12/04/2019
Evening

Answers & Solutions

for

JEE (MAIN)-2019 (Online) Phase-2

(Physics, Chemistry and Mathematics)

Time : 3 hrs. **M.M. : 360**

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test Booklet consists of **90** questions**.** The maximum marks are **360.**
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage.
4. Each question is allotted 4 **(four)** marks for each correct response. *¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
5. There is only one correct response for each question.

PART–A : PHYSICS

1. A uniform cylindrical rod of length L and radius r, is made from a material whose Young's modulus of Elasticity equals Y. When this rod is heated by temperature T and simultaneously subjected to a net longitudinal compressional force F, its length remains unchanged. The coefficient of volume expansion, of the material of the rod, is (nearly) equal to:

   (1) $9F/(\pi r^2 YT)$

   (2) $3F/(\pi r^2 YT)$

   (3) $F/(3\pi r^2 YT)$

   (4) $6F/(\pi r^2 YT)$

Answer (2)

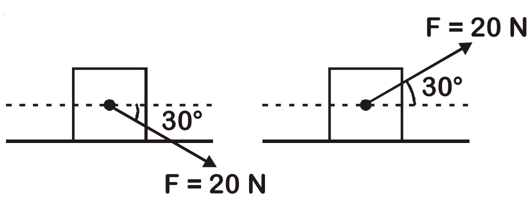
Sol. $Y = \frac{FL}{A|\Delta L|} \Rightarrow \Delta L = \frac{FL}{AY} = L\alpha T$

$\alpha = \frac{F}{AYT}$

$\gamma = 3\alpha = \frac{3F}{\pi r^2 YT}$

2. A block of mass 5 kg is (i) pushed in case (A) and (ii) pulled in case (B), by a force F = 20 N, making an angle of 30° with the horizontal, as shown in the figures. The coefficient of friction between the block and floor is μ = 0.2. The difference between the accelerations of the block, in case (B) and case (A) will be :

   ($g = 10\ ms^{-2}$)



(A) (B)

   (1) $0.4\ ms^{-2}$

   (2) $3.2\ ms^{-2}$

   (3) $0\ ms^{-2}$

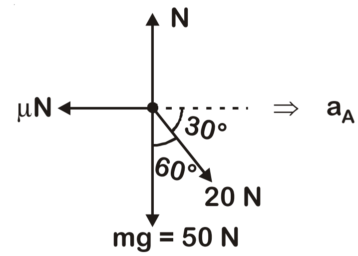
   (4) $0.8\ ms^{-2}$

Answer (4)

Sol. N = 60 N

F = 0.2 × 60 = 12 N

$$a_A = \frac{\left(\frac{20\sqrt{3}}{2} - 12\right)}{5} = \frac{5.3}{5} = 1.06\ m/s^2$$



For B N = 40 N

$$F = 8\,N \Rightarrow \frac{20\sqrt{3}}{2} - 8 = 5a_B$$

$$a_B = \frac{17.3-8}{5} = \frac{9.3}{5} = 1.86\ m/s^2$$

$a_B - a_A = 0.8\ m/s^2$

3. The electron in a hydrogen atom first jumps from the third excited state to the second excited state and subsequently to the first excited state. The ratio of the respective wavelengths, λ_1/λ_2, of the photons emitted in this process is

   (1) 7/5

   (2) 27/5

   (3) 9/7

   (4) 20/7

Answer (4)

Sol. $\frac{1}{\lambda_1} = R\left[\frac{1}{9} - \frac{1}{16}\right] = R\frac{7}{144}$ ...(i)

$\frac{1}{\lambda_2} = R\left[\frac{1}{4} - \frac{1}{9}\right] = R\frac{5}{36}$ ...(ii)

$$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{\lambda_1}{\lambda_2} = \frac{5\times 144}{36\times 7} = \frac{20}{7}$$

4. A moving coil galvanometer, having a resistance G, produces full scale deflection when a current I_g flows through it. This galvanometer can be converted into (i) an ammeter of range 0 to $I_0(I_0 > I_g)$ by connecting a shunt resistance R_A to it and (ii) into a voltmeter of range 0 to $V(V = GI_0)$ by connecting a series resistance R_V to it. Then,

(1) $R_A R_V = G^2 \text{and} \frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

(2) $R_A R_V = G^2\left(\frac{I_g}{I_0 - I_g}\right) \text{and} \frac{R_A}{R_V} = \left(\frac{I_0 - I_g}{I_g}\right)^2$

(3) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \frac{I_g}{(I_0 - I_g)}$

(4) $R_A R_V = G^2\left(\frac{(I_0 - I_g)}{I_g}\right) \text{and} \frac{R_A}{R_V} = \left(\frac{I_g}{(I_0 - I_g)}\right)^2$

Answer (1)

Sol. $V = I_g(R_v + G) = GI_0$ ...(1)

$(I_0 - I_g)R_A = I_g G$ ...(2)

From (1), $R_v = \frac{G(I_0 - I_g)}{I_g}$

From (2), $R_A = \frac{I_g G}{I_0 - I_g}$

$\Rightarrow R_A R_V = G^2$

$\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

5. A spring whose unstretched length is l has a force constant k. The spring is cut into two pieces of unstretched lengths l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio k_1/k_2 of the corresponding force constants, k_1 and k_2 will be

(1) n^2 (2) $\frac{1}{n^2}$

(3) n (4) $\frac{1}{n}$

Answer (4)

Sol. $l_1 = nl_2$

$\because k \propto \frac{1}{l}$

$\therefore \frac{k_1}{k_2} = \frac{l_2}{l_1} = \frac{1}{n}$

6. One kg of water, at 20°C, is heated in an electric kettle whose heating element has a mean (temperature averaged) resistance of 20 Ω. The rms voltage in the mains is 200 V. Ignoring heat loss from the kettle, time taken for water to evaporate fully, is close to

[Specific heat of water = 4200 J/(kg °C), Latent heat of water = 2260 kJ/kg]

(1) 16 minutes (2) 3 minutes

(3) 22 minutes (4) 10 minutes

Answer (3)

Sol. $\Delta Q = 1 \times 4200 \times 80 + 2260 \times 10^3$ J

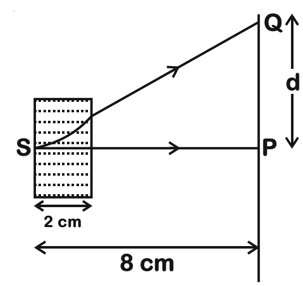
$= (336 + 2260) \times 10^3 \text{ J} = 2596 \times 10^3$ J

$\Delta Q = I_{rms} V_{rms} t = 200 \times \frac{200}{20} t = 2000t$

$\Rightarrow t = \frac{2596}{2} \text{s} \approx 21.6 \text{ minutes} \approx 22 \text{ minutes}$

7. An electron, moving along the x-axis with an initial energy of 100 eV, enters a region of magnetic field $\vec{B} = (1.5 \times 10^{-3} \text{ T})\hat{k}$ at S (See figure). The field extends between x = 0 and x = 2 cm. The electron is detected at the point Q on a screen placed 8 cm away from the point S. The distance d between P and Q (on the screen) is

(electron's charge = 1.6×10^{-19} C, mass of electron = 9.1×10^{-31} kg)



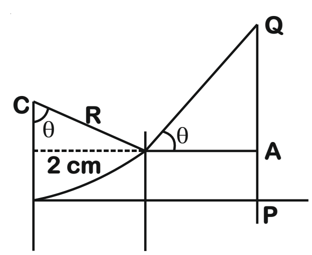
(1) 11.65 cm (2) 12.87 cm

(3) 2.25 cm (4) 1.22 cm

Answer (2)

Sol. Radius of path, $R = \frac{mv}{eB} = \frac{\sqrt{2m\,KE}}{eB}$

$= \frac{\sqrt{2 \times 9.1 \times 10^{-31} \times 100e}}{e \times 1.5 \times 10^{-3}} = \frac{\sqrt{2 \times 9.1 \times 10^{-29}}}{\sqrt{1.6 \times 10^{-19}} \times 1.5 \times 10^{-3}}$ m



$= \frac{3.37 \times 10^{-5}}{1.5 \times 10^{-3}} \times 100 \text{ cm} = 2.25 \text{ cm}$

$\sin\theta = \frac{2}{2.25} = \frac{8}{9}$

PQ = PA + AQ

= 2.25 [1 – cosθ] + 11.64

= 1.22 + 11.64 = 12.86 cm

8. A Carnot engine has an efficiency of $\frac{1}{6}$. When the temperature of the sink is reduced by 62°C, its efficiency is doubled. The temperatures of the source and the sink are, respectively,

(1) 99°C, 37°C (2) 37°C, 99°C

(3) 124°C, 62°C (4) 62°C, 124°C

Answer (1)

Sol. $\frac{1}{6} = 1 - \frac{T_C}{T_H} \quad \Rightarrow \quad \frac{T_C}{T_H} = \frac{5}{6}$ ...(i)

$\frac{1}{3} = 1 - \frac{(T_C - 62)}{T_H} \quad \Rightarrow \quad \frac{T_C - 62}{T_H} = \frac{2}{3}$ ...(ii)

$\frac{T_C - 62}{T_C} = \frac{2 \times 6}{3 \times 5} = \frac{4}{5}$

$\Rightarrow T_C = 310 \text{ K} = 37°\text{C}$

$T_H = 372 \text{ K} = 99°\text{C}$

9. A small speaker delivers 2 W of audio output. At what distance from the speaker will one detect 120 dB intensity sound?

[Given reference intensity of sound as 10^{-12} W/m^2]

(1) 30 cm (2) 40 cm

(3) 10 cm (4) 20 cm

Answer (2)

Sol. $120 = 10\log_{10}\frac{I}{10^{-12}}$

$\Rightarrow \frac{I}{10^{-12}} = 10^{12} \quad \Rightarrow \quad I = 1 \text{ W/m}^2$

$\frac{2}{4\pi r^2} = 1 \quad \Rightarrow \quad r = \sqrt{\frac{2}{4\pi}} \text{ m} = 0.399 \text{ m}$

$= 40$ cm

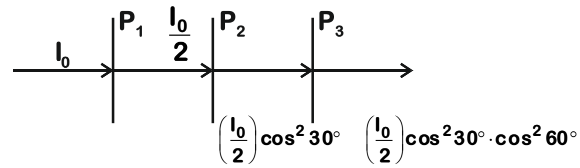
10. A system of three polarizers P_1, P_2, P_3 is set up such that the pass axis of P_3 is crossed with respect to that of P_1. The pass axis of P_2 is inclined at 60°C to the pass axis of P_3. When a beam of unpolarized light of intensity I_0 is incident on P_1, the intensity of light transmitted by the three polarizers is I. The ratio (I_0/I) equals (nearly) :

(1) 1.80 (2) 5.33

(3) 10.67 (4) 16.00

Answer (3)

Sol. Angle between pass axes of P_1 and P_2 is 30°.



