac{2}{5} = \frac{7}{5}$$

16. An object of mass m is suspended at the end of a massless wire of length L and area of cross-section A. Young modulus of the material of the wire is Y. If the mass is pulled down slightly its frequency of oscillation along the vertical direction is:

(1) $f = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$ (2) $f = \frac{1}{2\pi}\sqrt{\frac{mL}{YA}}$ (3) $f = \frac{1}{2\pi}\sqrt{\frac{YL}{mA}}$ (4) $f = \frac{1}{2\pi}\sqrt{\frac{mA}{YL}}$

Sol. **(1)**

$$Y = \frac{F/A}{\Delta L/L}$$

$$Y = \frac{FL}{A\Delta L}$$

$$F = \frac{YA\Delta L}{L}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{YA}{Lm}} \qquad \left(\frac{yA}{L} = k\right) \qquad \because T = 2\pi\sqrt{\frac{m}{k}}$$

17. An AC circuit has R= 100 Ω, C= 2 μF and L = 80 mH, connected in series. The quality factor of the circuit is:

(1) 20 (2) 2 (3) 0.5 (4) 400

Sol. **(2)**

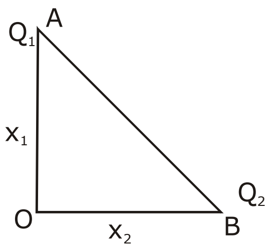
$$Q = \frac{\omega L}{R}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$= \frac{L}{R\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

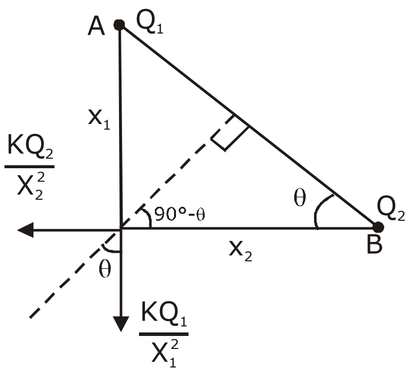
$$= \frac{1}{100}\sqrt{\frac{80 \times 10^{-3}}{2 \times 10^{-6}}} = 2$$

18. Charges Q_1 and Q_2 are at points A and B of a right angle triangle OAB (see figure). The resultant electric field at point O is perpendicular to the hypotenuse, then Q_1/Q_2 is proportional to:



(1) $\frac{x_2}{x_1}$ (2) $\frac{x_2^2}{x_1^2}$ (3) $\frac{x_1^3}{x_2^3}$ (4) $\frac{x_1}{x_2}$

Sol. **(4)**

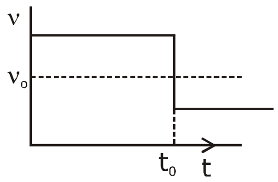


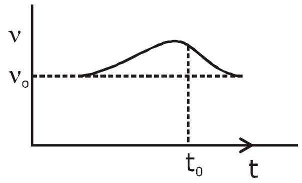
$$\tan\theta = \frac{kQ_2 / x_2^2}{kQ_1 / x_1^2} = \frac{x_1}{x_2}$$

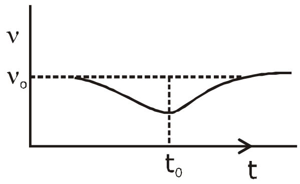
$$\frac{Q_2 \cdot x_1^2}{Q_1 \cdot x_2^2} = \frac{X_1}{X_2}$$

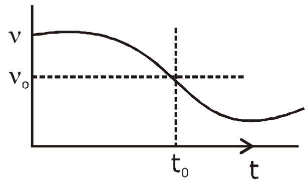
$$\frac{Q_1}{Q_2} = \frac{x_1}{x_2}$$

19. A sound source S is moving along a straight track with speed v, and is emitting, sound of frequency v_O (see figure). An observer is standing at a finite distance, at the point O, from the track. The time variation of frequency heard by the observer is best represented by: (t_0 represents the instant when the distance between the source and observer is minimum)

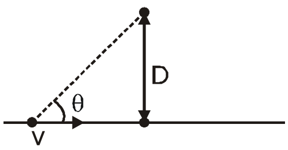
(1) 

(2) 

(3) 

(4) 

Sol. **(4)**



$$f_{observed} \Rightarrow \left(\frac{V_{sound}}{V_{sound} - v\cos\theta}\right) f_0$$

initially θ will be less $\Rightarrow \cos\theta$ more

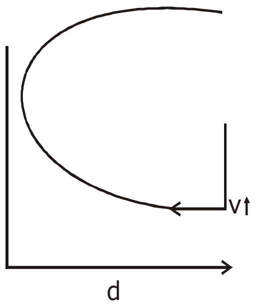
$\therefore$ $f_{observed}$ more, then it will decrease.

$\therefore$ Ans. 4

20. A particle of charge q and mass m is moving with a velocity $-v\hat{i}$ ($v \neq 0$) towards a large screen placed in the Y-Z plane at a distance d. If there is a magnetic field $\vec{B} = B_0\hat{k}$, the minimum value of v for which the particle will not hit the screen is:

(1) $\frac{qdB_0}{m}$ (2) $\frac{qdB_0}{3m}$ (3) $\frac{2qdB_0}{m}$ (4) $\frac{qdB_0}{2m}$

Sol. **(1)**



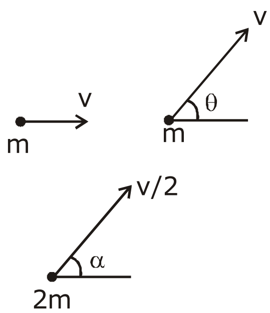
d > R

$d > \frac{mv}{qB_0}$

$v < \frac{qB_0 d}{m}$

21. Two bodies of the same mass are moving with the same speed, but in different directions in a plane. They have a completely inelastic collision and move together thereafter with a final speed which is half of their initial speed. The angle between the initial velocities of the two bodies (in degree) is _________.

21. **120**



∴ In Horizontal Direction
By Momentum conservation.

$$mv + mv\cos\theta = 2m\frac{v}{2}\cos\alpha$$

$$1 + \cos\theta = \cos\alpha \qquad ....(1)$$

In vertical direction
By Momentum conservation.

$$0 + mv\sin\theta = 2m\frac{v}{2}\sin\alpha$$

$$\sin\theta = \sin\alpha$$

$$1 + \cos\theta = \sqrt{1-\sin^2\theta}$$

$$\theta = 120^\circ$$

22. Suppose that intensity of a laser is $\left(\frac{315}{\pi}\right)$ W/m^2. The rms electric field, in units of V/m associated with this source is close to the nearest integer is __________.
(ϵ_0 = 8.86 × 10^{-12} C^{2}Nm^{-2}; c = 3 × 10^8 ms^{-1})

Sol. **275**

$$I = \frac{1}{2}\varepsilon_0 C\,E_{rms}^2$$

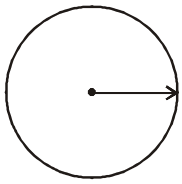
$$\frac{3.15}{\pi} = \frac{1}{2}\times 8.86\times 10^{-12}\times 3\times 10^8\times E_{rms}^2$$

$$E_{rms} = 275$$

23. The density of a solid metal sphere is determined by measuring its mass and its diameter. The maximum error in the density of the sphere is $\left(\frac{x}{100}\right)\%$. If the relative errors in measuring the mass and the diameter are 6.0% and 1.5% respectively, the value of x is_______.

