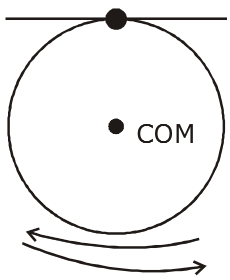
QUESTION PAPER WITH SOLUTION

PHYSICS _ 5 Sep. _ SHIFT - 2

1. A ring is hung on a nail. It can oscillate, without slipping or sliding (i) in its plane with a time period T_1 and, (ii) back and forth in a direction perpendicular to its plane, with a period T_2. The ratio $\frac{T_1}{T_2}$ will be:

(1) $\frac{3}{\sqrt{2}}$ (2) $\frac{\sqrt{2}}{3}$ (3) $\frac{2}{\sqrt{3}}$ (4) $\frac{2}{3}$

SOl. **3**



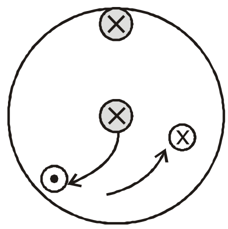
$$T_1 = 2\pi\sqrt{\frac{(mR^2 + mR^2)}{mgR}}$$

$$T_1 = 2\pi\sqrt{\frac{2R}{g}}$$

$$T_2 = 2\pi\sqrt{\frac{I}{mgL_{cm}}}$$

$$T_2 = 2\pi\sqrt{\frac{3mR^2/2}{mgR}} = 2\pi\sqrt{\frac{3R}{2g}}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$$



Back and forth

2. The correct match between the entries in column I and column II are:

I Radiation	**II** Wavelength
(a) Microwave	(i) 100 m
(b) Gamma rays	(ii) 10^{-15} m
(c) A.M. radio waves	(iii) 10^{-10} m
(d) X-rays	(iv) 10^{-3} m

(1) (a) - (ii), (b)-(i), (c)-(iv), (d)-(iii) (2) (a)-(iii), (b)-(ii), (c)-(i), (d)-(iv)
(3) (a)-(iv), (b)-(ii), (c)-(i), (d)-(iii) (4) (a)-(i),(b)-(iii), (c)-(iv), (d)-(ii)

Sol. **3**
By theory

3. In an experiment to verify Stokes law, a small spherical ball of radius r and density ρ falls under gravity through a distance h in air before entering a tank of water. If the terminal velocity of the ball inside water is same as its velocity just before entering the water surface, then the value of h is proportional to : (ignore viscosity of air)

(1) r^4 (2) r (3) r^3 (4) r^2

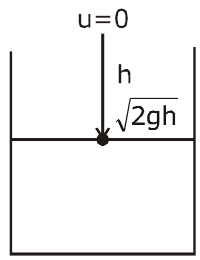
Sol. **1**

$V_T = \sqrt{2gh}$

$\frac{2}{9} r^2 \frac{(\rho_b - \rho_l)g}{\eta} = \sqrt{2gh}$

$r^2 \propto \sqrt{h} \Rightarrow r^4 \propto h$

$h \propto r^4$



4. Ten charges are placed on the circumference of a circle of radius R with constant angular separation between successive charges. Alternate charges 1, 3, 5, 7, 9 have charge (+q) each, while 2, 4, 6, 8, 10 have charge (–q) each. The potential V and the electric field E at the centre of the circle are respectively: (Take V= 0 at infinity)

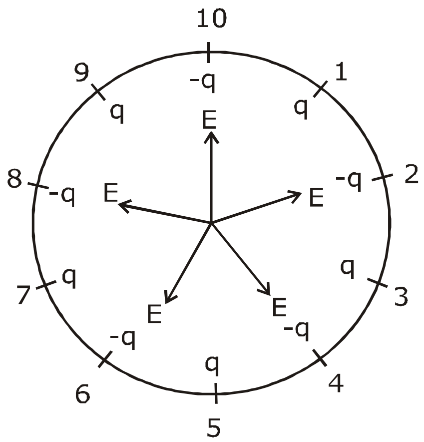
(1) V = 0; E = 0

(2) $V = \frac{10q}{4\pi\epsilon_0 R}; E = \frac{10q}{4\pi\epsilon_0 R^2}$

(3) $V = 0; E = \frac{10q}{4\pi\epsilon_0 R^2}$

(4) $V = \frac{10q}{4\pi\epsilon_0 R}; E = 0$

Sol. **1**



$v_{net} = 5\left(\frac{kq}{R}\right) + \left(\frac{5k(-q)}{R}\right)$

$v_{net} = 0$ [$Q_{net} = 0$]

$E_{net} = 0$ by symmetry

5. A spaceship in space sweeps stationary interplanetary dust. As a result, its mass increases at a rate $\frac{dM(t)}{dt}$ = bv^2 (t), where v(t) is its instantaneous velocity. The instantaneous acceleration of the satellite is :

(1) $-bv^3(t)$ (2) $-\frac{bv^3}{M(t)}$ (3) $-\frac{2bv^3}{M(t)}$ (4) $-\frac{bv^3}{2M(t)}$

Sol. **2**

$\frac{dM(t)}{dt} = -bv^2$

in free space
no external force
so there in only thrust force on rocket

$f_{in} = \frac{dM}{dt}(V_{rel})$

$Ma = \left(\frac{-bv^2}{(t)}\right)v$

$a = \frac{-bv^3}{M(t)}$

6. Two different wires having lengths L_1 and L_2, and respective temperature coefficient of linear expansion α_1 and α_2, are joined end-to-end. Then the effective temperature coefficient of linear expansion is:

(1) $\frac{\alpha_1 L_1 + \alpha_2 L_2}{L_1 + L_2}$ (2) $2\sqrt{\alpha_1\alpha_2}$

(3) $4\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\frac{L_2L_1}{(L_2+L_1)^2}$ (4) $\frac{\alpha_1+\alpha_2}{2}$

Sol. **1**

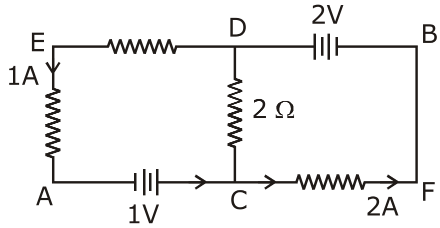
$L'_1 = L_1(1 + \alpha_1\Delta T)$
$L'_2 = L_2(1 + \alpha_2\Delta T)$
$L' + L_2' = L_1 + L_2 + L_1\alpha_1\Delta T + L_2\alpha_2\Delta T$

$= (L_1 + L_2)\left[1 + \left[\frac{L_1\alpha_1 + L_2\alpha_2}{L_1 + L_2}\right]\Delta T\right]$

$= (L_1 + L_2)[1 + \alpha_{eq}\Delta T]$

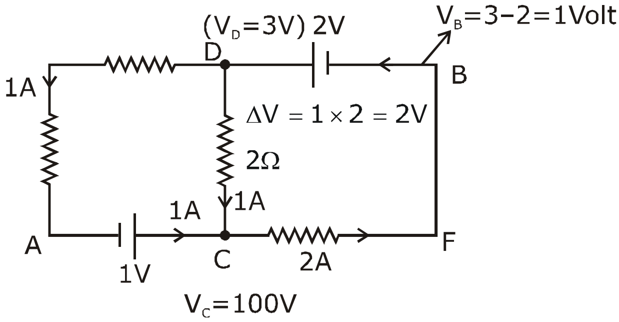
So, $\alpha_{eq} = \frac{L_1\alpha_1 + L_2\alpha_2}{L_1 + L_2}$

7. In the circuit, given in the figure currents in different branches and value of one resistor are shown. Then potential at point B with respect to the point A is:



(1) +2 V (2) −2 V (3) +1 V (4) −1 V

Sol. **3**

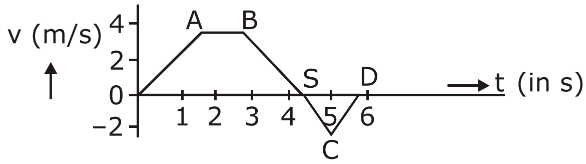


Let $V_A = 0$

$V_B - V_A = 1 - 0$

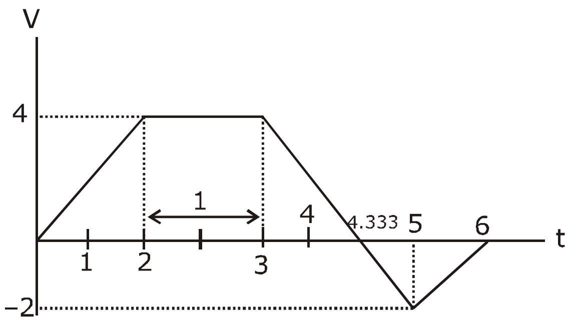
$= 1$ volt

8. The velocity (v) and time (t) graph of a body in a straight line motion is shown in the figure. The point S is at 4.333 seconds. The total distance covered by the body in 6 s is:



(1) $\frac{37}{3}$ m (2) $\frac{49}{4}$ m (3) 12 m (4) 11 m

Sol. **1**



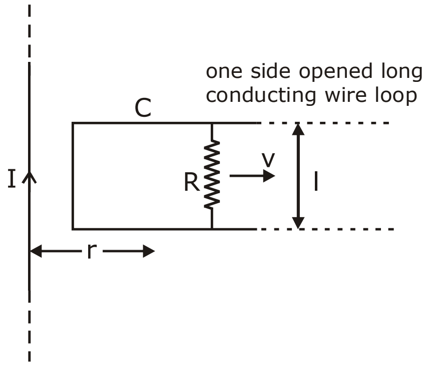
distance = area under graph

$= \frac{1}{2}(4)\left(\frac{13}{3}+1\right) + \left[\frac{1}{2}\left(6-\frac{13}{3}\right)\times 2\right]$

