QUESTION PAPER WITH SOLUTION

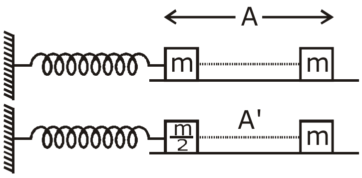
PHYSICS _ 3 Sep. _ SHIFT - 2

1. A block of mass m attached to a massless spring is performing oscillatory motion of amplitude 'A' on a frictionless horizontal plane. If half of the mass of the block breaks off when it is passing through its equilibrium point, the amplitude of oscillation for the remaining system become fA. The value of f is :

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2}$ (3) 1 (4) $\sqrt{2}$

Sol. **4**

$V_1 = V_{max} = A\omega$



$V_2 = V_{max} = A'\omega'$

$A\omega = A'\omega'$

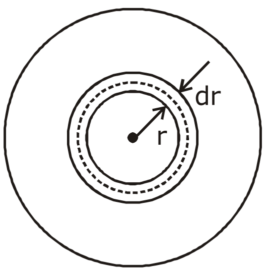
$A\sqrt{\frac{k}{m}} = A'\sqrt{\frac{2k}{m}}$

$A' = \frac{A}{\sqrt{2}}$

2. The mass density of a planet of radius R varies with the distance r from its centre as $\rho(r) = \rho_0 \left(1 - \frac{r^2}{R^2}\right)$. Then the gravitational field is maximum at :

(1) $r = \frac{1}{\sqrt{3}}R$ (2) $r = \sqrt{\frac{3}{4}}R$ (3) $r = R$ (4) $r = \sqrt{\frac{5}{9}}R$

Sol. **4**



$dm = \rho dv$

$\int dm = \int \rho_0 \left(1 - \frac{r^2}{R^2}\right) 4\pi r^2 dr$

$$M = 4\pi\rho_0\left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right)$$

$$E_g = \frac{GM}{r^2}$$

$$E_g = G4\pi\rho_0\left(\frac{r}{3} - \frac{r^3}{5R^2}\right)$$

$$\frac{dE_g}{dr} = \frac{1}{3} - \frac{3r^2}{5R^2} = 0$$

$$\frac{1}{3} = \frac{3}{5}\frac{r^2}{R^2}$$

$$r = \sqrt{\frac{5}{9}}R$$

3. Two sources of light emit X-rays of wavelength 1 nm and visible light of wavelength 500 nm, respectively. Both the sources emit light of the same power 200 W. The ratio of the number density of photons of X-rays to the number density of photons of the visible light of the given wavelengths is :

(1) $\frac{1}{500}$ (2) $\frac{1}{250}$ (3) 500 (4) 250

Sol. **1**

$$P = \frac{nhc}{\lambda}$$

$$\frac{n}{\lambda} = \text{const}$$

$$\frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = \frac{1\text{nm}}{500\text{nm}} = \frac{1}{500}$$

4. If a semiconductor photodiode can detect a photon with a maximum wavelength of 400 nm, then its band gap energy is :

Planck's constant $h = 6.63 \times 10^{-34}$ J.s. Speed of light $c = 3 \times 10^8$ m/s

(1) 1.5 eV (2) 2.0 eV (3) 3.1 eV (4) 1.1 eV

Sol. **3**

$$E = \frac{hc}{\lambda}$$

$$E = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{400 \times 10^{-9}}$$

$$E \approx \frac{1240}{400} eV$$

E = 3.1 eV

5. Amount of solar energy received on the earth's surface per unit area per unit time is defined a solar constant. Dimension of solar constant is :

(1) ML^0T^{-3} (2) MLT^{-2} (3) $M^2L^0T^{-1}$ (4) ML^2T^{-2}

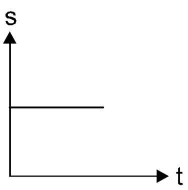
Sol. **1**

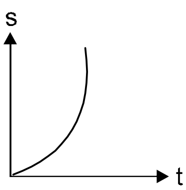
$$E = \frac{Q}{At}$$

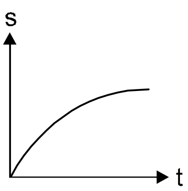
$$E = \frac{ML^2T^{-2}}{L^2T}$$

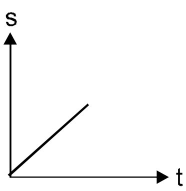
$E = MT^{-3}$

6. A particle is moving unidirectionally on a horizontal plane under the action of a constant power supplying energy source. The displacement (s) - time (t) graph that describes the motion of the particle is (graphs are drawn schematically and are not to scale) :

(1)


(2)


(3)


(4)


Sol. **2**

P = FV

$$P = m\frac{dv}{dt}v$$

$$vdv = \frac{P}{m}dt$$

$V^2 = k't$

$V = k''\sqrt{t}$

$s \alpha t^{3/2}$

7. Which of the following will NOT be observed when a multimeter (operating in resistance measuring mode) probes connected across a component, are just reversed ?

(1) Multimeter shows NO deflection in both cases i.e. before and after reversing the probes if the chosen component is metal wire.

(2) Multimeter shows a deflection, accompanied by a splash of light out of connected component in one direction and NO deflection on reversing the probes if the chosen component is LED.

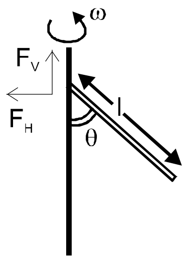
(3) Multimeter shows an equal deflection in both cases i.e. before and after reversing the probes if the chosen component is resistor.

(4) Multimeter shows NO deflection in both cases i.e. before and after reversing the probes if the chosen component is capacitor.

Sol. **4**

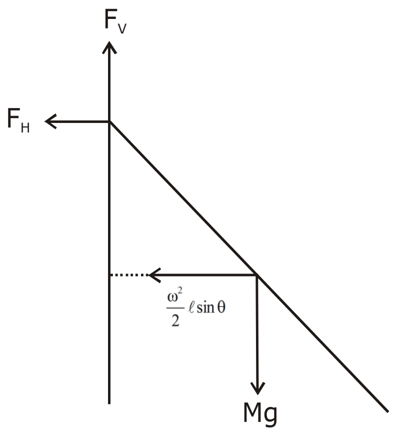
By Theory

8. A uniform rod of length 'l' is pivoted at one of its ends on a vertical shaft of negligible radius. When the shaft rotates at angular speed ω the rod makes an angle θ with it (see figure). To find θ equate the rate of change of angular momentum (direction going into the paper) $\frac{ml^2}{12}\omega^2 \sin\theta \cos\theta$ about the centre of mass (CM) to the torque provided by the horizontal and vertical forces F_H and F_V about the CM. The value of θ is then such that :



(1) $\cos\theta = \frac{2g}{3l\omega^2}$ (2) $\cos\theta = \frac{3g}{2l\omega^2}$ (3) $\cos\theta = \frac{g}{2l\omega^2}$ (4) $\cos\theta = \frac{g}{l\omega^2}$

Sol. **2**



$F_V = mg$

$F_H = m\omega^2 \frac{\ell}{2}\sin\theta$

$\therefore \tau_{net}$ about COM $= F_V.\frac{\ell}{2}\sin\theta - F_H\frac{\ell}{2}\cos\theta$

$= \frac{m\ell^2}{12}\omega^2\sin\theta\cos\theta$

$mg\frac{\ell}{2}\sin\theta - m\omega^2\frac{\ell}{2}\sin\theta\frac{\ell}{2}\cos\theta = \frac{m\ell^2}{12}\omega^2\sin\theta\cos\theta$

$\Rightarrow \frac{g\ell}{2} - \frac{\omega^2\ell^2}{4}\cos\theta = \frac{\ell^2}{12}\omega^2\cos\theta$

$\frac{g\ell}{2} = \omega^2\ell^2\cos\theta\left(\frac{1}{12}+\frac{1}{4}\right)$

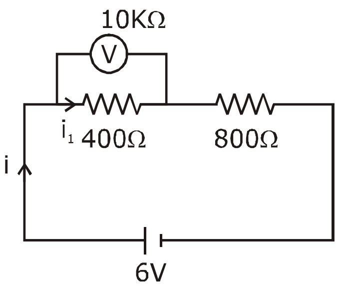
$\frac{g\ell}{2} = \frac{\omega^2\ell^2\cos\theta}{3}$

$\cos\theta = \frac{3g}{2\omega^2\ell}$

9. Two resistors 400Ω and 800Ω are connected in series across a 6 V battery. The potential difference measured by a voltmeter of 10 kΩ across 400 Ω resistor is close to :

(1) 2.05 V (2) 2 V (3) 1.95 V (4) 1.8 V

Sol. 3



$i = \frac{6}{800 + \frac{400\times10000}{400+10000}}$

$i = \frac{6}{800+\frac{40000}{104}}$

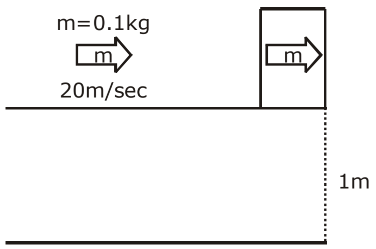
$i = \frac{6}{800+384.61} = \frac{6}{1184.61} = 0.00506$

$V_v = 6 - 800\times0.00506 = 6 - 4.05 = 1.95$

10. A block of mass 1.9 kg is at rest at the edge of a table, of height 1 m. A bullet of mass 0.1 kg collides with the block and sticks to it. If the velocity of the bullet is 20 m/s in the horizontal direction just before the collision then the kinetic energy just before the combined system strikes the floor, is [Take g = 10 m/s^2 . Assume there is no rotational motion and loss of energy after the collision is negligable.]