Sol. **1050**

$$\rho = \frac{m}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$



$$\rho = k\cdot\frac{m}{d^3}$$

$$\log\rho = \overset{o}{\cancel{\log k}} + \log m - 3\log d$$

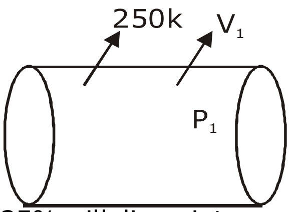
diff.

$$\frac{d\rho}{\rho} = \frac{dm}{m} - 3\cdot\frac{dd}{d}$$

= 6.0 + 3×1.5 = 10.5%
= x = 1050

24. Initially a gas of diatomic molecules is contained in a cylinder of volume V_1 at a pressure P_1 and temperature 250 K. Assuming that 25% of the molecules get dissociated causing a change in number of moles. The pressure of the resulting gas at temperature 2000 K, when contained in a volume $2V_1$ is given by P_2. The ratio P_2/P_1 is ________.

Sol. **5**



pv = nRT

$p_1v_1 = nR250$ (i)

25% will dissociate

out of 100

$\frac{3n}{4}$ molecules will remain same

S

$\frac{n}{4}$ mole become $\rightarrow \frac{n}{2}$

$\therefore$ Total molecules used

$\rightarrow \frac{3n}{4} + \frac{n}{2} = \frac{5n}{4}$

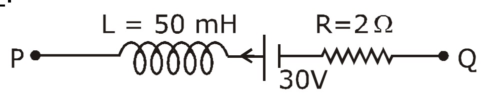
$P_2 2V_1 = \frac{5n}{4} \cdot R \cdot 2000 - \text{(ii)}$

Eq. (ii/i)

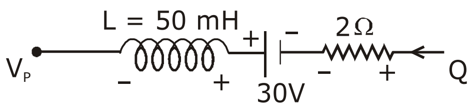
$\frac{2p_2v_1}{p_1v_1} = \frac{5nR \times 2000}{4nR \times 250}$

$\frac{P_2}{P_1} = 5$

25. A part of a complete circuit is shown in the figure. At some instant, the value of current I is 1A and it is decreasing at a rate of 10^2 A s^{-1}. The value of the potential difference $V_P - V_Q$, (in volts) at that instant, is __________.



Sol. **33**



$V_P + L \cdot \frac{di}{dt} - 30 + 2i = V_Q$

$V_P + 50 \times 10^{-3}(-10^2) - 30 + 2 \times 1 = v_Q$

$V_P - V_Q = 35 - 2 = 33$

QUESTION PAPER WITH SOLUTION

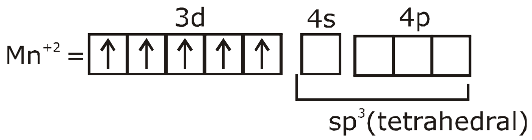
CHEMISTRY _ 6 Sep. _ SHIFT - 1

1. The INCORRECT statement is :
(1) Cast iron is used to manufacture wrought iron.
(2) Brass is an alloy of copper and nickel.
(3) German silver is an alloy of zinc, copper and nickel.
(4) Bronze is an alloy of copper and tin

Sol. **2**
Brass - (copper Zinc)
Bronze - (copper tin)

2. The species that has a spin-only magnetic moment of 5.9 BM, is : (T_d = tetrahedral)
(1) $[Ni(CN)_4]^{2-}$ (square planar)
(2) $Ni(CO)_4(T_d)$
(3) $[MnBr_4]^{2-}(T_d)$
(4) $[NiCl_4]^{2-}(T_d)$

Sol. **3**
$[MnBr_4]^{2-}$



$\mu = \sqrt{5(5+2)}$ = 5.9 BM

3. For the reaction

$$Fe_2N(s) + \frac{3}{2}H_2(g) \rightleftharpoons 2Fe(s) + NH_3(g)$$

(1) $K_c = K_p(RT)^{1/2}$
(2) $K_c = K_p(RT)^{-1/2}$
(3) $K_c = K_p(RT)^{\frac{3}{2}}$
(4) $K_c = K_p(RT)$

Sol. **1**

$$Fe_2N(s) + \frac{3}{2}H_2(g) \rightleftharpoons 2Fe(s) + NH_3(g)$$

$$\Delta n_g = 1 - \frac{3}{2} = \frac{-1}{2}$$

$$\frac{K_p}{K_c} = (RT)^{\Delta n_g} = (RT)^{-1/2}$$

$$K_c = \frac{K_p}{(RT)^{-1/2}} = K_p.(RT)^{1/2}$$

4. Consider the following reactions :

$$\underset{(C_7H_{14})}{\text{'A'}} \xrightarrow{\text{ozonolysis}} \text{'B'} + \text{'C'}$$

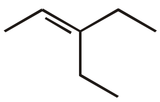
$$\text{'B'} \xrightarrow[\Delta]{(I_2+NaOH)} \text{yellow ppt}$$

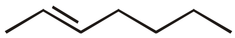
$$\text{'B'} \xrightarrow[\Delta]{Ag_2O} \text{silver mirror}$$

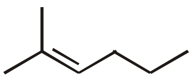
$$\text{'C'} \xrightarrow[\Delta]{(I_2+NaOH)} \text{no yellow ppt}$$

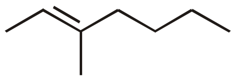
$$\text{'C'} \xrightarrow[\Delta]{LiAlH_4} \text{'D'} \xrightarrow[\text{\& Conc. HCl}]{\text{Anhydrous } ZnCl_2} \text{gives white turbidity within 5 minutes}$$

'A' is :

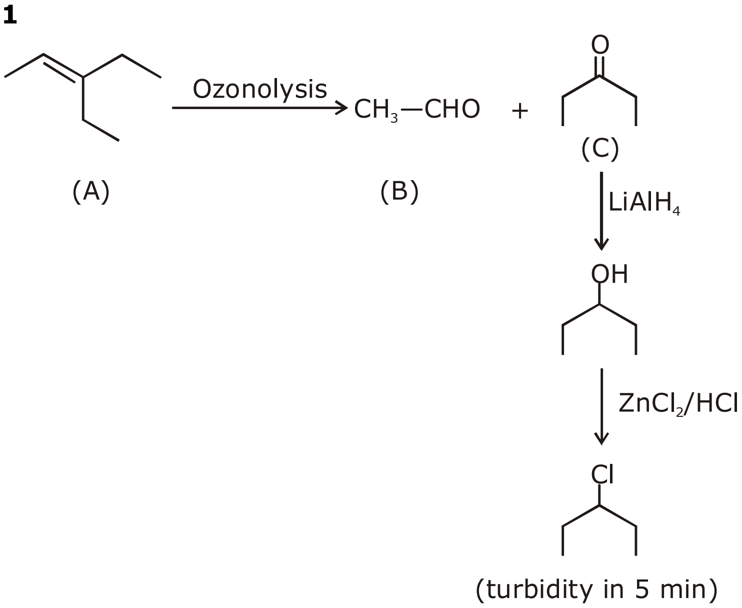
(1) 