$I = \frac{I_0}{2}\cdot\cos^2 30°\cdot\cos^2 60° = \frac{I_0}{2}\times\frac{3}{4}\times\frac{1}{4} = \frac{3I_0}{32}$

$\Rightarrow \frac{I_0}{I} = \frac{32}{3} = 10.67$

11. A solid sphere, of radius R acquires a terminal velocity v_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, v_2, when falling through the same fluid, the ratio $\left(\frac{v_1}{v_2}\right)$ equals :

(1) $\frac{1}{27}$ (2) 9

(3) $\frac{1}{9}$ (4) 27

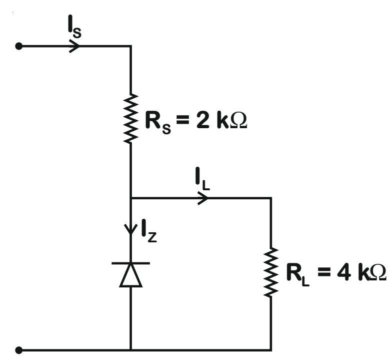
Answer (2)

Sol. $R^3 = 27r^3$

$\Rightarrow R = 3r$

$\frac{V_1}{V_2} = \left(\frac{R}{r}\right)^2 = 9$

12. Figure shows a DC voltage regulator circuit, with a Zener diode of breakdown voltage = 6 V. If the unregulated input voltage varies between 10 V to 16 V, then what is the maximum Zener current?



(1) 7.5 mA (2) 1.5 mA

(3) 2.5 mA (4) 3.5 mA

Answer (4)

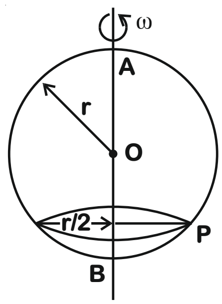
Sol. I_Z is maximum when input voltage is 16 V.

$$I_S = \frac{10}{2\times10^3} = 5\text{ mA}$$

$$I_L = \frac{6}{4\times10^3} = 1.5\text{ mA}$$

$$I_{Z(max)} = I_S - I_L = 3.5\text{ mA}$$

13. A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then the value of ω^2 is equal to:

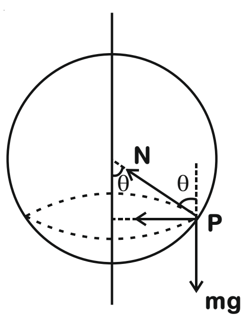


(1) $\frac{(g\sqrt{3})}{r}$ (2) $\frac{2g}{r}$

(3) $\frac{2g}{(r\sqrt{3})}$ (4) $\frac{\sqrt{3}g}{2r}$

Answer (3)

Sol.



$$\theta = \sin^{-1}\frac{1}{2} = 30^\circ$$

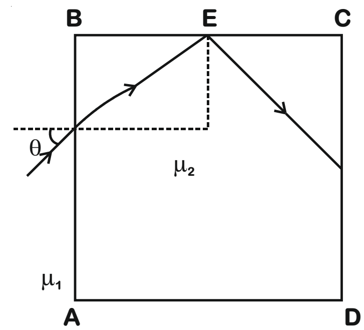
$$N\cos\theta = mg \quad \ldots(i)$$

$$N\sin\theta = \frac{m\omega^2 r}{2} \quad \ldots(ii)$$

$$\tan\theta = \frac{\omega^2 r}{2g}$$

$$\Rightarrow \quad \omega^2 = \frac{2g\tan\theta}{r} = \frac{2g}{r\sqrt{3}}$$

14. A transparent cube of side d, made of a material of refractive index μ_2, is immersed in a liquid of refractive index $\mu_1(\mu_1 < \mu_2)$. A ray is incident on the face AB at an angle θ(shown in the figure). Total internal reflection takes place at point E on the face BC.



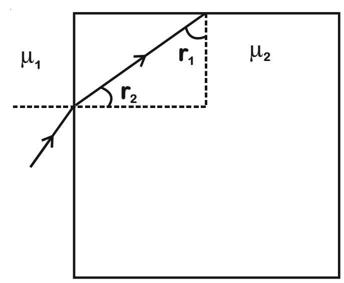
Then θ must satisfy:

(1) $\theta > \sin^{-1}\frac{\mu_1}{\mu_2}$ (2) $\theta < \sin^{-1}\frac{\mu_1}{\mu_2}$

(3) $\theta > \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}$ (4) $\theta < \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}$

Answer (4)

Sol. $\mu_2 \sin r_1 > \mu_1$



$$\Rightarrow \quad \sin r_1 > \frac{\mu_1}{\mu_2}$$

$$r_1 > \sin^{-1}\left[\frac{\mu_1}{\mu_2}\right]$$

$$\mu_1 \sin\theta = \mu_2 \sin r_2 = \mu_2 \sin(90^\circ - r_1)$$

$$\sin\theta = \frac{\mu_2}{\mu_1}\sin(90^\circ - r_1) = \frac{\mu_2}{\mu_1}\cos r_1$$

$$\Rightarrow \quad \theta < \sin^{-1}\left[\frac{\mu_2}{\mu_1}\frac{\sqrt{\mu_2^2 - \mu_1^2}}{\mu_2}\right]$$

$$\Rightarrow \quad \theta < \sin^{-1}\left[\sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}\right]$$

15. A particle is moving with speed $v = b\sqrt{x}$ along positive x-axis. Calculate the speed of the particle at time $t = \tau$ (assume that the particle is at origin at $t = 0$).

(1) $b^2\tau$ (2) $\frac{b^2\tau}{4}$

(3) $\frac{b^2\tau}{2}$ (4) $\frac{b^2\tau}{\sqrt{2}}$

Answer (3)

Sol. $v = \frac{dx}{dt} = b\sqrt{x}$ ...(i)

$\Rightarrow \int_0^x \frac{dx}{\sqrt{x}} = \int_0^\tau b\,dt$

$\Rightarrow 2\sqrt{x} = b\tau$ ...(ii)

$\Rightarrow v = b\cdot\frac{b\tau}{2} = \frac{b^2\tau}{2}$

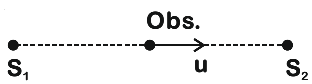
16. Two sources of sound S_1 and S_2 produce sound waves of same frequency 660 Hz. A listener is moving from source S_1 towards S_2 with a constant speed u m/s and he hears 10 beats/s. The velocity of sound is 330 m/s. Then, u equals:

(1) 10.0 m/s (2) 5.5 m/s

(3) 2.5 m/s (4) 15.0 m/s

Answer (3)

Sol.



$n_1 = \frac{v-u}{v}\times n_0$

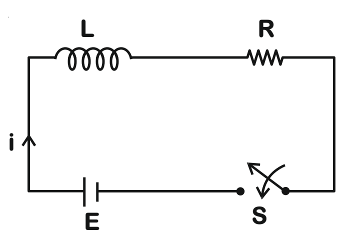
$n_2 = \frac{v+v}{v}\times n_0$

$\Rightarrow n_2 - n_1 = \frac{2u\times n_0}{v}$

$\Rightarrow 10 = \frac{2\times u\times 660}{330}$

$\Rightarrow$ u = 2.5 m/s

17. Consider the LR circuit shown in the figure. If the switch S is closed at t = 0 then the amount of charge that passes through the battery between t = 0 and $t = \frac{L}{R}$ is:



(1) $\frac{2.7EL}{R^2}$ (2) $\frac{EL}{2.7R^2}$

(3) $\frac{7.3\,EL}{R^2}$ (4) $\frac{EL}{7.3\,R^2}$

Answer (2)

Sol. $i = \frac{E}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$, $\tau = \frac{L}{R}$

$\Rightarrow \int dq = \frac{E}{R}\int\left(1 - e^{-\frac{t}{\tau}}\right)dt$

$\Rightarrow Q = \frac{E}{R}\times\left[t + e^{-\frac{t}{\tau}}\times\tau\right]_0^t$

$= \frac{E}{R}\times\left[\frac{L}{R} + \frac{L}{R}\cdot e^{-1} - \frac{L}{R}\right]$

$\Rightarrow Q = \frac{EL}{eR^2} = \frac{EL}{2.7R^2}$

18. The ratio of the weights of a body on the Earth's surface to that on the surface of a planet is 9 : 4. The mass of the planet is $\frac{1}{9}$th of that of the Earth. If 'R' is the radius of the Earth, what is the radius of the planet ? (Take the planets to have the same mass density)

(1) $\frac{R}{4}$ (2) $\frac{R}{3}$

(3) $\frac{R}{9}$ (4) $\frac{R}{2}$

Answer (4)

Sol. $\frac{g_e}{g_p} = \frac{9}{4}$

$\Rightarrow \frac{M_e\times R_P^2}{R_e^2\times M_p} = \frac{9}{4}$

$\Rightarrow 9\times\left(\frac{R_p}{R_e}\right)^2 = \frac{9}{4}$

$\Rightarrow R_p = \frac{R_e}{2}$

$\therefore R_p = \frac{R}{2}$

19. In an amplitude modulator circuit, the carrier wave is given by,

C(t) = 4 sin (20000 πt) while modulating signal is given by, m(t) = 2 sin (2000 πt). The values of modulation index and lower side band frequency are :

(1) 0.5 and 10 kHz

(2) 0.3 and 9 kHz

(3) 0.4 and 10 kHz

(4) 0.5 and 9 kHz

Answer (4)

Sol. Modulation index $= \frac{A_m}{A_c} = \frac{2}{4} = 0.5$

$$f_c = \frac{\omega_c}{2\pi} = \frac{20000\pi}{2\pi} = 10000 \text{ Hz}$$

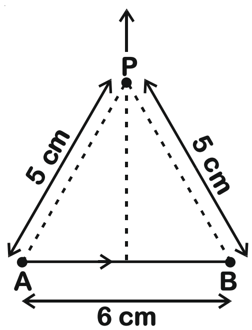
$$f_m = \frac{\omega_m}{2\pi} = \frac{2000\pi}{2\pi} = 1000 \text{ Hz}$$

$\therefore$ Lower band side = (10000 – 1000) Hz

= 9 kHz

20. Find the magnetic field at point P due to a straight line segment AB of length 6 cm carrying a current of 5 A. (See figure)

($\mu_0 = 4\pi \times 10^{-7}$ N-A^{-2})

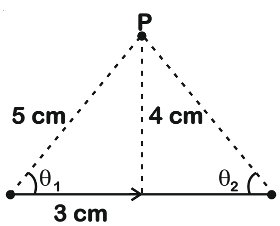


(1) 2.5×10^{-5} T (2) 1.5×10^{-5} T

(3) 3.0×10^{-5} T (4) 2.0×10^{-5} T

Answer (2)