(1) 23 J (2) 21 J (3) 20 J (4) 19 J

Sol. **2**



0.1×20 = (1.9+0.1)V

2 = 2 V

V = 1 m/sec

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}2\times(1)^2 = 1J$$

TE = KE + Mgh = 1+2×10×1 = 21J

11. A metallic sphere cools from 50°C to 40°C in 300 s. If atmospheric temperature around is 20°C, then the sphere's temperature after the next 5 minutes will be close to :

(1) 35°C (2) 31°C (3) 33°C (4) 28°C

Sol. **3**

$$\frac{\Delta T}{\Delta t} = k\left(\frac{T_f + T_i}{2} - T_0\right)$$

$$\frac{50-40}{300} = k\left(\frac{90}{2} - 20\right)$$

$$\frac{40-T}{300} = k\left(\frac{40+T}{2} - 20\right)$$

$$\frac{10}{40-T} = \frac{25\times 2}{40+T-40}$$

$$\frac{1}{40-T} = \frac{5}{T}$$

$T = 200 - 5T$

6T = 200

T = 33°C

12. To raise the temperature of a certain mass of gas by 50°C at a constant pressure, 160 calories of heat is required. When the same mass of gas is cooled by 100°C at constant volume, 240 calories of heat is released. How many degrees of freedom does each molecule of this gas have (assume gas to be ideal) ?

(1) 6 (2) 7 (3) 5 (4) 3

Sol. **1**

$$Q = nC_P\Delta T$$

$$160 = nC_P 50$$

$$240 = nC_v 100$$

$$\frac{16}{24} = \frac{C_P}{2C_v}$$

$$r = \frac{4}{3}$$

$$r = 1 + \frac{2}{f}$$

$$\frac{4}{3} - 1 = \frac{2}{f}$$

$$f = 6$$

13. The radius R of a nucleus of mass number A can be estimated by the formula $R = (1.3 \times 10^{-15})A^{1/3}$ m. It follows that the mass density of a nucleus is of the order of :

($M_{prot.} \cong M_{neut} \simeq 1.67 \times 10^{-27}$ kg)

(1) 10^{17} kg m^{-3} (2) 10^{10} kg m^{-3} (3) 10^{24} kg m^{-3} (4) 10^{3} kg m^{-3}

Sol. **1**

$$R = (1.3 \times 10^{-15})\,A^{1/3}$$

$$m = \rho V$$

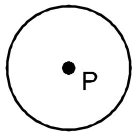
$$\rho = \frac{m}{V}$$

$$\rho = \frac{m_P A}{\frac{4}{3}\pi R^3}$$

$$\rho = \frac{m_P A}{\frac{4}{3}\pi \times (1.3 \times 10^{-15})^3 A}$$

$$\rho \approx 10^{17} \text{kg} / \text{m}$$

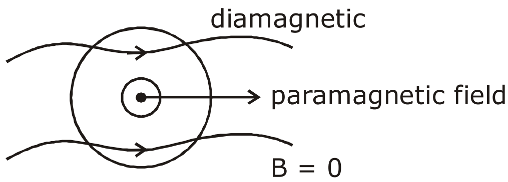
14. A perfectly diamagnetic sphere has a small spherical cavity at its centre, which is filled with a paramagnetic substance. The whole system is placed in a uniform magnetic field $\vec{B}$. Then the field inside the paramagnetic substance is :



(1) much large than $|\vec{B}|$ and parallel to $\vec{B}$ (2) zero

(3) $\vec{B}$ (4) much large than $|\vec{B}|$ but opposite to $\vec{B}$

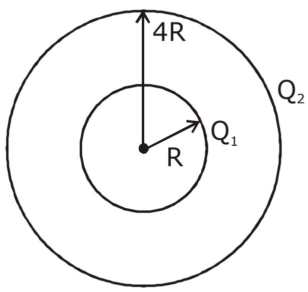
Sol. **2**



15. Concentric metallic hollow spheres of radii R and 4R hold charges Q_1 and Q_2 respectively. Given that surface charge densities of the concentric spheres are equal, the potential difference V(R) – V(4R) is :

(1) $\frac{3Q_2}{4\pi\varepsilon_0 R}$ (2) $\frac{3Q_1}{4\pi\varepsilon_0 R}$ (3) $\frac{3Q_1}{16\pi\varepsilon_0 R}$ (4) $\frac{Q_2}{4\pi\varepsilon_0 R}$

Sol. **3**



$$\sigma = \frac{Q_1}{4\pi R^2} = \frac{Q_2}{4\pi 16R^2}$$

$$16Q_1 = Q_2$$

$$V_R - V_{4R} = \frac{KQ_1}{R} + \frac{KQ_2}{4R} - \frac{KQ_1}{4R} - \frac{KQ_2}{4R}$$

$$= \frac{3KQ_1}{4R} = \frac{3Q_1}{16\pi\varepsilon_0 R}$$

16. The electric field of a plane electromagnetic wave propagating along the x direction in vacuum is $\vec{E} = E_0 \hat{j} \cos(\omega t - kx)$. The magnetic field $\vec{B}$, at the moment t = 0 is :

(1) $\vec{B} = \frac{E_0}{\sqrt{\mu_0 \epsilon_0}} \cos(kx) \hat{j}$ (2) $\vec{B} = \frac{E_0}{\sqrt{\mu_0 \epsilon_0}} \cos(kx) \hat{k}$

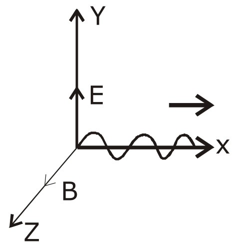
(3) $\vec{B} = E_0 \sqrt{\mu_0 \epsilon_0} \cos(kx) \hat{j}$ (4) $\vec{B} = E_0 \sqrt{\mu_0 \epsilon_0} \cos(kx) \hat{k}$

Sol. **4**

$E = E_0 \cos(\omega t - kx) \hat{j}$

$E = BC$

$B = \frac{E}{C} = \frac{E_0}{\frac{1}{\sqrt{\mu_0 \epsilon_0}}}$



$B = E_0 \sqrt{\mu_0 \epsilon_0}$

$B = E_0 \sqrt{\mu_0 \epsilon_0} \cos(\omega t - kx) \hat{k}$

at t = 0

$B = E_0 \sqrt{\mu_0 \epsilon_0} \cos(kx) \hat{k}$

17. A uniform magnetic field B exists in a direction perpendicular to the plane of a square loop made of a metal wire. The wire has a diameter of 4 mm and a total length of 30 cm. The magnetic field changes with time at a steady rate dB/dt = 0.032 Ts^{-1}. The induced current in the loop is close to (Resistivity of the metal wire is 1.23×10^{-8} Ωm)

(1) 0.53 A (2) 0.61 A (3) 0.34 A (4) 0.43 A

Sol. **2**

$\phi = BA$

$E = \frac{d\phi}{dt} = \frac{AdB}{dt}$

$E = \ell^2 \frac{dB}{dt}$

$i = \frac{E}{R} = \frac{\ell^2}{\rho \ell} \frac{dB}{dt} A$

$i - \frac{30}{4} \times \frac{30}{4} \times \frac{10^{-4} \times 0.032 \times 4 \times 10^{-6} \times \pi}{1.23 \times 10^{-8} \times 30 \times 10^{-2} \times 10^{3}}$

$i = \frac{240 \times \pi \times 10^{-10}}{1.23 \times 10^{-7}}$

$i = \frac{240 \times 3.14 \times 10^{-3}}{1.23} = \frac{753.6}{1.23} \times 10^{-3}$

$i = 612.68 \times 10^{-3} = 0.61 A$

18. Hydrogen ion and singly ionized helium atom are accelerated, from rest, through the same potential difference. The ratio of final speeds of hydrogen and helium ions is close to :

(1) 2 : 1 (2) 1 : 2 (3) 5 : 7 (4) 10 : 7

Sol. **1**

$$K = \frac{P^2}{2m}$$

$$qV = \frac{P^2}{2m} = \frac{m^2v^2}{2m}$$

$$V = \sqrt{\frac{2qv}{m}}$$

$$V \propto \sqrt{\frac{q}{m}}$$

$$\frac{V_H}{V_{He}} = \frac{\sqrt{\frac{e}{m}}}{\sqrt{\frac{e}{4m}}} = \frac{2}{1}$$

19. Two light waves having the same wavelength λ in vacuum are in phase initially. Then the first wave travels a path L_1 through a medium of refractive index n_1 while the second wave travels a path of length L_2 through a medium of refractive index n_2. After this the phase difference between the two waves is :

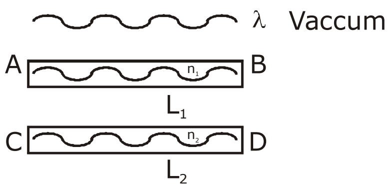
(1) $\frac{2\pi}{\lambda}(n_1L_1 - n_2L_2)$ (2) $\frac{2\pi}{\lambda}\left(\frac{L_1}{n_1} - \frac{L_2}{n_2}\right)$

