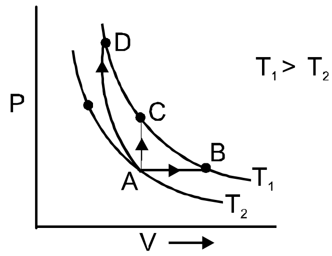
QUESTION PAPER WITH SOLUTION

PHYSICS _ 5 Sep. _ SHIFT - 1

1. Three different processes that can occur in an ideal monoatomic gas are shown in the P vs V diagram. The paths are labelled as $A \to B$, $A \to C$ and $A \to D$. The change in internal energies during these process are taken as E_{AB} , E_{AC} and E_{AD} and the workdone as W_{AB}, W_{AC} and E_{AD}. The correct relation between these parameters are :



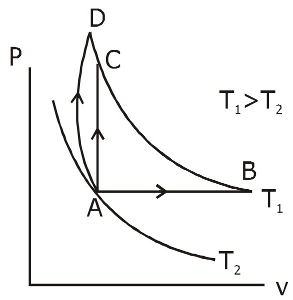
(1) $E_{AB} = E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} < 0$
(2) $E_{AB} > E_{AC} > E_{AD}$, $W_{AB} < W_{AC} < W_{AD}$
(3) $E_{AB} < E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} > W_{AD}$
(4) $E_{AB} = E_{AC} = E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} > 0$

Sol. **1 (Bonus)**

$E_{AB} = E_{AC} = E_{AD}$

$dU = \frac{nfR}{2}(T_f - T_i)$

$w_{AB} > 0\ (+)\ V\uparrow$
$w_{AC} = 0$ V const.
$w_{AD} < 0\ (-)\ V\downarrow$



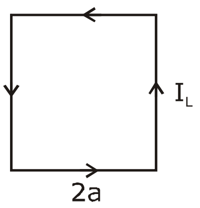
2. With increasing biasing voltage of a photodiode, the photocurrent magnitude :
(1) increases initially and saturates finally
(2) remains constant
(3) increases linearly
(4) increases initially and after attaining certain value, it decreases

Sol. **1**
By theory

3. A square loop of side 2a, and carrying current I, is kept in XZ plane with its centre at origin. A long wire carrying the same current I is placed parallel to the z-axis and passing through the point (0, b, 0), (b > > a). The magnitude of the torque on the loop about z-axis is given by :

(1) $\frac{2\mu_0 I^2 a^3}{\pi b^2}$ (2) $\frac{\mu_0 I^2 a^3}{2\pi b^2}$ (3) $\frac{2\mu_0 I^2 a^2}{\pi b}$ (4) $\frac{\mu_0 I^2 a^2}{2\pi b}$

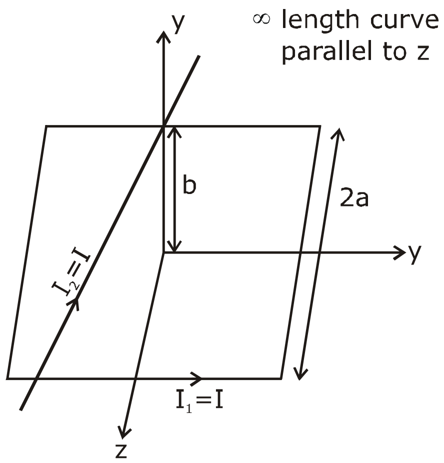
Sol. **3**



$M = I_2 (2a)^2 = 4a^2 I_2$
(magnetic moment)

$B = \frac{\mu_0 I_2}{2\pi b}$

$\tau = MB \sin\theta$



θ angle between B and M [$\theta = 90°$]

$\tau = 4 (a^2 I_2) \frac{\mu_0 I_1}{2\pi b}$

$\tau = \frac{2\mu_0 I_1 I_2 a^2}{\pi b} = \frac{2\mu_0 I^2 a^2}{\pi b}$

4. Assume that the displacement (s) of air is proportional to the pressure difference (Δp) created by a sound wave. Displacement(s) further depends on the speed of sound (ν), density of air (ρ) and the frequency (f). If $\Delta p \sim 10$Pa, $\nu \sim 300$ m/s , $\rho \sim 1$ kg/m^3 and f $\sim$ 1000 Hz, then s will be of the order of (take the multiplicative constant to be 1)

(1) 1 mm (2) 10 mm (3) $\frac{1}{10}$ mm (4) $\frac{3}{100}$ mm

Ans. **4**

$$S_0 = \frac{\Delta P}{\beta k} = \frac{\Delta P}{\rho v^2 \frac{\omega}{v}} = \frac{\Delta P}{\rho v \omega} = \frac{\Delta P}{\rho v 2\pi f}$$

$\therefore$ Proportionally constant = 1

$$S_0 = \frac{\Delta P}{\rho v f}$$

$$= \frac{10}{1 \times 300 \times 1000} \text{ m}$$

$$= \frac{10}{300} \text{ mm}$$

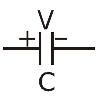
$$= \frac{3}{90}$$

$$\simeq \frac{3}{100} \text{ mm}$$

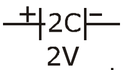
5. Two capacitors of capacitances C and 2C are charged to potential differences V and 2V, respectively. These are then connected in parallel in such a manner that the positive terminal of one is connected to the negative terminal of the other. The final energy of this configuration is :

(1) zero (2) $\frac{9}{2}$ CV^2 (3) $\frac{25}{6}$ CV^2 (4) $\frac{3}{2}$CV^2

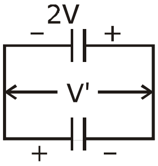
Sol. **4**



$$U_i = \frac{1}{2} Cv^2 + \frac{1}{2} (2C)(2v)^2$$



$$= \frac{9}{2} Cv^2$$



$$q_1 + q_2 = q_1' + q_2'$$

$$-CV + (2C(2V) = (C + 2C)V'$$

$$V' = \frac{3CV}{3C} = V$$

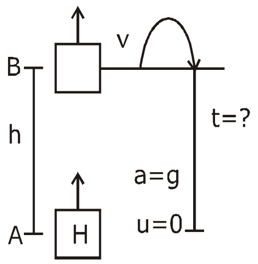
$$U_f = \frac{1}{2} CV^2 + \frac{1}{2} (2C)V^2$$

$$U_f = \frac{3}{2} CV^2$$

6. A helicopter rises from rest on the ground vertically upwards with a constant acceleration g. A food packet is dropped from the helicopter when it is at a height h. The time taken by the packet to reach the ground is close to [g is the acceleration due to gravity] :

(1) $t = 3.4\sqrt{\left(\frac{h}{g}\right)}$ (2) $t = \sqrt{\frac{2h}{3g}}$ (3) $t = \frac{2}{3}\sqrt{\left(\frac{h}{g}\right)}$ (4) $t = 1.8\sqrt{\frac{h}{g}}$

Sol. **1**



$V_B^2 = 0^2 + 2gh$

$V_B = \sqrt{2gh}$

$-h = (V_B)t - \frac{1}{2}gt^2$

$-h = \sqrt{2gh}t - \frac{1}{2}gt^2$

$gt^2 - 2\sqrt{2gh}t - 2h = 0$

$$t = \frac{\sqrt{2gh}t \pm \sqrt{8gh + 8gh}}{2g} = \frac{2\sqrt{2gh} \pm \sqrt{16}gh}{2g} = \frac{\sqrt{2gh} + 2\sqrt{gh}}{g}$$

$$t = \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{h}{g}} = \sqrt{\frac{h}{g}}\left(\sqrt{2} + 2\right) = 3.4\sqrt{\frac{h}{g}}$$

7. A bullet of mass 5 g, travelling with a speed of 210 m/s, strikes a fixed wooden target. One half of its kinetic energy is converted into heat in the bullet while the other half is converted into heat in the wood. The rise of temperature of the bullet if the specific heat of its material is 0.030 cal/(g – °C) (1 cal = 4.2×10^7 ergs) close to :

(1) 38.4°C (2) 87.5°C (3) 83.3°C (4)119.2°C

Sol. **2**

$\left(\frac{1}{2}m\upsilon^2\right) \times \frac{1}{2} = ms\Delta T$ $\qquad$ s = 0.03 cal/y°C

$\frac{\upsilon^2}{4} = 126 \times \Delta T$

$\upsilon^2 = 4 \times 126 \times \Delta T$

$(210)^2 = 4 \times 126 \times \Delta T$

$210 \times 210 = 4 \times 126 \times \Delta T$

$44100 = 504 \times \Delta T$

$\Delta T = \frac{44100}{504} = 87.5°C$

$= \frac{0.03 \times 4.2J}{10^{-3}kgC}$

$= 126$ J/kgC

8. A wheel is rotating freely with an angular speed ω on a shaft. The moment of inertia of the wheel is I and the moment of inertia of the shaft is negligible. Another wheel of moment of inertia 3I initially at rest is suddenly coupled to the same shaft. The resultant fractional loss in the kinetic energy of the system is :

(1) $\frac{3}{4}$ (2) 0 (3) $\frac{5}{6}$ (4) $\frac{1}{4}$

Sol. **1**

$k_i = \frac{1}{2} I\omega^2$

$k_f = \frac{1}{2} (4I) (\omega')^2$

$= 2I \left(\frac{\omega}{4}\right)^2 = \frac{1}{8} I\omega^2$

A.M.C

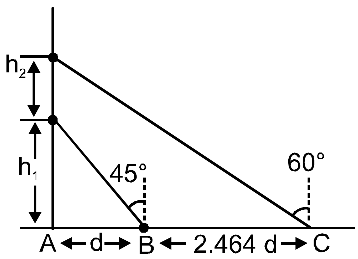
$I\omega = (I+3I)\omega'$

$\omega' = \frac{I\omega}{4I} = \frac{\omega}{4}$

$= \frac{K_i - K_f}{K_i} \Rightarrow \frac{\frac{1}{2}I\omega^2 - \frac{1}{8}I\omega^2}{\frac{1}{2}I\omega^2}$