$= 2 \times \frac{16}{3} + \frac{5}{3}$

$= \frac{32}{3} + \frac{5}{3} = \frac{37}{3}$ m

9. An infinitely long straight wire carrying current I, one side opened rectangular loop and a conductor C with a sliding connector are located in the same plane, as shown in the figure. The connector has length l and resistance R. It slides to the right with a velocity v. The resistance of the conductor and the self inductance of the loop are negligible. The induced current in the loop, as a function of separation r, between the connector and the straight wire is:



(1) $\frac{\mu_0}{2\pi}\frac{Ivl}{Rr}$ (2) $\frac{\mu_0}{\pi}\frac{Ivl}{Rr}$ (3) $\frac{2\mu_0}{\pi}\frac{Ivl}{Rr}$ (4) $\frac{\mu_0}{4\pi}\frac{Ivl}{Rr}$

Sol. **1**

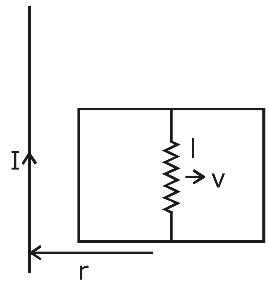
$B = \left(\frac{\mu_0 I}{2\pi r}\right)$

induced emf

e = Bvl

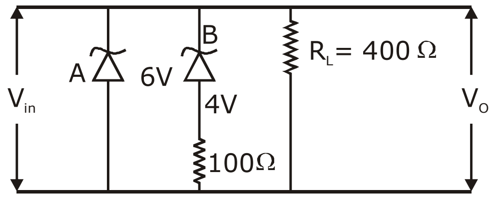
$= \frac{\mu_0 I}{2\pi r}$ V.l

$= \frac{\mu_0 I v l}{2\pi r}$

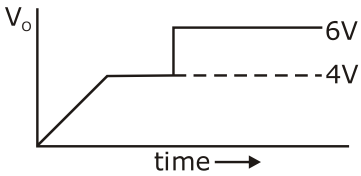


induced current $i = \frac{e}{R} = \frac{\mu_0 I v l}{2\pi r R}$

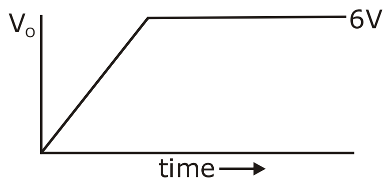
10. Two zener diodes (A and B) having breakdown voltages of 6 V and 4 V respectively, are connected as shown in the circuit below. The output voltage V_O variation with input voltage linearly increasing with time, is given by: (V_{input} = 0 V at t = 0)
(figures are qualitative)



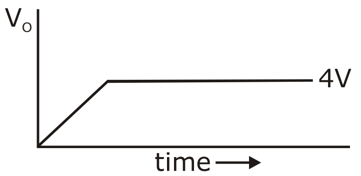
(1)



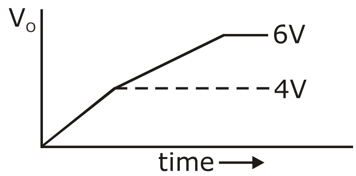
(2)



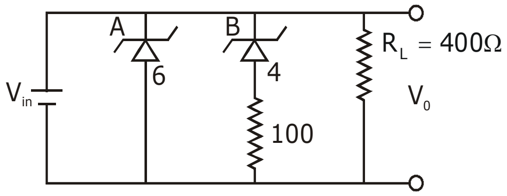
(3)

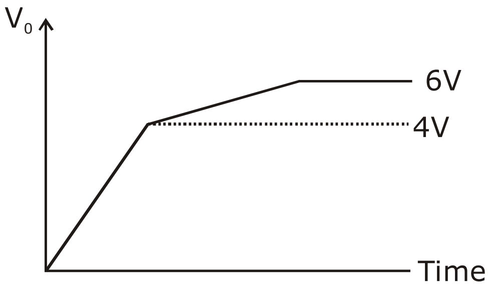


(4)



Sol. **(4)**
t = 0
$V_i = 0$
$V_i \propto t$
Given
$\because$ Zenerdiode maintain constant breakdown voltage.





11. In an adiabatic process, the density of a diatomic gas becomes 32 times its initial value. The final pressure of the gas is found to be n times the initial pressure. The value of n is:

(1) 32 (2) $\frac{1}{32}$ (3) 326 (4) 128

Sol. **4**

PV^r = const.

p (ρ^{-r}) = const.

$P_1\rho_1^{-r} = p_2\rho_2^{-r}$ $\quad r = \frac{7}{5}$ for diatomic

$p_0\rho_0^{-7/5} = (np_0)\,(32\rho_0)^{-7/5}$

$\rho_0^{-7/5} = \frac{n}{(32)^{7/5}}\,(\rho_0^{-7/5})$

$n = (2^5)^{7/5} = 2^7 = 128$

12. A galvanometer is used in laboratory for detecting the null point in electrical experiments. If, on passing a current of 6 mA it produces a deflection of 2°, its figure of merit is close to:

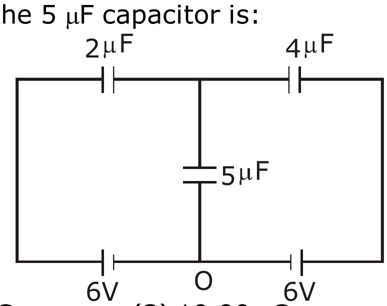
(1) 6×10^{-3} A/div. (2) 3×10^{-3} A/div. (3) 666° A/div. (4) 333° A/div.

Sol. **2**

figure of merit = $\frac{I}{\theta} \Rightarrow$ A/div.

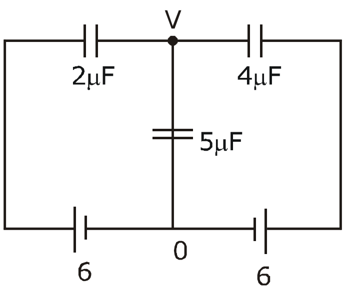
$= \frac{6 \times 10^{-3}}{2} = 3 \times 10^{-3}$ A/div.

13. In the circuit shown, charge on the 5 μF capacitor is:



(1) 5.45 μC (2) 18.00 μC (3) 10.90 μC (4) 16.36 μC

Sol. **2**



(V − 6) × 2 + (V − 0) × 5 + (V − 6) 4 = 0
2V − 12 + 5V + 4V − 24 = 0
11V = 36

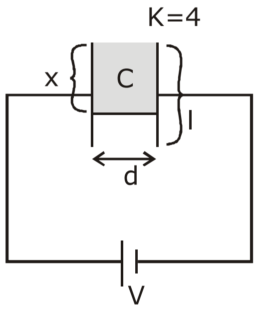
$V = \frac{36}{11}$

$q = CV = 5 \times \frac{36}{11} \approx 18.00\ \mu C$

14. A parallel plate capacitor has plate of length 'l', width 'w' and separation of plates is 'd'. It is connected to a battery of emf V.A dielectric slab of the same thickness 'd' and of dielectric constant k=4 is being inserted between the plates of the capacitor. At what length of the slab inside plates, will the energy stored in the capacitor be two times the initial energy stored?

(1) 2l/3 (2) l/2 (3) l/4 (4) l/3

Sol. **4**

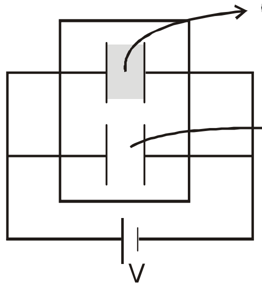


area of plate = lw

$C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 l\omega}{d}$

$U_1 = \frac{1}{2} cv^2 = \frac{\frac{1}{2}\varepsilon_0 l\omega}{d} v^2$

$C_{eq} = C_1 + C_2$



$C_1 = \frac{\varepsilon_0 \omega \times k}{d}$

$C_2 = \frac{\varepsilon_0 \omega (l - x)}{d}$

$$C_{eq} = \frac{\varepsilon_0 \omega x k}{d} + \frac{\varepsilon_0 \omega (l - x)}{d}$$

$$C_{eq} = \frac{\varepsilon_0 \omega}{d} [kx + l - x]$$

$$U_f = \frac{1}{2} C_{eq} V^2$$

$$U_f = 2U_i \Rightarrow \frac{1}{2} \frac{\varepsilon_0 \omega}{d} [kx + l - x] v^2 = 2 \times \frac{1}{2} \frac{\varepsilon_0 l \omega}{d} v^2$$

$kx + l - x = 2l$

$4x - x = l$

$3x = l$

$$x = \frac{l}{3}$$

15. A radioactive nucleus decays by two different processes. The half life for the first process is 10 s and that for the second is 100 s. The effective half life of the nucleus is close to:

(1) 55 sec. (2) 6 sec. (3) 12 sec. (4) 9 sec.