(3) $\frac{2\pi}{\lambda}\left(\frac{L_2}{n_1} - \frac{L_1}{n_2}\right)$ (4) $\frac{2\pi}{\lambda}(n_2L_1 - n_1L_2)$

Sol. **1**

$$\lambda_{n_1} = \frac{\lambda}{n_1}$$

$$\lambda_{n_2} = \frac{\lambda}{n_2}$$



$$(\Delta\phi)_1 = \frac{2\pi}{\lambda_{n_1}} L_1$$

$$(\Delta\phi)_2 = \frac{2\pi}{\lambda_{n_2}} L_2$$

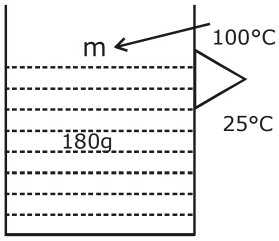
$$(\Delta\phi_1 - \Delta\phi_2) = \frac{2\pi}{\lambda}(n_1L_1 - n_2L_2)$$

20. A calorimeter of water equivalent 20 g contains 180 g of water at 25°C. 'm' grams of steam at 100°C is mixed in it till the temperature of the mixure is 31°C. The value of 'm' is close to (Latent heat of water = 540 cal g^{-1} , specific heat of water = 1 cal g^{-1} $°C^{-1}$)

(1) 2 (2) 2.6 (3) 4 (4) 3.2

Sol. **1**

$m_C s_C = 20g$



Temp of mixture $\rightarrow$ 31°C

180×1×(31-25)+20×(31-25)= m×540+m×1×(100-31)

180×6+20×6 = 540m + 100 m - 31m

1080+120 = 640 m - 31m

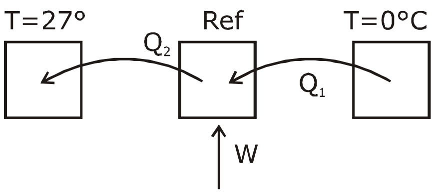
1200 = 609m

$$m = \frac{1200}{609} = 1.97$$

21. If minimum possible work is done by a refrigerator in converting 100 grams of water at 0°C to ice, how much heat (in calories) is released to the surroundings at temperature 27°C (Latent heat of ice = 80 Cal/gram) to the nearest integer ?

Sol. **8791**

$Q_1 = mL = 8000cal$



$Q_1 = W + Q_1$

$$\text{C.O.P.} = \frac{Q_1}{W} = \frac{Q_1}{Q_2 - Q_1} = \frac{T_1}{T_2 - T_1}$$

$$\frac{Q_1}{W} = \frac{273}{300 - 273}$$

$$\frac{Q_1}{W} = \frac{273}{27}$$

$$W = \frac{27}{273} Q_1$$

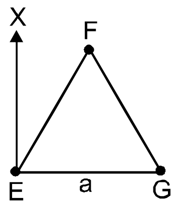
$$W = \frac{27}{273} mL$$

$$W = \frac{27}{273} \times 80 \times 100$$

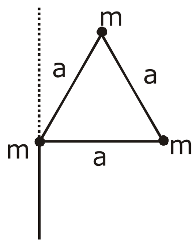
$$Q_2 = \frac{27}{273} \times 80 \times 100 + 80 \times 100$$

$= 8791.2$ cal

22. An massless equilateral triangle EFG of side 'a' (As shown in figure) has three particles of mass m situated at its vertices. The moment of inertia of the system about the line EX perpendicular to EG in the plane of EFG is $\frac{N}{20} ma^2$ where N is an integer. The value of N is ______.



Sol. **25**



$$I = ma^2 + \frac{ma^2}{4} = \frac{5}{4} ma^2$$

$$\frac{5}{4} \times ma^2 = \frac{N}{20} ma^2$$

N = 25

23. A galvanometer coil has 500 turns and each turn has an average area of 3×10^{-4} m². If a torque of 1.5 Nm is required to keep this coil parallel to a magnetic field when a current of 0.5 A is flowing through it, the strength of the field (in T) is _______.

Sol. **20**

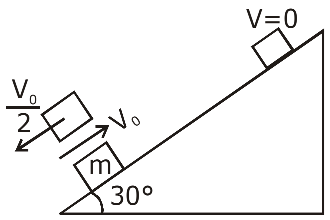
$\tau = BINA\ Sin\theta$

$1.5 = B\times0.5\times500\times3\times10^{-4}$

$B = \frac{10000}{500} = 20\text{Tesla}$

24. A block starts moving up an inclined plane of inclination 30° with an initial velocity of v_0. It comes back to its initial position with velocity $\frac{v_0}{2}$. The value of the coefficient of kinetic friction between the block and the inclined plane is close to $\frac{I}{1000}$. The nearest integer to I is_____.

Sol. **346**



$a = g\sin30 + \mu g\cos30$

$V_0^2 = 2ad$

$d = \frac{V_0^2}{2a}$

$W_f = k_f - k_i$

$-2\mu mg\cos30\frac{V_0^2}{2a} = \frac{1}{2}m\frac{V_0^2}{4} - \frac{1}{2}mV_0^2$

$\frac{+2\mu g\cos30}{a} = +\frac{3}{4}$

$8\mu g\cos30 = 3g\sin30 + 3\mu g\cos30$

$5\mu g\cos30 = 3g\sin30$

$\mu = \frac{3\tan30}{5} = \frac{\sqrt{3}}{5}$

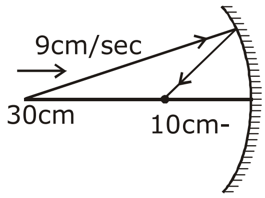
$\frac{\sqrt{3}}{5} = \frac{I}{1000}$

I = 346

25. When an object is kept at a distance of 30 cm from a concave mirror, the image is formed at a distance of 10 cm from the mirror. If the object is moved with a speed of 9 cms^{-1}, the speed (in cms^{-1}) with which image moves at that instant is _____.

Sol. 1

$$V_I = -\frac{V^2}{u^2} V_0$$



$$V_I = -\frac{10 \times 10}{30 \times 30} \times 9$$

$$V_I = 1cm / sec$$

QUESTION PAPER WITH SOLUTION

CHEMISTRY _ 3 Sep. _ SHIFT - 2

1. The five successive ionization enthalpies of an element are 800, 2427, 3658, 25024 and 32824 kJ mol^{-1}. The number of valence electrons in the element is:
(1) 2 (2) 4 (3) 3 (4) 5

Sol. **3**
Fourth & Fifth I.E. are very high (periodic properties) indicating presence of three valence shell electrons

2. The incorrect statement is:
(1) Manganate and permanganate ions are tetrahedral
(2) In manganate and permanganate ions, the π-bonding takes place by overlap of p-orbitals of oxygen and d-orbitals of manganese
(3) Manganate and permanganate ions are paramagnetic
(4) Manganate ion is green in colour and permanganate ion is purple in colour

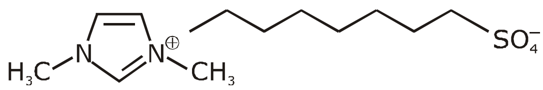
Sol. **3**
$\overset{+7}{Mn}O_4^-$ $d^0 \rightarrow$ dimagnetic
$\overset{+6}{Mn}O_4^{2-}$ $d^1 \rightarrow$ Paramagnetic

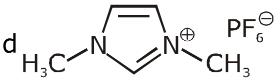
3. Match the following drugs with their therapeutic actions:

(i)	Ranitidine	(a)	Antidepressant
(ii)	Nardil (Phenelzine)	(b)	Antibiotic
(iii)	Chloramphenicol	(c)	Antihistamine
(iv)	Dimetane (Brompheniramine)	(d)	Antacid
		(e)	Analgesic

(1) (i)-(d); (ii)-(a); (iii)-(b); (iv)-(c) (2) (i)-(d); (ii)-(c); (iii)-(a); (iv)-(e)
(3) (i)-(a); (ii)-(c); (iii)-(b); (iv)-(e) (4) (i)-(e); (ii)-(a); (iii)-(c); (iv)-(d)

Sol. **1**

4. An ionic micelle is formed on the addition of:
(1) liquid diethyl ether to aqueous NaCl solution
(2) sodium stearate to pure toluene
(3) excess water to liquid 

(4) excess water to liquid 

Sol. **3**
ionic micelles formed by addition of water to soap {sodium stearate}
Ans. (3)

5. Among the statements (I–IV), the correct ones are:
(I) Be has smaller atomic radius compared to Mg.
(II) Be has higher ionization enthalpy than Al.
(III) Charge/radius ratio of Be is greater than that of Al.
(IV) Both Be and Al form mainly covalent compounds.
(1) (I), (II) and (IV) (2) (I), (II) and (III)
(3) (II), (III) and (IV) (4) (I), (III) and (IV)

Sol. **1**
Refer S-Block

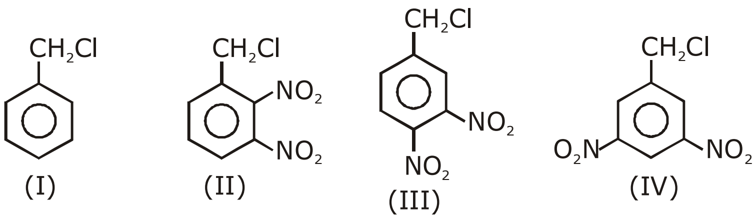
6. Complex A has a composition of $H_{12}O_6Cl_3Cr$. If the complex on treatment with conc.H_2SO_4 loses 13.5% of its original mass, the correct molecular formula of A is:
[Given: atomic mass of Cr = 52 amu and Cl = 35 amu]
(1) $[Cr(H_2O)_5Cl]Cl_2.H_2O$
(2) $[Cr(H_2O)_4Cl_2]Cl.2H_2O$
(3) $[Cr(H_2O)_3Cl_3].3H_2O$
(4) $[Cr(H_2O)_6]Cl_3$