$\frac{\frac{3}{8}I\omega^2}{\frac{1}{2}I\omega^2} = \frac{3}{4}$

9. A balloon is moving up in air vertically above a point A on the ground. When it is at a height h_1 , a girl standing at a distance d(point B) from A (see figure) sees it at an angle 45° with respect to the vertical. When the balloon climbs up a further height h_2, it is seen at an angle 60° with respect to the vertical if the girl moves further by a distance 2.464 d (point C). Then the height h_2 is (given tan 30° = 0.5774) :



(1) 0.464 d (2) d (3) 0.732 d (4) 1.464 d

Sol. **2**

ΔABD

$$\tan 45 = \frac{h_1}{d}$$

$$\Rightarrow 1 = \frac{h_1}{d} \Rightarrow h_1 = d$$

ΔACE

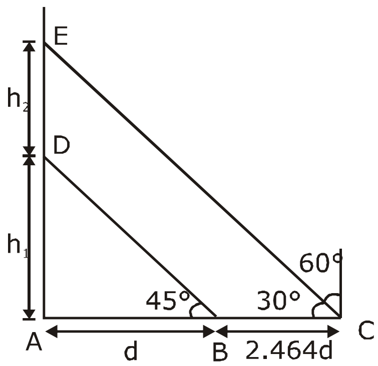
$$\tan 30 = \frac{h_1 + h_2}{d + 2.464d}$$

$$0.5774 = \frac{d + h_2}{3.464d}$$

$d + h_2 = 0.5774 \times 3.464 \times d$

$h_2 = 2.0001136d - d$

$h_2 = 2.000d - d = d$



10. An electrical power line, having a total resistance of $2\,\Omega$, delivers 1 kW at 220 V. The efficiency of the transmission line is approximately :

(1) 72% (2) 91 % (3) 85% (4) 96%

Sol. 4

$$\eta = \frac{P_{out}}{(P_{out} + P_{loss})} \times 100$$

$$I = \frac{P}{V}$$

$$= \frac{100}{220} = \frac{5}{11}\ A$$

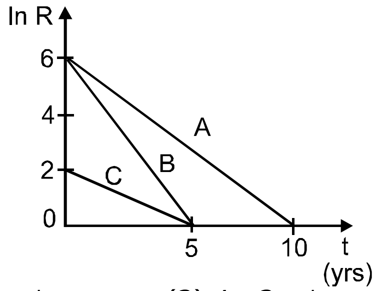
$P_{loss} = I^2R$

$$= \left(\frac{50}{11}\right)^2 \times 2 = 41.322$$

$$\eta = \frac{1000}{1000 + 41.322} \times 100$$

$\eta = 96\%$

11. Activities of three radioactive substances A, B and C are represented by the curves A, B and C, in the figure. Then their half-lives $T_{\frac{1}{2}}(A) : T_{\frac{1}{2}}(B) : T_{\frac{1}{2}}(C)$ are in the ratio :



(1) 3 : 2 : 1 (2) 2 : 1 : 1 (3) 4 : 3 : 1 (4) 2 : 1 : 3

Sol. 4

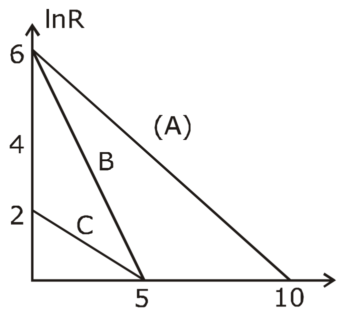
$R_A = R_0Ae^{-\frac{\ln 2}{T}(t)}$

$\ln(R_A) = \ln(R_0A) - \lambda t$ (λ = slope of graph)

$$\lambda_A = \frac{6}{10} = \frac{\ln 2}{T_A}$$

$$\lambda_B = \frac{6}{5} = \frac{\ln 2}{T_B}$$

$$\lambda_C = \frac{2}{5} = \frac{\ln 2}{T_C}$$



$$\left.\begin{array}{l} T_A = \frac{5}{3}\ln 2 \\ T_B = \frac{5}{6}\ln 2 \\ T_C = \frac{5}{2}\ln 2 \end{array}\right\} \Rightarrow T_A : T_B : T_C = \frac{1}{3} : \frac{1}{6} : \frac{1}{2}$$

$= 2 : 1 : 3$

12. The value of the acceleration due to gravity is g_1 at a height $h = \frac{R}{2}$ (R = radius of the earth) from the surface of the earth. It is again equal to g_1 at a depth d below the surface of the earth. The ratio $\left(\frac{d}{R}\right)$ equals :

(1) $\frac{4}{9}$ (2) $\frac{1}{3}$ (3) $\frac{5}{9}$ (4) $\frac{7}{9}$

Sol. **3**

$g_{\text{at high}} = g_{\text{at depth}}$

$g_{\text{surface}} = \frac{GM}{R^2}$

$$g\left(1 - \frac{d}{R}\right) = \frac{GMe}{(R+h)^2}$$

$$g\left(1 - \frac{d}{R}\right) = \frac{GM}{R^2\left(1+\frac{h}{R}\right)^2} = \frac{g}{\left(1+\frac{R}{2R}\right)^2} = \frac{4g}{9}$$

$$\frac{d}{R} = 1 - \frac{4}{9} = \frac{5}{9}$$

13. A hollow spherical shell at outer radius R floats just submerged under the water surface. The inner radius of the shell is r. If the specific gravity of the shell material is $\frac{27}{8}$ w.r.t water, the value of r is:

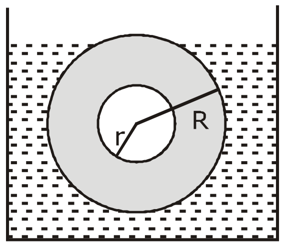
(1) $\frac{4}{9}$ R (2) $\frac{8}{9}$R (3) $\frac{1}{3}$R (4) $\frac{2}{3}$R

Sol. **2**

$F_B = mg$

$\rho_\ell V_{body} g$ (displaced water) $=$ $\rho_b V_b g$ where mater present

$$\frac{4}{3}\pi R^3 = \frac{\rho_b}{\rho_\ell}\left(\frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3\right)$$



$$R^3 = \frac{27}{8}(R^3 - r^3)$$

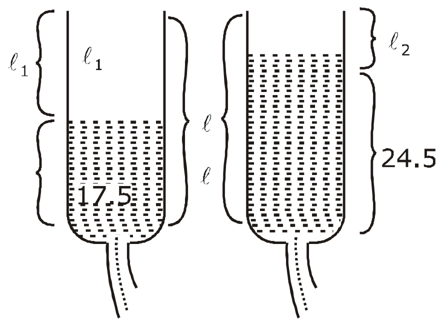
$$\frac{8}{27}R^3 = R^3 - r^3 \Rightarrow r^3 = R^3 - \frac{8R^3}{27} = \frac{19}{27}R^3$$

$$r = \frac{(19)^{1/3}}{3}R \approx 0.88 \approx \frac{8}{9}R$$

14. In a resonance tube experiment when the tube is filled with water up to a height of 17.0 cm from bottom, it resonates with a given tuning fork. When the water level is raised the next resonance with the same tuning fork occurs at a height of 24.5 cm. If the velocity of sound in air is 330 m/s, the tuning fork frequency is :

(1) 2200 Hz (2) 550 Hz (3) 3300 Hz (4) 1100 Hz

Sol. **1**

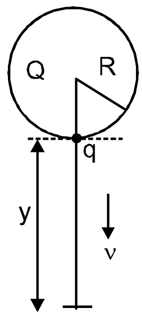


$\ell_1 = \ell - 17$
$\ell_2 = \ell - 24.5$
$v = 2f(\ell_1 - \ell_2)$
$330 = 2 \times f \times [(f \times [(\ell - 17) - (\ell - 24.5)] \times 10^{-2}$
$165 = f \times 7.5 \times 10^{-2}$

$$f = \frac{165 \times 1000}{7.5}$$

$f = 2200$ Hz

15. A solid sphere of radius R carries a charge Q + q distributed uniformly over its volume. A very small point like piece of it of mass m gets detached from the bottom of the sphere and falls down vertically under gravity. This piece carries charge q. If it acquires a speed v when it has fallen through a vertical height y (see figure), then : (assume the remaining portion to be spherical).



(1) $\nu^2 = 2y\left[\dfrac{qQ}{4\pi\in_0 R(R+y)m}+g\right]$

(2) $\nu^2 = 2y\left[\dfrac{QqR}{4\pi\in_0 (R+y)^3 m}+g\right]$

(3) $\nu^2 = y\left[\dfrac{qQ}{4\pi\in_0 R(R+y)m}+g\right]$

(4) $\nu^2 = y\left[\dfrac{qQ}{4\pi\in_0 R^2 ym}+g\right]$

Sol. **1**

M.E.C.