Sol.



$$B_P = \frac{\mu_0 I}{4\pi d}(\cos\theta_1 + \cos\theta_2)$$

$$= \frac{10^{-7} \times 5}{0.04} \times \left(2 \times \frac{3}{5}\right) = 1.5 \times 10^{-5} \text{ T}$$

21. Consider an electron in a hydrogen atom, revolving in its second excited state (having radius 4.65 Å). The de-Broglie wavelength of this electron is :

(1) 3.5 Å

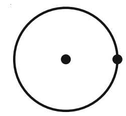
(2) 12.9 Å

(3) 9.7 Å

(4) 6.6 Å

Answer (3)

Sol. For n = 3,



$2\pi r = 3 \times \lambda$

$$\Rightarrow \lambda = \frac{2\pi \times 4.65}{3} \text{ Å} = 9.7 \text{ Å}$$

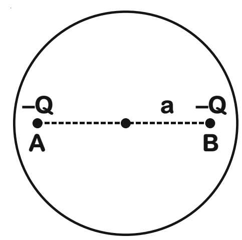
22. Let a total charge 2 Q be distributed in a sphere of radius R, with the charge density given by $\rho(r) = kr$, where r is the distance from the centre. Two charges A and B, of –Q each, are placed on diametrically opposite points, at equal distance, a, from the centre. If A and B do not experience any force, then :

(1) $a = \frac{3R}{2^{1/4}}$ (2) $a = R/\sqrt{3}$

(3) $a = 2^{-1/4}R$ (4) $a = 8^{-1/4}R$

Answer (4)

Sol. $E \times 4\pi a^2 = \frac{q_{in}}{\varepsilon_0}$



$$q_{in} = \int_0^a (kr) \times 4\pi r^2 dr$$

$$= 4\pi k \cdot \left(\frac{a^4}{4}\right)$$

$$\therefore E \times 4\pi a^2 = \frac{4\pi k a^4}{4\varepsilon_0} \Rightarrow E = \frac{ka^2}{4\varepsilon_0}$$

$$\therefore \frac{ka^2}{4\varepsilon_0} \times Q = \frac{Q^2}{4\pi\varepsilon_0 \times 4a^2}$$

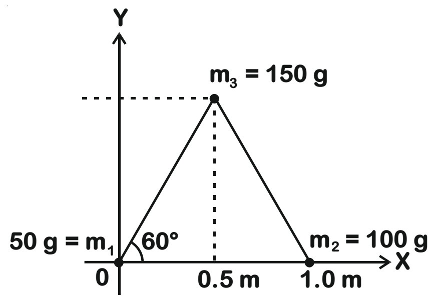
$$\frac{2Q}{\pi R^4} \cdot \frac{a^2 Q}{4\varepsilon_0} = \frac{Q^2}{4\pi\varepsilon_0 \times 4a^2} \Rightarrow 8a^4 = R^4$$

$$\Rightarrow 8^{1/4} \cdot a = R$$

$$\Rightarrow a = \frac{R}{8^{1/4}}$$

As, $2Q = \frac{4\pi k R^2}{4} \Rightarrow k = \frac{2Q}{\pi R^4}$

23. Three particles of masses, 50 g, 100 g and 150 g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be :

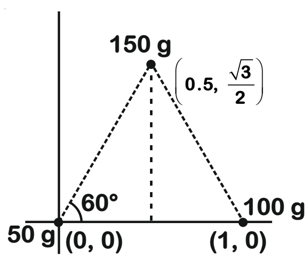


(1) $\left(\frac{\sqrt{3}}{8}\text{ m}, \frac{7}{12}\text{ m}\right)$ (2) $\left(\frac{\sqrt{3}}{4}\text{ m}, \frac{5}{12}\text{ m}\right)$

(3) $\left(\frac{7}{12}\text{ m}, \frac{\sqrt{3}}{8}\text{ m}\right)$ (4) $\left(\frac{7}{12}\text{ m}, \frac{\sqrt{3}}{4}\text{ m}\right)$

Answer (4)

Sol.



$$X_{CM} = \frac{m\times 0 + 2m\times 1 + 3m\times\left(\frac{1}{2}\right)}{6\text{ m}} = \frac{7}{12}\text{ m}$$

$$Y_{CM} = \frac{m\times 0 + 2m\times 0 + 3m\times\left(\frac{\sqrt{3}}{2}\right)}{6\text{ m}} = \frac{3\sqrt{3}}{12}\text{ m}$$

$$= \frac{\sqrt{3}}{4}\text{ m}$$

24. Half lives of two radioactive nuclei A and B are 10 minutes and 20 minutes, respectively. If, initially a sample has equal number of nuclei, then after 60 minutes, the ratio of decayed numbers of nuclei A and B will be :

(1) 9 : 8 (2) 3 : 8

(3) 8 : 1 (4) 1 : 8

Answer (1)

Sol. $N_A = \frac{N_0}{2^{\left(\frac{60}{20}\right)}} = \frac{N_0}{64}$

$N_B = \frac{N_0}{2^{\left(\frac{60}{20}\right)}} = \frac{N_0}{8}$

$\therefore$ Required ratio = $\frac{N_0 - \frac{N_0}{64}}{N_0 - \frac{N_0}{8}}$

$= \frac{63\times 8}{64\times 7} = \frac{9}{8}$

25. A tuning fork of frequency 480 Hz is used in an experiment for measuring speed of sound (v) in air by resonance tube method. Resonance is observed to occur at two successive lengths of the air column, l_1 = 30 cm and l_2 = 70 cm. Then, v is equal to

(1) 332 ms^{-1} (2) 384 ms^{-1}

(3) 379 ms^{-1} (4) 338 ms^{-1}

Answer (2)

Sol. l_1 = 30 cm, l_2 = 70 cm

$\therefore \frac{\lambda}{2} = (l_2 - l_1) = 40\text{ cm}$

$\Rightarrow \lambda$ = 80 cm

$\therefore$ U = $\nu\lambda$ = 480 × (0.8) m/s

= 384 m/s

26. The number density of molecules of a gas depends on their distance r from the origin as, $n(r) = n_0 e^{-\alpha r^4}$. Then the total number of molecules is proportional to

(1) $n_0\alpha^{-3/4}$ (2) $\sqrt{n_0}\,\alpha^{1/2}$

(3) $n_0\alpha^{1/4}$ (4) $n_0\alpha^{-3}$

Answer (1)

Sol. $n = n_0 e^{-\alpha r^4}$

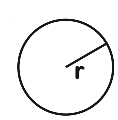
$\Rightarrow \int dN = \int n_0 e^{-\alpha r^4} \times 4\pi r^2 dr$

$\Rightarrow N = 4\pi n_0 \cdot \int_0^\infty r^2 e^{-\alpha r^4} dr$

Put $\sqrt{\alpha}\, r^2 = t$

$2\sqrt{\alpha}\, r\, dr = dt$

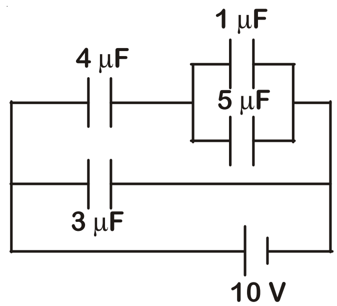
$N = \frac{4\pi n_0}{2\sqrt{\alpha}} \int_0^\infty \frac{t^{\frac{1}{2}} e^{-t^2}}{\alpha^{\frac{1}{4}}} dt$



$= \frac{4\pi n_0}{2\alpha^{\frac{3}{4}}} \int t^{\frac{1}{2}} e^{-t^2} dt$

$N \propto n_0 \alpha^{-\frac{3}{4}}$

27. In the given circuit, the charge on 4 μF capacitor will be

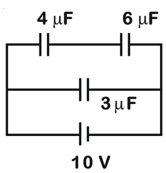


(1) 5.4 μC (2) 9.6 μC

(3) 13.4 μC (4) 24 μC

Answer (4)

Sol.



$$Q_{(4\,\mu F)} = \left(\frac{4\times6}{4+6}\right)\times10\,\mu C$$

$= 24\ \mu C$

28. Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights, h_1 and h_2. Which of the following is correct?

(1) $R^2 = 4h_1h_2$ (2) $R^2 = 16h_1h_2$

(3) $R^2 = 2h_1h_2$ (4) $R^2 = h_1h_2$

Answer (2)

Sol. At complementary angles, ranges are equal.

$$\therefore\quad h_1 = \frac{u^2\sin^2\theta}{2g},\ h_2 = \frac{u^2\cos^2\theta}{2g}$$

$$\therefore\quad h_1\times h_2 = \left(\frac{2u^2\sin\theta\cos\theta}{g}\right)^2\times\left(\frac{1}{16}\right)$$

$\Rightarrow\ 16h_1h_2 = R^2$

29. A diatomic gas with rigid molecules does 10 J of work when expanded at constant pressure. What would be the heat energy absorbed by the gas, in this process?

(1) 30 J (2) 35 J

(3) 25 J (4) 40 J

Answer (2)

Sol. $W = nR\Delta T = 10$ J

$\Delta Q = (\Delta Q)_P = nC_P\Delta T$

$$\therefore\quad \Delta Q = n\times\frac{7}{2}R\times\Delta T$$

$$= \frac{7}{2}\times(10)$$

$= 35$ J

30. A plane electromagnetic wave having a frequency $\nu = 23.9$ GHz propagates along the positive z-direction in free space. The peak value of the electric field is 60 V/m. Which among the following is the acceptable magnetic field component in the electromagnetic wave?

(1) $\vec{\ } = 0\sin(0.5\times10^3x+1.5\times10^{11}t)\hat{k}$

(2) $\vec{B} = 2\times10^7\sin(0.5\times10^3z+1.5\times10^{11}t)\hat{i}$

(3) $\vec{B} = 2\times10^{-7}\sin(0.5\times10^3z-1.5\times10^{11}t)\hat{i}$

(4) $\vec{B} = 2\times10^{-7}\sin(1.5\times10^2x+0.5\times10^{11}t)\hat{j}$

Answer (3)

Sol. $B_0 = \frac{E_0}{c} = \frac{60}{3\times10^8} = 2\times10^{-7}$ T

$\nu = 23.9\times10^9$ Hz

$\therefore\ \omega = 2\pi\nu = 2\times3.142\times23.9\times10^9$

$= 1.5\times10^{11}\ S^{-1}$

$$\because\quad c = \frac{\omega}{k}\ \Rightarrow\ k = \frac{\omega}{c} = \frac{1.5\times10^{11}}{3\times10^8}$$

$= 0.5\times10^3$

$$\therefore\quad \vec{B} = 2\times10^{-7}\sin\left(0.5\times10^3z-1.5\times10^{11}t\right)\hat{i}$$

PART–B : CHEMISTRY

1. The INCORRECT statement is :

(1) Lithium is least reactive with water among the alkali metals.