(2) 

(3) 

(4) 

Sol. **1**



5. Arrange the following solutions in the decreasing order of pOH :

(A) 0.01 M HCl (B) 0.01 M NaOH

(C) 0.01 M CH_3COONa (D) 0.01 M NaCl

(1) (A) > (C) > (D) > (B) (2) (B) > (D) > (C) > (A)

(3) (B) > (C) > (D) > (A) (4) (A) > (D) > (C) > (B)

Sol. **4**

(i) 10^{-2} M HCl $\Rightarrow [H^+] = 10^{-2}$ M $\rightarrow$ pH = 2

(ii) 10^{-2} M NaOH $\Rightarrow [OH^-] = 10^{-2}$ M $\rightarrow$ pOH = 2

(iii) 10^{-2} M $CH_3COO^-Na^+ \Rightarrow [OH^+] > 10^{-7} \Rightarrow$ pOH < 7

(iv) 10^{-2} M NaCl $\Rightarrow$ Neutral pOH = 7

(i) > (iv) > (iii) > (ii)

6. The variation of equilibrium constant with temperature is given below :

Temperature	**Equilibrium Constant**
$T_1 = 25^oC$	$K_1 = 10$
$T_2 = 100^oC$	$K_2 = 100$

The value of ΔH^0, ΔG^0 at T_1 and ΔG^0 at T_2 (in Kj mol^{-1}) respectively, are close to [use R = 8.314JK^{-1} mol^{-1}]

(1) 28.4, -7.14 and -5.71 (2) 0.64, - 7.14 and -5.71

(3) 28.4, - 5.71 and -14.29 (4) 0.64, - 5.71 and -14.29

Sol. **3**

$$\ln\left[\frac{k_2}{k_1}\right] = \frac{\Delta H^\circ}{R}\left\{\frac{1}{T_1} - \frac{1}{T_2}\right\}$$

$$\ln(10) = \frac{\Delta H^\circ}{R}\left\{\frac{1}{298} - \frac{1}{373}\right\}$$

$$\frac{373 \times 298 \times 8.314 \times 2.303}{75} = \Delta H^\circ = 28.37 \text{ kJ mol}^{-1}$$

$$\Delta G^\circ_{T_1} = -RT_1\ln(K_1) = -298R\ln(10) = -5.71 \text{ kJ mol}^{-1}$$

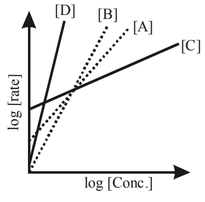
$$\Delta G^\circ_{T_2} = -RT_2\ln(K_2) = -373R\ln(100)$$

$= -14.283$ kJ/mol

7. Consider the following reactions

$A \rightarrow P1;\ B \rightarrow P2;\ C \rightarrow P3;\ D \rightarrow P4,$

The order of the above reactions are a,b,c and d, respectively. The following graph is obtained when log[rate] vs. log[conc.] are plotted :



Among the following the correct sequence for the order of the reactions is :

(1) c > a > b > d (2) d > a > b > c

(3) d > b > a > c (4) a > b > c > d

Sol. **3**

$A \rightarrow P1$ $B \rightarrow P2$ $C \rightarrow P3$ $D \rightarrow P4$

Rate = K (conc.)$^{\text{order}}$

log(rate) = log(K) + order log (case)

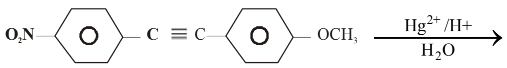
$$\underbrace{y \qquad\quad c \quad + \; m.x}_{\text{Straight line}}$$

Slope = order

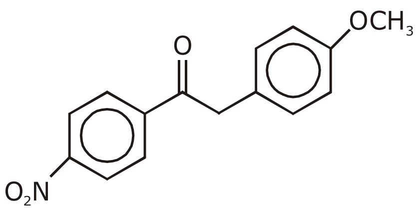
According graph

d > b > a > c order of slope

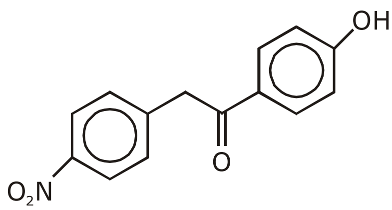
8. The major product obtained from the following reactions is :



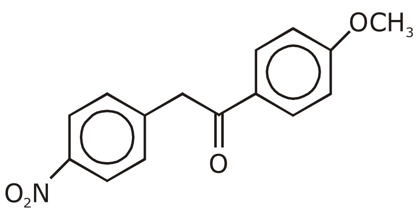
(1)



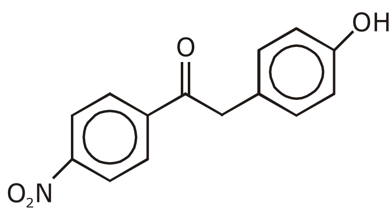
(2)



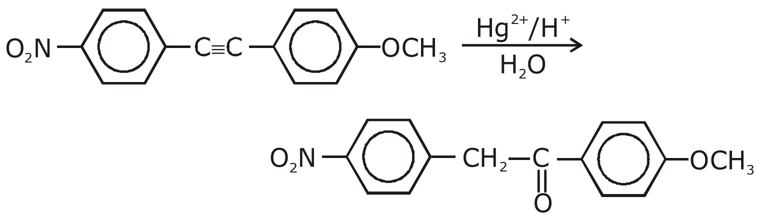
(3)



(4)



Sol. **3**

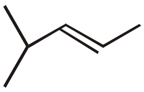


9. Which of the following compounds shows geometrical isomerism ?

(1) 2-methylpent-1-ene (2) 4-methylpent-2-ene

(3) 2-methylpent-2-ene (4) 4-methylpent-1-ene

Sol. **2**



4-Methylpent-2-ene

Can show G.I.

10. The lanthanoid that does NOT shows +4 oxidation state is :

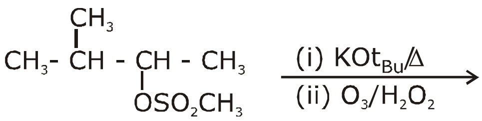
(1) Dy (2) Ce

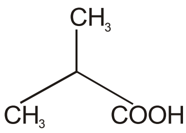
(3) Tb (4) Eu

Sol. **4**

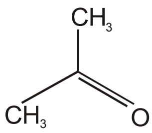
Fact

11. The major products of the following reactions are :

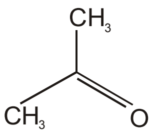


(1) 

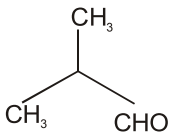
+ HCOOH

(2) 

+ CH_3CHO

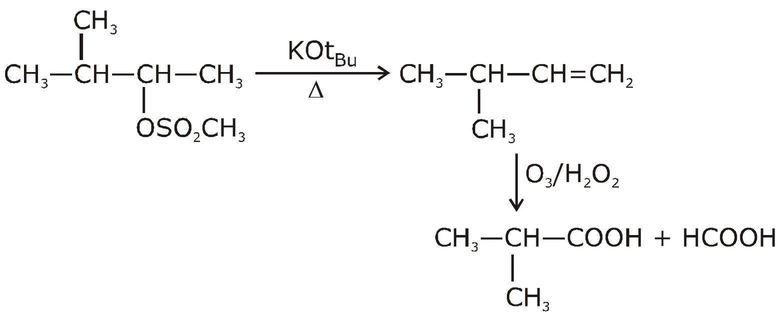
(3) 