$K_A + U_A = K_B + U_B$

$0 + mgy + qV_A = \dfrac{1}{2}mv^2 + 0 + (+qv_B)$

$mgy + qV_A = \dfrac{1}{2}mv^2 + q(V_B)$

$mgy + \dfrac{qk(Q)}{R} = \dfrac{1}{2}mv^2 + \dfrac{qk(Q)}{R+y}$

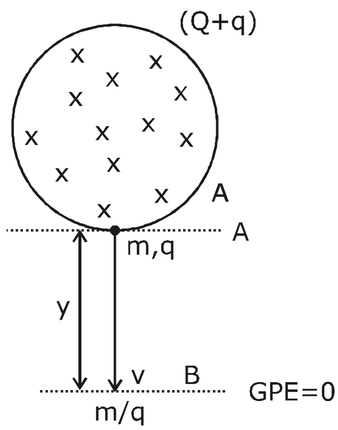
$\dfrac{1}{2}mv^2 = -\dfrac{kq(Q)}{R+y} + \dfrac{kq(Q)}{R} + mgy$

$\dfrac{mv^2}{2} = \dfrac{-kq(Q)R + kq(Q)(R+y)}{R(R+y)} + mgy$

$v^2 = \dfrac{2}{m}\left[\dfrac{-kqQR + kqQR + kqQ}{R(R+y)} + mgy\right]$

$v^2 = \dfrac{2}{m}\left[\dfrac{kq(Q)y}{R(R+y)} + mgy\right]$

$v^2 = 2y\left[\dfrac{q(Q)}{4\pi\varepsilon_0 R(R+y)m} + g\right] = 2y\left[\dfrac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right]$



potential at A

$V_A = \dfrac{k(Q+q)}{R}$

$V_B = \dfrac{k(Q+q)}{R+y}$

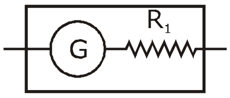
16. A galvanometer of resistance G is converted into a voltmeter of range 0 – 1V by connecting a resistance R_1 in series with it. The additional resistance that should be connected in series with R_1 to increase the range of the voltmeter to 0 – 2V will be :

(1) G (2) R_1 (3) $R_1 - G$ (4) $R_1 + G$

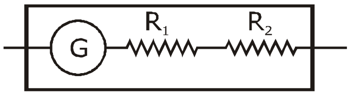
Sol. **4**

$V = I(R_1 + G)$

$$\frac{1}{2} = \frac{I(R_1 + G).....(i)}{I(R_1 + R_2 + G).....(ii)}$$



$$\frac{1}{2} = \frac{R_1 + G}{R_1 + R_2 + G}$$



$R_1 + R_2 + G = 2R_1 + 2G$

$R_2 = R_1 + G$

17. Number of molecules in a volume of 4 cm^3 of a perfect monoatomic gas at some temperature T and at a pressure of 2 cm of mercury is close to ? (Given, mean kinetic energy of a molecule (at T) is 4×10^{-14} erg, g = 980 cm/s^2, density of mercury = 13.6 g/cm^3)

(1) 4.0×10^{18} (2) 4.0×10^{6} (3) 5.8×10^{16} (4) 5.8×10^{18}

Sol. **1**

$KE = \frac{3}{2}kT \Rightarrow \left(T = \frac{2E}{3k}\right)$, PV = NkT

$P = \rho gh$, $V = 4cm^3$

$13.6 \times 10^3 \times 9.8 \times 2 \times 10^{-2} \times 4 \times 10^{-6}$

$= Nk \times \frac{2E}{3k} = \frac{N \times 2}{3} \times 4 \times 10^{-14} \times 10^7$

$$N = \frac{13.6 \times 19.6 \times 4 \times 10^{-5} \times 3 \times 10}{8}$$

$N = 399.84 \times 10^{16}$

$= 3.99 \times 10^{18}$

$N = 4 \times 10^{18}$

18. An electron is constrained to move along the y-axis with a speed of 0.1 c (c is the speed of light) in the presence of electromagnetic wave, whose electric field is $\vec{E} = 30\,\hat{j}\,\sin(1.5 \times 10^7 t - 5 \times 10^{-2} x)$ V/m. The maximum magnetic force experienced by the electron will be :

(given $c = 3 \times 10^8\ ms^{-1}$ and electron charge = 1.6×10^{-19}C)

(1) 2.4×10^{-18} N (2) 4.8×10^{-19} N (3) 3.2×10^{-18} N (4) 1.6×10^{-19} N

Sol. **2**

v_e = 0.1C along y-axis direction of emwave - along (x)

$\vec{E} = \vec{v} \times \vec{B}$

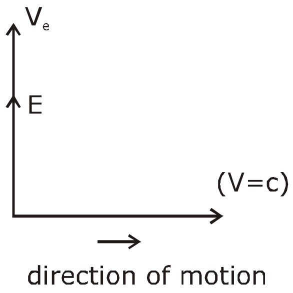
E = CB $\Rightarrow$ B = E/C

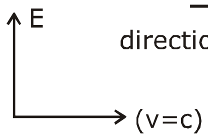
$\therefore$ force on e^- will be max.

If B is $\perp$ to y-along z-axis

[$\because$ E also $\perp$ B, B also $\perp$ to direction of motion of wave]

$\therefore$ B $\rightarrow$ along Bz ($-$z) as





$$B = \frac{30}{C} \sin(1.5 \times 10^7 t - 5 \times 10^{-2} x)$$

$$B_{max} = \frac{30}{3 \times 10^8} = 10^{-7}\ T$$

θ = 90 between v_e & B so F_{max} = qvB

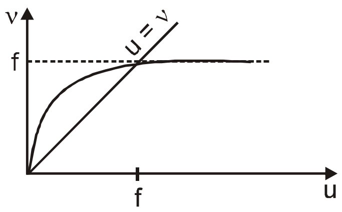
$$F_{max} = e \times (0.1 \times C) \times \frac{30}{C}$$

$= 1.6 \times 10^{-19} \times 3$

$F_{max} = 4.8 \times 10^{-19}$ N

19. For a concave lens of focal length f, the relation between object and image distances u and v, respectively, from its pole can best be represented by (u = v is the reference line) :

(1)



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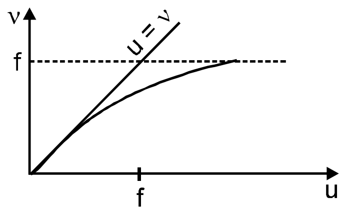

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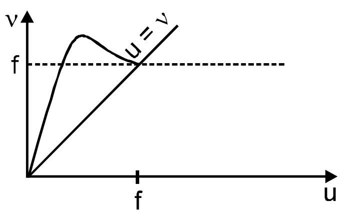
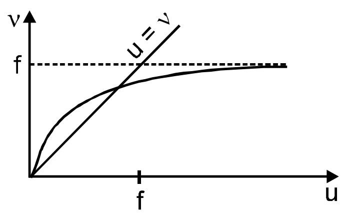
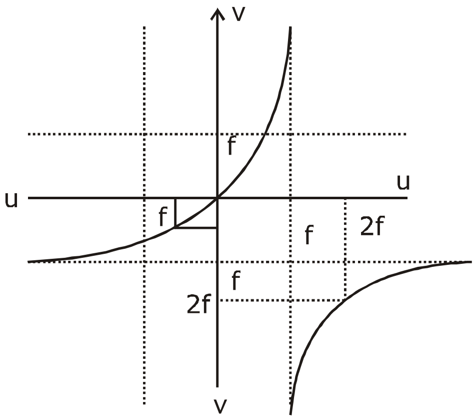


(4)



Sol. **4**

Concave lens graph u v/s v by u-v graph theory



20. A physical quantity z depends on four observables a, b, c and d, as $z = \frac{a^2 b^{\frac{2}{3}}}{\sqrt{c}\, d^3}$. The percentages of error in the measurement of a, b, c and d are 2% , 1.5%, 4% and 2.5% respectively. The percentage of error in z is :

(1) 16.5 % (2) 12.25 % (3) 13.5% (4) 14.5%

Sol. **4**

$z = a^2 b^{2/3} c^{-1/2} d^{-3}$

$$100 \times \frac{dz}{z} = \left(2\frac{da}{a} + \frac{2}{3}\frac{db}{b} + \frac{1}{2}\frac{dc}{c} + 3\frac{d(d)}{(d)}\right) \times 100$$

% error in z

$$= \left(2 \times 2 + \frac{2}{3} \times 1.5 + \frac{1}{2} \times 4 + 3 \times 2.5\right)\%$$

$= 4 + 1 + 2 + 7.5$

$= 14.5\ \%$

21. A particle of mass 200 Me V/c^2 collides with a hydrogen atom at rest. Soon after the collision the particle comes to rest, and the atom recoils and goes to its first excited state. The initial kinetic energy of the particle (in eV) is $\frac{N}{4}$. The value of N is :

(Given the mass of the hydrogen atom to be 1 GeV/c^2)______.

Sol. **51**

m_H = 1GeVC^2 = 1000 MeV/C^2, $m_{particle}$ = 200 meV/c^2 = m

$\therefore mv_0 + 0 = 0 + 5\,mV' \Rightarrow v' = \frac{v_0}{5}$

loss in KE

$= \frac{1}{2}\ mv_0{}^2 - \frac{1}{2}\ (5m)\left(\frac{v_0}{5}\right)^2$

$= \frac{4}{5}\left(\frac{mv_0^2}{2}\right) = \frac{4}{5}\ k$

$\frac{4}{5}\ k = 10.2$

$k = 12.75\ eV = \frac{12.75}{100} = \frac{51}{4}$

so = n = 51

22. Two concentric circular coils, C_1 and C_2, are placed in the XY plane. C_1 has 500 turns, and a radius of 1 cm. C_2 has 200 turns and radius of 20 cm. C_2 carries a time dependent current $I(t) = (5t^2-2t + 3)$ A where t is in s. The emf induced in C_1 (in mV), at the instant t = 1s is $\frac{4}{x}$. The value of x is_____.