Sol. **4**

$$T_1 = 10 \text{ sec} \quad \lambda_1 = \frac{\ln 2}{T_1}$$

$$T_2 = 100\text{s}, \lambda_2 = \frac{\ln 2}{T_2}, \lambda_{eq} = \frac{\ln 2}{T_{eq}}$$

we know

$\lambda_{eq} = \lambda_1 + \lambda_2$

$$\frac{\ln 2}{T_{eq}} = \frac{\ln 2}{T_1} + \frac{\ln 2}{T_2}$$

$$\frac{1}{T_{eq}} = \frac{1}{10} + \frac{1}{100} = \frac{10+1}{100} = \frac{11}{100}$$

$$T_{eq} = \frac{100}{11} = 9 \text{ s}$$

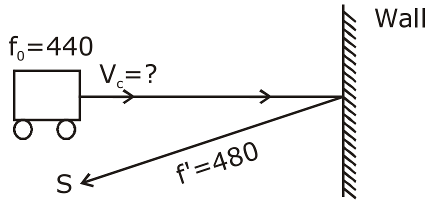
16. A driver in a car, approaching a vertical wall notices that the frequency of his car horn, has changed from 440 Hz to 480 Hz, when it gets reflected from the wall. If the speed of sound in air is 345 m/s, then the speed of the car is:

(1) 24 km/hr (2) 36 km/hr (3) 54 km/hr (4) 18 km/hr

Sol. **3**

car towards

$f_1 = \left(\dfrac{v - 0}{v - v_C}\right) f_0 \quad \text{....(i)}$



$480 = \left(\dfrac{v + v_c}{v - 0}\right) f_i \Rightarrow \left(\dfrac{v + v_c}{v}\right)\left(\dfrac{v}{v - v_c}\right) f_0$

$480 = (350 + V_C) \times \left(\dfrac{440}{350 - V_C}\right)$

$12 = \left(\dfrac{350 + V_C}{350 - V_C}\right) \times 11$

$12 \times 350 - 12 \times V_C = 350 \times 11 + 11\, V_C$

$23V_C = 4200 - 3850 = 350$

$V_C = \dfrac{350}{23}$ m

$V_C = \dfrac{350}{23} \times \dfrac{18}{5}$ km/h

$= \dfrac{70 \times 18}{23}$

= 54.78

= 54 km/hr

17. An iron rod of volume $10^{-3} m^3$ and relative permeability 1000 is placed as core in a solenoid with 10 turns/cm. If a current of 0.5 A is passed through the solenoid, then the magnetic moment of the rod will be:

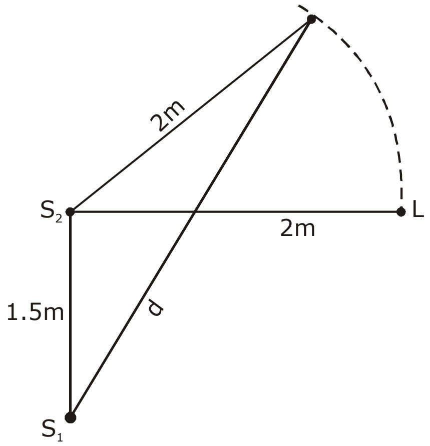
(1) 0.5×10^2 Am^2 (2) 50×10^2 Am^2 (3) 5×10^2 Am^2 (4) 500×10^2 Am^2

Sol. **3**

magnetic moment $\vec{M} = NIA(\mu_r - 1)$

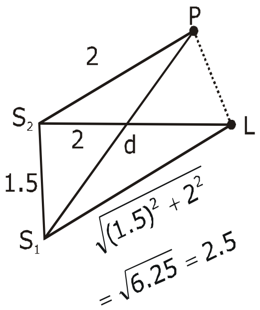
n = 10 turns/cm	$= (nl)\, IA\, (\mu_r - 1)$
$= \dfrac{10}{10^{-2}}$ turn/m	$= nI\, (Al)\, (\mu_r - 1)$
= 1000 turn/m	$= 1000 \times 0.5 \times 10^{-3}\, (1000 - 1)$
$V = 10^{-3} m^3 = Al$	$= 0.5 \times (999) = 499.5$
I = 0.5A, μ_r	= 500
N = nl	$= 5 \times 10^2$

18. Two coherent sources of sound, S_1 and S_2, produce sound waves of the same wavelength, $\lambda = 1$ m, in phase. S_1 and S_2 are placed 1.5 m apart (see fig). A listener, located at L, directly in front of S_2 finds that the intensity is at a minimum when he is 2 m away from S_2. The listener moves away from S_1, keeping his distance from S_2 fixed. The adjacent maximum of intensity is observed when the listener is at a distance d from S_1. Then, d is :



(1) 12 m (2) 2 m (3) 3 m (4) 5 m

Sol. **3**



For min at (L)

$S_1L - S_2L = \Delta x = \frac{\lambda}{2}(2n + 1)$;(n = 0, 12)

$2.5 - 2 = \frac{1}{2}(2n + 1)$

$0.5 \times 2 = (2n + 1)$

$2n = 0$

n = 0 (first minima)

so at 'p' $\rightarrow$ first maxima

$S_1P - S_2P = \lambda$ [n = 1] for first maxima

$S_1 P - 2 = 1$

$S_1P = 1 + 2$

d = 3 m

19. The quantities $x = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$, $y = \frac{E}{B}$ and $z = \frac{l}{CR}$ are defined where C-capacitance, R-Resistance, l-length, E-Electric field, B-magnetic field and ϵ_0, μ_0 -free space permittivity and permeability respectively. Then:

(1) Only y and z have the same dimension (2) x, y and z have the same dimension

(3) Only x and y have the same dimension (4) Only x and z have the same dimension

Sol. **2**

$x = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$ = (speed)

$[x] = LT^{-1}$ $\qquad$ $y = \frac{E}{B}$ = speed

$Z = \frac{l}{CR} = \frac{m}{sec}$ = m/s $\qquad$ $[y] = LT^{-1}$

[RC = T]

$[Z] = LT^{-1}$

So, x,y,z has same dimension

20. The acceleration due to gravity on the earth's surface at the poles is g and angular velocity of the earth about the axis passing through the pole is ω. An object is weighed at the equator and at a heigh h above the poles by using a spring balance. If the weights are found to be same, then h is : (h<<R, where R is the radius of the earth)

(1) $\frac{R^2\omega^2}{g}$ (2) $\frac{R^2\omega^2}{8g}$ (3) $\frac{R^2\omega^2}{4g}$ (4) $\frac{R^2\omega^2}{2g}$ \

Sol. **4**

$\because$ weight same at poles and at h (so $g_1 = g_2$)

$g_1 = g - R\omega^2$

$g_2 = g\left(1 - \frac{2h}{R}\right)$

$\because g_1 = g_2$

$g - R\omega^2 = g\left(1 - \frac{2h}{R}\right) \Rightarrow g - \frac{2gh}{R}$

$R\omega^2 = \frac{2gh}{R}$

$h = \frac{R^2\omega^2}{2g}$

21. Nitrogen gas is at 300° C temperature.The temperature (in K) at which the rms speed of a H_2 molecule would be equal to the rms speed of a nitrogen molecule, is _________.
(Molar mass of N_2 gas 28 g).

Sol. **41**

$$V_{rms} = \sqrt{\frac{3RT}{m}}$$

$$V_{N_2} = \sqrt{\frac{3R(573)}{28}}$$

$$V_{H_2} = \sqrt{\frac{3RT}{2}}$$

$$V_{H_2} = V_{N_2}$$

$$\sqrt{\frac{3RT}{2}} = \sqrt{\frac{3R(573)}{28}}$$

$$\frac{T}{2} = \frac{573}{28}$$

T = 41 K

22. The surface of a metal is illuminated alternately with photons of energies E_1 = 4 eV and E_2 = 2.5 eV respectively. The ratio of maximum speeds of the photoelectrons emitted in the two cases is 2. The work function of the metal in (eV) is _________.

Sol. **2**

$$\frac{\frac{1}{2}mV_1^2}{\frac{1}{2}mV_2^2} = \frac{E_1 - \phi_0}{2.5 - \phi_0} = \frac{4 - \phi_0}{2.5 - \phi_0}$$

$$\left(\frac{V_1}{V_2}\right)^2 = \frac{4 - \phi_0}{2.5 - \phi_0}$$

$$(2)^2 = \frac{4 - \phi_0}{2.5 - \phi_0}$$

$10 - 4\phi_0 = 4 - \phi_0$
$3\phi_0 = 10 - 4 = 6$
$\phi_0 = 2\text{eV}$

23. A prism of angle A= 1° has a refractive index μ = 1.5. A good estimate for the minimum angle of deviation (in degrees) is close to N/10. Value of N is

Sol. **5**

A = 1°

$\delta = (\mu - 1)A$

$= (1.5 - 1)A$

$= 0.5 \times 1$

$= \frac{5}{10} = \frac{N}{10}$ so N = 5

24. A body of mass 2 kg is driven by an engine delivering a constant power of 1 J/s. The body starts from rest and moves in a straight line. After 9 seconds, the body has moved a distance (in m) ____________.