+ CH_3COOH

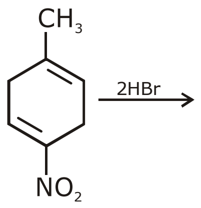
(4) 

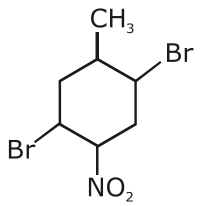
+ HCHO

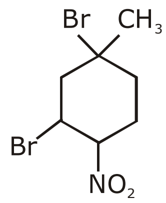
Sol. **1**

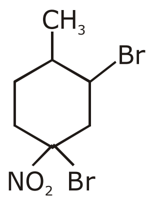


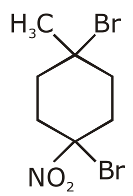
12. The major product of the following reaction is :



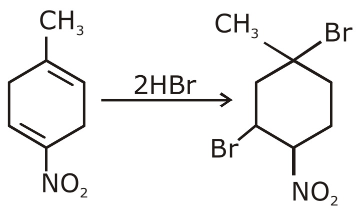
(1) 

(2) 

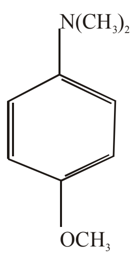
(3) 

(4) 

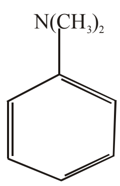
Sol. **2**



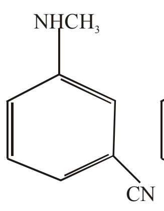
13. The increasing order of pK_b values of the following compounds is :



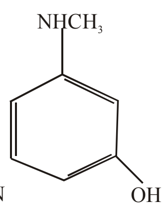
(I)



(II)



(III)



(IV)

(1) I < II < III < IV

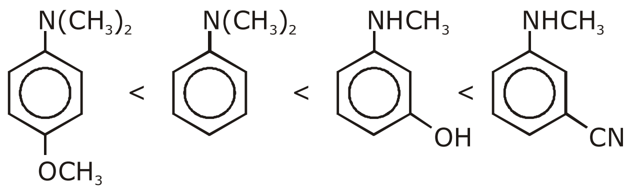
(2) II < IV < III < I

(3) I < II < IV < III

(4) II < I < III < IV

Sol. **3**

Order of pK_b



14. kraft temperature is the temperature :

(1) Above which the aqueous solution of detergents starts boiling

(2) Below which the formation of micelles takes place.

(3) Above which the formation of micelles takes place.

(4) Below which the aqueous solution of detergents starts freezing.

Sol. **3**

T_K + temp. above which formation of micelles takes place.

15. The set that contains atomic numbers of only transition elements, is ?

(1) 9, 17, 34, 38 (2) 21, 25, 42, 72

(3) 37, 42, 50, 64 (4) 21, 32, 53, 64

Sol. **2**

Tranition elements = 21 to 30
37 to 48
57 & 72 to 80

Ans. 21, 25, 42 & 72

16. Consider the Assertion and Reason given below.

Assertion (A) : Ethene polymerized in the presence of Ziegler Natta Catalyst at high temperature and pressure is used to make buckets and dustbins.

Reason (R) : High density polymers are closely packed and are chemically inert.

Choose the correct answer from the following :

(1) (A) and (R) both are wrong.

(2) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(3) (A) is correct but (R) is wrong

(4) Both (A) and (R) are correct but (R) is not the correct explanation of (A).

Sol. **2**

From ziegler - Natta catalyst HDPE is produced, HDPE is closely packed and are chemically inert, so used to make backet and dustbin.

17. A solution of two components containing n_1 moles of the 1^{st} component and n_2 moles of the 2^{nd} component is prepared. M_1 and M_2 are the molecular weights of component 1 and 2 respectively. If d is the density of the solution in g mL^{-1}, C_2 is the molarity and x_2 is the mole fraction of the 2^{nd} component, then C_2 can be expressed as :

(1) $C_2 = \dfrac{dx_1}{M_2 + x_2(M_2 - M_1)}$

(2) $C_2 = \dfrac{1000x_2}{M_1 + x_2(M_2 - M_1)}$

(3) $C_2 = \dfrac{dx_2}{M_2 + x_2(M_2 - M_1)}$

(4) $C_2 = \dfrac{1000dx_2}{M_1 + x_2(M_2 - M_1)}$

Sol. **4**

$$C_2 = \frac{x_2}{[x_2M_1 + (1 - x_2)M_2]/d} \times 1000$$

$$C_2 = \frac{1000\,dx_2}{M_1 + (M_2 - M_1)x_2}$$

18. The correct statement with respect to dinitrogen is ?

(1) Liquid dinitrogen is not used in cryosurgery.

(2) N_2 is paramagnetic in nature

(3) It can combine with dioxygen at 25°C

(4) It can be used as an inert diluent for reactive chemicals.

Sol. **4**

(1) Liquid nitrogen is used as a refrigerant to preserve biological material food items and in cryosurgery.

(2) N_2 is diamagnetic, with no unpaired elctrons.

(3) N_2 does not combine with oxygen, hydrogen or most other elements. Nitrogen will combine with oxygen, however ; in the presence of lightining or a spark.

(4) In iron and chemical Industry inert diluent for reactive chemicals.

19. Among the sulphates of alkaline earth metals, the solubilities of $BeSO_4$ and $MgSO_4$ in water, respectively, are :

(1) Poor and high

(2) High and high

(3) Poor and poor

(4) High and poor

Sol. **2**

Order of solubility of sulphate of Alkaline earth metals

$BeSO_4 > MgSO_4 > CaSO_4 > SrSO_4 > BaSO_4$

20. The presence of soluble fluoride ion upto 1ppm concentration in drinking water, is :

(1) Harmful to skin (2) Harmful to bones

(3) Safe for teeth (4) Harmful for teeth

Sol. **3**

Environmental chemistry - safe for teeth

21. A spherical balloon of radius 3cm containing helium gas has a pressure of 48×10^{-3} bar. At the same temperature, the pressure, of a spherical balloon of radius 12cm containing the same amount of gas will be........................$\times 10^{-6}$ bar.

Sol. **750**

$$\text{moles} = \frac{48 \times 10^{-3} \times \frac{4}{3\pi}(3\text{cm})^3}{R \times T}$$

$$\text{moles} = \frac{P \times \frac{4}{3\pi}(12\text{cm})^3}{R\,T}$$

$P \times 144 \times 12 = 48 \times 9 \times 3 \times 10^{-3}$

$$P = \frac{27}{36} \times 10^{-3}$$

$$P = \frac{27000}{36} \times 10^{-6}$$

$$P = \frac{3000}{4} \times 10^{-6}$$

$P = 750 \times 10^{-6}$ bar

22. The elevation of boiling point of 0.10m aqueous $CrCl_3 x NH_3$ solution is two times that of 0.05 m aqueous $CaCl_2$ solution. The value of x is................ .