Sol. **5**

$B_2 = \frac{\mu_0 I_2 N_2}{2R_2}$

$\phi = N_1B_2\pi R_1{}^2 = N_1N_2\ \frac{\mu_0 I}{2R_2}\ \pi R_1{}^2$

$e = \frac{d\phi}{dt}$

$\phi = \frac{500\times 200\times 4\pi\times 10^{-7}\times(5t^2-2t-3)\pi(10^{-2})^2}{2\times 20\times 10^{-2}}$

$\frac{10^5\times 4\pi^2\times 10^{-7}(5t^2-2t+3)\times 10^{-4}}{40\times 10^{-2}}$

$\phi = (5t^2 - 2t + 3) \times 10^{-4}$

$e = \left|\frac{d\phi}{dt}\right| = (10t-2) \times 10^{-4}$

t = 1sec

$e = 8 \times 10^{-4} = 0.8\ mV = \frac{0.8}{10} = \frac{4}{5}$

x = 5

23. A beam of electrons of energy E scatters from a target having atomic spacing of 1Å. The first maximum intensity occurs at $\theta = 60°$ Then E (in eV) is ______.
(Planck constant $h = 6.64 \times 10^{-34}$ Js, 1 eV = 1.6×10^{-19} J, electron mass $m = 9.1 \times 10^{-31}$ kg)

Sol. **5**

$$2d\sin\theta = n\lambda = 1 \times \sqrt{\frac{150}{v} \times 10^{-10}}\ , \theta = 90 - \frac{\phi}{2}$$

$$2 \times 10^{-10} \times \sin 60 = \sqrt{\frac{150}{V}} \times 10^{-10}\ , \theta = 90 - \frac{60}{2} = 60$$

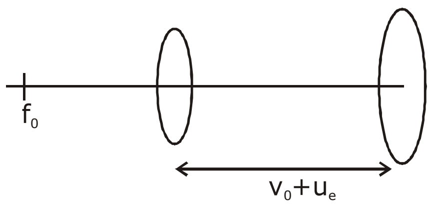
$$2 \times \frac{\sqrt{3}}{2} = \sqrt{\frac{150}{V}}$$

$$V = \frac{150}{3} = 50 \text{ volt}$$

$E = ev = 50ev$

24. A compound microscope consists of an objective lens of focal length 1 cm and an eye piece of focal length 5 cm with a separation of 10 cm. The distance between an object and the objective lens, at which the strain on the eye is minimum is $\frac{n}{40}$ cm. The value of n is _____.

Sol. **50**



$f_0 = 1$cm, $f_e = 5$ cm, $u_0 = ?$
final image at (∞)
($v_e = \infty$)

$v_0 + u_e = 10$cm ...(i) $\qquad L = v_0 + u_e = 10$ cm

$v_0 + 5 = 10 \qquad \frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$

$v_0 = 5$ cm $\qquad \frac{1}{\infty} - \frac{1}{u_e} = \frac{1}{5}$

$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0} \qquad u_e = -5$cm

$\frac{1}{5} - \frac{1}{u_0} = \frac{1}{1} \qquad |u_e| = 5$

$$\frac{1}{u_0} = \frac{1}{5} - 1 = -\frac{4}{5} \Rightarrow u_0 = -\frac{4}{5}$$

$$|u_0| = \frac{5}{4} = \frac{50}{40} = \frac{n}{40}$$

$\therefore$ n = 50

25. A force $\vec{F} = (\hat{i} + 2\hat{j} + 3\hat{k})$ N acts at a point $(4\hat{i} + 3\hat{j} - \hat{k})$ m. Then the magnitude of torque about the point $(\hat{i} + 2\hat{j} + \hat{k})$ m will be $\sqrt{x}$ N–m. The value of x is _______.

Sol. **195**

$$\vec{\tau} = \vec{r} \times F = (3\hat{i} + \hat{j} - 2\hat{k}) \times (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= \begin{vmatrix} i & j & k \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix}$$

$$\hat{i}(3 + 4) - \hat{j}(9 + 2) + \hat{k}(6 - 1)$$

$$\vec{\tau} = 7\hat{j} - 11\hat{j} + 5\hat{k}$$

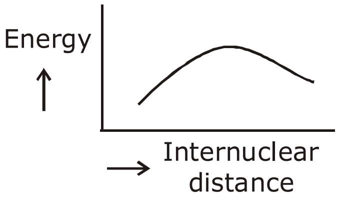
$$|\vec{\tau}| = \sqrt{49 + 121 + 25} = \sqrt{195}$$

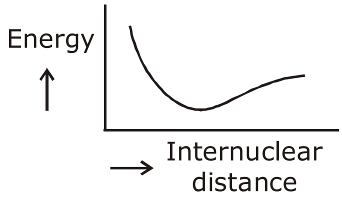
x = 195

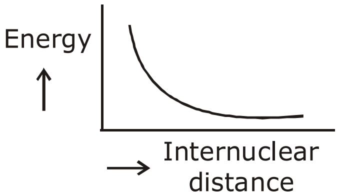
QUESTION PAPER WITH SOLUTION

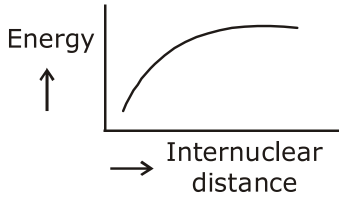
CHEMISTRY _ 5 Sep. _ SHIFT - 1

1. The potential energy curve for the H_2 molecule as a function of internuclear distance is:

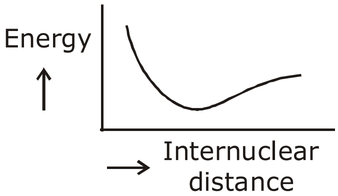
(1) 

(2) 

(3) 

(4) 

Sol. **2**



2. The most appropriate reagent for conversion of C_2H_5CN into $CH_3CH_2CH_2NH_2$ is:
(1) $NaBH_4$ (2) $Na(CN)BH_3$ (3) CaH_2 (4) $LiAlH_4$

Sol. **4**

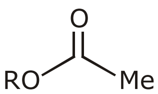
$CH_3CH_2CN \xrightarrow{LiAlH_4} CH_3CH_2CH_2NH_2$

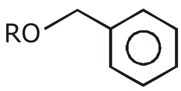
3. Which of the following is not an essential amino acid?
(1) Valine (2) Tyrosine (3) Lysine (4) Leucine

Sol. **2**

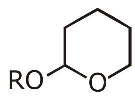
Tyrosine in not an essential amino acid

4. Which of the following derivatives of alcohols is unstable in an aqueous base?

(1) 

(2) 

(3) RO–CMe_3

(4) 

Sol. **1**

Hydrolysis of ester occurs in basic medium.

5. The structure of PCl_5 in the solid state is:
(1) Square planar $[PCl_4]^+$ and octahedral $[PCl_6]^-$
(2) Tetrahedral $[PCl_4]^+$ and octahedral $[PCl_6]^-$
(3) Trigonal bipyramidal
(4) Square pyramidal

Sol. **2**

In solid state PCl_5 exist in Ionpair i.e. (PCl_4^+) and (PCl_6^-)

PCl_4^+ (sp^3_ tetrahedral)

PCl_6^- (sp^3d^2) – octahedral)

6. A diatomic molecule X_2 has a body-centred cubic (bcc) structure with a cell edge of 300 pm. The density of the molecule is 6.17 g cm^{-3}. The number of molecules present in 200 g of X_2 is:(Avogadro constant $(N_A) = 6 \times 10^{23}$ mol^{-1})

(1) 8 N_A (2) 2 N_A (3) 40 N_A (4) 4 N_A

Sol. **4**

$X_2 \rightarrow$ BCC

a = 300pm

$$d = 6.17 g/cm^3 = \frac{2 \times GMM}{6 \times 10^{23} \times (300 \times 10^{-10})^3}$$

$$GMM = \frac{6.17 \times 6 \times 9 \times 3 \times 10^{-1}}{2}$$

$$GMM = 81 \times 6.17 \times 10^{-1}$$

$$= 49.97 \text{ g/mol}$$

$$\text{No. of molecules} = \frac{200g}{49.97 g/mol} = 4 \text{ mol}$$

$$= 4N_A$$

7. The equation that represents the water-gas shift reaction is:

(1) $CO(g) + H_2O(g) \xrightarrow[\text{Catalyst}]{673 K} CO_2(g) + H_2(g)$

(2) $2C(s) + O_2(g) + 4N_2(g) \xrightarrow{1273 K} 2CO(g) + 4N_2(g)$

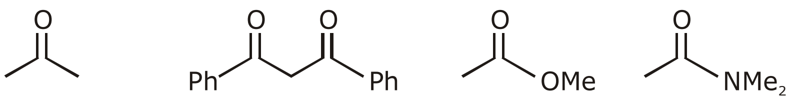
(3) $C(s) + H_2O(g) \xrightarrow{1270 K} CO(g) + H_2(g)$

(4) $CH_4(g) + H_2O(g) \xrightarrow[\text{Ni}]{1270 K} CO(g) + 3H_2(g)$

Sol. **1**

Fact

8. The increasing order of the acidity of the α-hydrogen of the following compounds is:



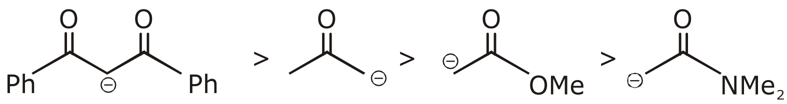
(A) (B) (C) (D)

(1) (D) < (C) < (A) < (B) (2) (A) < (C) < (D) < (B)

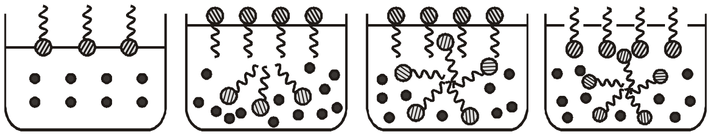
(3) (C) < (A) < (B) < (D) (4) (B) < (C) < (A) < (D)

Sol. 1

Stability order



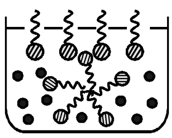
9. Identify the correct molecular picture showing what happens at the critical micellar concentration (CMC) of an aqueous solution of a surfactant (⊘ polar head; ∿∿ non-polar tail; • water).