Sol. **18**

$P = Fv = mav$

$a = \frac{p}{mv}$

$\frac{dv}{dt} = \frac{p}{mv}$

$\int_0^u v\,dv = \frac{p}{m}\int_0^t dt$

$\frac{u^2}{2} = \frac{p}{m}t$

$u = \sqrt{\frac{2p}{m}}\ \sqrt{t}$

$\frac{dx}{dt} = \sqrt{\frac{2p}{m}}\sqrt{t}$

$\int_0^x dx = \sqrt{\frac{2p}{m}}\ \int_0^9 \sqrt{t}\,dt$

$x = \frac{2}{3}\left[(9)^{1/2}\right]^3$

$= \frac{2}{3} \times 27$

$x = 18$

II-method

$Pt = w = \frac{1}{2}mv^2 - 0$

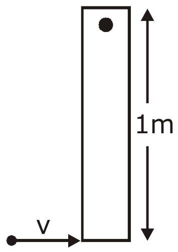
$1\times t = \frac{1}{2} \times 2 \times u^2$

$u = \sqrt{t}$

$$\frac{dx}{dt} = \sqrt{t} = \int_0^1 dx = \int_0^9 \sqrt{t}dt$$

$$x = \frac{\left[t^{3/2}\right]_0^9}{\frac{3}{2}} = 18 \text{ m}$$

25. A thin rod of mass 0.9 kg and length 1 m is suspended, at rest, from one end so that it can freely oscillate in the vertical plane. A particle of mass 0.1 kg moving in a straight line with velocity 80 m/s hits the rod at its bottom most point and sticks to it (see figure). The angular speed (in rad/s) of the rod immediately after the collision will be ___________.



Sol. **20**

$L_i = L_f$

$$0.1 \times 80 \times 1 = \frac{0.9 \times 1^2}{3} \times \omega + (0.1)\, 1^2\omega$$

$8 = (0.3 + 0.1)\, \omega$

$8 = (0.4)\, \omega$

$$\omega = \frac{80}{4} = 20$$

QUESTION PAPER WITH SOLUTION

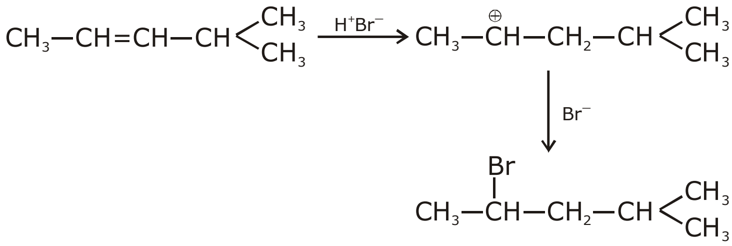
CHEMISTRY _ 5 Sep. _ SHIFT - 2

1. The major product formed in the following reaction is :

$CH_3CH = CHCH(CH_3)_2 \xrightarrow{HBr}$

(1) $CH_3CH(Br)CH_2CH(CH_3)_2$ (2) $CH_3CH_2CH_2C(Br)(CH_3)_2$
(3) $CH_3CH_2CH(Br)CH(CH_3)_2$ (4) $Br(CH_2)_3CH(CH_3)_2$

Sol. **1**



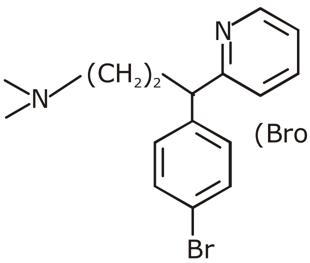
2. Hydrogen peroxide, in the pure state, is :
(1) Linear and blue in color (2) Linear and almost colorless
(3) Non-planar and almost colorless (4) Planar and bluein color

Sol. **3**
H_2O_2 has openbook structure it is non planar

3. Boron and silicon of very high purity can be obtained through :
(1) Liquation (2) Electrolytic refining
(3) Zone refining (4) Vapour phase refining

Sol. **3**
Fact

4. The following molecule acts as an :

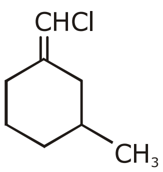


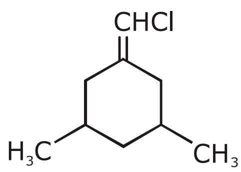
(Brompheniramine)

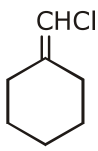
(1) Anti-histamine (2) Antiseptic (3) Anti-depressant (4) Anti-bacterial

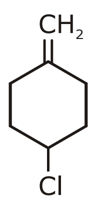
Sol. **1**
Anti-histamine

5. Among the following compounds, geometrical isomerism is exhibited by :

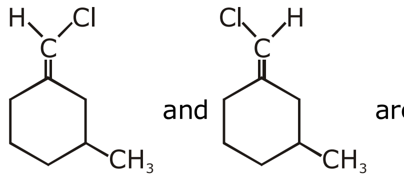
(1) 

(2) 

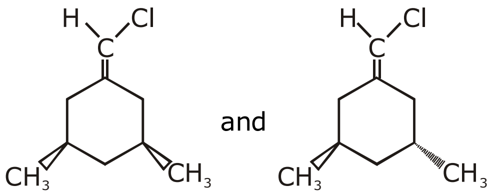
(3) 

(4) 

Sol. **1 & 2**

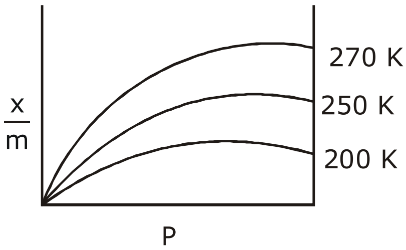


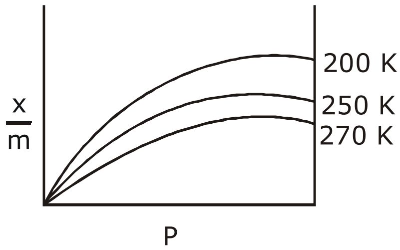
are geometrical isomer

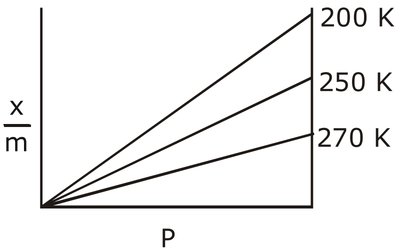


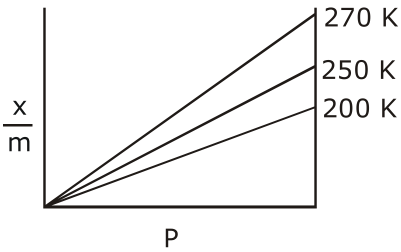
are geometrical isomer

6. Adsorption of a gas follows Freundlich adsorption isotherm. If x is the mass of the gas adsorbed on mass m of the adsorbent, the correct plot of $\frac{x}{m}$ versus p is :

(1)


(2)


(3)


(4)


Sol. **2**

As temp. increases extent of Adsorption decreases

Therefore correct option (2)

$\frac{x}{m} = KP^{1/n}$

$\frac{x}{m}$ v/s P $\rightarrow$ non linear curve

7. The compound that has the largest H–M–H bond angle (M=N, O, S, C) is :

(1) CH_4 (2) H_2S (3) NH_3 (4) H_2O

Sol. **1**

CH_4	>	NH_3	>	H_2O	>	H_2S
$Sp^3(\ell p = 0)$		$Sp^3(\ell p = 1$		$Sp^3(\ell p = 2)$		Sp^3 $(\ell p = 2)$
BA $107^{o}28^{1}$		BA = 107^{o}		BA = $104^{o}5^{1}$		BA =92^{o}

8. The correct statement about probability density (except at infinite distance from nucleus) is :

(1) It can be zero for 3p orbital (2) It can be zero for 1s orbital

(3) It can never be zero for 2s orbital (4) It can negative for 2p orbital

Sol. **1**

$\psi^2_{R/S} > 0$ always

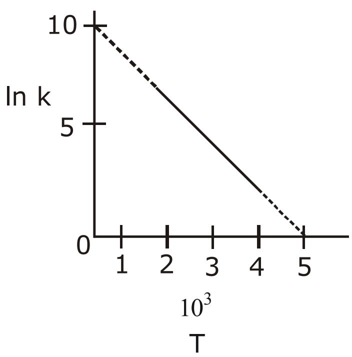
$\psi^2_{R/S}$ can be = 0; As '2s' has 1 Radial Node

ψ^2_{R} can never be negative

ψ^2_{R} (3P) can be = 0 as 3P has Radial Nodes

Ans. Option (1)

9. The rate constant (k) of a reaction is measured at differenct temperatures (T), and the data are plotted in the given figure. The activation energy of the reaction in kJ mol^{-1} is : (R is gas constant)



(1) R (2) 2/R (3) 1/R (4) 2R

Sol. **4**

$$\ln(k) = \ln(A) - \frac{Ea}{R}\left(\frac{1}{T}\right)$$

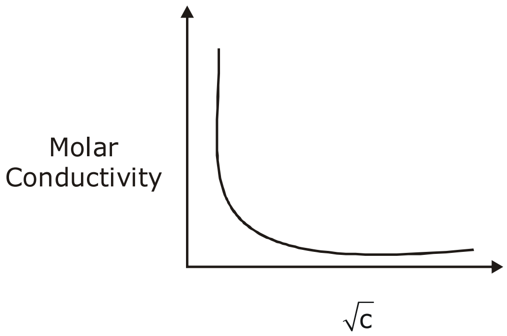
$\ln(A) = 10$

$$\text{Slope} = \frac{-Ea}{R} \times 10^{-3} = -10/5$$

E_a = 2000R J/mol

E_a = 2R KJ/mol

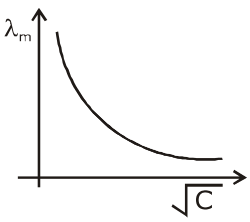
10. The variation of molar conductivity with concentration of an electrolyte (X) in aqueous solution is shown in the given figure.