[Assume 100% ionisation of the complex and $CaCl_2$, coordination number of Cr as 6, and that all NH_3 molecules are present inside the coordination sphere]

Sol. **5**

$\Delta T_b = i \times K_b \times m$

$i \times 0.1 \times K_b = 3 \times 0.05 \times K_b \times 2$

$i = 3$

$[Cr(NH_3)_5.\ Cl]\ Cl_2 \rightarrow [Cr(NH_3)_5Cl]^{+2} + 2Cl^-$

$x = 5$

23. Potassium chlorate is prepared by the electrolysis of KCl in basic solution

$6OH^- + Cl^- \longrightarrow ClO_3^- + 3H_2O + 6e^-$

If only 60% of the current is utilized in the reaction, the time (rounded to the nearest hour) required to produce 10g of $KClO_3$ using a current of 2A is ....

(Given : F = 96,500 C mol^{-1}; molar mass of $KClO_3$=122g mol^{-1})

Sol. **11**

$$\frac{10}{122}\times 6 = \frac{2\times t(hr)\times 3600\times 60\%}{96500}$$

$$t(hr) = \frac{96500}{122\times 72} = 10.98\,hr$$

= 11 hours

24. In an estimation of bromine by Carius method, 1.6 g of an organic compound gave 1.88 g of AgBr. The mass percentage of bromine in the compound is ....(Atomic mass, Ag=108, Br=80 g mol^{-1})

Sol. **50 %**

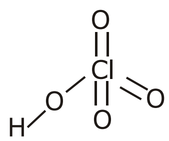
Carius method

$$\% \text{ of Br} = \frac{\text{wt of AgBr}}{\text{wt. of organic compound}} \times 100 \times \frac{\text{molar mass of Br}}{\text{AgBr}}$$

$$= \frac{1.88}{1.6}\times\frac{80}{188} \times 100 = \frac{15040}{300.8} = 50\%$$

25. The number of Cl = O bonds in perchloric acid is, "..............."

Sol. **3**



QUESTION PAPER WITH SOLUTION

MATHEMATICS _ 6 Sep. _ SHIFT - 1

Q.1 The region represented by $\{z = x + iy \in C : |z| - Re(z) \leq 1\}$ is also given by the inequality:
$\{z = x + iy \in C : |z| - Re(z) \leq 1\}$

(1) $y^2 \leq 2\left(x + \frac{1}{2}\right)$ (2) $y^2 \leq x + \frac{1}{2}$ (3) $y^2 \geq 2(x + 1)$ (4) $y^2 \geq x + 1$

Sol. **1**

$\{z = x + iy \in c : |z| - Re(z) \leq 1\}$

$|z| = \sqrt{x^2 + y^2}$

$Rc(z) = x$

$|z| - Re(z) \leq 1$

$\Rightarrow \sqrt{x^2 + y^2} - x \leq 1$

$\Rightarrow \sqrt{x^2 + y^2} \leq 1 + x$

$\Rightarrow x^2 + y^2 \leq 1 + x^2 + 2x$

$\Rightarrow y^2 \leq 2\left(x + \frac{1}{2}\right)$

Q.2 The negation of the Boolean expression $p \vee (\sim p \wedge q)$ is equivalent to:

(1) $p \wedge \sim q$ (2) $\sim p \vee \sim q$ (3) $\sim p \vee q$ (4) $\sim p \wedge \sim q$

Sol. **4**

$p \vee (\sim p \wedge q)$

$(p \wedge \sim p) \wedge (p \vee q)$

$t \wedge (p \vee q)$

$p \vee q$

$\sim (p \vee (\sim p \wedge q)) = \sim (P \vee q)$

$= (\sim P) \wedge (\sim q)$

Q.3 The general solution of the differential equation $\sqrt{1 + x^2 + y^2 + x^2y^2} + xy\frac{dy}{dx} = 0$ is:

(where C is a constant of integration)

(1) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + C$

(2) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + C$

(3) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$

(4) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$

Sol. **3**

$$\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$$

$$\sqrt{(1+x^2)(1+y^2)} + xy\frac{dy}{dx} = 0$$

$$\frac{\sqrt{(1+x^2)}dx}{x} = -\frac{y}{\sqrt{1+y^2}}dy$$

Integrate the equation

$$\int \frac{\sqrt{1+x^2}}{x}dx = -\int \frac{y}{\sqrt{1+y^2}}dy$$

$1 + x^2 = t^2$ $\qquad$ $1 + y^2 = z^2$

$2xdx = 2tdt$

$dx = \frac{t}{x}dt$ $\qquad$ $2ydy = 2zdz$

$$\int \frac{t.tdt}{t^2-1} = -\int \frac{zdx}{z}$$

$$\int \frac{t^2-1+1}{t^2-1}dt = -z + c$$

$$\int 1dt + \int \frac{1}{t^2-1}dt = -z + c$$

$$t + \frac{1}{2}\ln\left(\frac{t-1}{t+1}\right) = -z + c$$

$$\sqrt{1+x^2} + \frac{1}{2}\ln\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) = -\sqrt{1+y^2} + c$$

$$\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\ln\left(\frac{\sqrt{x^2+1}+1}{\sqrt{x^2+1}-1}\right) + c$$

Q.4 Let L_1 be a tangent to the parabola $y^2 = 4(x + 1)$ and L_2 be a tangent to the parabola $y^2 = 8(x + 2)$ such that L_1 and L_2 intersect at right angles. Then L_1 and L_2 meet on the straight line:

(1) $x + 2y = 0$ (2) $x + 2 = 0$ (3) $2x + 1 = 0$ (4) $x + 3 = 0$

Sol. **4**

Let tangent of $y^2 = 4(x + 1)$

$L_1 : t_1y = (x + 1) + t_1^2$ ....(i)

and tangent of $y^2 = 8 (x + 2)$

$L_2 : t_2y = (x + 2) + 2 t_2^2$

$L_1 \perp L_2$

$$\frac{1}{t_1}\cdot\frac{1}{t_2} = -1$$

$t_1t_2 = -1$

t_2(i) − t_1 (ii)

$$\begin{array}{l} t_1t_2y = t_2\,(x + 1) + t_2.\ t_1^2 \\ t_1t_2y = t_1\,(x + 2) + 2t_2^2.\ t_1 \\ - \qquad - \qquad - \\ \hline \end{array}$$

$(t_2-t_1)\,x + (t_2-2t_1) + t_2t_1(t_1-2t_2) = 0$

$(t_2 - t_1)\,x + (t_2 - 2t_1) - (t_1 - 2t_2) = 0$

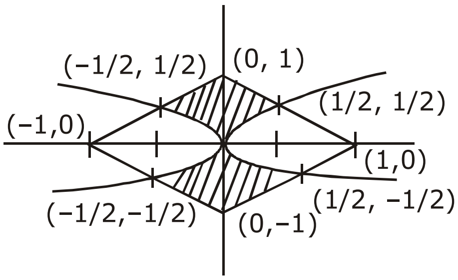
$(t_2 - t_1)\,x + 3t_2 - 3t_1 = 0$

$\Rightarrow x + 3 = 0$

Q.5 The area (in sq. units) of the region $A = \{(x, y): |x| + |y| \le 1,\ 2y^2 \ge |x|\}$