Sol. 2
Let x molecule of water are lost then

$$13.5 = \left[\frac{x \times 18}{6 \times 18 + 3 \times 35 + 52}\right] \times 100$$

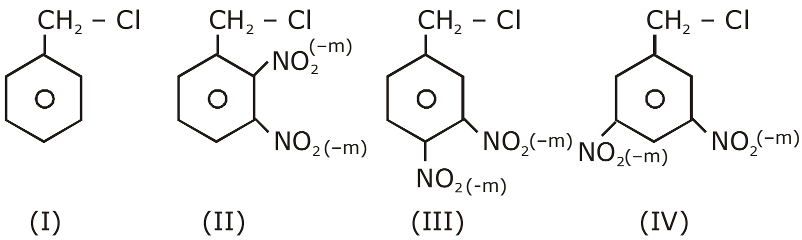
$x = 1.99 \simeq 2$
so, complex is $[Cr(H_2O)_4Cl_2].2H_2O$

7. The decreasing order of reactivity of the following compounds towards nucleophilic substitution (S_N2) is:



(1) (III) > (II) > (IV) > (I)
(2) (IV) > (II) > (III) > (I)
(3) (II) > (III) > (IV) > (I)
(4) (II) > (III) > (I) > (IV)

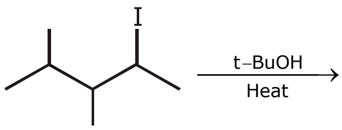
Sol. 3

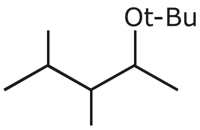
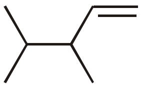
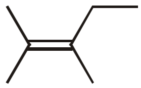
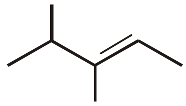


8. The increasing order of the reactivity of the following compounds in nucleophilic addition reaction is:
Propanal, Benzaldehyde, Propanone, Butanone
(1) Benzaldehyde < Propanal < Propanone < Butanone
(2) Propanal < Propanone < Butanone < Benzaldehyde
(3) Butanone < Propanone < Benzaldehyde < Propanal
(4) Benzaldehyde < Butanone < Propanone < Propanal

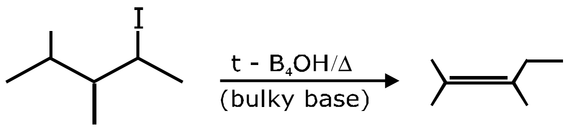
Sol. 3
Rate of Nucleophilic addition $\Rightarrow$ Aldehyde > Ketone
Aliphatic aldehyde > Aromatic aldehyde

9. The major product in the following reaction is:



(1)  (2)  (3)  (4) 

Sol. **3**



10. The incorrect statement(s) among (a) – (d) regarding acid rain is (are):

(a) It can corrode water pipes.

(b) It can damage structures made up of stone.

(c) It cannot cause respiratory ailments in animals

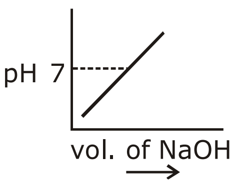
(d) It is not harmful for trees

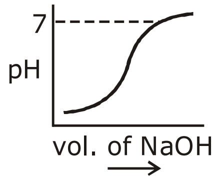
(1) (a), (b) and (d) (2) (a), (c) and (d) (3) (c) and (d) (4) (c) only

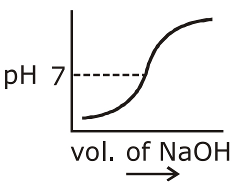
Sol. **3**

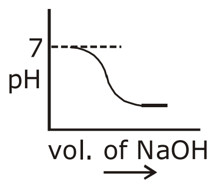
Acid rain can cause respiratory ailments in animals and also harmful for trees and plant.

11. 100 mL of 0.1 M HCl is taken in a beaker and to it 100 mL of 0.1 M NaOH is added in steps of 2 mL and the pH is continuously measured. Which of the following graphs correctly depicts the change in pH?

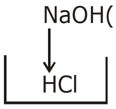
(1) 

(2) 

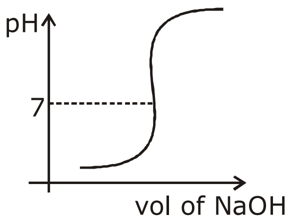
(3) 

(4) 

Sol. **3**



initially pH will be acidic < 7
at eq pH pH = 7
& finally pH will be basic > 7



option (3)

12. Consider the hypothetical situation where the azimuthal quantum number, l, takes values 0, 1, 2, ... n + 1, where n is the principal quantum number. Then, the element with atomic number:

(1) 13 has a half-filled valence subshell (2) 9 is the first alkali metal

(3) 8 is the first noble gas (4) 6 has a 2p-valence subshell

Sol. **1**

(1) $_{13}X = 1s^2\ 1p^6\ 1d^5$ - half filled
(2) $_{9}X = 1s^2\ 1p^6\ 1d^1$ - not alkali metal
(3) $_{8}X = 1s^2\ 1p^6$ - Second nobel gas
Option (1)

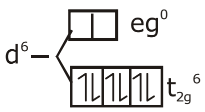
13. The d-electron configuration of $[Ru(en)_3]Cl_2$ and $[Fe(H_2O)_6]Cl_2$, respectively are:

(1) $t_{2g}^4\, e_g^2$ and $t_{2g}^6\, e_g^0$ (2) $t_{2g}^6\, e_g^0$ and $t_{2g}^6\, e_g^0$

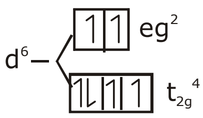
(3) $t_{2g}^4\, e_g^2$ and $t_{2g}^4\, e_g^2$ (4) $t_{2g}^6\, e_g^0$ and $t_{2g}^4\, e_g^2$

Sol. **4**

$[\overset{2+}{Ru}(en)_3]Cl_2$
Low spin
complex

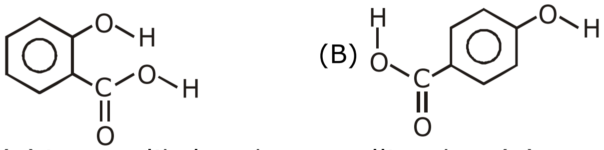


$[\overset{2+}{Fe}(H_2O)_6]Cl_2$
High spin
complex



14. Consider the following molecules and statements related to them:

(A) (B)



(a) (B) is more likely to be crystalline than (A)
(b) (B) has higher boiling point than (A)
(c) (B) dissolves more readily than (A) in water
Identify the correct option from below:

(1) (a) and (c) are true
(2) only (a) is true
(3) (b) and (c) are true
(4) (a) and (b) are true

Sol. Bonus
All answer are correct

15. The strengths of 5.6 volume hydrogen peroxide (of density 1 g/mL) in terms of mass percentage and molarity (M), respectively, are:
(Take molar mass of hydrogen peroxide as 34 g/mol)

(1) 0.85 and 0.5
(2) 0.85 and 0.25
(3) 1.7 and 0.25
(4) 1.7 and 0.5

Sol. **4**
Volume strength = 5.6V

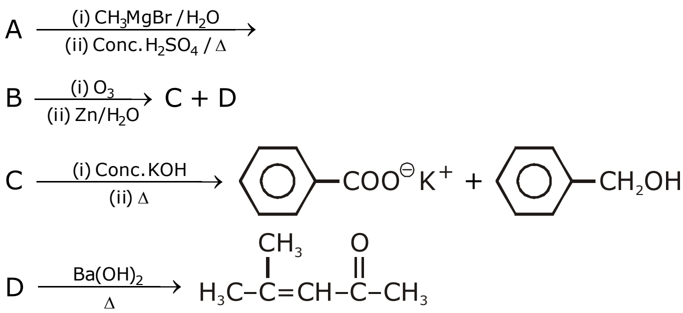
$$\text{molarity} = \frac{5.6}{11.2} = 0.5 \text{ mol / l}$$

$$\text{mass \%} = \left[\frac{0.5 \times 34}{10}\right] \times \frac{1}{1\text{g / ml}}$$

$$= 1.7 \%$$

Ans. 1.7 & 0.5 option (4)