The electrolyte X is :

(1) HCl (2) CH_3COOH (3) NaCl (4) KNO_3

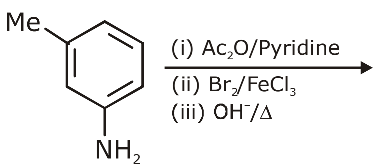
Sol. **2**

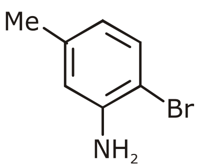


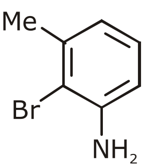
Such type of variation is always for weak electrolyte

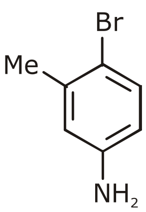
Hence Ans (2) CH_3COOH

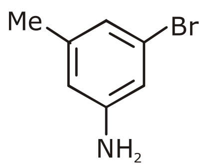
11. The final major product of the following reaction is :



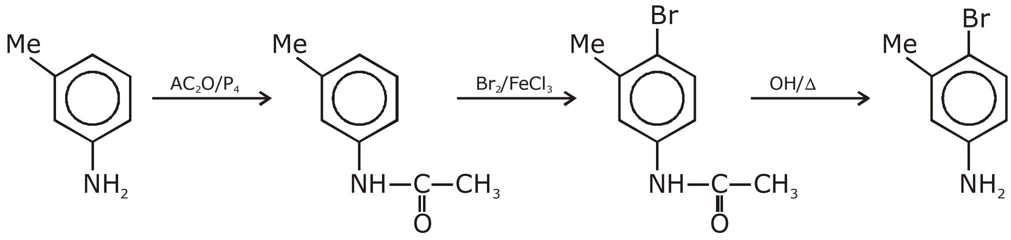
(1) 

(2) 

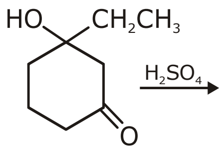
(3) 

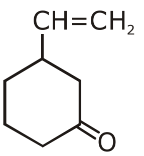
(4) 

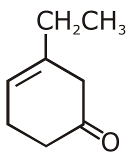
Sol. 3

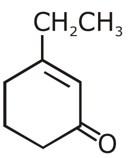


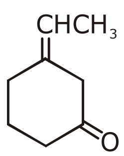
12. The major product of the following reaction is :



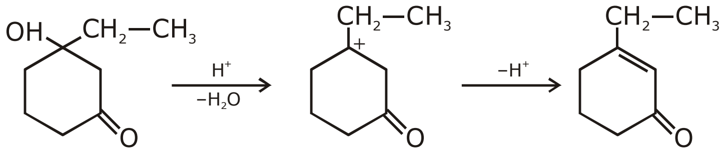
(1) 

(2) 

(3) 

(4) 

Sol. 3



13. Lattice enthalpy and enthalpy of solution of NaCl are 788 kJ mol^{-1}, and 4 kJ mol^{-1}, respectively. The hydration enthalpy of NaCl is :

(1) −780 kJ mol^{-1}
(2) 784 kJ mol^{-1}
(3) −784 kJ mol^{-1}
(4) 780 kJ mol^{-1}

Sol. 3

$\Delta H_{sol} = L.E. + \Delta H_{hyd}$

$4 = 788 + \Delta H_{Hyd}$

$\Delta H_{Hyd} = -784$ KJ/mol Ans

14. Reaction of ammonia with excess Cl_2 gives :

(1) NH_4Cl and N_2
(2) NH_4Cl and HCl
(3) NCl_3 and HCl
(4) NCl_3 and NH_4Cl

Sol. **3**

(1) $NH_3 + 3Cl_2 \xrightarrow{\quad} NCl_3 + 3HCl$
(excess)

(2) $8NH_3 + 3Cl_2 \xrightarrow{\quad} 6NH_4Cl + N_2$
(excess)

15. Which one of the following polymers is not obtained by condensation polymerisation ?
(1) Bakelite
(2) Nylon 6
(3) Buna-N
(4) Nylon 6, 6

Sol. **2**

$$n\ \underset{\text{1,3-Butadiene}}{CH_2{=}CH{-}CH{=}CH_2} + n\ \underset{\text{Acrylo nitrile}}{CH_2{=}\overset{\overset{\displaystyle CN}{|}}{C}H} \longrightarrow \underset{\text{Buna - n}}{\left[CH_2{-}CH{=}CH{-}CH_2{-}\overset{\overset{\displaystyle CN}{|}}{C}H_2{-} \right]_n}$$

16. Consider the comples ions,
trans-$[Co(en)_2Cl_2]^+$ (A) and
cis-$[Co(en)_2Cl_2]^+$ (B)
The correct statement regarding them is :
(1) Both (A) and (B) can be optically active.
(2) (A) can be optically active, but (B) cannot be optically active.
(3) Both (A) and (B) cannot be optically active.
(4) (A) cannot be optically active, but (B) can be optically active.

Sol. **4**
Due to presence of Pos (A) cannot be optically active, but (B) can be optically active

17. An element crystallises in a face-centred cubic (fcc) unit cell with cell edge a. The distance between the centres of two nearest octahedral voids in the crystal lattice is :

(1) a
(2) $\frac{a}{2}$
(3) $\sqrt{2}a$
(4) $\frac{a}{\sqrt{2}}$

Sol. **4**
Nearest octahedral voids
One along edge center & other at Body centre

$$\text{Distance} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \sqrt{2}\ \frac{a}{2}$$

$$= \frac{a}{\sqrt{2}} \text{ Ans.}$$

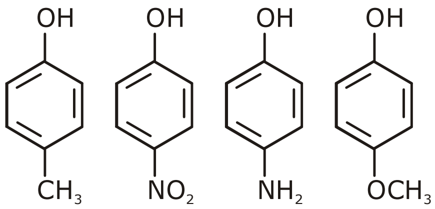
18. The correct order of the ionic radii of O^{2-}, N^{3-}, F^-, Mg^{2+}, Na^+ and Al^{3+} is :
(1) $N^{3-} < O^{2-} < F^- < Na^+ < Mg^{2+} < Al^{3+}$
(2) $N^{3-} < F^- < O^{2-} < Mg^{2+} < Na^+ < Al^{3+}$
(3) $Al^{3+} < Na^+ < Mg^{2+} < O^{2-} < F^- < N^{3-}$
(4) $Al^{3+} < Mg^{2+} < Na^+ < F^- < O^{2-} < N^{3-}$

Sol. **4**
all are Isoelectronic

(1) $\dfrac{N^{3-}O^{2-}F^-Na^+Mg^{2+}Al^{3+}}{Z\uparrow, Zeff\uparrow, \text{Ionic Radii}\downarrow}$

(2) $Al^{3+} < Mg^{2+} < Na^+ < F^- < O^{2-} < N^{3-}$

19. The increasing order of boiling points of the following compounds is :

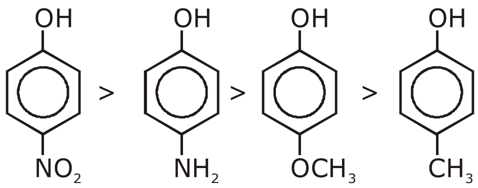


I II III IV

(1) I < III < IV < II (2) IV < I < II < III

(3) I < IV < III < II (4) III < I < II < IV

Sol. **3**



(II) (III) (IV) (I)

20. The one that is NOT suitable for the removal of permanent hardness of water is :

(1) Ion-exchange method (2) Calgon's method

(3) Treatment with sodium carbonate (4) Clark's method

Sol. **4**

Clark's method is used for Removal of Temporary hardness

$Ca(HCO_3)_2 + Ca(OH)_2 \rightarrow 2CaCO_3\downarrow + 2H_2O$

$Mg(HCO_3)_2 + 2Ca(OH)_2 \rightarrow 2CaCP_3 + Mg(OH)_2\downarrow + 2H_2O$

21. For a reaction $X + Y \rightleftharpoons 2Z$, 1.0 mol of X, 1.5 mol of Y and 0.5 mol of Z were taken in a 1 L vessel and allowed to react. At equilibrium, the concentration of Z was 1.0 mol L^{-1}. The equilibrium constant of reaction is ________ $\frac{x}{15}$. The value of x is _________.

Sol. **16**

	$x +$	$y \rightleftharpoons$	$2Z$	
$t = 0$	1mol	$\frac{3}{2}$mol	$\frac{1}{2}$mol	
t_{eq}	–	–	1 mol	$2x = \frac{1}{2}$
t_{eq}	$1 - x$	$\frac{3}{2} - x$	$\frac{1}{2} + 2x$	$x = \frac{1}{4}$
t_{eq}	$\frac{3}{4}$mol	$\frac{5}{4}$mol	1 mol	

$$K_{eq} = \frac{(1)^2}{\frac{5}{4} \times \frac{3}{4}} = \frac{16}{15}$$

x = 16 Ans.