(1) $\frac{1}{6}$ (2) $\frac{5}{6}$ (3) $\frac{1}{3}$ (4) $\frac{7}{6}$

Sol. **2**



Total area = $4\int_0^{1/2}\left[(1-x)-\left(\sqrt{\frac{x}{2}}\right)\right]dx$

$$= 4\left[x - \frac{x^2}{2} - \frac{1}{\sqrt{2}}\frac{x^{3/2}}{3/2}\right|_0^{1/2}\Bigg]$$

$$= 4\left[\frac{1}{2} - \frac{1}{8} - \frac{\sqrt{2}}{3}\left(\frac{1}{2}\right)^{3/2}\right]$$

$$= 4 \times \frac{5}{24} = \frac{5}{6}$$

Q.6 The shortest distance between the lines $\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$ and $x + y + z + 1 = 0$, $2x - y + z + 3 = 0$ is:

(1) 1 (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{1}{2}$

Sol. **3**

Plane through line of intersection is

$x + y + z + 1 + \lambda(2x - y + z + 3) = 0$

It should be parallel to given line

$0(1 + 2\lambda) - 1(1 - \lambda) + 1(1 + \lambda) = 0 \Rightarrow \lambda = 0$

Plane: $x + y + z + 1 = 0$

Shortest distance of (1, −1, 0) from this plane

$= \frac{|1 - 1 + 0 + 1|}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}$

Q.7 Let a, b, c, d and p be any non zero distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$. Then:

(1) a, c, p are in G.P. (2) a, b, c, d are in G.P.

(3) a, b, c, d are in A.P. (4) a, c, p are in A.P.

Sol. **2**

$(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + b^2 + c^2 + d^2) = 0$

$(a^2p^2 - 2abp + b^2] + [b^2p^2 - 2bcp + c^2] + [c^2p^2 - 2cdp + d^2]$

$(ap - b)^2 + (bp - c)^2 + (cp - d)^2 = 0$

$ap = b$ $\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = p$

$bp = c$

$cp = d$ a, b, c, d are in G.P.

Q.8 Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated?

(1) 2! 3! 4! (2) $(3!)^3 \cdot (4!)$ (3) $3!\ (4!)^3$ (4) $(3!)^2 \cdot (4!)$

Sol. **2**

$F_1 \to$ 3 members

$F_2 \to$ 3 members

$F_3 \to$ 4 members

No. of ways can they be seated so that the same family members are not separated

$= 3! \times 3! \times 3! \times 4! = (3!)^3 .\ 4!$

Q.9 The values of λ and μ for which the system of linear equations

$x + y + z = 2$

$x + 2y + 3z = 5$

$x + 3y + \lambda z = \mu$

has infinitely many solutions are, respectively:

(1) 6 and 8 (2) 5 and 8 (3) 5 and 7 (4) 4 and 9

Sol. **2**

$x + y + z = 2$

$x + 2y + 3z = 5$

$x + 3y + \lambda z = \mu$

has infinitely many solutions

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & \lambda - 1 \end{vmatrix} = 0$$

$(\lambda -1-4) = 0$

$\Rightarrow \lambda = 5$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 1 & 3 & \mu \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\begin{vmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 2 & \mu - 2 \end{vmatrix} = 0$$

$(\mu - 2)-6) = 0$

$\Rightarrow \mu = 8$

$\lambda = 5, \mu = 8$

Q.10 Let m and M be respectively the minimum and maximum values of

$$\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$$

Then the ordered pair (m, M) is equal to:

(1) (–3, –1) (2) (–4, –1) (3) (1, 3) (4) (–3, 3)

Sol. **1**

$$\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$$

$R_1 \to R_1 - R_2$, $R_3 \to R_3 - R_2$

$$\begin{vmatrix} -1 & 1 & 0 \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ -1 & 0 & 1 \end{vmatrix}$$

$\Rightarrow -1(\sin^2 x) - 1(1 + \cos^2 x + \sin 2x)$

$\Rightarrow -\sin^2 x - \cos^2 x - 1 - \sin 2x$

$= -2 - \sin 2x$

$\therefore$ minimum value when $\sin 2x = 1$

$m = -2-1 = -3$

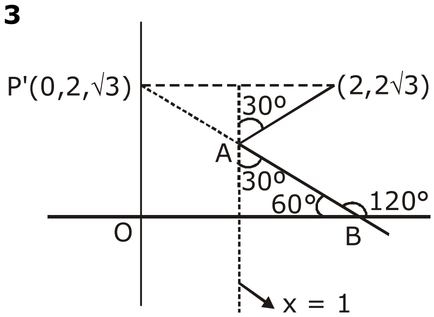
$\therefore$ Maximum value when $\sin 2x = -1$

$(m, M) = (-3, -1)$

Q.11 A ray of light coming from the point $(2, 2\sqrt{3})$ is incident at an angle 30° on the line x = 1 at the point A. The ray gets reflected on the line x = 1 and meets x-axis at the point B. Then, the line AB passes through the point:

(1) $(4, -\sqrt{3})$ (2) $\left(3, -\frac{1}{\sqrt{3}}\right)$ (3) $(3, -\sqrt{3})$ (4) $\left(4, -\frac{\sqrt{3}}{2}\right)$

Sol. **3**



Equation of P'B $\rightarrow$ y $-2\sqrt{3}$ = tan 120° (x – 0)

$\sqrt{3}$ x + y = 2$\sqrt{3}$

(3, $-\sqrt{3}$) satisfy the line

Q.12 Out of 11 consecutive natural numbers if three numbers are selected at random (without repetition), then the probability that they are in A.P. with positive common difference, is:

(1) $\frac{10}{99}$ (2) $\frac{5}{33}$ (3) $\frac{15}{101}$ (4) $\frac{5}{101}$

Sol. **2**

Case-1

E, O, E, O, E, O, E, O, E, O, E

2b = a + c $\rightarrow$ Even

$\Rightarrow$ Both a and c should be either even or odd.

$$P = \frac{{}^6C_2 + {}^5C_2}{{}^{11}C_3} = \frac{5}{33}$$

Case -2

O, E, O, E, O, E, O, E, O, E, O

$$P = \frac{{}^5C_2 + {}^6C_2}{{}^{11}C_3} = \frac{5}{33}$$

Total probability = $\frac{1}{2} \times \frac{5}{33} + \frac{1}{2} \times \frac{5}{33} = \frac{5}{33}$

Q.13 If f(x + y) = f(x) f(y) and $\sum_{x=1}^{\infty} f(x) = 2$, x, y$\in$N, where N is the set of all natural number, then the value of $\frac{f(4)}{f(2)}$ is :

(1) $\frac{2}{3}$ (2) $\frac{1}{9}$ (3) $\frac{1}{3}$ (4) $\frac{4}{9}$

Sol. **4**

f(x + y) = f(x) f(y)