16. The compound A in the following reactions is:



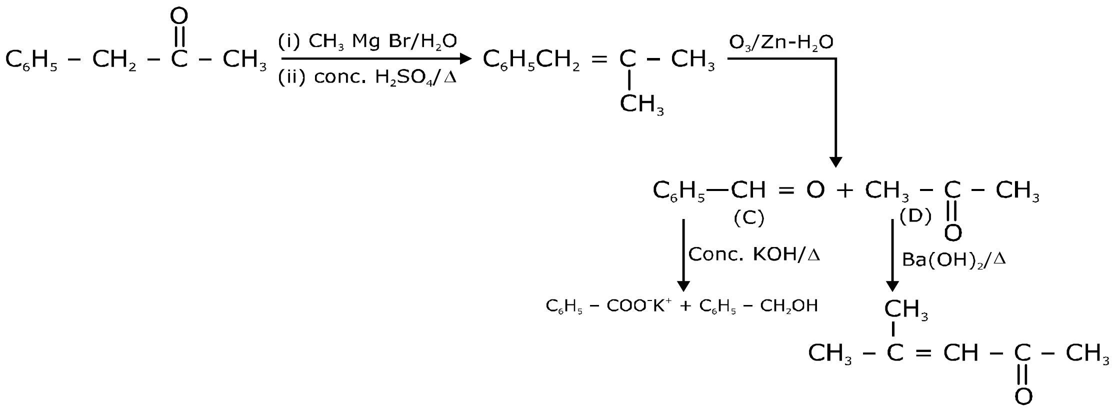
(1) $C_6H_5-CH_2-\overset{O}{\overset{||}{C}}-CH_3$

(2) $C_6H_5-\overset{O}{\overset{||}{C}}-CH_3$

(3) $C_6H_5-\overset{O}{\overset{||}{C}}-CH(CH_3)_2$

(4) $C_6H_5-\overset{O}{\overset{||}{C}}-CH_2CH_3$

Sol. **1**



17. A mixture of one mole each of H_2, He and O_2 each are enclosed in a cylinder of volume V at temperature T. If the partial pressure of H_2 is 2 atm, the total pressure of the gases in the cylinder is:

(1) 6 atm (2) 14 atm (3) 38 atm (4) 22 atm

Sol. **1**

$p_{H_2} = 2 \text{ atm} = xH_2 \times p_{total}$

$2 \text{ atm} = \frac{1}{1+1+1} \times P_{total}$

$P_{total} = 6$ atm

Ans. option (1)

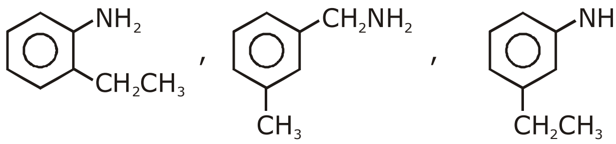
18. Three isomers A, B and C (mol. formula $C_8H_{11}N$) give the following results:

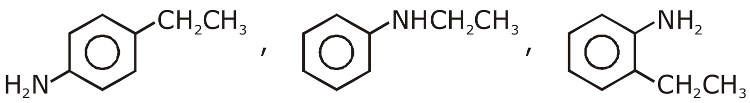
A and C $\xrightarrow{\text{Diazotization}}$ P + Q $\xrightarrow[\text{(ii) Oxidation (KMnO}_4\text{+H}^+\text{)}]{\text{(i) Hydrolysis}}$ R (product of A) + S (product of C)

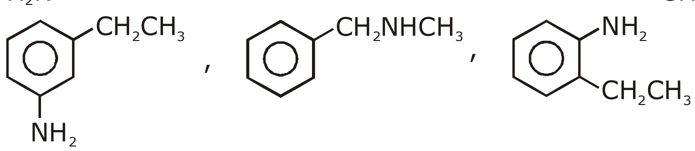
R has lower boiling point than S

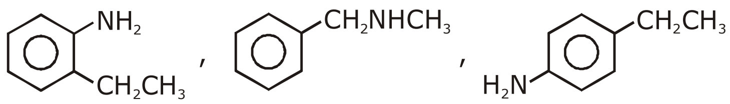
B $\xrightarrow{C_6H_5SO_2Cl}$ alkali-insoluble product

A, B and C, respectively are:

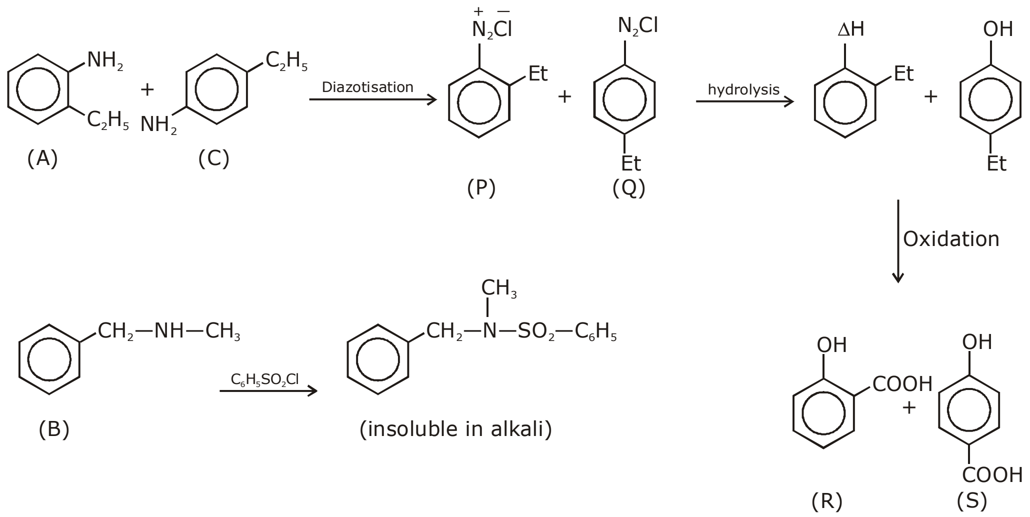
(1) 

(2) 

(3) 

(4) 

Sol. **2**



19. For the reaction $2A + 3B + \frac{3}{2}C \rightarrow 3P$,

which statement is correct?

(1) $\frac{dn_A}{dt} = \frac{dn_B}{dt} = \frac{dn_C}{dt}$

(2) $\frac{dn_A}{dt} = \frac{3}{2}\frac{dn_B}{dt} = \frac{3}{4}\frac{dn_C}{dt}$

(3) $\frac{dn_A}{dt} = \frac{2}{3}\frac{dn_B}{dt} = \frac{4}{3}\frac{dn_C}{dt}$

(4) $\frac{dn_A}{dt} = \frac{2}{3}\frac{dn_B}{dt} = \frac{3}{4}\frac{dn_C}{dt}$

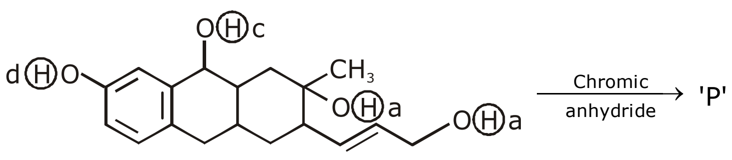
Sol. **3**

$$2A + 3B + \frac{3}{2}C \longrightarrow 3P$$

$$ROR = \frac{1}{2}\left[\frac{-d[n_A]}{dt}\right] = \frac{1}{3}\left[\frac{-d[n_B]}{dt}\right] = \frac{2}{3}\left[\frac{-d[n_c]}{dt}\right] = \frac{1}{3}\left[\frac{+d[n_e]}{dt}\right]$$

$$\left[\frac{-dn_A}{dt}\right] = \frac{2}{3}\left[\frac{-d[n_B]}{dt}\right] = \frac{4}{3}\left[\frac{-d[n_e]}{dt}\right]$$

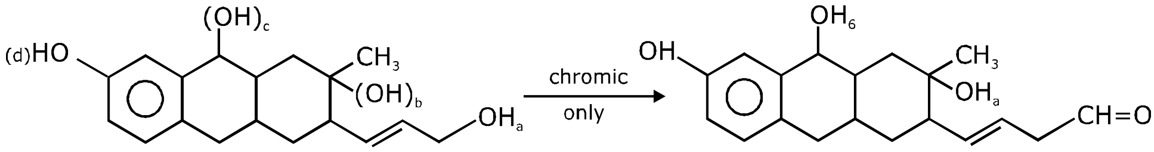
20. Consider the following reaction:



The product 'P' gives positive ceric ammonium nitrate test. This is because of the presence of which of these –OH group(s)?

(1) (b) only (2) (b) and (d) (3) (c) and (d) (4) (d) only

Sol. **1**



21. The volume (in mL) of 0.1 N NaOH required to neutralise 10 mL of 0.1 N phosphinic acid is ___________.

Sol. **10 ml**

$NaOH + H_3PO_2 \longrightarrow NaH_2PO_2 + H_2O$

Phosphinic

Vol. × 0.1 = 0.1 × 10

vol = 10 ml Ans.

22. An acidic solution of dichromate is electrolyzed for 8 minutes using 2A current. As per the following equation

$Cr_2O_7^{2-} + 14H^+ + 6e^- \longrightarrow 2Cr^{3+} + 7H_2O$

The amount of Cr^{3+} obtained was 0.104 g. The efficiency of the process (in %) is (Take: F = 96000 C, At. mass of chromium = 52) _______.

Sol. **60 %**

$[\text{moles of } Cr^{3+}] \times 3 = \frac{8 \times 60 \times 2}{96000}$

$\text{moles of } Cr^{3+} = \frac{8 \times 4}{9600} = \frac{1}{300} \text{ mol}$

$\text{mass of } Cr^{3+} = \frac{52}{300} \text{ g}$

$\% \text{ efficiency} = \frac{\text{Actual obtained Amt}}{\text{Theo. obtained Amt}} \times 100$

$= \frac{0.104}{\frac{52}{300}} \times 100 \qquad = 30 \times \frac{104}{52} = 60\%$

23. If 250 cm^3 of an aqueous solution containing 0.73 g of a protein A is isotonic with one litre of another aqueous solution containing 1.65 g of a protein B, at 298 K, the ratio of the molecular masses of A and B is ______ × 10^{-2} (to the nearest integer).