(2) $LiNO_3$ decomposes on heating to give $LiNO_2$ and O_2.

(3) Lithium is the strongest reducing agent among the alkali metals.

(4) LiCl crystallises from aqueous solution as $LiCl.2H_2O$.

Answer (2)

Sol. $4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 4NO_2 + O_2$

2. The pair that has similar atomic radii is :

(1) Mo and W (2) Ti and Hf

(3) Sc and Ni (4) Mn and Re

Answer (1)

Sol. Mo and W belong to group-6 and period 5 (4d series) and 6 (5d series) respectively.

Due to lanthanoid contraction, radius of Mo and W are almost same.

3. The C–C bond length is maximum in :

(1) graphite (2) C_{60}

(3) diamond (4) C_{70}

Answer (3)

Sol. Carbon-carbon bond length is maximum in diamond

Species	C – C bond length
Diamond	154 pm
Graphite	141.5 pm
C_{60}	138.3 pm and 143.5 pm (double bond) (single bond)

4. The decreasing order of electrical conductivity of the following aqueous solutions is:

0.1 M Formic acid (A),

0.1 M Acetic acid (B),

0.1 M Benzoic acid (C),

(1) A > B > C (2) A > C > B

(3) C > B > A (4) C > A > B

Answer (2)

Sol. HCOOH (A) CH_3COOH (B) C_6H_5COOH (C)

Order of acidic strength

$$HCOOH > C_6H_5COOH > CH_3COOH$$

pK_a 3.8 4.2 4.8

More the acidic strength more will be the dissociation of acid into ions and more will be conductivity.

∴ Order of conductivity:

$$HCOOH > C_6H_5COOH > CH_3COOH$$

$\Rightarrow$ A > C > B

5. The compound used in the treatment of lead poisoning is :

(1) EDTA (2) desferrioxime B

(3) Cis-platin (4) D-penicillamine

Answer (1)

Sol. EDTA is used in the treatment of lead poisoning.

6. An 'Assertion' and a 'Reason' are given below. Choose the correct answer from the following options :

Assertion (A) : Vinyl halides do not undergo nucleophilic substitution easily.

Reason (R) : Even though the intermediate carbocation is stabilized by loosely held π-electrons, the cleavage is difficult because of strong bonding.

(1) Both (A) and (R) are correct statements and (R) is the correct explanation of (A).

(2) Both (A) and (R) are correct statements but (R) is not the correct explanation of (A).

(3) Both (A) and (R) are wrong statements.

(4) (A) is a correct statement but (R) is a wrong statement.

Answer (4)

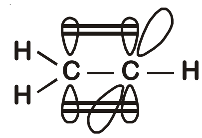
Sol. $CH_2 = CH - Cl$ $CH_2 = \overset{\oplus}{C} - H$ or 

Fig. (1) Fig. (2) Fig. (3)

Also,

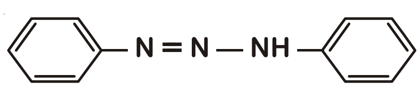
$\overset{}{C}H_2 = CH - \ddot{C}l \longleftrightarrow \overset{\ominus}{C}H_2 - CH = \overset{+}{C}l$

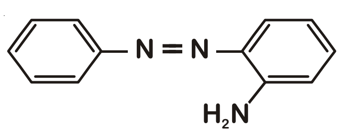
Due to partial double bond character of C – Halogen bond, Halogen leaves with great difficulty, if at all it does. Hence, vinyl halides do not undergo nucleophilic substitution easily.

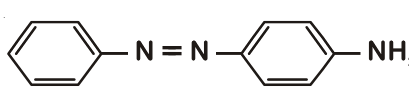
Assertion is correct.

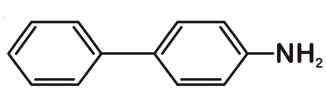
Intermediate carbocation is not stabilised by loosely held π-electrons because empty orbital (see Fig. (3)), being at 90°, cannot overlap with p-orbitals of π bond. Reason is wrong.

7. Benzene diazonium chloride on reaction with aniline in the presence of dilute hydrochloric acid gives :

(1) 

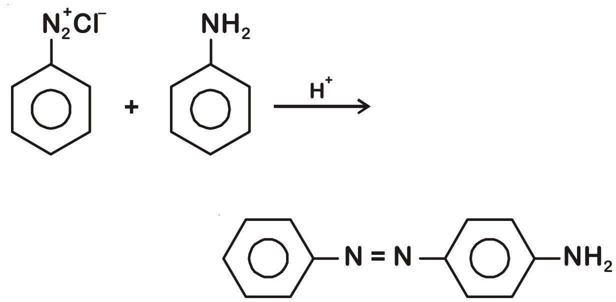
(2) 

(3) 

(4) 

Answer (3)

Sol.



p-aminoazobenzene

8. 25 g of an unknown hydrocarbon upon burning produces 88 g of CO_2 and 9 g of H_2O. This unknown hydrocarbon contains :

(1) 22 g of carbon and 3 g of hydrogen

(2) 24 g of carbon and 1 g of hydrogen

(3) 20 g of carbon and 5 g of hydrogen

(4) 18 g of carbon and 7 g of hydrogen

Answer (2)

Sol. $C_xH_y + \left(x + \frac{y}{4}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$

(excess)

25 g	88 gm	9 g
	2 moles	$\frac{1}{2}$ moles
	$\Rightarrow x = 2$	$\Rightarrow y = 1$

$\therefore$ x = 2 and y = 1

Hydrocarbon : $(C_2H)_n$

2 mol carbon contains 24 g

1 mol hydrogen contains 1 g

9. In which one of the following equilibria, $K_P \neq K_C$?

(1) $2HI(g) \rightleftharpoons H_2(g) + I_2(g)$

(2) $2NO(g) \rightleftharpoons N_2(g) + O_2(g)$

(3) $NO_2(g) + SO_2(g) \rightleftharpoons NO(g) + SO_3(g)$

(4) $2C(s) + O_2(g) \rightleftharpoons 2CO(g)$

Answer (4)

Sol. $\because$ $K_P = K_C \cdot (RT)^{\Delta n_g}$

$\therefore$ If $\Delta n_g \neq 0$ then $K_P \neq K_C$

$2C(s) + O_2(g) \rightleftharpoons 2CO(g)$

$\Delta n_g = +1$

$\Rightarrow K_P = K_C \cdot (RT)^1$

10. Among the following, the INCORRECT statement about colloids is

(1) They can scatter light.

(2) The range of diameters of colloidal particles is between 1 and 1000 nm.

(3) The osmotic pressure of a colloidal solution is of higher order than the true solution at the same concentration.

(4) They are larger than small molecules and have high molar mass.

Answer (3)

Sol. The osmotic pressure of a colloidal solution is of lower order than that of true solution at the same concentration due to association of solute molecule till they acquire colloidal dimensions.

$\pi = iCRT$

i is less in colloidal solution than true solution

11. The primary pollutant that leads to photochemical smog is

(1) Nitrogen oxides (2) Sulphur dioxide

(3) Ozone (4) Acrolein

Answer (1)

Sol. In photochemical smog :

Primary pollutants $\Rightarrow$ NO_2, hydrocarbons

Secondary pollutants $\Rightarrow$ Ozone, acrolein

12. The INCORRECT match in the following is

(1) $\Delta G° = 0, K = 1$ (2) $\Delta G° < 0, K < 1$

(3) $\Delta G° > 0, K < 1$ (4) $\Delta G° < 0, K > 1$

Answer (2)

Sol. $\Delta G° = -RT \ln K$

$\therefore$ If K > 1 then $\Delta G° < 0$

If K < 1 then $\Delta G° > 0$

If K = 1 then $\Delta G° = 0$

13. NO_2 required for a reaction is produced by the decomposition of N_2O_5 in CCl_4 as per the equation,

$2N_2O_5(g) \rightarrow 4NO_2(g) + O_2(g)$.

The initial concentration of N_2O_5 is 3.00 mol L^{-1} and it is 2.75 mol L^{-1} after 30 minutes. The rate of formation of NO_2 is

(1) 1.667×10^{-2} mol L^{-1} min^{-1}

(2) 4.167×10^{-3} mol L^{-1} min^{-1}

(3) 8.333×10^{-3} mol L^{-1} min^{-1}

(4) 2.083×10^{-3} mol L^{-1} min^{-1}

Answer (1)

Sol. $2N_2O_5 \rightarrow 4NO_2 + O_2$

$$\text{rate} = \frac{-1}{2}\frac{d[N_2O_5]}{dt} = \frac{1}{4}\frac{d[NO_2]}{dt} = \frac{d[O_2]}{dt}$$

Since, instant of finding rate of formation of NO_2 is not mentioned, hence

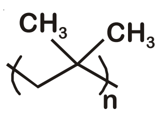
$$\because \quad \frac{-\Delta[N_2O_5]}{\Delta t} = -\frac{(2.75-3)}{30} = \frac{0.25}{30}\ \text{M min}^{-1}$$

$$\therefore \quad \frac{\Delta[NO_2]}{\Delta t} = 2 \times \frac{-\Delta[N_2O_5]}{\Delta t}$$

$$= 2 \times \frac{0.25}{30}$$

$$= 1.67 \times 10^{-2}\ \text{M min}^{-1}$$

14. The correct name of the following polymer is

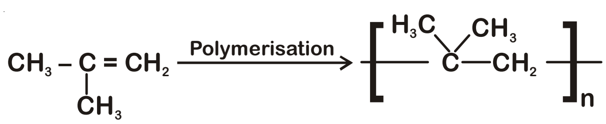


(1) Polyisobutane (2) Polyisoprene

(3) Polyisobutylene (4) Polytert-butylene

Answer (3)

Sol.



iso-butylene Poly isobutylene

15. A solution is prepared by dissolving 0.6 g of urea (molar mass = 60 g mol^{-1}) and 1.8 g of glucose (molar mass = 180 g mol^{-1}) in 100 mL of water at 27°C. The osmotic pressure of the solution is

($R = 0.08206$ L atm K^{-1} mol^{-1})

(1) 1.64 atm (2) 2.46 atm

(3) 8.2 atm (4) 4.92 atm

Answer (4)

Sol. Osmotic pressure (π) = CRT

Solute : urea and glucose

$$\therefore \quad \pi = (C_1 + C_2)\,RT$$

$$= \left(\frac{0.6}{60 \times 0.1} + \frac{1.8}{180 \times 0.1}\right) \times 0.0821 \times 300$$

$$= 0.2 \times 0.0821 \times 300$$

$$= 4.926 \text{ atm}$$

16. The ratio of number of atoms present in a simple cubic, body centered cubic and face centered cubic structure are, respectively

(1) 4 : 2 : 3 (2) 4 : 2 : 1

(3) 8 : 1 : 6 (4) 1 : 2 : 4

Answer (4)

Sol. No. of atoms in

Simple cubic unit cell $= \frac{1}{8} \times 8 = 1$

B.C.C. unit cell $= \frac{1}{8} \times 8 + 1 \times 1 = 2$

FCC unit cell $= \frac{1}{8} \times 8 + \frac{1}{2} \times 6 = 4$

17. In comparison to boron, berylium has

(1) Greater nuclear charge and lesser first ionisation enthalpy.