* Put x = 1, y = 1

$f(2) = (f(1))^2$

* Put x = 2, y = 1

$f(3) = f(2).\ f(1) = f((1))^3$

* Put x = 2, y = 2

$f(4) = f((2))^2 = f((1))^4$

$f(n) = (f(1))^n$

$$\sum_{x=1}^{\infty} f(x) = f(1) + f(2) + f(3) + ...f(\infty) = 2$$

$\Rightarrow$ f(1) + f((1))^2 + f((1))^3.......= 2

$$\frac{f(1)}{1-f(1)} = 2$$

f(1) = 2/3

$$f(2) = \left(\frac{2}{3}\right)^2, \; f(4) = \left(\frac{2}{3}\right)^4$$

$$\frac{f(4)}{f(2)} = \frac{(2/3)^4}{(2/3)^2} = \frac{4}{9}$$

Q.14 If {p} denotes the fractional part of the number p, then $\left\{\frac{3^{200}}{8}\right\}$, is equal to :

(1) $\frac{5}{8}$ (2) $\frac{1}{8}$ (3) $\frac{7}{8}$ (4) $\frac{3}{8}$

Sol. **2**

$$\left\{\frac{3^{200}}{8}\right\} = \left\{\frac{9^{100}}{8}\right\} = \left\{\frac{(8+1)^{100}}{8}\right\}$$

$$\left\{\frac{{}^{100}C_0 1^{100} + {}^{100}C_1 (8) 1^{99} + {}^{100}C_2 (8^2) 1^{98} + \ldots + {}^{100}C_{100} 8^{100}}{8}\right\}$$

$$= \left\{\frac{{}^{100}C_0 1^{100} + 8k}{8}\right\}$$

$$= \left\{\frac{1+8k}{8}\right\} = \left\{\frac{1}{8} + k\right\} K \in I$$

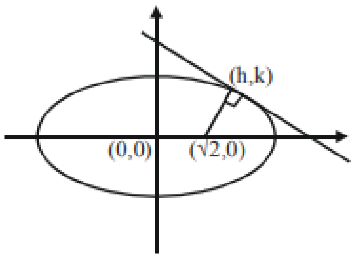
$$= \frac{1}{8}$$

Q.15 Which of the following points lies on the locus of the foot of perpedicular drawn upon any tangent to the ellipse, $\frac{x^2}{4} + \frac{y^2}{2} = 1$ from any of its foci ?

(1) $(-1, \sqrt{3})$ (2) $(-2, \sqrt{3})$ (3) $(-1, \sqrt{2})$ (4) (1, 2)

Sol. **4**

Let foot of perpendicular is (h,k)



$\frac{x^2}{4}+\frac{y^2}{2}=1$ (Given $)$

$a=2, b=\sqrt{2}, e=\sqrt{1-\frac{2}{4}}=\frac{1}{\sqrt{2}}$

$\therefore$ Focus $(ae, 0)=(\sqrt{2}, 0)$

Equation of tangent

$y=mx+\sqrt{a^2m^2+b^2}$

$y=mx+\sqrt{4m^2+2}$

Passes throguh (h,k) $(k-mh)^2=4m^2+2$

line perpendicular to tangent will have slope

$-\frac{1}{m}$

$y-0=-\frac{1}{m}(x-\sqrt{2})$

$my=-x+\sqrt{2}$

$(h+mk)^2=2$

Add equaiton (1) and (2) $k^2\left(1+m^2\right)+h^2\left(1+m^2\right)=4\left(1+m^2\right)$

$h^2+k^2=4$

$x^2+y^2=4$ (Auxilary circle)

$\therefore (-1, \sqrt{3})$ lies on the locus.

Q.16 $\lim\limits_{x \to 1} \left(\dfrac{\int_0^{(x-1)^2} t\cos(t^2)dt}{(x-1)\sin(x-1)} \right)$

(1) is equal to 1 (2) is equal to $\frac{1}{2}$ (3) does not xist (4) is equal to $-\frac{1}{2}$

Sol **Bouns**

$\lim\limits_{x \to 1} \left(\dfrac{\int_0^{(x-1)^2} t\cos(t^2)dt}{(x-1)\sin(x-1)} \right)$

Using L-Hopital rule

$$= \lim_{x \to 1} \frac{2(x-1) \cdot (x-1)^2 \cos(x-1)^4 - 0}{(x-1) \cdot \cos(x-1) + \sin(x-1)} \left(\frac{0}{0} \right)$$

$$= \lim_{x \to 1} \frac{2(x-1)^3 \cdot \cos(x-1)^4}{(x-1)\left[\cos(x-1) + \dfrac{\sin(x-1)}{(x-1)}\right]}$$

$$= \lim_{x \to 1} \frac{2(x-1)^2 \cos(x-1)^4}{(x-1)\left[\cos(x-1) + \dfrac{\sin(x-1)}{(x-1)}\right]}$$

$$= \lim_{x \to 1} \frac{2(x-1)^2 \cos(x-1)^4}{\cos(x-1) + \dfrac{\sin(x-1)}{(x-1)}}$$

on taking limit

$$= \frac{0}{1+1} = 0$$

Q.17 If $\sum_{i=1}^{n}(x_i - a) = n$ and $\sum_{i=1}^{n}(x_i - a)^2 = na$, (n, a > 1) then the standard deviation of n observations $x_1, x_2, \ldots, x_n$ is :

(1) $n\sqrt{a-1}$ (2) $\sqrt{na-1}$ (3) $a - 1$ (4) $\sqrt{a-1}$

Sol. **4**

S.D. = $\sqrt{\frac{\Sigma(x_i - a)^2}{n} - \left(\frac{\Sigma(x_i - a)}{n}\right)^2}$

$= \sqrt{\left(\frac{na}{n}\right) - \left(\frac{n}{n}\right)^2} = \sqrt{a-1}$

Q.18 If α and β be two roots of the equation $x^2 - 64x + 256 = 0$. Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$ is :

(1) 1 (2) 3 (3) 2 (4) 4

Sol. **3**

$x^2 - 64x + 256 = 0$

$\alpha + \beta = 64$

$\alpha\beta = 256$

$$\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$$

$$= \frac{\alpha + \beta}{(\alpha\beta)^{5/8}} = \frac{64}{(256)^{5/8}} = \frac{64}{32} = 2$$

Q.19 The position of a moving car at time t is given by $f(t) = at^2 + bt + c$, $t > 0$, where a, b and c are real numbers greater than 1. Then the average speed of the car over the time interval $[t_1, t_2]$ is attained at the point :

(1) $(t_1 + t_2)/2$ (2) $2a(t_1 + t_2) + b$ (3) $(t_2 - t_1)/2$ (4) $a(t_2 - t_1) + b$

Sol. **1**

$$f'(t) = V_{av} = \frac{f(t_2) - f(t_1)}{t_2 - t_1}$$

$$= \frac{a(t_2^2 - t_1^2) + b(t_2 - t_1)}{t_2 - t_1}$$

$= a(t_1 + t_2) + b = 2at + b$

$$t = \frac{t_1 + t_2}{2}$$

Q.20 If $I_1 = \int_0^1 (1-x^{50})^{100}\,dx$ and $I_2 = \int_0^1 (1-x^{50})^{101}\,dx$ such that $I_2 = \alpha I_1$ then α equals to :

(1) $\frac{5050}{5049}$ (2) $\frac{5050}{5051}$ (3) $\frac{5051}{5050}$ (4) $\frac{5049}{5050}$