Sol. **177**

$\frac{0.73}{M_A} \times \frac{1000}{250} = \frac{1.65}{M_B}$

$\frac{M_A}{M_B} = \frac{73 \times 4}{165} = 1.769$

$= 176.9 \times 10^{-2}$

$= 177 \times 10^{-2}$

24. 6.023×10^{22} molecules are present in 10 g of a substance 'x'. The molarity of a solution containing 5 g of substance 'x' in 2 L solution is ______ $\times 10^{-3}$.

Sol. **25**

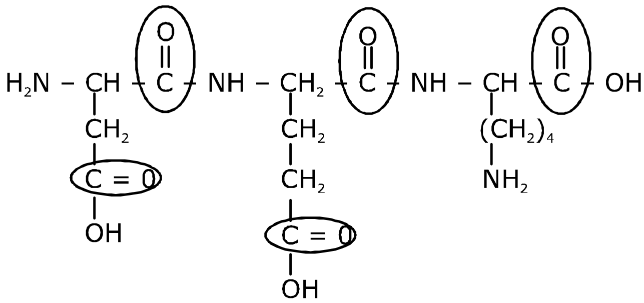
$$\text{Mol. wt of 'x'} = \frac{10}{6.023 \times 10^{22}} \times 6.023 \times 10^{23}$$
$$= 100 \text{ g/mol}$$

$$M = \frac{5/100}{2} = \left(\frac{5}{200} \times 1000\right) \times 10^{-3}$$

$M = 25 \times 10^{-3}$ mol/lit

25. The number of $>C{=}O$ groups present in a tripeptide Asp–Glu–Lys is _____.

Sol. **5**



QUESTION PAPER WITH SOLUTION

MATHEMATICS _ 3 Sep. _ SHIFT - 2

Q.1 If $x^3dy+xy\,dx=x^2dy+2y\,dx$; $y(2)=e$ and $x>1$, then $y(4)$ is equal to:

(1) $\frac{\sqrt{e}}{2}$ (2) $\frac{3}{2}\sqrt{e}$ (3) $\frac{1}{2}+\sqrt{e}$ (4) $\frac{3}{2}+\sqrt{e}$

Sol. **2**

$(x^3 - x^2)dy = (2 - x)\,ydx$

$$\int \frac{dy}{y} = \int \frac{2-x}{x^2(x-1)}dx$$

$$\int \frac{dy}{y} = -\int \frac{x-1-1}{x^2(x-1)}dx$$

$$\int \frac{dy}{y} = -\int \frac{dx}{x^2} = \int \frac{x^2-1-x^2}{x^2(x-1)}$$

$$= \frac{1}{x} - \int \frac{x+1}{x^2}dx + \int \frac{dx}{x-1}$$

$$\ln|y| = \frac{2}{x} - \ln|x| + \ln|x-1| + c$$

$x = 2, y = e$

$1 = 1 - \ln 2 + c \Rightarrow c = \ln 2$

$$\ln|y| = \frac{2}{x} - \ln|x| + \ln|x-1| + \ln 2$$

put $x = 4$

$$\ln|y| = \frac{1}{2} - 2\ln 2 + \ln 3 + \ln 2$$

$$\ln y = \ln\left(\frac{3}{2}\right) + \frac{1}{2}$$

$$y = \frac{3}{2} \cdot e^{\frac{1}{2}} = \frac{3}{2}\sqrt{e}$$

Q.2 Let A be a 3×3 matrix such that $\text{adj}\,A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix}$ and B=adj(adj A).

If $|A| = \lambda$ and $|(B^{-1})^T| = \mu$, then the ordered pair, $(|\lambda|, \mu)$ is equal to:

(1) $\left(9, \frac{1}{81}\right)$ (2) $\left(9, \frac{1}{9}\right)$ (3) $\left(3, \frac{1}{81}\right)$ (4) (3, 81)

Sol. **3**

adjA = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix}$ $\Rightarrow$ |adjA| = 9

$\Rightarrow$ $|A|^2 = 9 \Rightarrow |A| = 3 = |\lambda|$
B = adj (adjA) = |A|. A = 3A

$|(B^T)^{-1}| = \frac{1}{|B^T|} = \frac{1}{|B|} = \frac{1}{|3A|} = \frac{1}{27 \times 3} = \frac{1}{81} = \mu$

$|\lambda|, \mu = \left(3, \frac{1}{81}\right)$

Q.3 Let a,b,c $\in \mathbb{R}$ be such that $a^2+b^2+c^2 = 1$, If $a\cos\theta = b\cos\left(\theta + \frac{2\pi}{3}\right) = c\cos\left(\theta + \frac{4\pi}{3}\right)$, where $\theta = \frac{\pi}{9}$,

then the angle between the vectors $a\hat{i} + b\hat{j} + c\hat{k}$ and $b\hat{i} + c\hat{j} + a\hat{k}$ is

(1) $\frac{\pi}{2}$ (2) $\frac{2\pi}{3}$ (3) $\frac{\pi}{9}$ (4) 0

Sol. **1**

$\cos\alpha = \frac{ab + bc + ca}{a^2 + b^2 + c^2}$

$a\cos\theta = b\cos(\theta + \frac{2\pi}{3}) = c\cos\left(\theta + \frac{4\pi}{3}\right) = \lambda$

$$\frac{1}{a}=\frac{\cos\theta}{\lambda},\frac{1}{b}=\frac{\cos\left(\theta+2\frac{\pi}{3}\right)}{\lambda},\frac{1}{c}=\frac{\cos\left(\theta+\frac{4\pi}{3}\right)}{\lambda}$$

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{\lambda}\left[\cos\theta+\cos\left(\theta+\frac{2\pi}{3}\right)+\cos\left(\theta+\frac{4\pi}{3}\right)\right]$$

$$=\frac{1}{\lambda}\frac{\sin\left[(3)\left(\frac{\pi}{3}\right)\right]}{\sin\left(\frac{\pi}{3}\right)}\ .\cos\left[\frac{\theta+\theta+\frac{4\pi}{3}}{2}\right]$$

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$$

$\sum ab = 0$

$\cos\alpha = 0$

$\alpha = \frac{\pi}{2}$

Q.4 Suppose f(x) is a polynomial of degree four, having critical points at –1,0,1. If $T=\{x\in R\,|\,f(x)=f(0)\}$, then the sum of squares of all the elements of T is:

(1) 6 (2) 2 (3) 8 (4) 4

Sol. **4**

$f'(x) = k\,(x + 1)x(x-1)$

$f'(x) = k\,[\,x^3 - x]$

Integrating both sides

$$f(x) = k\left[\frac{x^4}{4}-\frac{x^2}{2}\right]+c$$

$f(0) = c$

$$f(x) = f(0) \Rightarrow k\left(\frac{x^4}{4}-\frac{x^2}{2}\right)+c=c$$

$$\Rightarrow k\frac{x^2}{4}\left(x^2-2\right)=0$$

$\Rightarrow x = 0, \pm\sqrt{2}$

sum of all of squares of elements = $o^2 + (\sqrt{2})^2 + (-\sqrt{2})^2$

$= 4$

Q.5 If the value of the integral $\int_0^{1/2} \frac{x^2}{(1-x^2)^{3/2}} dx$ is $\frac{k}{6}$, then k is equal to:

(1) $2\sqrt{3}+\pi$ (2) $3\sqrt{2}+\pi$ (3) $3\sqrt{2}-\pi$ (4) $2\sqrt{3}-\pi$

Sol. **4**

$$\int_0^{\frac{1}{2}} \frac{x^2}{(1-x^2)^{\frac{3}{2}}} dx$$

x = sinθ

$$\int_0^{\frac{\pi}{6}} \frac{\sin^2\theta}{\cos^3\theta}.\cos\theta d\theta$$

$$\int_0^{\frac{\pi}{6}} \tan^2\theta d\theta = [\tan\theta - \theta]_0^{\frac{\pi}{6}}$$

$$\Rightarrow \left(\frac{1}{\sqrt{3}} - \frac{\pi}{6}\right) = \frac{k}{6}$$

$$\frac{2\sqrt{3}-\pi}{6} = \frac{k}{6}$$

k = $2\sqrt{3} - \pi$

Q.6 If the term independent of x in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is k, then 18 k is equal to:

(1) 5 (2) 9 (3) 7 (4) 11

Sol. **3**

$$T_{r+1} = {}^9C_r \left(\frac{3}{2}x^2\right)^{9-r} \left(\frac{-1}{3x}\right)^r$$

$$= {}^9C_r \frac{3^{9-2r}}{2^{9-r}} (-1)^r . x^{18-3r}$$

18 - 3r = 0

$\Rightarrow$ r=6

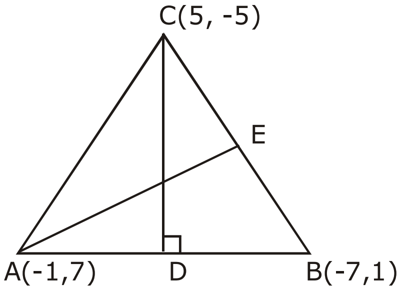
$$= {}^9C_r \left(\frac{3^{-3}}{2^3}\right) = k$$

$$= \frac{7}{18} = k \Rightarrow 18k = 7$$

7. If a ΔABC has vertices A(−1,7), B(−7,1) and C(5,−5), then its orthocentre has coordinates:

(1) (−3,3) (2) $\left(-\frac{3}{5},\frac{3}{5}\right)$ (3) $\left(\frac{3}{5},-\frac{3}{5}\right)$ (4) (3,−3)