(2) Greater nuclear charge and greater first ionisation enthalpy.

(3) Lesser nuclear charge and greater first ionisation enthalpy.

(4) Lesser nuclear charge and lesser first ionisation enthalpy.

Answer (3)

Sol. Nuclear charge B > Be

Ionisation energy Be > B

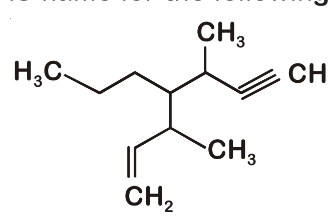
↓

(due to ns^2 outer electronic configuration)

Be = $1s^2\ 2s^2$ (more stable)

B = $1s^2\ 2s^2\ 2p^1$

18. The IUPAC name for the following compound is



(1) 3-methyl-4-(1-methylprop-2-ynyl)-1-heptene

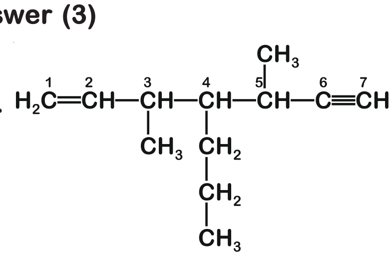
(2) 3-methyl-4-(3-methylprop-1-enyl)-1-heptyne

(3) 3,5-dimethyl-4-propylhept-1-en-6-yne

(4) 3,5-dimethyl-4-propylhept-6-en-1-yne

Answer (3)

Sol.



3, 5-dimethyl-4-propylhept-1-en-6-yne

19. The molar solubility of $Cd(OH)_2$ is 1.84×10^{-5} M in water. The expected solubility of $Cd(OH)_2$ in a buffer solution of pH = 12 is:

(1) 1.84×10^{-9} M (2) 6.23×10^{-11} M

(3) $\frac{2.49}{1.84} \times 10^{-9}$ M (4) 2.49×10^{-10} M

Answer (4)

Sol. $Cd(OH)_2 \rightleftharpoons \underset{S}{Cd^{2+}} + \underset{2S}{2OH^-}$

At equilibrium, $K_{sp} = S(2S)^2$

$= 4S^3$

$\Rightarrow K_{sp} = 4 \times (1.84 \times 10^{-5})^3$

Solubility in buffer solution having pH = 12

$[OH^-] = 10^{-2}$

$Cd(OH)_2 \rightleftharpoons \underset{S'}{Cd^{2+}} + \underset{2S'+10^{-2} \approx 10^{-2}}{2OH^-}$

$\therefore K_{sp} = 4 \times (1.84 \times 10^{-5})^3 = S'(10^{-2})^2$

$\Rightarrow S' = \frac{2.49 \times 10^{-15}}{10^{-4}} = 2.49 \times 10^{-10}$ M

20. The temporary hardness of a water sample is due to compound X. Boiling this sample converts X to compound Y. X and Y, respectively are:

(1) $Ca(HCO_3)_2$ and $Ca(OH)_2$

(2) $Mg(HCO_3)_2$ and $Mg(OH)_2$

(3) $Mg(HCO_3)_2$ and $MgCO_3$

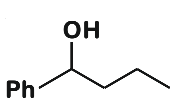
(4) $Ca(HCO_3)_2$ and CaO

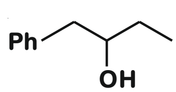
Answer (2)

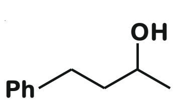
Sol. Temporary hardness is caused by bicarbonates of calcium and magnesium

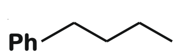
$$\underset{(X)}{Mg(HCO_3)_2(aq)} \xrightarrow{\text{Boiling}} \underset{(Y)}{Mg(OH)_2} \downarrow + 2CO_2 \uparrow$$

21. Heating of 2-chloro-1-phenylbutane with EtOK/EtOH gives X as the major product. Reaction of X with $Hg(OAc)_2/H_2O$ followed by $NaBH_4$ gives Y as the major product. Y is:

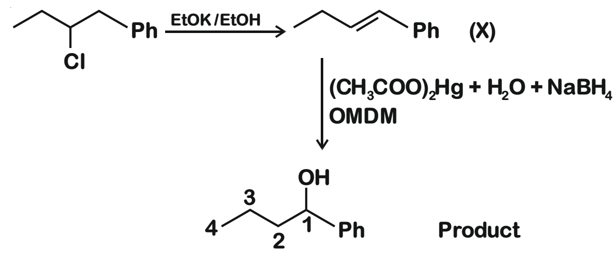
(1) 

(2) 

(3) 

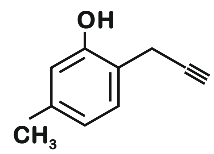
(4) 

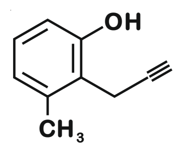
Answer (1)

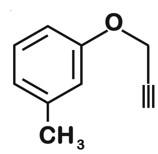
Sol. 

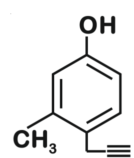
1-phenylbutanol

22. What will be the major product when m-cresol is reacted with propargyl bromide ($HC \equiv C{-}CH_2Br$) in presence of K_2CO_3 in acetone ?

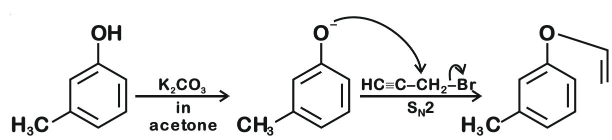
(1) 

(2) 

(3) 

(4) 

Answer (3)

Sol. 

Meta cresol is neutralised by K_2CO_3. The phenoxide ion attacks at the C-atom carrying Br-atom to give ether following S_N2 mechanism.

23. Among the following, the energy of 2s orbital is lowest in:

(1) Li (2) K

(3) H (4) Na

Answer (2)

Sol. As the value of Z (atomic number) increases, energy of orbitals decreases (becomes more –ve value)

$\therefore$ order of energy of 2s orbital is

H > Li > Na > K

24. Thermal decomposition of a Mn compound (X) at 513 K results in compound Y, MnO_2 and a gaseous product. MnO_2 reacts with NaCl and concentrated H_2SO_4 to give a pungent gas Z. X, Y and Z respectively are:

(1) K_2MnO_4, $KMnO_4$ and Cl_2

(2) K_3MnO_4, K_2MnO_4 and Cl_2

(3) K_2MnO_4, $KMnO_4$ and SO_2

(4) $KMnO_4$, K_2MnO_4 and Cl_2

Answer (4)

Sol. $$\underset{(X)}{2KMnO_4} \xrightarrow{513\,K} \underset{(Y)}{K_2MnO_4} + MnO_2 + \underset{gas}{O_2}$$

$$MnO_2 + 4NaCl + \underset{Conc.}{4H_2SO_4} \rightarrow MnCl_2 + 4NaHSO_4 + \underset{\substack{(Z)\\ \text{pungent gas}}}{Cl_2} + 2H_2O$$

25. The correct statement is

(1) Pig iron is obtained from cast iron.

(2) Leaching of bauxite using concentrated NaOH solution gives sodium aluminate and sodium silicate.

(3) The blistered appearance of copper during the metallurgical process is due to the evolution of CO_2.

(4) The Hall-Heroult process is used for the production of aluminium and iron.

Answer (2)

Sol. During metallurgy of aluminium, when bauxite (powdered ore) is treated with NaOH (conc.), sodium aluminate ($Na[Al(OH)_4]$) is formed and impurity SiO_2 dissolves by forming sodium silicate (Na_2SiO_3)

$$Al_2O_3(s) + 2NaOH(aq) + 3H_2O(l) \rightarrow 2Na[Al(OH)_4](aq)$$

26. Which of the given statements is INCORRECT about glycogen?

(1) It is present in some yeast and fungi.

(2) It is a straight chain polymer similar to amylose.

(3) It is present in animal cells.

(4) Only α-linkages are present in the molecule.

Answer (2)

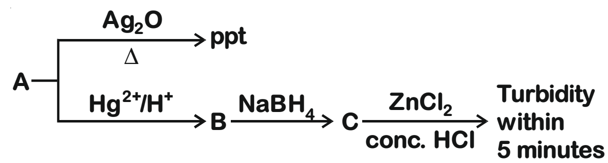
Sol. Structure of glycogen is similar to amylopectin glycogen

– contains α-glycosidic linkages

– is stored in animal body

– is found in yeast and fungi

27. Consider the following reactions :



'A' is

(1) $CH_3 - C \equiv CH$

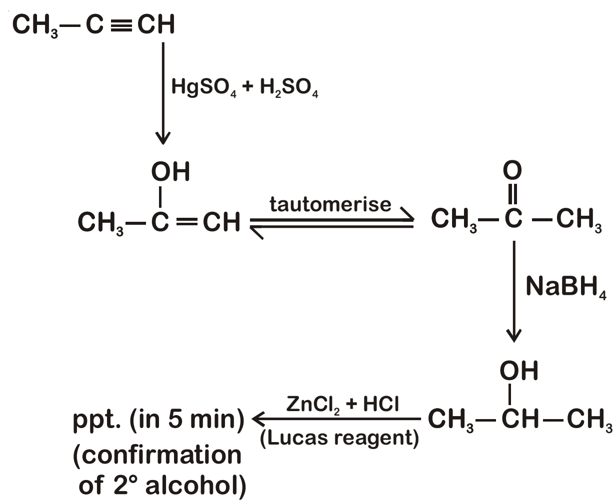
(2) $CH_3 - C \equiv C - CH_3$

(3) $CH_2 = CH_2$

(4) $CH \equiv CH$

Answer (1)

Sol.