Sol. **2**

$I_1 = \int_0^1 (1-x^{50})^{100}\,dx$

$I_2 = \int_0^1 (1-x^{50})(1-x^{50})^{100}dx$

$= \int_0^1 (1-x^{50})^{100}\,dx - \int_0^1 x^{50}(1-x^{50})^{100}\,dx$

$I_2 = I_1 - \int_0^1 \underset{\mathrm{I}}{x} - \underbrace{x^{49}(1-x^{50})^{100}}_{\mathrm{II}}dx$

By using by parts

$1 - x^{50} = t$

$\Rightarrow x^{49}\,dx = \frac{-dt}{50}$

$I_2 = I_1 - \left[x\left(\frac{-1}{50}\right)\frac{(1-x^{50})^{101}}{101}\right]_0^1 + \int_0^1 \left(\frac{-1}{50}\right)\frac{(1-x^{50})^{101}}{101}$

$I_2 = I_1 - 0 + \frac{\int_0^1 (1-X^{50})^{101}}{(-5050)}\,dx$

$I_2 = I_1 - \frac{I_2}{5050}$

$\frac{5051}{5050} I_2 = I_1$

$I_2 = \frac{5050}{5051} I_1$

$\alpha = \frac{5050}{5051}$

Q.21 If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value of $\sqrt{3}\,\left|\vec{a}+\vec{b}\right| + \left|\vec{a}-\vec{b}\right|$ is_____.

Sol. **4**

$\sqrt{3}\,|\vec{a}+\vec{b}\,| + |\vec{a}-\vec{b}\,|$

$= \sqrt{3}(\sqrt{2+2\cos\theta}) + \sqrt{2-2\cos\theta}$

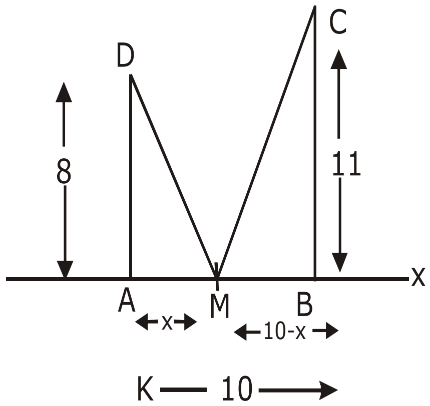
$= \sqrt{6}(\sqrt{1+\cos\theta}) + \sqrt{2}(\sqrt{1-\cos\theta})$

$$= 2\sqrt{3}\left|\cos\frac{\theta}{2}\right| + 2\left|\sin\frac{\theta}{2}\right|$$

$$\le \sqrt{(2\sqrt{3})^2 + (2)^2} = 4$$

Q.22 Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If AD = 8 m, BC = 11 m and AB = 10 m; then the distance (in meters) of a point M on AB from the point A such that $MD^2 + MC^2$ is minimum is ______.

Sol. **5**



$(MD)^2 = x^2 + 8^2 = x^2 + 64$
$(MC)^2 = (10-x)^2 + (11)^2 = (x-10)^2 + 121$
$f(x) = (MD)^2 + (MC)^2 = x^2 + 64 + (x-10)^2 + (2)$
Differentiate
$f'(x) = 0$
$2x + 2(x-10) = 0$
$4x = 20 \Rightarrow x = 5$
$f''(x) = 4 > 0$
at x = 5 point of minima

Q.23 Let $f : R \to R$ be defined as

$$f(x) = \begin{cases} x^5 \sin\left(\frac{1}{x}\right) + 5x^2, & x < 0 \\ 0, & x = 0 \\ x^5 \cos\left(\frac{1}{x}\right) + \lambda x^2, & x > 0 \end{cases}$$

The value of λ for which $f''(0)$ exists, is _______.

Sol. **5**

$$f(x) = \begin{cases} x^5 \sin\left(\frac{1}{x}\right) + 5x^2, & x < 0 \\ 0, & x = 0 \\ x^5 \cos\left(\frac{1}{x}\right) + \lambda x^2, & x > 0 \end{cases}$$

$$f'(x) \begin{cases} 5x^4 \sin\left(\frac{1}{x}\right) - x^3 \cos\left(\frac{1}{x}\right) + 10x, & x < 0 \\ 0, & x = 0 \\ 5x^4 \cos\left(\frac{1}{x}\right) + x^3 \sin\left(\frac{1}{x}\right) + 2\lambda x, & x > 0 \end{cases}$$

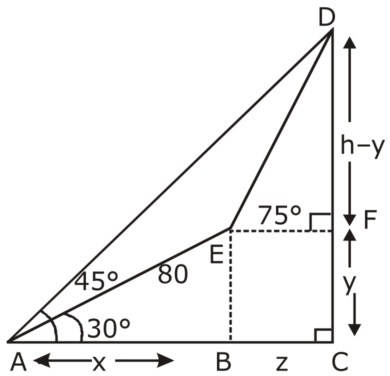
$$f''(x) = \begin{cases} 20x^3 \sin\left(\frac{1}{x}\right) - 5x^2 \cos\left(\frac{1}{x}\right) - 3x^2 \cos\left(\frac{1}{x}\right) - x\sin\left(\frac{1}{x}\right) + 10, & x < 0 \\ 0, & x = 0 \\ 20x^3 \cos\left(\frac{1}{x}\right) + 5x^2 \sin\left(\frac{1}{x}\right) + 3x^2 \sin\left(\frac{1}{x}\right) - x\cos\left(\frac{1}{x}\right) + 2\lambda & \\ , x > 0 \end{cases}$$

$f''(0^+) = f''(0^-)$
$2\lambda = 10 \Rightarrow \lambda = 5$

Q.24 The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45°. After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30° to the horizontal plane, the angle of elevation of the top of the hill becomes 75°. Then the height of the hill (in meters) is _______.

Sol. **80**

x =



$x = 80 \cos 30° = 40\sqrt{3}$
$y = 80 \sin 30° = 40$
In ΔADC

$\tan 45° = \dfrac{h}{x+z} \Rightarrow h = x + z$

$\Rightarrow h = 40\sqrt{3} + z$ ....(i)
In ΔEDF

$\tan 75° \dfrac{h-y}{z}$

$2 + \sqrt{3} = \dfrac{h-40}{z} \Rightarrow z = \dfrac{h-40}{2+\sqrt{3}}$ .....(ii)
Put the value of z from (i)

$h - 40\sqrt{3} = \dfrac{h-40}{2+\sqrt{3}}$

$h(1 + \sqrt{3}) = 40(2\sqrt{3} + 3 - 1)$

$h(1 + \sqrt{3}) = 80(1 + \sqrt{3})$
$h = 80$

Q.25 Set A has m elements and set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of m.n is ______.
Sol. **28**
A & B are set
No. of subset of A = 2^m
No. of subset of B = 2^n
$2^m = 2^n + 112$
$2^m - 2^n = 112$
$2^n(2^{m-n} - 1) = 112$
$2^n(2^{m-n} - 1) = 2^4(2^3 - 1)$
$n = 4$ $\quad\quad$ $m - n = 3$
$m - 4 = 3 \Rightarrow m = 7$
$m.n = 28$