Sol. **1**



equation of CD
y + 5 = -1 (x - 5)
x + y = 0 .....(1)
equation of AE
y - 7 = 2 (x + 1)
2x - y = -9 ....(2)
from (1) & (2)
x = -3, y = 3
Othocentre = (-3, 3)

Q.8. Let e_1 and e_2 be the eccentricities of the ellipse, $\frac{x^2}{25}+\frac{y^2}{b^2}=1$ (b<5) and the hyperbola, $\frac{x^2}{16}-\frac{y^2}{b^2}=1$ respectively satisfying $e_1e_2=1$. If α and β are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair (α,β) is equal to:

(1) (8,12) (2) $\left(\frac{24}{5},10\right)$ (3) $\left(\frac{20}{3},12\right)$ (4) (8,10)

Sol. **4**

$$\left.\begin{array}{l}\alpha = 10e_1 \\ \beta = 8e_2\end{array}\right]$$

$$\left.\begin{array}{l}b^2 = 25\left(1-e_1^2\right) \\ b^2 = 16\left(e_2^2-1\right)\end{array}\right]$$

$(e_1e_2)^2 = 1$

$$\left(1-\frac{b^2}{25}\right)\left(1+\frac{b^2}{16}\right)=1$$

$$\Rightarrow 1+\frac{b^2}{25}-\frac{b^2}{25}-\frac{b^4}{400}=1$$

$$\Rightarrow \frac{9}{16.25}b^2 = \frac{b^4}{400} \Rightarrow b^2 = 9$$

$$\left.\begin{matrix} e_1 = \frac{4}{5} \\ e_2 = \frac{5}{4} \end{matrix}\right] = \left.\begin{matrix} \alpha = 2ae_1 = 10\times\frac{4}{5} = 8 \\ \beta = 2ae_2 = 8\times\frac{5}{4} = 10 \end{matrix}\right] = (\alpha, \beta) = (8, 10)$$

Q.9 If z_1, z_2 are complex numbers such that $Re(z_1)=|z_1-1|$, $Re(z_2)=|z_2-1|$ and $\arg(z_1-z_2)=\frac{\pi}{6}$, then $Im(z_1+z_2)$ is equal to:

(1) $2\sqrt{3}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{\sqrt{3}}{2}$

Sol. **1**

$z_1 = x_1 + iy_1$, $z_2 = z_2 + iy_2$

$x_1^2 = (x_1 - 1)^2 + y_1^2$ .....(1)

$\Rightarrow y_1^2 - 2x_1 + 1 = 0$

$x_2^2 = (x_2 - 1)^2 + y_2^2$

$y_2^2 - 2x_2 - 1 = 0$ ....(2)

from equation (1) – (2)

$(y_1^2 - y_2^2) + 2(x_2 - x_1) = 0$

$(y_1 + y_2)(y_1 - y_2) = 2(x_1 - x_2)$

$$y_1 + y_2 = 2\left(\frac{x_1 - x_2}{y_1 - y_2}\right)$$

$$\arg(z_1 - z_2) = \frac{\pi}{6}$$

$$\tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \frac{\pi}{6}$$

$$\Rightarrow \frac{y_1 - y_2}{x_1 - x_2} = \frac{1}{\sqrt{3}}$$

$$\therefore y_1 + y_2 = 2\sqrt{3}$$

Q.10 The set of all real values of λ for which the quadratic equations, $\left(\lambda^2+1\right)x^2-4\lambda x+2=0$ always have exactly one root in the interval (0,1) is:

(1) (–3,–1) (2) (2,4] (3) (1,3] (4) (0,2)

Sol. **3**

f(0) f(1) ≤ 0

$\Rightarrow$ (2) $[\lambda^2 - 4\lambda + 3] \le 0$

$(\lambda - 1)(\lambda - 3) \le 0$

$\Rightarrow \lambda \in [1, 3]$

at $\lambda = 1$

$2x^2 - 4x + 2 = 0$

$\Rightarrow (x - 1)^2 = 0$

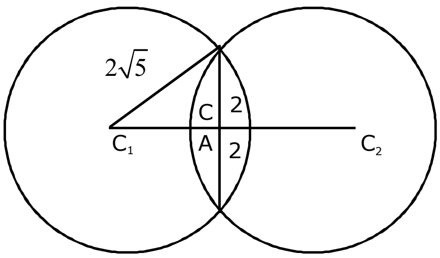
x = 1, 1

$\therefore \lambda \in (1, 3]$

Q.11 Let the latus ractum of the parabola $y^2=4x$ be the common chord to the circles C_1 and C_2 each of them having radius $2\sqrt{5}$. Then, the distance between the centres of the circles C_1 and C_2 is:

(1) 8 (2) $8\sqrt{5}$ (3) $4\sqrt{5}$ (4) 12

Sol. **1**



$C_1C_2 = 2C_1A$

$(C_1A)^2 + 4 = (2\sqrt{5})^2$

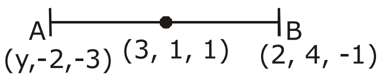
$C_1A = 4$

$C_1C_2 = 8$

Q.12 The plane which bisects the line joining the points (4,–2,3) and (2,4,–1) at right angles also passes through the point:

(1) (0,–1,1) (2) (4,0,1) (3) (4,0,–1) (4) (0,1,–1)

Sol. **3**



a = 2, b = -6

c = 4

equation of plane

$2(x - 3) + (-6)(y - 1) + 4(z - 1) = 0$

$\Rightarrow$ 2x - 6y + 4z = 4
passes through (4, 0, -1)

Q.13 $\lim_{x\to a}\frac{(a+2x)^{\frac{1}{3}}-(3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}}-(4x)^{\frac{1}{3}}}(a\neq 0)$ is equal to :

(1) $\left(\frac{2}{9}\right)^{\frac{4}{3}}$ (2) $\left(\frac{2}{3}\right)^{\frac{4}{3}}$ (3) $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$ (4) $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$

Sol. **3**

Apply L-H Rule

$$\lim_{x\to a}\frac{\frac{2}{3}(a+2x)^{\frac{-2}{3}}-3^{\frac{1}{3}}.\frac{1}{3}x^{-\frac{2}{3}}}{\frac{1}{3}(3a+x)^{\frac{-2}{3}}-4^{\frac{1}{3}}.\frac{1}{3}x^{-\frac{2}{3}}}$$

$$\Rightarrow \frac{\frac{2}{3}(3a)^{\frac{-2}{3}}-\frac{1}{3^{\frac{2}{3}}}.\left(a^{-\frac{2}{3}}\right)}{\frac{1}{3}(4a)^{\frac{-2}{3}}-\frac{1}{3}.4^{\frac{1}{3}}\left(a^{-\frac{2}{3}}\right)}$$

$$=\frac{2}{3}.\left(\frac{2}{9}\right)^{\frac{1}{3}}$$

Q.14 Let $x_i (1\leq i\leq 10)$ be ten observations of a random variable X. If $\sum_{i=1}^{10}(x_i-p)=3$ and $\sum_{i=1}^{10}(x_i-p)^2=9$ where $0\neq p\in R$, then the standard deviation of these observations is :

(1) $\frac{7}{10}$ (2) $\frac{9}{10}$ (3) $\sqrt{\frac{3}{5}}$ (4) $\frac{4}{5}$

Sol. **2**

Standard deviation
is free from shifting
of origin

S . D = $\sqrt{\text{variance}}$

$$= \sqrt{\frac{9}{10} - \left(\frac{3}{10}\right)^2}$$

$$= \sqrt{\frac{9}{10} - \frac{9}{100}}$$

$$= \sqrt{\frac{81}{100}} = \frac{9}{10}$$

Q.15 The probability that a randomly chosen 5–digit number is made from exactly two digits is :

(1) $\frac{134}{10^4}$ (2) $\frac{121}{10^4}$ (3) $\frac{135}{10^4}$ (4) $\frac{150}{10^4}$

Sol. **3**

Total case = $9(10^4)$

fav. case = ${}^9C_2\ (2^5 - 2) + {}^9C_1\ (2^4 - 1)$

= 1080 + 135 = 1215

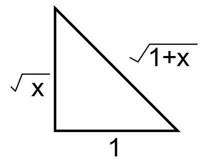
Prob = $\frac{1215}{9 \times 10^4} = \frac{135}{10^4}$

Q.16 If $\int \sin^{-1}\left(\sqrt{\frac{x}{1+x}}\right)dx = A(x)\tan^{-1}\left(\sqrt{x}\right) + B(x) + C$, where C is a constant of integration, then the ordered pair (A(x),B(x)) can be:

(1) $\left(x+1, -\sqrt{x}\right)$ (2) $\left(x-1, -\sqrt{x}\right)$ (3) $\left(x+1, \sqrt{x}\right)$ (4) $\left(x-1, \sqrt{x}\right)$