28. The coordination numbers of Co and Al in $[Co(Cl)(en)_2]Cl$ and $K_3[Al(C_2O_4)_3]$, respectively, are (en = ethane-1, 2-diamine)

(1) 3 and 3

(2) 5 and 3

(3) 5 and 6

(4) 6 and 6

Answer (3)

Sol. $[Co(en)_2Cl]Cl$

Cl^- - monodentate ligand

en - bidentate ligand

$\therefore$ Co-ordination No. of Co = (2 × 2) + 1 = 5

$K_3[Al(C_2O_4)_3]$

$C_2O_4^{2-}$- bidentate ligand

$\therefore$ Co-ordination No. of Al = 2 × 3 = 6

29. Which one of the following is likely to give a precipitate with $AgNO_3$ solution?

(1) $CH_2 = CH - Cl$ (2) $CHCl_3$

(3) CCl_4 (4) $(CH_3)_3CCl$

Answer (4)

Sol. Carbocation is formed on reaction with Ag^+

$$R—Cl + Ag^+ \longrightarrow R^+ + \underset{ppt.}{AgCl\downarrow}$$

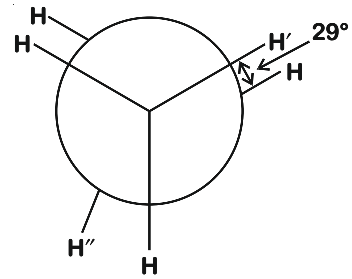
The more the stability of R^+, the more R — Cl is likely to give precipitate

$$CH_3—\overset{\overset{CH_3}{|}}{\underset{\underset{CH_3}{|}}{C}}\oplus$$ is most stable carbocation

compared to other options:

$$CH_2{=}\overset{\oplus}{C}H,\ \overset{\oplus}{C}HCl_2,\ \overset{\oplus}{C}Cl_3$$

30. In the following skew conformation of ethane, H′ – C – C – H″ dihedral angle is

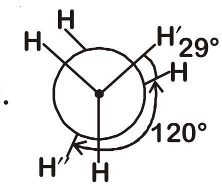


(1) 120° (2) 58°

(3) 151° (4) 149°

Answer (4)

Sol.



∴ Angle between H′ and H″ = 120° + 29°

= 149°

PART–C : MATHEMATICS

1. An ellipse, with foci at (0, 2) and (0, –2) and minor axis of length 4, passes through which of the following points?

(1) $(\sqrt{2}, 2)$

(2) $(2, 2\sqrt{2})$

(3) $(1, 2\sqrt{2})$

(4) $(2, \sqrt{2})$

Answer (1)

Sol. Let the equation of ellipse : $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\because$ be = 2 and a = 2 (Given)

$\because$ $a^2 = b^2(1 - e^2)$

$\Rightarrow$ $4 = b^2 - 4$

$\Rightarrow$ $b = 2\sqrt{2}$

Equation of ellipse will be $\frac{x^2}{4}+\frac{y^2}{8}=1$

Only $(\sqrt{2}, 2)$ satisfies this equation.

2. For an initial screening of an admission test, a candidate is given fifty problems to solve. If the probability that the candidate can solve any problem is $\frac{4}{5}$, then the probability that he is unable to solve less than two problems is:

(1) $\frac{316}{25}\left(\frac{4}{5}\right)^{48}$

(2) $\frac{54}{5}\left(\frac{4}{5}\right)^{49}$

(3) $\frac{201}{5}\left(\frac{1}{5}\right)^{49}$

(4) $\frac{164}{25}\left(\frac{1}{5}\right)^{48}$

Answer (2)

Sol. Let p is the probability that candidate can solve a problem.

$p = \frac{4}{5}$; $q = \frac{1}{5}$ $(\because\ p + q = 1)$

Probability that candidate is able to solve either 50 or 49 problems $= {}^{50}C_{50}p^{50}\cdot q^0 + {}^{50}C_{49}\cdot p^{49}\cdot q^1$

$= p^{49}\,[p + 50q]$

$= \left(\frac{4}{5}\right)^{49}\cdot\left(\frac{4}{5}+\frac{50}{5}\right)$

$= \frac{54}{5}\cdot\left(\frac{4}{5}\right)^{49}$

3. The derivative of $\tan^{-1}\left(\frac{\sin x-\cos x}{\sin x+\cos x}\right)$, with respect to $\frac{x}{2}$, where $\left(x\in\left(0,\frac{\pi}{2}\right)\right)$ is :

(1) $\frac{1}{2}$

(2) $\frac{2}{3}$

(3) 2

(4) 1

Answer (3)

Sol. $f(x)=\tan^{-1}\left(\frac{\tan x-1}{\tan x+1}\right)=-\tan^{-1}\left(\tan\left(\frac{\pi}{4}-x\right)\right)$

$\because\ \frac{\pi}{4}-x\in\left(-\frac{\pi}{4},\frac{\pi}{4}\right)$

So, $f(x)=-\left(\frac{\pi}{4}-x\right)=x-\frac{\pi}{4}$

Let $y=\frac{x}{2}$

$$\frac{d}{dy}f(x)=\frac{\frac{d}{dx}f(x)}{\frac{d}{dx}y}=\frac{1}{1/2}=2$$

4. The tangents to the curve $y = (x - 2)^2 - 1$ at its points of intersection with the line $x - y = 3$, intersect at the point :

(1) $\left(\frac{5}{2},1\right)$ (2) $\left(-\frac{5}{2},1\right)$

(3) $\left(\frac{5}{2},-1\right)$ (4) $\left(-\frac{5}{2},-1\right)$

Answer (3)

Sol. Equation of chord is T = 0

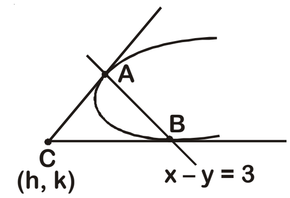
$\Rightarrow \frac{1}{2}(y+k)=(x-2)(h-2)-1$

$\Rightarrow \frac{y+k}{2}=xh-2x-2h+3$

$\Rightarrow (2h - 4)x - y - 4h + 6 - k = 0$

Given $x - y - 3 = 0$

$\Rightarrow \frac{2h-4}{1}=\frac{4h-6+k}{3}=1$



$h=\frac{5}{2},\ k=-1$

5. If the area (in sq. units) bounded by the parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$, $\lambda > 0$, is $\frac{1}{9}$, then λ is equal to :

(1) 48 (2) 24

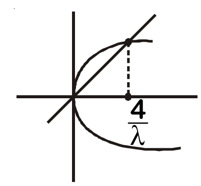
(3) $4\sqrt{3}$ (4) $2\sqrt{6}$

Answer (2)

Sol. $y^2 = 4\lambda x$ and $y = \lambda x$

On solving ; $(\lambda x)^2 = 4\lambda x$

$x = 0,\ \frac{4}{\lambda}$



Required area $=\int_0^{\frac{4}{\lambda}}\left(2\sqrt{\lambda x}-\lambda x\right)dx$

$=\left.\frac{2\sqrt{\lambda}\cdot x^{3/2}}{3/2}-\frac{\lambda x^2}{2}\right|_0^{4/\lambda}$

$=\frac{32}{3\lambda}-\frac{8}{\lambda}=\frac{8}{3\lambda}=\frac{1}{9}$

$\lambda = 24$

6. A group of students comprises of 5 boys and n girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then n is equal to :

(1) 24 (2) 27

(3) 25 (4) 28

Answer (3)

Sol. Number of ways of selecting three persons such that there is atleast one boy and atleast one girl in the selected persons

$= {}^{n+5}C_3 - {}^{n}C_3 - {}^{5}C_3 = 1750$

$\Rightarrow \frac{(n+5)(n+4)(n+3)}{6}-\frac{n(n-1)(n-2)}{6}=1760$

$\Rightarrow n^2 + 3n - 700 = 0$

$\Rightarrow n = -28$ (rejected) or $n = 25$

7. If $a_1, a_2, a_3, ...$ are in A.P. such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this A.P. is :

(1) 150

(2) 280

(3) 200

(4) 120

Answer (3)

Sol. Let the common difference is 'd'.

$a_1 + a_7 + a_{16} = 40$

$\Rightarrow a_1 + a_1 + 6d + a_1 + 15d = 40$

$\Rightarrow 3a_1 + 21d = 40$

$\Rightarrow a_1 + 7d = \frac{40}{3}$

$S_{15} = \frac{15}{2}[2a_1 + 14d]$

$= 15(a_1 + 7d)$

$= 15\left(\frac{40}{3}\right)$

$= 200$

8. The angle of elevation of the top of a vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30 m vertically above the point A. If the angle of elevation of the top of the tower from B be 30°, then the distance (in m) of the foot of the tower from the point A is :

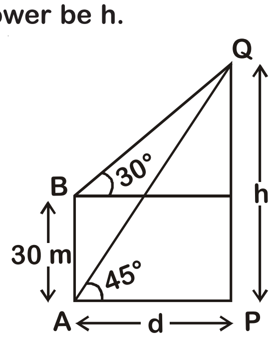
(1) $15(3+\sqrt{3})$ (2) $15(1+\sqrt{3})$

(3) $15(3-\sqrt{3})$ (4) $15(5-\sqrt{3})$

Answer (1)

Sol. Let the height of the tower be h.

Refer to diagram ;



$\tan 45^\circ = \frac{h}{d} = 1$

$h = d$ ...(i)

$\tan 30^\circ = \frac{h-30}{d}$

$\sqrt{3}(h-30) = d$ ...(ii)

from (i) and (ii)

$\sqrt{3}d = d + 30\sqrt{3}$

$d = \frac{30\sqrt{3}}{\sqrt{3}-1} = 15\sqrt{3}\left(\sqrt{3}+1\right) = 15\left(3+\sqrt{3}\right)$

9. If [x] denotes the greatest integer $\le$ x, then the system of linear equations $[\sin\theta]x + [-\cos\theta]y = 0$

$[\cot\theta]x + y = 0$

(1) Has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and have infinitely many solutions if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$.

(2) Has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$.

(3) Have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and has a unique solution if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$.

(4) Have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$.

Answer (3)

Sol. There are two cases.