22. The volume, in mL, of 0.02 M $K_2Cr_2O_7$ solution required to react with 0.288 g of ferrous oxalate in acidic medium is ________.
(Molar mass of Fe= 56 g mol^{-1})

Sol. **50 ml**

$K_2Cr_2O_7 + FeC_2O_4 \rightarrow Cr^{3+} + Fe^{3+} + CO_2$

$$\frac{0.02 \times vol \times 6}{1000} = 3 \times \frac{0.288}{144} \times 100$$

$$Vol. = \frac{200}{4} = 50 \text{ ml Ans.}$$

23. Considering that $\Delta_0 > P$, the magnetic moment (in BM) of $[Ru(H_2O)_6]^{2+}$ would be ________.

Sol. **0**

$[Ru(H_2O)_6)^{2+}$
$Ru^{2+} = 3d^6$ $(\Delta_0 > P)$
$= t_2g^6\ eg^o$
$n = 0$, $u = 0$

24. For a dimerization reaction, $2A(g) \rightarrow A_2(g)$ at 298 K, $\Delta U^{\ominus} = -20$ kJ mol^{-1}, $\Delta S^{\ominus} = -30$ kJ mol^{-1}, then the $\Delta G^{\ominus}$ will be __________ J.

Sol. **–13538 J**

$2A \longrightarrow A_2$

$\Delta U^{\ominus} = -20$ kJ

$\Delta H^{\ominus} = -20000 + (-1)\, R \times 298$

$\Delta G^{\ominus} = -20000 - 298R + 30 \times 298$

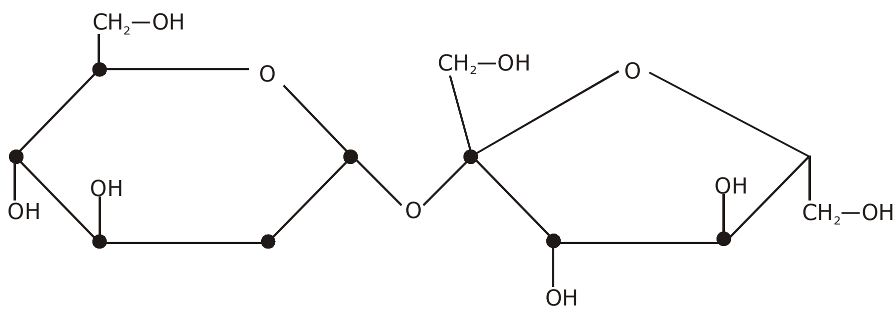
$$\Delta G^{\ominus} = -20,000 + 298\left(\frac{90-25}{3}\right)$$

$$\Delta G^{\ominus} = 20,000 + \frac{298 \times 65}{3}$$

$\Delta G^{\ominus} = -13538$ J

25. The number of chiral carbons present in sucrose is _______.

Sol. **9**



QUESTION PAPER WITH SOLUTION

MATHEMATICS _ 5 Sep. _ SHIFT - 2

Q.1 If x=1 is a critical point of the function $f(x)=(3x^2+ax-2-a)e^x$, then:

(1) x=1 is a local minima and $x=-\frac{2}{3}$ is a local maxima of f.

(2) x=1 is a local maxima and $x=-\frac{2}{3}$ is a local minima of f.

(3) x=1 and $x=-\frac{2}{3}$ are local minima of f.

(4) x=1 and $x=-\frac{2}{3}$ are local maxima of f.

Sol. **1**

$f(x) = (3x^2+ax-2-a)e^x$

$f'(x) = (3x^2+ax-2-a)e^x + (6x+a)e^x = 0$

$e^x\left[3x^2+(a+6)x-2\right]=0$

at x = 1, 3 +a+6 - 2 = 0

a=–7

$f(x) = (3x^2 - 7x + 5)e^x$

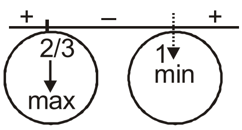
$f'(x) = (6x-7)e^x+(3x^2-7x+5)e^x$

$= e^x(3x^2-x-2) = 0$

$= 3x^2 - 3x + 2x - 2 = 0$

$= (3x+2)(x-1) = 0$

x = 1, -2/3



Q.2 $\lim\limits_{x\to 0}\frac{x\left(e^{\left(\sqrt{1+x^2+x^4}-1\right)/x}-1\right)}{\sqrt{1+x^2+x^4}-1}$

(1) is equal to $\sqrt{e}$ (2) is equal to 1 (3) is equal to 0 (4) does not exist

Sol. **2**

$$\lim_{x\to 0}\frac{x\left[e^{\left(\sqrt{1+x^2+x^4}-1\right)/x}-1\right]}{\left(\sqrt{1+x^2+x^4}-1\right)}$$

$$\lim_{x\to 0}\frac{x\left[e^{\left[\frac{\left(\sqrt{1+x^2+x^4}\right)^2-1}{x\times 2}\right]}-1\right]\times\left(\sqrt{1+x^2+x^4}+1\right)}{\left(x^2+x^4\right)}$$

$$x\lim_{x\to\infty}\frac{e^{\left(\frac{x^3+x}{2}\right)}-1}{\left(\frac{x^3+x}{2}\right)\times 2}\times 2$$

$= 1$

Q.3 The statement $(p \to (q \to p)) \to (p \to (p \vee q))$ is:

(1) equivalent to $(p \vee q) \wedge (\sim p)$

(2) equivalent to $(p \wedge q) \vee (\sim p)$

(3) a contradiction

(4) a tautology

Sol. **4**

p	$q \to p$	$p \to (q \to p)$
T	T	T
T	T	T
F	F	T
F	T	T

$\Rightarrow$ (Tautology)

p	$p \vee q$	$p \to (p \vee q)$
T	T	T
T	T	T
F	T	T
F	F	T

Q.4 If $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$ and $M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$, then:

(1) $M = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

(2) $M = \frac{1}{4\sqrt{2}} + \frac{1}{4}\cos\frac{\pi}{8}$

(3) $L = -\frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

(4) $L = \frac{1}{4\sqrt{2}} - \frac{1}{4}\cos\frac{\pi}{8}$

Sol. **1**

$$\ell = \sin\left(\frac{3\pi}{16}\right)\sin\left(\frac{-\pi}{16}\right)$$

$$\ell = \frac{-1}{2}\left[\cos\frac{\pi}{8} - \cos\frac{\pi}{4}\right]$$

$$\ell = \frac{1}{2\sqrt{2}} - \frac{1}{2}\cos\frac{\pi}{8}$$

$$M = \cos\left(\frac{3\pi}{16}\right)\cos\left(\frac{\pi}{16}\right)$$

$$M = \frac{1}{2}\left[\cos\frac{\pi}{4} + \cos\frac{\pi}{8}\right]$$

$$M = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8} \text{....(1)}$$

Q.5 If the sum of the first 20 terms of the series $\log_{\left(7^{1/2}\right)} x + \log_{\left(7^{1/3}\right)} x + \log_{\left(7^{1/4}\right)} x + \cdots$ is 460, then x is equal to:

(1) $7^{1/2}$ (2) 7^{2} (3) e^{2} (4) $7^{46/21}$

Sol. **2**

$(2 + 3 + 4 + \ldots + 21)\log_7 x = 460$

$$\Rightarrow \frac{20\times(21+2)}{2}\log_7 x = 460$$

$\Rightarrow 230 \log_7 x = 460 \Rightarrow \log_7 x = 2 \Rightarrow x = 7^2$

Q.6 There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is:

(1) 2250 (2) 2255 (3) 1500 (4) 3000

Sol. **1**

	S—1	S—2	S—3
	1,2,3,4,5	1,2,3,4,5	1,2,3,4,5
6 case	1	1	3
	1	2	2
	1	3	1
	2	2	1
	2	1	2
	3	1	1

$3(5_{c_1} \times 5_{c_1} \times 5_{c_3}) + 3(5_{c_1} \times 5_{c2} \times 5_{c_2}) = 3(25 \times 10) + (100\times5)3 = 750 + 1500 = 2250$

Q.7 If the mean and the standard deviation of the data 3,5,7,a,b are 5 and 2 respectively, then a and b are the roots of the equation:

(1) $x^2-20x+18=0$ (2) $x^2-10x+19=0$ (3) $2x^2-20x+19=0$ (4) $x^2-10x+18=0$

Sol. **2**

$$\text{S.D.} = \sqrt{\frac{\Sigma x_i^2}{n} - (\bar{x})^2}$$

$$(2)^2 = \frac{83 + a^2 + b^2}{5} - \left(\frac{15 + a + b}{5}\right)^2$$

$$4 = \frac{83 + a^2 + b^2}{5} - 25$$

$29\times5 - 83 = a^2+b^2 \Rightarrow a^2+b^2 = 62$ (1)

$\frac{a + b + 15}{5} = 5 \Rightarrow \boxed{a + b = 10}$

$2ab = 100 - 62 = 38$

$\boxed{ab = 19}$ (2)

Q.8 The derivative of $\tan^{-1}\left(\frac{\sqrt{1 + x^2} - 1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1 - x^2}}{1 - 2x^2}\right)$ at $x = \frac{1}{2}$ is:

(1) $\frac{2\sqrt{3}}{3}$ (2) $\frac{2\sqrt{3}}{5}$ (3) $\frac{\sqrt{3}}{12}$ (4) $\frac{\sqrt{3}}{10}$

Sol. **4**

$x = \tan\theta$

$$u = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right) = \tan^{-1}(\tan\theta/_2) = \frac{\theta}{2} = \frac{\tan^{-1} x}{2}$$

$$v = \tan^{-1}\left(\frac{2\sin\theta\cos\theta}{\cos 2\theta}\right) = 2\theta$$