Sol. **1**

$$\int \sin^{-1}\sqrt{\frac{x}{1+x}}dx$$



$$\int \underset{I}{\tan^{-1}\sqrt{x}}.\ \underset{II}{1}\,dx$$

$$\left(\tan^{-1}\sqrt{x}\right).x - \int \frac{x}{1+x}.\frac{1}{2\sqrt{x}}dx$$

put $x = t^2 \Rightarrow dx = 2t\ dt$

$= x\tan^{-1}\sqrt{x} - \int \frac{(t^2)(2tdt)}{(1+t^2)(2t)}$

$= x\tan^{-1}\sqrt{x} - t + \tan^{-1} t + c$

$= x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + c$

$A(x) = x + 1$, $B(x) = -\sqrt{x}$

Q.17 If the sum of the series $20+19\frac{3}{5}+19\frac{1}{5}+18\frac{4}{5}+\ldots$ upto n^{th} term is 488 and the n^{th} term is negative, then:

(1) n=60 (2) n=41 (3) n^{th} term is -4 (4) n^{th} term is $-4\frac{2}{5}$

Sol. **3**

$20+\frac{98}{5}+\frac{96}{5}+\ldots\ldots$

$S_n = 488$

$\Rightarrow \frac{n}{2}\left[2\times 20+(n-1)\left(\frac{-2}{5}\right)\right] = 488$

$\Rightarrow 20n - \frac{n^2}{5} + \frac{n}{5} = 488$

$\Rightarrow 100n - n^2 + n = 2440$

$= n^2 - 101n + 2440 = 0$

$\Rightarrow n = 61$ or 40

for $n = 40$, $T_n = 20 + 39\left(\frac{-2}{5}\right) = +ve$

$n = 61$, $T_n = 20 + 60\left(\frac{-2}{5}\right) = 20 - 24 = -4$

Q.18 Let p, q, r be three statements such that the truth value of $(p \wedge q) \rightarrow (\sim p \vee r)$ is F. Then the truth values of p, q, r are respectively :

(1) F, T, F (2) T, F, T (3) T, T, F (4) T, T, T

Sol. **3**

$(p \wedge q) \rightarrow (\sim q \vee r)$

Possible when

$p \wedge q \rightarrow T$

$\sim q \vee r \rightarrow F$

$$\left.\begin{array}{l} p \rightarrow T \\ q \rightarrow T \\ r \rightarrow F \end{array}\right] \qquad \begin{array}{l} p \wedge q \Rightarrow T \\ \sim q \vee r \rightarrow F \vee F \Rightarrow F \\ T \rightarrow F \Rightarrow F \end{array}$$

Q.19 If the surface area of a cube is increasing at a rate of 3.6 cm^2/sec, retaining its shape; then the rate of change of its volume (in cm^3/sec), when the lenght of a side of the cube is 10cm, is :

(1) 9 (2) 10 (3) 18 (4) 20

Sol. **1**

$A = 6a^2$

$a \rightarrow$ side of cube

$$\frac{dA}{dt} = 6\left(2a\frac{da}{dt}\right) \Rightarrow 3.6 = 12 \times 10\ \frac{da}{dt} \Rightarrow \frac{da}{dt} = \frac{3}{100}$$

$v = a^3$

$$\frac{dV}{dt} = 3a^2\frac{da}{dt}$$

$$= 3 \times 100 \times \frac{3}{100}$$

$= 9cm^3$ / sec

Q.20 Let R_1 and R_2 be two relations defined as follows:

$R_1 = \{(a, b) \in R^2 : a^2 + b^2 \in Q\}$ and

$R_2 = \{(a, b) \in R^2 : a^2 + b^2 \notin Q\}$, where Q is the set of all rational numbers. Then :

(1) R_1 is transitive but R_2 is not transitive
(2) R_1 and R_2 are both transitive
(3) R_2 is transitive but R_1 is not transitive
(4) Neither R_1 nor R_2 is transitive

Sol. **4**

for R_1

Let $a = 1 + \sqrt{2}$, $b = 1 - \sqrt{2}$, $c = 8^{\frac{1}{4}}$

aR_1b $a^2 + b^2 = (1 + \sqrt{2})^2 + (1 - \sqrt{2})^2 = 6 \in Q$

bR$_1$c $b^2 + c^2 = (1 - \sqrt{2})^2 + \left(8^{\frac{1}{4}}\right)^2 = 3 \in Q$

aR$_1$c $\Rightarrow a^2+c^2 = \left(1+\sqrt{2}\right)+\left(8^{1/4}\right)^2 = 3+4\sqrt{2} \notin Q$

R$_1$ is not transitive

R$_2$

let $a = 1 + \sqrt{2}$, $b = \sqrt{2}$, $c = 1- \sqrt{2}$

aR$_2$b $a^2 + b^2 = 5 + 2\sqrt{2} \notin Q$

bR$_2$c $b^2 + c^2 = 5 - 2\sqrt{2} \notin Q$

aR$_2$c $a^2 + c^2 = 6 \in Q$

R$_2$ is not transitive

Q.21 If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4th A.M. is equal to 2nd G.M., then m is equal to____

Sol. **39**

3, ..., 243
m A.M.

$$d = \frac{b-a}{n+1} = \frac{243-3}{m+1} = \frac{240}{m+1}$$

4^{tn} A.M = 3 + 4d = $3+4\left(\frac{240}{m+1}\right)$

$$3+\frac{960}{m+1} = 27$$

$$= \frac{960}{m+1} = 24$$

$\Rightarrow$ m = 39

3,...........,243
3 G.M

$243 = 3(r)^4$

r = 3

2nd G.M. = $ar^2 = 27$

Q.22 Let a plane P contain two lines $\vec{r} = \hat{i}+\lambda\left(\hat{i}+\hat{j}\right), \lambda \in R$ and $\vec{r} = -\hat{j}+\mu\left(\hat{j}-\hat{k}\right), \mu \in R$. If $Q(\alpha,\beta,\gamma)$ is the foot of the perpendicular drawn from the point M(1,0,1) to P,then $3(\alpha+\beta+\gamma)$ equals _____

Sol. **5**

$$\left.\begin{array}{l}\vec{r} = \hat{i}+\lambda\left(\hat{i}+\hat{j}\right) \\ \vec{r} = -\hat{j}+\mu\left(\hat{j}-\hat{k}\right)\end{array}\right]$$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}$$

= (-1, 1 ,1)

equation of plane

-1(x - 1) + 1(y - 0) + 1(z - 0) = 0

$\Rightarrow$ x - y - z - 1 = 0

foot of $\perp^r$ from m(1, 0, 1)

$$\frac{x-1}{1} = \frac{y-0}{-1} = \frac{z-1}{-1} = -\frac{(1-0-1-1)}{3}$$

$$x-1 = \frac{1}{3} \quad \left|\frac{y}{-1} = \frac{1}{3}\right| \quad = \frac{z-1}{-1} = \frac{1}{3}$$

$$x = \frac{4}{2}, y = \frac{-1}{3}, z = \frac{2}{3}$$

$$\Rightarrow \left.\begin{matrix} \alpha = \frac{4}{3} \\ \beta = \frac{-1}{3} \\ \gamma = \frac{2}{3} \end{matrix}\right]$$

$$\alpha+\beta+\gamma = \frac{4}{3} - \frac{1}{3} + \frac{2}{3} = \frac{5}{3}$$

3($\alpha+\beta+\gamma$) = 5

Q.23 Let S be the set of all integer solutions, (x, y, z), of the system of equations

x – 2y + 5z = 0

–2x + 4y + z = 0

–7x + 14y + 9z = 0

such that $15 \le x^2 + y^2 + z^2 \le 150$. Then, the number of elements in the set S is equal to ____

Sol. **8**

x - 2y + 5z = 0 .....(1)

-2x + 4y + z = 0 ....(2)

-7x + 14y + 9z = 0 .....(3)

2.(1) + (2) we get z = 0, x = 2y

$15 \le 4y^2 + y^2 \le 150$
$\Rightarrow 3 \le y^2 \le 30$

$y \in \left[-\sqrt{30}, -\sqrt{3}\right] \cup \left[\sqrt{3}, \sqrt{30}\right]$

$y = \pm 2, \pm 3, \pm 4, \pm 5$
no. of integer's in S is 8

Q.24 The total number of 3–digit numbers, whose sum of digits is 10, is _____
Sol. **54**
Let xyz be 3 digit number
$x + y + z = 0$ where $x \ge 1$, $y \ge 0$, $z \ge 0$

$\Rightarrow t + y + z = 9$ $\qquad \left.\begin{matrix} x - 1 \ge 0 \\ t \ge 0 \end{matrix}\right] x - 1 = t$

${}^{9+3-1}C_{3-1} = 11c_2 = 55$
but for $t = 9$, $x = 10$ not possible
total numbers = 55 - 1 = 54

Q.25 If the tangent to the curve, $y=e^x$ at a point (c,e^C) and the normal to the parabola, $y^2=4x$ at the point (1,2) intersect at the same point on the x-axis, then the value of c is _____
Sol. **4**
Tangent at (c, e^c) $\quad y - e^c = e^c (x - c)$ $\quad$ ....(1)
normal to parabola $y - 2 = -1 (x - 1)$
$x + y = 3$ $\quad$ ...(2)
at x-axis $y = 0$ $\quad$ at x-axis $y = 0$
in (1), $x = c - 1$ $\quad$ in (2), $x = 3$
$c - 1 = 3 \Rightarrow c = 4$