Case 1 : $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

So ; $[\sin\theta] = 0, [-\cos\theta] = 0, [\cot\theta] = -1$

The system of equations will be ;

$0\cdot x + 0\cdot y = 0$ and $-x + y = 0$

(Infinitely many solutions)

Case 2 : $\theta \in \left(\pi, \frac{7\pi}{6}\right)$

So ; $[\sin\theta] = -1, [-\cos\theta] = 0,$

The system of equations will be ;

$-x + 0\cdot y = 0$ and $[\cot\theta]x + y = 0$

Clearly x = 0 and y = 0 (unique solution)

10. A value of $\theta \in (0, \pi/3)$, for which

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0, \text{ is:}$$

(1) $\frac{\pi}{18}$

(2) $\frac{7\pi}{36}$

(3) $\frac{7\pi}{24}$

(4) $\frac{\pi}{9}$

Answer (4)

Sol. $C_1 \to C_1 + C_2$

$$\begin{vmatrix} 2 & \sin^2\theta & 4\cos 6\theta \\ 2 & 1+\sin^2\theta & 4\cos 6\theta \\ 1 & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & \sin^2\theta & (1+4\cos 6\theta) \end{vmatrix} = 0$$

$\Rightarrow$ $2 + 4\cos 6\theta = 0$

$\cos 6\theta = \frac{-1}{2}$

$\because$ $6\theta \in (0, 2\pi)$

So, $6\theta = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$

$\Rightarrow$ $\theta = \frac{\pi}{9}$ or $\frac{2\pi}{9}$

11. The Boolean expression $\sim(p \Rightarrow (\sim q))$ is equivalent to

(1) $(\sim p) \Rightarrow q$ (2) $p \vee q$

(3) $p \wedge q$ (4) $q \Rightarrow \sim p$

Answer (3)

Sol. $\sim(p \Rightarrow (\sim q))$ $\{\because p \Rightarrow q$ is same as $\sim p \vee q\}$

$\equiv$ $\sim((\sim p) \vee (\sim q))$

$\equiv$ $p \wedge q$

12. The equation of a common tangent to the curves, $y^2 = 16x$ and $xy = -4$, is :

(1) $x + y + 4 = 0$ (2) $2x - y + 2 = 0$

(3) $x - 2y + 16 = 0$ (4) $x - y + 4 = 0$

Answer (4)

Sol. $y^2 = 16x$ and $xy = -4$

Equation of tangent to the given parabola;

$y = mx + \frac{4}{m}$

If this is common tangent, then

$x\left(mx + \frac{4}{m}\right) + 4 = 0$

$\Rightarrow$ $mx^2 + \frac{4}{m}x + 4 = 0$

$D = 0$

$\frac{16}{m^2} = 16m$

$\Rightarrow$ $m^3 = 1$ $\Rightarrow m = 1$

Equation of common tangent is $y = x + 4$

13. The general solution of the differential equation $(y^2 - x^3)\,dx - xydy = 0$ $(x \neq 0)$ is :

(where c is a constant of integration)

(1) $y^2 + 2x^3 + cx^2 = 0$ (2) $y^2 - 2x^2 + cx^3 = 0$

(3) $y^2 - 2x^3 + cx^2 = 0$ (4) $y^2 + 2x^2 + cx^3 = 0$

Answer (1)

Sol. $y^2dx - xydy = x^3dx$

$\Rightarrow$ $\frac{(ydx - xdy)y}{x^2} = xdx$

$\Rightarrow$ $-yd\left(\frac{y}{x}\right) = xdx$

$\Rightarrow$ $-\frac{y}{x}\cdot d\left(\frac{y}{x}\right) = dx$

$\Rightarrow$ $-\frac{1}{2}\left(\frac{y}{x}\right)^2 = x + c_1$

$\Rightarrow$ $2x^3 + cx^2 + y^2 = 0$

14. A plane which bisects the angle between the two given planes $2x - y + 2z - 4 = 0$ and $x + 2y + 2z - 2 = 0$, passes through the point :

(1) $(1, -4, 1)$ (2) $(2, -4, 1)$

(3) $(1, 4, -1)$ (4) $(2, 4, 1)$

Answer (2)

Sol. Equation of angle bisectors;

$$\frac{x + 2y + 2z - 2}{3} = \pm\frac{2x - y + 2z - 4}{3}$$

$\Rightarrow$ $x - 3y - 2 = 0$ or $3x + y + 4z - 6 = 0$

Only $(2, -4, 1)$ lies on the second plane

15. Let S be the set of all $\alpha \in R$ such that the equation, $\cos 2x + \alpha \sin x = 2\alpha - 7$ has a solution. Then S is equal to :

(1) $[1, 4]$ (2) R

(3) $[2, 6]$ (4) $[3, 7]$

Answer (3)

Sol. $\cos 2x + \alpha\sin x = 2\alpha - 7$

$1 - 2\sin^2 x + \alpha\sin x = 2\alpha - 7$

$\Rightarrow$ $2\sin^2 x - \alpha\sin x + (2\alpha - 8) = 0$

$\Rightarrow$ $\sin x = \frac{\alpha \pm \sqrt{\alpha^2 - 8(2\alpha - 8)}}{4}$

$\Rightarrow$ $\sin x = \frac{\alpha \pm (\alpha - 8)}{4}$

$\Rightarrow$ $\sin x = 2$ (rejected) or $\sin x = \frac{\alpha - 4}{2}$

$\because$ Equation has solution, then $\frac{\alpha - 4}{2} \in [1, 1]$

$\Rightarrow$ $\alpha \in [2, 6]$

16. Let $\alpha \in R$ and the three vectors

$\vec{a} = \alpha\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}$ and

$\vec{c} = \alpha\hat{i} - 2\hat{j} + 3\hat{k}$. Then the set

$S = \{\alpha : \vec{a}, \vec{b}$ and $\vec{c}$ are coplanar$\}$

(1) contains exactly two numbers only one of which is positive

(2) is singleton

(3) contains exactly two positive numbers

(4) is empty

Answer (4)

Sol. If $\vec{a}, \vec{b}, \vec{c}$ are coplanar, then

$$\left[\vec{a}, \vec{b}, \vec{c}\right] = 0$$

$$\Rightarrow \begin{vmatrix} \alpha & 1 & 3 \\ 2 & 1 & -\alpha \\ \alpha & -2 & 3 \end{vmatrix} = 0$$

$\Rightarrow \alpha^2 + 6 = 0$

No value of 'α' exist

Set S is an empty set.

17. The length of the perpendicular drawn from the point (2, 1, 4) to the plane containing the lines

$\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and

$\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - 2\hat{k})$ is :

(1) $\frac{1}{3}$

(2) 3

(3) $\frac{1}{\sqrt{3}}$

(4) $\sqrt{3}$

Answer (4)

Sol. Equation of plane containing two given lines;

$$\begin{vmatrix} x-1 & y-1 & z \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix} = 0$$

$\Rightarrow x - y - z = 0$

The length of perpendicular from (2, 1, 4) to

this plane $= \left|\frac{2-1-4}{\sqrt{1^2+1^2+1^2}}\right| = \sqrt{3}$

18. Let $\alpha \in (0, \pi/2)$ be fixed. If the integral $\int \frac{\tan x + \tan\alpha}{\tan x - \tan\alpha} dx =$

$A(x) \cos 2\alpha + B(x) \sin 2\alpha + C$, where C is a constant of integration, then the functions A(x) and B(x) are respectively :

(1) $x - \alpha$ and $\log_e|\cos(x - \alpha)|$

(2) $x + \alpha$ and $\log_e|\sin(x - \alpha)|$

(3) $x - \alpha$ and $\log_e|\sin(x - \alpha)|$

(4) $x + \alpha$ and $\log_e|\sin(x + \alpha)|$

Answer (3)

Sol. $\int \frac{\tan x + \tan\alpha}{\tan x - \tan\alpha} dx$

$= \int \frac{\sin(x+\alpha)}{\sin(x-\alpha)} dx$ let $x - \alpha = t$

$\Rightarrow dx = dt$

$= \int \frac{\sin(t+2\alpha)}{\sin t} dt$

$= [\cos 2\alpha + \sin 2\alpha \cdot \cot t]dt$

$= \cos 2\alpha \cdot t + \sin 2\alpha \cdot \ln|\sin t| + c$

$= (x - \alpha) \cdot \cos 2\alpha + \sin 2\alpha \cdot \ln|\sin(x - \alpha)| + c$

19. If ${}^{20}C_1 + (2^2)\,{}^{20}C_2 + (3^2)\,{}^{20}C_3 + \ldots\ldots + (20^2)\,{}^{20}C_{20} = A(2^\beta)$, then the ordered pair (A, β) is equal to :

(1) (420, 19)

(2) (380, 19)

(3) (420, 18)

(4) (380, 18)

Answer (3)

Sol. ${}^{20}C_1 + 2^2 \cdot {}^{20}C_2 + 3^2 \cdot {}^{20}C_3 + \ldots + 20^2 \cdot {}^{20}C_{20}$

$$= \sum_{r=1}^{20} r^2 \cdot {}^{20}C_r$$

$$= 20 \sum_{r=1}^{20} r \cdot {}^{19}C_{r-1}$$

$$= 20\left[\sum_{r=1}^{20} (r-1)\,{}^{19}C_{r-1} + \sum_{r=1}^{20} {}^{19}C_{r-1}\right]$$

$$= 20\left[19 \sum_{r=2}^{20} {}^{18}C_{r-2} + 2^{19}\right]$$

$= 20[19 \cdot 2^{18} + 2^{19}]$

$= 20 \times 21 \times 2^{18}$

$= 420 \times 2^{18}$

So; A = 420 and $\beta = 18$

20. A triangle has a vertex at (1, 2) and the mid points of the two sides through it are (–1, 1) and (2, 3). Then the centroid of this triangle is :

(1) $\left(\frac{1}{3}, 2\right)$ (2) $\left(\frac{1}{3}, \frac{5}{3}\right)$

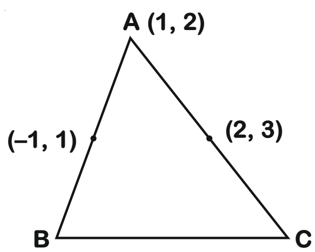
(3) $\left(\frac{1}{3}, 1\right)$ (4) $\left(1, \frac{7}{3}\right)$

Answer (1)

Sol. Co-ordinates of vertex B and C are B(–3, 0) and C(3, 4)

Centroid $G\left(\frac{3-3+1}{3}, \frac{0+4+2}{3}\right)$

$G\left(\frac{1}{3}, 2\right)$



21. If α, β and γ are three consecutive terms of a non-constant G.P. such that the equations $\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root, then $\alpha(\beta + \gamma)$ is equal to :

(1) 0 (2) $\alpha\gamma$

(3) $\beta\gamma$ (4) $\alpha\beta$

Answer (3)

Sol. $\beta^2 = \alpha\gamma$ so roots of the equation $\alpha x^2 + 2\beta x + \gamma = 0$

are $\frac{-2\beta \pm 2\sqrt{\beta^2 - \alpha\gamma}}{2\alpha} = -\frac{\beta}{\alpha}$

This root satisfy the equation $x^2 + x - 1 = 0$

$\beta^2 - \alpha\beta - \alpha^2 = 0$

$\Rightarrow \alpha\gamma - \alpha\beta - \alpha^2 = 0$

$\Rightarrow \alpha + \beta = \gamma$

Now, $\alpha(\beta + \gamma) = \alpha\beta + \alpha\gamma = \alpha\beta + \beta^2$