$= 2\sin^{-1}x$

$$\frac{du}{dv} = \frac{1}{2(1 + x^2)} \times \frac{\sqrt{1 - x^2}}{2}$$

$$= \frac{\sqrt{3}}{2 \times 2} \times \frac{4}{5 \times 2} = \frac{\sqrt{3}}{10}$$

Q.9 If $\int \frac{\cos\theta}{5+7\sin\theta-2\cos^2\theta}d\theta = A\log_e|B(\theta)|+C$ where C is a constant of integration, then $\frac{B(\theta)}{A}$ can be:

(1) $\frac{5(2\sin\theta+1)}{\sin\theta+3}$ (2) $\frac{5(\sin\theta+3)}{2\sin\theta+1}$ (3) $\frac{2\sin\theta+1}{\sin\theta+3}$ (4) $\frac{2\sin\theta+1}{5(\sin\theta+3)}$

Sol. **1**

$$\int \frac{\cos\theta}{5+7\sin\theta-2+2\sin^2\theta}d\theta$$

$$\int \frac{dt}{2t^2+7t+3}$$

$$= \frac{1}{2}\int \frac{dt}{t^2+\frac{7t}{2}+\frac{3}{2}} = \frac{1}{2}\int \frac{dt}{t^2+\frac{7}{2}t+\left(\frac{7}{4}\right)^2-\frac{49}{16}+\frac{24}{16}}$$

$$= \frac{1}{2}\int \frac{dt}{(t+7/4)^2-(5/4)^2}$$

$$\frac{1}{2}\times\frac{1}{2\cdot\frac{5}{4}}\ln\left|\left[\frac{t+7/4-5/4}{t+7/4+5/4}\right]\right|$$

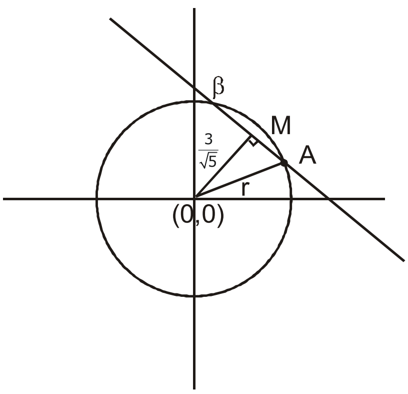
$$\frac{1}{5}\ln\left|\left(\frac{\sin\theta+1/2}{\sin\theta+3}\right)\right|+C$$

$$\frac{B(\theta)}{A} = 5\left(\frac{2\sin\theta+1}{\sin\theta+3}\right)$$

Q.10 If the length of the chord of the circle, $x^2+y^2 = r^2(r>0)$ along the line, $y-2x=3$ is r, then r^2 is equal to:

(1) 12 (2) $\frac{24}{5}$ (3) $\frac{9}{5}$ (4) $\frac{12}{5}$

Sol. **4**



$AB = 2\sqrt{r^2 - 9/5} = r$

$r^2 - 9/5 = \frac{r^2}{4}$

$3r^2/4 = 9/5$

$\boxed{r^2 = \frac{12}{5}}$

Q.11 If α and β are the roots of the equation, $7x^2-3x-2=0$, then the value of $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$ is equal to:

(1) $\frac{27}{32}$ (2) $\frac{1}{24}$ (3) $\frac{27}{16}$ (4) $\frac{3}{8}$

Sol. 3

$\alpha + \beta = 3/7$, $\alpha\beta = -2/7$

$$\frac{(\alpha+\beta) - \alpha\beta(\alpha+\beta)}{1-(\alpha^2+\beta^2)+(\alpha\beta)^2}$$

$$\frac{\frac{3}{7}+\frac{2}{7}\times\frac{3}{7}}{1-\left\{\frac{9}{49}+\frac{4}{7}\right\}+\frac{4}{49}}$$

$$\frac{\left(\frac{21+6}{49}\right)}{\frac{16}{49}} \Rightarrow \frac{27}{16}$$

Q.12 If the sum of the second, third and fourth terms of a positive term G.P. is 3 and the sum of its sixth, saventh and eighth terms is 243, then the sum of the first 50 terms of this G.P. is:

(1) $\frac{2}{13}\left(3^{50}-1\right)$ (2) $\frac{1}{26}\left(3^{49}-1\right)$ (3) $\frac{1}{13}\left(3^{50}-1\right)$ (4) $\frac{1}{26}\left(3^{50}-1\right)$

Sol. **4**

$$\frac{ar+ar^2+ar^3}{ar^5+ar^6+ar^7} = \frac{3}{243}$$

$$\frac{1+r+r^2}{r^4\left(1+r+r^2\right)} = \frac{1}{81}$$

$\boxed{r=3}$

a(3+9+27) = 3

$a = \frac{3}{39} = \boxed{\frac{1}{13}}$

$S_{50} = a\left(\frac{r^{50}-1}{r-1}\right)$

$= \frac{1}{13}\left\{\frac{3^{50}-1}{2}\right\}$.........(4)

Q.13 If the line y=mx+c is a common tangent to the hyperbola $\frac{x^2}{100} - \frac{y^2}{64} = 1$ and the circle $x^2+y^2=36$, then which one of the following is true?

(1) $4c^2=369$ (2) $c^2=369$ (3) 8m+5=0 (4) 5m=4

Sol. **1**

$c = \pm\sqrt{a^2m^2 - b^2}$

$c = \pm\sqrt{100m^2 - 64}$

$y = mx \pm\sqrt{100m^2 - 64}$

$d|_{(0,0)} = 6$

$\left|\frac{\sqrt{100m^2 - 64}}{\sqrt{m^2+1}}\right| = 6$

$100m^2 - 64 = 36m^2 + 36$

$64m^2 = 100$

$m = 10/8$

$c^2 = 100 \times \frac{100}{64} - 64 \Rightarrow \frac{(164)(36)}{64}$ $\boxed{4c^2 = 369}$

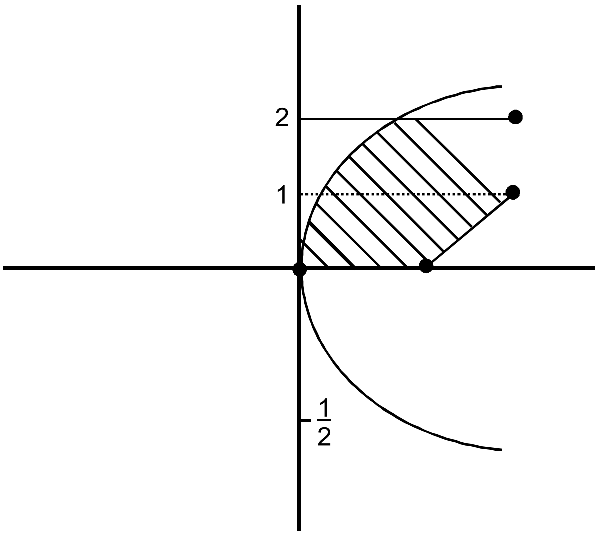
Q.14 The area (in sq. units) of the region $A = \{(x, y) : (x-1)[x] \le y \le 2\sqrt{x}, 0 \le x \le 2\}$ where [t] denotes the greatest integer function, is:

(1) $\frac{4}{3}\sqrt{2} - \frac{1}{2}$ (2) $\frac{8}{3}\sqrt{2} - \frac{1}{2}$ (3) $\frac{8}{3}\sqrt{2} - 1$ (4) $\frac{4}{3}\sqrt{2} + 1$

Sol. **2**

$y = f(x) = (x - 1)[x] = \begin{cases} 0 & 0 \le x < 1 \\ x-1 & 1 \le x < 2 \\ 2(x-1) & x = 2 \end{cases}$

$y^2 \le 4x$



$$\int_0^1 \left(2\sqrt{x}-0\right)+\int_1^2\left(2\sqrt{x}-(x-1)\right)$$

$$\frac{2}{3}\times 2x^{3/2}\Big|_0^1+\left(\frac{4}{3}x^{3/2}-\frac{x^2}{2}+x\right)_1^2$$

$$\frac{4}{3}+\left\{\left(\frac{4}{3}\times 2\sqrt{2}-2+2\right)-\left(\frac{4}{3}+\frac{1}{2}\right)\right\}$$

$$\frac{4}{3}+\frac{8\sqrt{2}}{3}-\frac{4}{3}-\frac{1}{2} = \frac{8\sqrt{2}}{3}-\frac{1}{2}$$

Q.15 If a+x=b+y=c+z+1, where a,b,c,x,y,z are non-zero distinct real numbers. then $\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$ is equal to:

(1) y(a−b) (2) 0 (3) y(b−a) (4) y(a−c)

Sol. **1**

$$\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix}+\begin{vmatrix} x & y & x+a \\ y & y & y+b \\ z & y & z+c \end{vmatrix}$$

$$\begin{vmatrix} x & y & a \\ y & y & b \\ z & y & c \end{vmatrix} \Rightarrow y\begin{vmatrix} x & 1 & a \\ y & 1 & b \\ z & 1 & c \end{vmatrix}$$

$$y\begin{vmatrix} x & 1 & a \\ y-x & 0 & b-a \\ z-x & 0 & c-a \end{vmatrix}$$

$$yx \times 0 - 1\{(y-x)(c-a)-(b-a)(z-x)\} + a \times 0\}$$

$$y[bz - bx - az + ax - (cy - ay - cx + ax)]$$

$$y[bz - bx - az - cy + ay + cx]$$

$$y[b(z-x) + a(y-z) + c(x-y)]$$

$$y[b\{a-c-1\} + a(c-b+1) + c(b-a)]$$

$$y[ab - bc - b + ac - ab + a + bc - ac]$$

$$\boxed{y(a-b)}$$

Q.16 If for some $\alpha \in R$, the lines $L_1 : \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$ and $L_2 : \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$ are coplanar, then the line L_2 passes through the point:

(1) (2, –10, –2) (2) (10, –2, –2) (3) (10, 2, 2) (4) (–2, 10, 2)

Sol. **1**

A (-1,2,1), B(-2,-1, -1)

$$[\overrightarrow{AB}\ \vec{b_1}\ \ \vec{b_2}] = 0$$

$$\begin{vmatrix} -1 & -3 & -2 \\ 2 & -1 & 1 \\ \alpha & 5-\alpha & 1 \end{vmatrix} = 0$$

$-1(-1+\alpha-5) + 3(2-\alpha)-2(10-2\alpha+\alpha)=0$

$6-\alpha + 6-3\alpha + 2\alpha - 20 = 0$

$-8 - 2\alpha = 0$

$$\boxed{\alpha = -4}$$

$$L_2 : \frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$$

any point on L_2 is

$(-4\lambda-2, 9\lambda-1, \lambda-1) = A$

Q.17 The value of $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30}$ is:

(1) $2^{15}i$ (2) -2^{15} (3) $-2^{15}i$ (4) 6^5

Sol. **3**

$$\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30} \Rightarrow \left[\left(\frac{-1+i\sqrt{3}}{2}\right)(1+i)\right]^{30}$$

$$\omega^{30}(1+i)^{30} = 2^{15}(-i)$$

Q.18 Let y=y(x) be the solution of the differential equation $\cos x\frac{dy}{dx} + 2y\sin x = \sin 2x, x \in \left(0, \frac{\pi}{2}\right)$.

If $y(\pi/3) = 0$, then $y(\pi/4)$ is equal to:

(1) $2+\sqrt{2}$ (2) $\sqrt{2}-2$ (3) $\frac{1}{\sqrt{2}}-1$ (4) $2-\sqrt{2}$

Sol. **2**

$\frac{dy}{dx}+(2\tan x)y=2\sin x$

I.F. = $e^{2\ln(\sec x)} = \sec^2 x$

$y(\sec^2 x) = 2\int \frac{\sin x}{\cos^2 x}dx$

$=2\int \sec x \tan x\, dx = 2\sec x + c$

$y\left(\frac{\pi}{3}\right) = 0$

0 = 2 × 2 + c = C = -4

$y(\sec^2 x) = 2\sec x - 4$

x = $\pi/4$

2y = $2\sqrt{2}-4$

y = $\boxed{\sqrt{2}-2}$

Q.19 If the system of linear equations

x+y+3z=0

$x+3y+k^2z=0$

3x+y+3z=0

has a non-zero solution (x,y,z) for some $k \in R$, then $x+\left(\frac{y}{z}\right)$ is equal to:

(1) –9 (2) 9 (3) –3 (4) 3

Sol. **3**

$$\begin{vmatrix} 1 & 1 & 3 \\ 1 & 3 & k^2 \\ 3 & 1 & 3 \end{vmatrix} = 0$$

$(9-k^2)-(3-3k^2) + 3(-8)=0$

$9-k^2-3+3k^2 - 24 = 0$

$2k^2-18 = 0$

$K^2 = 9$

$\boxed{K = 3, -3}$

$$\begin{array}{r} x+y+3z=0 \\ x+3y+9z=0 \\ \hline 2y+6z=0 \end{array}$$

$\boxed{y = -3z}$

$\boxed{y / z = -3}$

$2x=0$

$\boxed{x = 0}$

$x + \left(\frac{y}{z}\right) = -3$

Q.20 Which of the following points lies on the tangent to the curve $x^4e^y + 2\sqrt{y+1} = 3$ at the point (1,0)?

(1) (2,6) (2) (2,2) (3) (−2,6) (4) (−2,4)

Sol. **3**

$4x^3e^y + x^4e^y y' + \frac{2y'}{2\sqrt{y+1}} = 0$

at (1,0)

$4 + y' + \frac{2y'}{2} = 0$

$2y' = -4 \Rightarrow y' = -2$

E.O.T. :

$y = -2(x-1)$

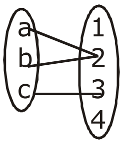
$\boxed{2x + y = 2}$

Q.21 Let A={a,b,c} and B={1,2,3,4}. Then the number of elements in the set $C = \{f : A \rightarrow B \mid 2 \in f(A)$ and f is not one−one} is________

Sol. **19**

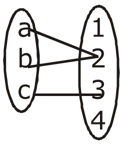
case - I

set B only have '2'



= 1

case - II
set B have more element with 2



$= {}^3C_1 \frac{\lfloor 3}{\lfloor 2 . \lfloor 1} . \lfloor 2 = 18$

total 18 + 1 = 19

Q.22 The coefficient of x^4 in the expansion of $(1+x+x^2+x^3)^6$ in powers of x, is ______

Sol. **120**

$(1+x)^6(1+x^2)^6$

$6_{c_r} x^r \quad 6_{c_r} x^{2S}$

$6_{c_r} 6_{c_r} \quad x^{r+2S}$

r	s
0	2
4	0
2	1

$\Rightarrow 6_{c_0} 6_{c_2} + 6_{c_4} 6_{c_0} + 6_{c_2} 6_{c_1}$

$\Rightarrow 15+15+15\times6$

$\Rightarrow 120$

Q.23 Let the vectors $\vec{a}, \vec{b}, \vec{c}$ be such that $|\vec{a}| = 2, |\vec{b}| = 4$ and $|\vec{c}| = 4$. If the projection of $\vec{b}$ on $\vec{a}$ is equal to the projection of $\vec{c}$ on $\vec{a}$ and $\vec{b}$ is perpendicular to $\vec{c}$, then the value of $|\vec{a} + \vec{b} - \vec{c}|$ is_______

Sol. **6**

$\frac{\vec{b}.\vec{a}}{2} = \frac{\vec{c}.\vec{a}}{2} \quad \boxed{\vec{b}.\vec{a} = \vec{c}.\vec{a}}$

$\boxed{\vec{b}.\vec{c} = 0}$

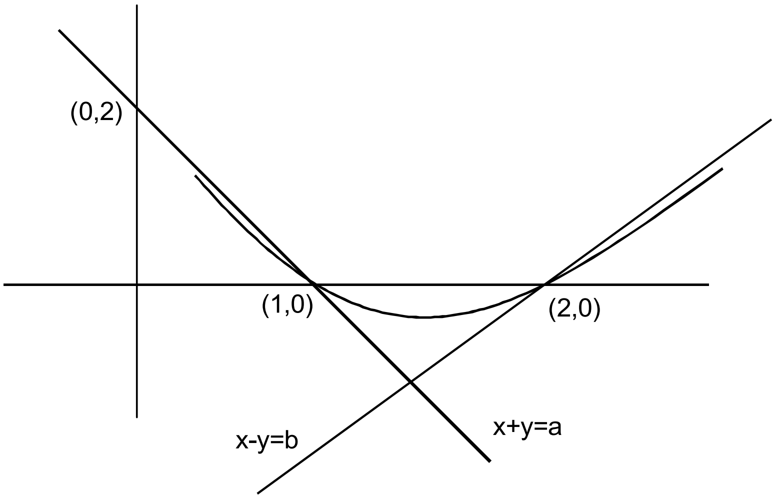
$\left|\vec{a} + \vec{b} - \vec{c}\right| = \sqrt{a^2 + b^2 + c^2 + 2\vec{a}.\vec{b} - 2\vec{b}.\vec{c} - 2\vec{a}.\vec{c}}$

$= \sqrt{4 + 16 + 16}$

$= 6$

Q.24 If the lines x+y=a and x-y=b touch the curve $y=x^2-3x+2$ at the points where the curve intersects the x–axis, then $\frac{a}{b}$ is equal to________

Sol. **0.5**



$y - 0 = -1(x-1)$

$x + y = 1 \Rightarrow a = 1$

$y - 0 = x - 2$

$x - y = 2 = b = 2$

$$\boxed{\frac{a}{b} = \frac{1}{2}}$$

Q.25 In a bombing attack, there is 50% chance that a bomb will hit the target. At least two independent hits are required to destroy the target completely. Then the minimum number of bombs, that must be dropped to ensure that there is at least 99% chance of completely destroying the target, is________

Sol. **11**

Let 'n is total no. of bombs being dropped

at least 2 bombs should hit

$\Rightarrow$ prob $\geq$ 0.99

$P(x\geq 2) \geq 0.99$

$1 - p(x<2) \geq 0.99$

$1 - (p(x=0) + p(x=1)) \geq 0.99$

$$1-\left[4_{c_0}(p)^0 q^n + {}^nC_1 (P)^1 (q)^{n-1}\right] \geq 0.99$$

$$1-\left[q^n + pnq^{n-1}\right] \geq 0.99$$

$$1-\left[\frac{1}{2^n} + \frac{I}{2} \times \frac{1}{2^{n-1}}\right] \geq 0.99$$

$$1 - \frac{1}{2^n}(n+1) \geq 0.99$$

$$0.01 \geq \frac{1}{2^n}(n+1)$$

$2^n \geq 100 + 100n$

$n \geq 11$