(A) (B) (C) (D)

(1) (B) (2) (A) (3) (C) (4) (D)

Sol. 4



10. If a person is suffering from the deficiency of nor-adrenaline, what kind of drug can be suggested?

(1) Antihistamine (2) Antidepressant

(3) Anti-inflammatory (4) Analgesic

Sol. 2

If nor-adrenaline is low, person may suffer from depression. Hence, anti depressant drug is suggested.

11. The values of the crystal field stabilization energies for a high spin d^6 metal ion in octahedral and tetrahedral fields, respectively, are:

(1) $-2.4\ \Delta_o$ and $-0.6\ \Delta_t$ (2) $-1.6\ \Delta_o$ and $-0.4\ \Delta_t$

(3) $-0.4\ \Delta_o$ and $-0.27\ \Delta_t$ (4) $-0.4\ \Delta_o$ and $-0.6\ \Delta_t$

Sol. 4

d^6(octahedral) $\rightarrow$ high spin complex

$= t_{2g^4}\ eg^2$

$$\text{CFSE} = \left(-\frac{2}{5}\times 4+\frac{3}{5}\times 2\right)\Delta_0$$

$$= \left(\frac{-8+6}{5}\right)\Delta_0$$

$= -0.4\Delta_0$

d^6 (tetrahedral) $\rightarrow$ high spin complex

$= eg^3\, t_{2g^3}$

$$\text{CFSE} = \left(-\frac{3}{5}\times 3+\frac{2}{5}\times 3\right)\Delta_t$$

$= -0.6\Delta_t$

12. A flask contains a mixture of compounds A and B. Both compounds decompose by first-order kinetics. The half-lives for A and B are 300 s and 180 s, respectively. If the concentrations of A and B are equal initially, the time required for the concentration of A to be four times that of B (in s) is: (Use ln 2 = 0.693)

(1) 180 (2) 300 (3) 120 (4) 900

Sol. **4**

$A_t = A_0 . e^{-k_1 t}$

$B_t = B_0 . e^{-k_2 t}$

$k_1 = \frac{\ln 2}{300}$

$k_2 = \frac{\ln 2}{180}$

A_t and B_t are related as [A] = 4[B]

$A_0 . e^{-k_1 t} = 4 \times B_0 . e^{-k_2 t}$

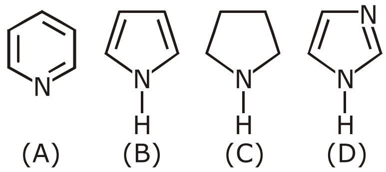
$\frac{t}{180} - \frac{t}{300} = 2$

$\frac{t}{3} - \frac{t}{5} = 120$

$\frac{2t}{15} = 120$

t = 900 sec

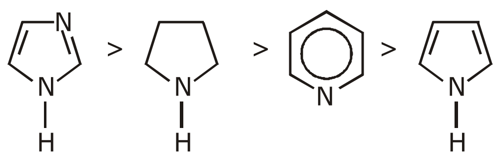
13. The increasing order of basicity of the following compounds is:



(1) (D) < (A) < (B) < (C) (2) (A) < (B) < (C) < (D)
(3) (B) < (A) < (D) < (C) (4) (B) < (A) < (C) < (D)

Sol. **4**

Correct order of basicity



14. The condition that indicates a polluted environment is:
(1) pH of rain water to be 5.6 (2) BOD value of 5 ppm
(3) 0.03% of CO_2 in the atmosphere (4) eutrophication

Sol **4**
Eutrophication is the condition in which excessive richness of nutrients in a lake or water body, which causes dense growth of plant life and BOD increases.

15. In the sixth period, the orbitals that are filled are:
(1) 6s, 5d, 5f, 6p (2) 6s, 4f, 5d, 6p (3) 6s, 6p, 6d, 6f (4) 6s, 5f, 6d, 6p

Sol. **2**
(Fact) $\rightarrow$ energy order of orbital's according to Aufbau principle
6s < 4f < 5d < 6p

16. The difference between the radii of 3rd and 4th orbits of Li^{2+} is ΔR_1. The difference between the radii of 3rd and 4th orbits of He^+ is ΔR_2. Ratio $\Delta R_1 : \Delta R_2$ is:
(1) 8 : 3 (2) 3 : 8 (3) 3 : 2 (4) 2 : 3

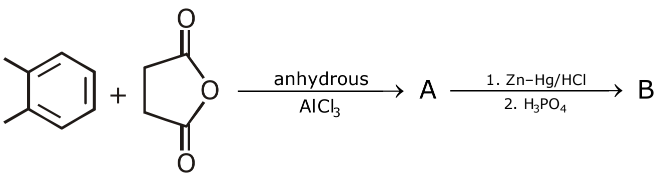
Sol. **4**

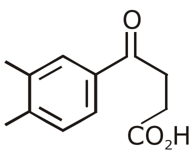
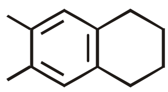
$$(R_4 - R_3)_{Li^{+2}} = \frac{0.529}{3}\left\{4^2 - 3^2\right\} = \Delta R_1$$

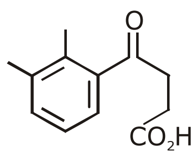
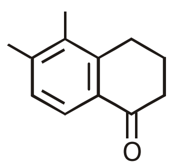
$$(R_4 - R_3)_{He^{+2}} = \frac{0.529}{2}\left\{4^2 - 3^2\right\} = \Delta R_2$$

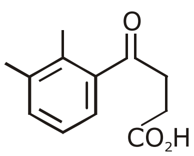
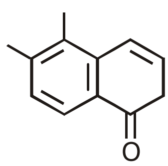
$$\frac{\Delta R_1}{\Delta R_2} = \frac{1/3}{1/2} = \frac{2}{3}$$

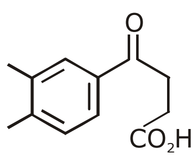
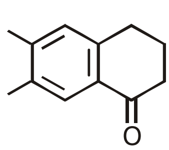
17. In the following reaction sequence the major products A and B are:



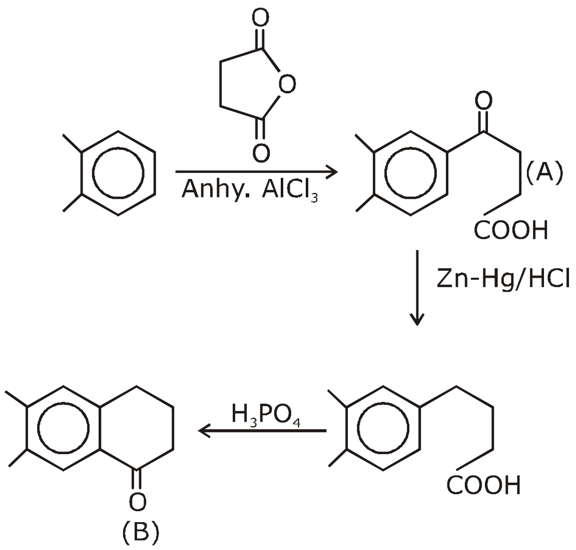
(1) A =  ; B = 

(2) A =  ; B = 

(3) A =  ; B = 

(4) A =  ; B = 

Sol. **4**



18. The correct electronic configuration and spin-only magnetic moment (BM) of Gd^{3+} (Z = 64), respectively, are:

(1) [Xe] $5f^7$ and 7.9 (2) [Xe] $4f^7$ and 7.9 (3) [Xe] $5f^7$ and 8.9 (4) [Xe] $4f^7$ and 8.9

Sol. **2**

Gd $\rightarrow [Xe]^{54}\ 4f^7\ 5d^1\ 6s^2$

Z=64

$\Big\downarrow -3e^{\ominus}$

$Gd^{+3} = [Xe]^{54}\ 4f^7$

$\mu = \sqrt{7(7+2)} = \sqrt{63}$

= 7.9 BM

19. An Ellingham diagram provides information about:

(1) The pressure dependence of the standard electrode potentials of reduction reactions involved in the extraction of metals.

(2) The conditions of pH and potential under which a species is thermodynamically stable.

(3) The kinetics of the reduction process.

(4) The temperature dependence of the standard Gibbs energies of formation of some metal oxides.

Sol. **4**

Fact

20. Consider the following reaction:

$N_2O_4\ (g) \rightleftharpoons 2NO_2\ (g);\ \Delta H^o = +58\ kJ$

For each of the following cases (a, b), the direction in which the equilibrium shifts is:

(a) Temperature is decreased.

(b) Pressure is increased by adding N_2 at constant T.