$= (\alpha + \beta)\beta = \beta\gamma$

22. $\lim_{x\to 0} \frac{x + 2\sin x}{\sqrt{x^2 + 2\sin x + 1} - \sqrt{\sin^2 x - x + 1}}$ is :

(1) 3

(2) 6

(3) 1

(4) 2

Answer (4)

Sol. $\lim_{x\to 0} \frac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x-x+1}}$

$= \lim_{x\to 0} \frac{(x+2\sin x)\left[\sqrt{x^2+2\sin x+1}+\sqrt{\sin^2 x-x+1}\right]}{(x^2-\sin^2 x)+(x+2\sin x)}$

$= \lim_{x\to 0} \frac{\left[1+2\left(\frac{\sin x}{x}\right)\right]\left[\sqrt{x^2+2\sin x+1}+\sqrt{\sin^2 x-x+1}\right]}{\left(x-\frac{\sin^2 x}{x}\right)+\left(1+2\left(\frac{\sin x}{x}\right)\right)}$

$= \frac{3\times 2}{3} = 2$

23. A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular from the origin to this line makes an angle of 60° with the line x + y = 0. Then an equation of the line L is :

(1) $\left(\sqrt{3}+1\right)x + \left(\sqrt{3}-1\right)y = 8\sqrt{2}$

(2) $\left(\sqrt{3}-1\right)x + \left(\sqrt{3}+1\right)y = 8\sqrt{2}$

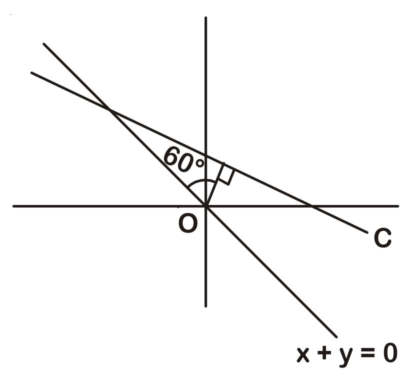
(3) $\sqrt{3}x + y = 8$

(4) $x + \sqrt{3}y = 8$

Answer (1) or (2)

Sol. If perpendicular makes an angle of 60° with the line x + y = 0.

Then the perpendicular makes an angle of 15° or 75° with x-axis. So the equation of line will be



$x\cos 75° + y\sin 75° = 4$ or $x\cos 15° + y\sin 15° = 4$

$\left(\sqrt{3}-1\right)x + \left(\sqrt{3}+1\right)y = 8\sqrt{2}$ or

$\left(\sqrt{3}+1\right)x + \left(\sqrt{3}-1\right)y = 8\sqrt{2}$

By rotating the normal towards the line x + y = 0 in anticlockwise sense we get the answer (2).

24. The term independent of x in the expansion of $\left(\frac{1}{60}-\frac{x^8}{81}\right)\cdot\left(2x^2-\frac{3}{x^2}\right)^6$ is equal to :

(1) –108 (2) –36

(3) –72 (4) 36

Answer (2)

Sol. $\left(\frac{1}{60}-\frac{x^8}{81}\right)\left(2x^2-\frac{3}{x^2}\right)^6$

$= \frac{1}{60}\left(2x^2-\frac{3}{x^2}\right)^6 - \frac{x^8}{81}\left(2x^2-\frac{3}{x^2}\right)^6$

Coefficient of x° in $\frac{1}{60}\left(2x^2-\frac{3}{x^2}\right)^6 - \frac{1}{81}$ coefficient of x^{-8} in $\left(2x^2-\frac{3}{x^2}\right)^6$

$= \frac{-1}{60}\,{}^6C_3\,(2)^3(3)^3 + \frac{1}{81}\,{}^6C_5\,(2)(3)^5$

= –72 + 36

= –36

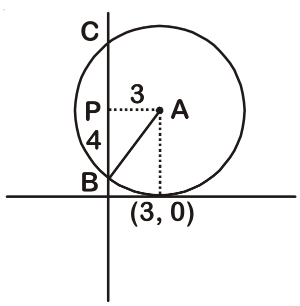
25. A circle touching the x-axis at (3, 0) and making an intercept of length 8 on the y-axis passes through the point :

(1) (2, 3) (2) (1, 5)

(3) (3, 5) (4) (3, 10)

Answer (4)

Sol. Let centre of circle is A and circle cuts the y axis at B and C. Let mid point of chord BC is P.



$AB = \sqrt{PA^2 + PB^2}$

$= 5 =$ radius of circle

Equation of circle is : $(x-3)^2 + (y-5)^2 = 5^2$

Only (3, 10) satisfies this equation.

Although there will be another circle satisfying the same conditions that will lie below the x-axis having equation $(x-3)^2 + (y-5)^2 = 5^2$

26. A person throws two fair dice. He wins Rs. 15 for throwing a doublet (same numbers on the two dice), wins Rs. 12 when the throw results in the sum of 9, and loses Rs. 6 for any other outcome on the throw. Then the expected gain/loss (in Rs.) of the person is :

(1) $\frac{1}{2}$ loss (2) 2 gain

(3) $\frac{1}{2}$ gain (4) $\frac{1}{4}$ loss

Answer (1)

Sol. Let X be the random variable which denotes the Rs gained by the person.

$P(X=15) = \frac{6}{36} = \frac{1}{6}$ {Total cases = 36 favourable cases are (1,1), (2,2), (3, 3), (4, 4), (5, 5), (6, 6)}

$P(X=12) = \frac{4}{36} = \frac{1}{9}$ {Favourable cases are (6, 3), (5, 4), (4, 5), (3, 6)}

$P(X=-6) = \frac{26}{36} = \frac{13}{18}$

X	15	12	–6
P(X)	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{13}{18}$
X·P(X)	$\frac{5}{2}$	$\frac{4}{3}$	$\frac{-13}{3}$

$E(X) = \sum X\cdot P(X) = \frac{5}{2}+\frac{4}{3}-\frac{13}{3} = -\frac{1}{2}$

27. Let A, B and C be sets such that $\phi \neq A \cap B \subseteq C$. Then which of the following statements is not true?

(1) $B \cap C \neq \phi$

(2) $(C \cup A) \cap (C \cup B) = C$

(3) If $(A - C) \subseteq B$, then $A \subseteq B$

(4) If $(A - B) \subseteq C$, then $A \subseteq C$

Answer (3)

Sol. $\because$ $A \cap B \subseteq C$ and $A \cap B \neq \phi$

(1) $B \cap C \neq \phi$ is correct

(2) $(C \cup A) \cap (C \cup B) = C \cup (A \cap B) = C$ (correct) (becasue $A \cap B \subseteq C$)

(3) If A = C then $A - C = \phi$

Clearly $\phi \subseteq B$ but $A \subseteq B$ is not always true.

(4) $\because$ $A - B \subseteq C$ and $A \cap B \subseteq C$ so $A \subseteq C$ (correct)

28. A value of α such that

$$\int_{\alpha}^{\alpha+1} \frac{dx}{(x+\alpha)(x+\alpha+1)} = \log_e\left(\frac{9}{8}\right) \text{ is :}$$

(1) $\frac{1}{2}$ (2) -2

(3) $-\frac{1}{2}$ (4) 2

Answer (2)

Sol. $\int_{\alpha}^{\alpha+1} \frac{dx}{(x+\alpha)(x+\alpha+1)} = \int_{\alpha}^{\alpha+1}\left[\frac{1}{x+\alpha} - \frac{1}{x+\alpha+1}\right]dx$

$$= \ln\left(\frac{x+\alpha}{x+\alpha+1}\right)\Bigg|_{\alpha}^{\alpha+1}$$

$$= \ln\left(\frac{2\alpha+1}{2\alpha+2}\cdot\frac{2\alpha+1}{2\alpha}\right) = \ln\frac{9}{8}$$

So, $\frac{(2\alpha+1)^2}{\alpha(\alpha+1)} = \frac{9}{2}$

$\Rightarrow$ $8\alpha^2 + 8\alpha + 2 = 9\alpha^2 + 9\alpha$

$\Rightarrow$ $\alpha^2 + \alpha - 2 = 0$

$\Rightarrow$ $\alpha = 1, -2$

29. Let $z \in C$ with $\text{Im}(z) = 10$ and it satisfies $\frac{2z-n}{2z+n} = 2i - 1$ for some natural number n. Then :

(1) n = 20 and Re(z) = 10

(2) n = 20 and Re(z) = –10

(3) n = 40 and Re(z) = –10

(4) n = 40 and Re(z) = 10

Answer (3)

Sol. Let $z = x + 10i$

$2z - n = (2i - 1)(2z + n)$

$(2x - n) + 20i = (2i - 1)((2x + n) + 20i)$

Comparing real and imaginary part

$-(2x + n) - 40 = 2x - n$ and $20 = 4x + 2n - 20$

$\Rightarrow$ $4x = -40$ $\quad$ $40 = -40 + 2n$

$\Rightarrow$ $x = -10$ $\quad$ $n = 40$

$\Rightarrow$ $\text{Re}(z) = -10$

30. Let $f(x) = 5 - |x - 2|$ and $g(x) = |x + 1|$, $x \in R$. If f(x) attains maximum value at α and g(x) attains minimum value at β, then

$$\lim_{x \to -\alpha\beta} \frac{(x-1)(x^2-5x+6)}{x^2-6x+8} \text{ is equal to :}$$

(1) 1/2

(2) –1/2

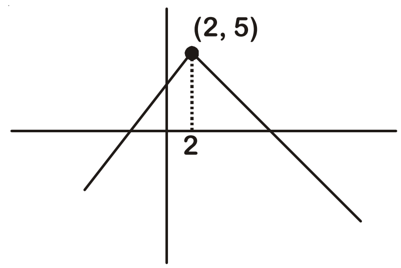
(3) –3/2

(4) 3/2

Answer (1)

Sol. $f(x) = 5 - |x - 2|$

Graph of $y = f(x)$

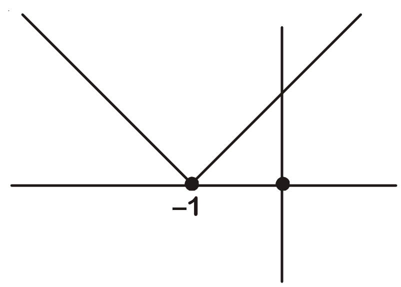


f(x) is maximum at x = 2

$\alpha = 2$

$g(x) = |x + 1|$

Graph of $y = g(x)$



g(x) is minimum at x = –1

$\beta = -1$

$$\lim_{x\to 2}\frac{(x-1)(x-2)(x-3)}{(x-2)(x-4)} = \lim_{x\to 2}\frac{(x-1)(x-3)}{x-4}$$

$$= \frac{1}{2}$$

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