(1) (a) towards reactant, (b) towards product

(2) (a) towards reactant, (b) no change

(3) (a) towards product, (b) towards reactant

(4) (a) towards product, (b) no change

Sol. **2**

$N_2O_4(g) \rightleftharpoons 2NO_2(g)$

$\Delta H^o = +\ 58\ kJ$

(towards reactant)

(a) temp $\downarrow \Rightarrow$ Backward shift as it is endothermic reaction

(b) As 'N_2' will not react with both N_2O_4 & NO_2, as moles increases in reactants, as much as in products, a = hence there is no change in equilibria.

$\therefore$ no change

21. The minimum number of moles of O_2 required for complete combustion of 1 mole of propane and 2 moles of butane is ______.

Sol. **18**

$C_3H_8 + 5O_2 \rightarrow 3CO_2 + 4H_2O$

1 mol 5 mol

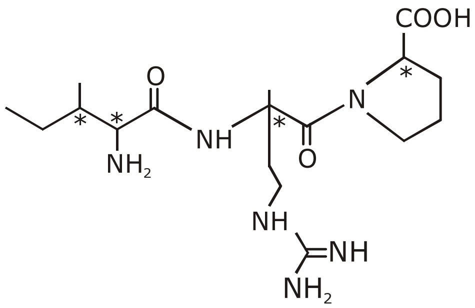
$$C_4H_{10} + \frac{13}{2}O_2 \rightarrow 4CO_2 + 5H_2O$$

2 mol 13 mol

Total required mol of O_2 = 5 + 13 = 18

22. The number of chiral carbon(s) present in piptide, Iie-Arg-Pro, is _______ .

Sol. **4**



23. A soft drink was bottled with a partial pressure of CO_2 of 3 bar over the liquid at room temperature. The partial pressure of CO_2 over the solution approaches a value of 30 bar when 44 g of CO_2 is dissolved in 1 kg of water at room temperature. The approximate pH of the soft drink is _______ $\times 10^{-1}$.

(First dissociation constant of $H_2CO_3 = 4.0 \times 10^{-7}$; log 2 = 0.3; density of the soft drink = 1 g mL^{-1})

Sol. **37**

$CO_2 + H_2O \rightarrow H_2CO_3$

30 bar ....$\rightarrow$ 1 m/lit.

3 bar ....$\rightarrow$ 0.1 m/lit

$$H_2CO_3 \rightleftharpoons H^{\oplus} + HCO_3^-$$

t = 0	0.1	0	0
Eq.	$0.1(1-\alpha)$	0.1α	0.1α

$$4 \times 10^{-7} = \frac{0.1\alpha^2}{1-\alpha}$$

$(1-\alpha) \simeq 1$

$\alpha^2 = 4 \times 10^{-6}$

$\alpha = 2 \times 10^{-3}$

$[H^+] = 2 \times 10^{-4}M$

$pH = -[-4 \times \log(2)] = 3.7 = 37 \times 10^{-1}$

24. An oxidation-reduction reaction in which 3 electrons are transferred has a ΔG^o of 17.37 kJ mol^{-1} at 25ºC. The value of E°_{cell} (in V) is ______ $\times 10^{-2}$.

(1 F = 96,500 C mol^{-1})

Sol. **6**

$\Delta G^\circ = -nFE^\circ$

$17.37 \times 1000 = -3 \times 96500 \times E^\circ$

$$E^\circ = \frac{17370}{3 \times 96500}$$

$$E^\circ = \frac{579}{9650} \text{ volt}$$

$= 0.06 = 6 \times 10^{-2}$ volt

Ans. 6

25. The total number of coordination sites in ethylenediaminetetraacetate ($EDTA^{4-}$) is _____.

Sol. **6**

$EDTA^{4-}$ is hexadentate ligand

QUESTION PAPER WITH SOLUTION

MATHEMATICS _ 5 Sep. _ SHIFT - 1

Q.1 If the volume of a parallelopiped, whose coterminus edges are given by the vectors $\vec{a} = \hat{i} + \hat{j} + n\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - n\hat{k}$ and $\vec{c} = \hat{i} + n\hat{j} + 3\hat{k}$ $(n \geq 0)$, is 158 cu. units, then:

(1) $\vec{a}.\vec{c} = 17$ (2) $\vec{b}.\vec{c} = 10$ (3) n=9 (4) n=7

Sol. **2**

$$\begin{vmatrix} 1 & 1 & n \\ 2 & 4 & -n \\ 1 & n & 3 \end{vmatrix} = 158$$

$(12 + n^2) - (6+n) + n(2n-4) = 158$
$3n^2 - 5n + 6 - 158 = 0$
$3n^2 - 5n - 152 = 0$
$3n^2 - 24n + 19n - 152 = 0$
$(3n + 19)(n-8) = 0$
$\Rightarrow n = 8$
$\Rightarrow \vec{b}.\vec{c} = 10$

Q.2 A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be:
(1) 63 (2) 54 (3) 38 (4) 36

Sol. 4

n(coffee) = $\frac{73}{100}$

n(tea) = $\frac{65}{100}$

$n(T \cap C) = \frac{x}{100}$

$n(C \cup T)$ = n(C) + n(T) − x $\leq$ 100
= 73 + 65 − x $\leq$ 100
$\Rightarrow x \geq 38$
Ans. 36

Q.3 The mean and variance of 7 observations are 8 and 16, respectively. If five observations are 2,4,10,12,14, then the absolute difference of the remaining two observations is:
(1) 1 (2) 4 (3) 3 (4) 2

Sol. **4**

$$\text{Var}(x) = \sum \frac{x_i^2}{n} - \left(\bar{x}\right)^2$$

$16 = \dfrac{x_1^2 + x_2^2 + x_4^2 + x_5^2 + x_6^2 + x_7^2}{7} - 64$

$80 \times 7 = x_1^2 + x_2^2 + x_3^2 + ... + x_7^2$

Now, $x_2^6 + x_7^2 = 560 - (x_1^2 + ... x_5^2)$

$x_6^2 + x_7^2 = 560 - (4 + 16 + 100 + 144 + 196)$

$x_6^2 + x_7^2 = 100$ ......(1)

Now, $\dfrac{x_1 + x_2 + ... + x_7}{7} = 8$

$x_6 + x_7 = 14$ ......(2)

from (1) & (2)

$(x_6 + x_7)^2 - 2x_6 x_7 = 100$

$2x_6x_7 = 96 \Rightarrow x_6 x_7 = 48$ ......(3)

Now, $|x_6 - x_7| = \sqrt{(x_6 + x_7)^7 - 4x_6 x_7}$

$= \sqrt{196 - 192} = 2$

Q.4 If $2^{10}+2^9.3^1+2^8.3^2+.....+2.3^9+3^{10}=S-2^{11}$, then S is equal to:

(1) 3^{11} (2) $\dfrac{3^{11}}{2} + 2^{10}$ (3) 2.3^{11} (4) $3^{11} - 2^{12}$

Sol. **1**

let

$S' = 2^{10} + 2^9\ 3^1 + 2^8\ 3^2 + ------ + 2.3^9 + 3^{10}$

$\dfrac{3 \times S'}{2} = 2^9 \times 3^1 + 2^8.\ 3^2 + ---- + 3^{10} + \dfrac{3^{11}}{2}$

$\dfrac{-S'}{2} = 2^{10} - \dfrac{3^{11}}{2}$

$S' = 3^{11} - 2^{11}$

Now $S' = S - 2^{11}$

$S = 3^{11}$

Q.5 If $3^{2\sin 2\alpha - 1}$, 14 and $3^{4-2\sin 2\alpha}$ are the first three terms of an A.P. for some α, then the sixth terms of this A.P. is:

(1) 65 (2) 81 (3) 78 (4) 66

Sol. **4**

$28 = 3^{2\sin 2\alpha - 1} + 3^{4 - 2\sin 2\alpha}$

$28 = \dfrac{9^{\sin 2\alpha}}{3} + \dfrac{81}{9^{\sin 2\alpha}}$

Let $9^{\sin 2\alpha} = t$

$28 = \dfrac{t}{3} + \dfrac{81}{t}$

$t^2 - 84t + 243 = 0$

$t = 81, 3$

$9^{\sin 2\alpha} = 9^2$ or 3

$\sin 2\alpha = 2$ or $\sin 2\alpha = 1/2$

(Not possible)

Now three terms in A.P. are

1, 14, 27

Next term are

40,53,66

Q.6 If the common tangent to the parabolas, $y^2=4x$ and $x^2=4y$ also touches the circle, $x^2+y^2 = c^2$, then c is equal to:

(1) $\dfrac{1}{2}$ (2) $\dfrac{1}{4}$ (3) $\dfrac{1}{\sqrt{2}}$ (4) $\dfrac{1}{2\sqrt{2}}$

Sol. **3**

$y = mx + \dfrac{1}{m}$

$x^2 = 4\left(mx + \dfrac{1}{m}\right)$

$x^2 - 4mx - \dfrac{4}{m} = 0$

$D = 0$

$16m^2 + \dfrac{16}{m} = 0$

$16\left(\dfrac{m^3+1}{m}\right) = 0$

$m = -1$

$\Rightarrow y + x = -1$

Now, $\left|\dfrac{-1}{\sqrt{2}}\right| = c$

$c = \dfrac{1}{\sqrt{2}}$

Q.7 If the minimum and the maximum values of the function $f:\left[\frac{\pi}{4},\frac{\pi}{2}\right]\to R$, defined by

$$f(\theta)=\begin{vmatrix}-\sin^2\theta & -1-\sin^2\theta & 1\\ -\cos^2\theta & -1-\cos^2\theta & 1\\ 12 & 10 & -2\end{vmatrix}$$ are m and M respectively, then the ordered pair (m,M) is equal to :

(1) (0,4)
(2) (-4,0)
(3) (-4,4)
(4) $\left(0,2\sqrt{2}\right)$

Sol. **2**

$$f(\theta)=\begin{vmatrix}-\sin^2\theta & -1-\sin^2\theta & 1\\ -\cos^2\theta & -1-\cos^2\theta & 1\\ 12 & 10 & -2\end{vmatrix}$$

$C_1 \to C_1 - C_2$, $C_3 \to C_3 + C_2$

$$\begin{vmatrix}1 & -1-\sin^2\theta & -\sin^2\theta\\ 1 & -1-\cos^2\theta & -\cos^2\theta\\ 2 & 10 & 8\end{vmatrix}$$

$C_2 \to C_2 - C_3$

$$\begin{vmatrix}1 & -1 & -\sin^2\theta\\ 1 & -1 & -\cos^2\theta\\ 2 & 2 & 8\end{vmatrix}$$

$1(2\cos^2\theta-8)+(8+2\cos^2\theta)-4\sin^2\theta$
$f(\theta)=4\cos 2\theta$

Q.8 Let $\lambda \in R$. The system of linear equations
$2x_1-4x_2+\lambda x_3=1$
$x_1-6x_2+x_3=2$
$\lambda x_1-10x_2+4x_3=3$
is inconsistent for:
(1) exactly two values of λ
(2) exactly one negative value of λ.
(3) every value of λ.
(4) exactly one positive value of λ.

Sol. **2**

$$D = \begin{vmatrix} 2 & -4 & \lambda \\ 1 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix} = 0$$

$2(-14) + 4(4 - \lambda) + \lambda(6\lambda - 10) = 0$
$-28 + 16 - 4\lambda + 6\lambda^2 - 10\lambda = 0$
$6\lambda^2 - 14\lambda - 12 = 0$
$3\lambda^2 - 7\lambda - 6 = 0$
$3\lambda^2 - 9\lambda + 2\lambda - 6 = 0$
$(3\lambda + 2)(\lambda - 3) = 0$
$\lambda = -2/3, 3$

$$D_1 = \begin{vmatrix} 1 & -4 & \lambda \\ 2 & -6 & 1 \\ 3 & -10 & 4 \end{vmatrix}$$

$\Rightarrow -14 + 4(5) + \lambda(-2)$
$\Rightarrow -2\lambda + 6$

$$D_2 = \begin{vmatrix} 2 & 1 & \lambda \\ 1 & 2 & 1 \\ \lambda & 3 & 4 \end{vmatrix}$$

$\Rightarrow 2(5) -1(4 -\lambda) + \lambda(3 - 2\lambda)$
$\Rightarrow 10 - 4 + \lambda + 3\lambda - 2\lambda^2$
$\Rightarrow -2\lambda^2 + 4\lambda + 6$
$\Rightarrow -2(\lambda^2 - 2\lambda - 3)$
$\Rightarrow -2[\lambda^2 - 3\lambda + \lambda - 3]$
$\Rightarrow -2(\lambda - 3)(\lambda + 1)$

$$D_3 = \begin{vmatrix} 2 & -4 & 1 \\ 1 & -6 & 2 \\ \lambda & -10 & 3 \end{vmatrix} \Rightarrow 2(-18 + 20) + 4(3 - 2\lambda) + 1(-10 + 6\lambda)$$

$= 4 + 12 - 8\lambda - 10 + 6\lambda$
$= - 2\lambda + 6$
$\Rightarrow \lambda = - 2/3$ is answer

Q.9 If the point P on the curve, $4x^2+5y^2=20$ is farthest from the point Q(0, -4), then PQ^2 is equal to:

(1) 48 (2) 29 (3) 21 (4) 36

Sol. **4**

Let P be $\left(\sqrt{5}\cos\theta, 2\sin\theta\right)$

Now, PQ = $\sqrt{\left(\sqrt{5}\cos\theta\right)^2 + \left(2\sin\theta + 4\right)^2}$

PQ = $\sqrt{5\cos^2\theta + \left(2\sin\theta + 4\right)^2}$

$\frac{d(PQ)}{d\theta} = 0 \Rightarrow -10\sin\theta\cos\theta + (4\sin\theta + 8)\cos\theta = 0$

$\Rightarrow -6\sin\theta\cos\theta + 8\cos\theta = 0$

$\cos\theta = 0$ or $\sin\theta = \frac{4}{3}$

Not possible

So P is either (0,2) or (0,-2)

$PQ^2 = 36$

Q.10 The product of the roots of the equation $9x^2-18|x|+5=0$ is :

(1) $\frac{25}{81}$
(2) $\frac{5}{9}$
(3) $\frac{5}{27}$
(4) $\frac{25}{9}$

Sol. **1**

$9t^2 - 18t + 5 = 0$

$9t^2 - 15t - 3t + 5 = 0$

$(3t - 5)(3t - 1) = 0$

$|x| = \frac{5}{3}, \frac{1}{3}$

$\Rightarrow \quad x = \frac{5}{3}, \frac{-5}{3}, \frac{1}{3}, \frac{-1}{3}$

$\Rightarrow \quad P = \frac{25}{81}$

Q.11 If y=y(x) is the solution of the differential equation $\frac{5+e^x}{2+y}\cdot\frac{dy}{dx} + e^x = 0$ satisfying y(0)=1, then a value of $y(\log_e 13)$ is:

(1) 1
(2) 0
(3) 2
(4) -1

Sol. **4**

$\frac{dy}{dx} + \left(e^x \times \frac{y+2}{e^x+5}\right) = 0$

$$\frac{dy}{dx}+\left(\frac{e^x}{e^x+5}\right)y=\frac{-2e^x}{e^x+5}$$

$$\text{I.F.} = e^{\int \frac{e^x}{e^x+5}dx}$$

$$= e^{\int\left(1-\frac{5}{e^x+5}\right)dx}$$

$$= e^{\int\left(1-\frac{5e^{-x}}{1+5e^{-x}}\right)dx}$$

$$= e^{x+\ln(1+5e-x)}$$

$= e^x.(1+5e^{-x}) \Rightarrow (e^x + 5)$

$y(e^x + 5) = -\int 2e^x dx$

$y(e^x + 5) = -2e^x + C$

$\Downarrow x=0$

$(6) = -2 + C \Rightarrow C = 8$

$$y(\ln 13) = \frac{8-2\times 13}{13+5} = \frac{-18}{18} = -1$$

Q.12 If S is the sum of the first 10 terms of the series $\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right)+\tan^{-1}\left(\frac{1}{21}\right)+.....,$ then tan(S) is equal to :

(1) $\frac{5}{11}$ (2) $\frac{5}{6}$

(3) $-\frac{6}{5}$ (4) $\frac{10}{11}$

Sol. **2**

$$S = \tan^{-1}\left(\frac{1}{1+1\times 2}\right)+\tan^{-1}\left(\frac{1}{1+2\times 3}\right)+....$$

$$T_r = \tan^{-1}\left(\frac{1}{1+r(r+1)}\right)$$

$T_r = \tan^{-1}(r + 1) - \tan^{-1}r$

$T_1 = \tan^{-1}2 - \tan^{-1}1$

$T_2 = \tan^{-1}3 - \tan^{-1}2$

$T_3 = \tan^{-1}4 - \tan^{-1}3$

$T_{10} = \tan^{-1}11 - \tan^{-1}10$

$\Rightarrow S = \tan^{-1}11 - \tan^{-1}1$

$\Rightarrow \tan S = \frac{10}{12} = \frac{5}{6}$

Q.13 The value of $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1+e^{\sin x}} dx$ is:

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{4}$ (3) π (4) $\frac{3\pi}{2}$

Sol. **1**

$I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1+e^{\sin x}} dx$

$I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{e^{\sin x}}{1+e^{\sin x}} dx$ $\Rightarrow 2I = \pi$

$I = \frac{\pi}{2}$

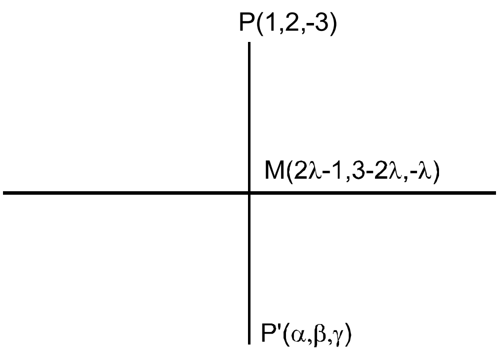
Q.14 If (a, b, c) is the image of the point (1,2,-3) in the line, $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$, then a+b+c is

(1) 2 (2) 3 (3) -1 (4) 1

Sol. **1**

$\overrightarrow{PM} \perp (2\hat{i} - 2\hat{j} - \hat{k})$
$\Rightarrow (2\lambda-2).2 + (1-2\lambda)(-2) + (3-\lambda)(-1) = 0$
$\Rightarrow 4\lambda - 4 + 4\lambda - 2 + \lambda - 3 = 0$
$\Rightarrow 9\lambda = 9 \Rightarrow \lambda = 1$

$\Rightarrow m(1,1,-1)$
Now, p' = 2M – P
= 2(1,1,–1)—(1, 2, –3)
= (1, 0, 1)
a + b + c = 2



Q.15 If the function $f(x)=\begin{cases}k_1(x-\pi)^2-1 & , x\le\pi\\ k_2\cos x & , x>\pi\end{cases}$ is twice differentiable, then the ordered pair (k_1,k_2) is equal to:

(1) (1,1) (2) (1,0) (3) $\left(\frac{1}{2},-1\right)$ (4) $\left(\frac{1}{2},1\right)$

Sol. **4**

$$f(x) = \begin{cases}2k_1(x-\pi); & x\le\pi\\ -k_2\sin x & ;x>\pi\end{cases}$$

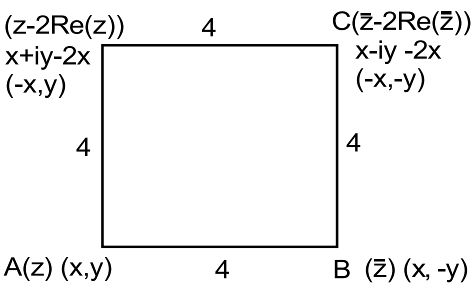
$$f''(x) = \begin{cases}2k_1 & ;x\le\pi\\ -k_2\cos x; & x>\pi\end{cases}$$

$2k_1 = k_2$

Q.16 If the four complex numbers $z,\bar{z}$, $\bar{z}$-2Re($\bar{z}$) and z-2Re(z) represent the vertices of a square of side 4 units in the Argand plane, then |z| is equal to:

(1) 2 (2) 4 (3) $4\sqrt{2}$ (4) $2\sqrt{2}$

Sol. **4**



Let z = x + iy
$CA^2 = AB^2 + BC^2$
$2^2x^2 + 2^2y^2 = 32$
$x^2 + y^2 = 8$
$\sqrt{x^2+y^2} = 2\sqrt{2}$

Q.17 If $\int\left(e^{2x}+2e^x-e^{-x}-1\right)e^{\left(e^x+e^{-x}\right)}dx = g(x)e^{\left(e^x+e^{-x}\right)}+c$, where c is a constant of integration, then g(0) is equal to :

(1) 2 (2) e
(3) 1 (4) e^2

Sol. **1**

$$\int\left(e^{2x}+2e^{x}-e^{-x}-1\right)e^{\left(e^{x}+e^{-x}\right)}dx$$

$$\int\left(e^{2x}+e^{x}-1\right)e^{\left(e^{x}+e^{-x}\right)}dx+\int\left(e^{x}-e^{-x}\right)e^{\left(e^{x}+e^{-x}\right)}dx$$

$$\int\left(e^{x}+1-e^{-x}\right)e^{\left(e^{x}+e^{-x}+x\right)}dx+\int\left(e^{x}-e^{-x}\right)e^{\left(e^{x}+e^{-x}\right)}dx$$

$$e^{\left(e^{x}+e^{-x}+x\right)}+e^{e^{x}+e^{-x}}+C$$

$$\left(e^{e^{x}+e^{-x}}\right)\left[e^{x}+1\right]+C$$

$$\Downarrow$$

$$g(x)$$

$\Rightarrow$ g(0) = 2

Q.18 The negation of the Boolean expression $x \leftrightarrow \sim y$ is equivalent to :

(1) $(x \wedge y) \wedge (\sim x \vee \sim y)$

(2) $(x \wedge y) \vee (\sim x \wedge \sim y)$

(3) $(x \wedge \sim y) \vee (\sim x \wedge y)$

(4) $(\sim x \wedge y) \vee (\sim x \wedge \sim y)$

Sol. **2**

As we know

$\sim(p \leftrightarrow q) = (p \wedge \sim q) \vee (\sim p \wedge q)$

$\Rightarrow$ so, $\sim(x \leftrightarrow \sim y) = (x \wedge y) \vee (\sim x \wedge \sim y)$

Q.19 If α is positive root of the equation, p(x) =x²-x-2=0, then $\lim\limits_{x\to\alpha^+}\dfrac{\sqrt{1-\cos(p(x))}}{x+\alpha-4}$ is equal to :

(1) $\dfrac{1}{2}$ (2) $\dfrac{3}{\sqrt{2}}$ (3) $\dfrac{3}{2}$ (4) $\dfrac{1}{\sqrt{2}}$

Sol. **2**

$f(x) = x^2 - x - 2 \left\langle {2 \atop -1} \right. = \alpha$

$$\lim_{x\to 2^+}\frac{\sqrt{1-\cos(x-2)(x+1)}}{x+\alpha-4}$$

$$\lim_{x\to 2+} \frac{\sqrt{1-\cos(x-2)(x+1)}}{(x-2)}$$

$$\lim_{h\to 0} \frac{\sqrt{1-\cos(h\times(h+3))}}{h}$$

$$\lim_{h\to 0} \sqrt{\frac{1-\cos(h\,(h+3))}{h^2\times(h+3)^2}\times(h+3)^2} \Rightarrow \sqrt{\frac{1}{2}\times 9} = \frac{3}{\sqrt{2}}$$

Q.20 If the co-ordinates of two points A and B are $(\sqrt{7},0)$ and $(-\sqrt{7},0)$ respectively and P is any point on the conic, $9x^2+16y^2=144$, then PA+PB is equal to :

(1) 6 (2) 16

(3) 9 (4) 8

Sol. **4**

$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

$$e = \sqrt{1-\frac{9}{16}} = \frac{\sqrt{7}}{4}$$

$$F_1(\sqrt{7},0), F_2(-\sqrt{7},0)$$

$PF_1 + PF_2 = 2a$

$PA + PB = 2 \times 4 = 8$

Q.21 The natural number m, for which the coefficient of x in the binomial expansion of $\left(x^m+\frac{1}{x^2}\right)^{22}$ is 1540, is ....

Sol. 13

$$T_{r+1} = {}^{22}C_r \left(x^m\right)^{22-r}\left(\frac{1}{x^2}\right)^r$$

$$= {}^{22}C_r\,(x)^{22m-mr-2r}$$

Given ${}^{22}C_r = 1540 = {}^{22}C_{19} \Rightarrow r=19$

$\because 22m - rm - 2r = 1$

$$\Rightarrow m = \frac{2r+1}{22-r}$$

m = 13(At r=19)

Q.22 Four fair dice are thrown independently 27 times. Then the expected number of times, at least two dice show up a three or a five, is ....

Sol. **11**

(atteat 2 or 3) = ${}^4C_2\left(\frac{2}{6}\right)^2\left(\frac{4}{6}\right)^2 + {}^4C_3\left(\frac{2}{6}\right)^3\left(\frac{4}{6}\right)^1 + {}^4C_4\left(\frac{2}{6}\right)^4$

= 6 × $\frac{1}{9}\times\frac{4}{9}$ + 4 × $\frac{1}{27}\times\frac{2}{3}+\frac{1}{81}$

$=\frac{33}{81}=\frac{11}{27}$ $\Rightarrow$ nP $\Rightarrow$ 11

Q.23 Let $f(x) = x.\left[\frac{x}{2}\right]$, for-10<x<10, where [t] denotes the greatest integer function. Then the number of points of discontinuity of f is equal to...........

Sol. **8**

f(x) = $x\left[\frac{x}{2}\right]$, $-10 < x < 10$

$-5<\frac{x}{2}<5$

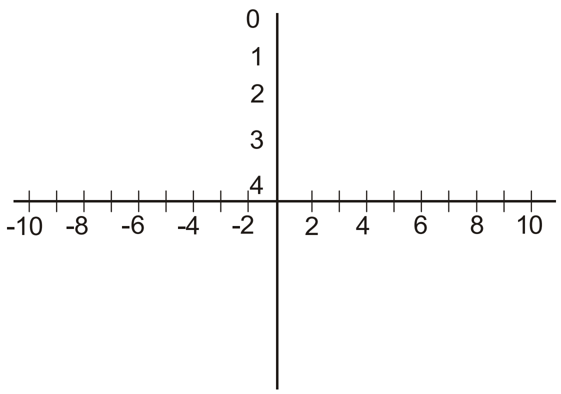
$-5x$ $-5<\frac{x}{2}<-4$

$-4x$ $-4<\frac{x}{2}<3$

$-3x$ $-3 < x/2 < -2$

$-2x$ $-2 < x/2 < -1$

$-x$ $-1 < x/2 < 0$



Number of point of discontinuity = 8

Q.24 The number of words, with or without meaning, that can be formed by taking 4 lettersat a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is

Sol. **240**

SS, Y, LL, A, B, U

$\boxed{S}\ \boxed{S}\ \boxed{}\ \boxed{}$ $\Rightarrow {}^5C_2 \times \frac{4!}{2!} \times {}^2C_1$

$\Rightarrow 120 \times 2$

$= 240$

Q.25 If the line, 2x-y+3=0 is at a distance $\frac{1}{\sqrt{5}}$ and $\frac{2}{\sqrt{5}}$ from the lines $4x-2y+\alpha=0$ and $6x-3y+\beta=0$, respectively, then the sum of all possible values of α and β is

Sol. **30**

$L_1 : 2x - y + 3 = 0$

$L_2 : 4x - 2y + \alpha = 0$

$L_3 : 6x - 3y + \beta = 0$

$$\frac{\left|\frac{\alpha}{2} - 3\right|}{\sqrt{5}} = \frac{1}{\sqrt{5}} \qquad \Rightarrow \frac{\alpha}{2} - 3 = 1, -1$$

$\Rightarrow \alpha = 8, 4$

$$\frac{\left|\frac{\beta}{3} - 3\right|}{\sqrt{5}} = \frac{2}{\sqrt{5}} \qquad \Rightarrow \frac{\beta}{3} - 3 = 2, -2$$

$\Rightarrow \beta = 15, 3$