10/04/2019
Evening

Answers & Solutions

for

JEE (MAIN)-2019 (Online) Phase-2

(Physics, Chemistry and Mathematics)

Time : 3 hrs. **M.M. : 360**

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test Booklet consists of **90** questions**.** The maximum marks are **360.**
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage.
4. Each question is allotted 4 **(four)** marks for each correct response. *¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
5. There is only one correct response for each question.

PART–A : PHYSICS

1. One mole of an ideal gas passes through a process where pressure and volume obey the relation $P = P_0\left[1-\frac{1}{2}\left(\frac{V_0}{V}\right)^2\right]$. Here P_0 and V_0 are constants. Calculate the change in the temperature of the gas if its volume changes from V_0 to $2V_0$.

(1) $\frac{1}{4}\frac{P_0V_0}{R}$ (2) $\frac{5}{4}\frac{P_0V_0}{R}$

(3) $\frac{1}{2}\frac{P_0V_0}{R}$ (4) $\frac{3}{4}\frac{P_0V_0}{R}$

Answer (2)

Sol. If $V_1 = V_0 \Rightarrow P_1 = P_0\left[1-\frac{1}{2}\right] = \frac{P_0}{2}$

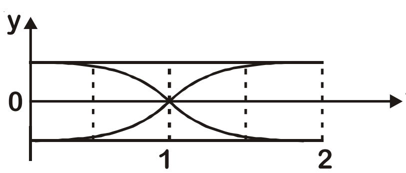
If $V_2 = 2V_0 \Rightarrow P_2 = P_0\left[1-\frac{1}{2}\left(\frac{1}{4}\right)\right] = \left(\frac{7P_0}{8}\right)$

$\left(T = \frac{PV}{nR}\right) \Rightarrow \Delta T = \left|\frac{P_1V_1}{nR} - \frac{P_2V_2}{nR}\right|$

$\Delta T = \left|\left(\frac{1}{nR}\right)(P_1V_1 - P_2V_2)\right| = \left(\frac{1}{nR}\right)\left|\left(\frac{P_0V_0}{2} - \frac{7P_0V_0}{4}\right)\right|$

$= \frac{5P_0V_0}{4nR}$

2. The correct figure that shows, schematically, the wave pattern produced by superposition of two waves of frequencies 9 Hz and 11 Hz, is :

(1)


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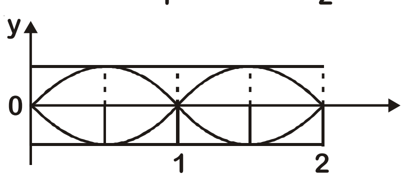

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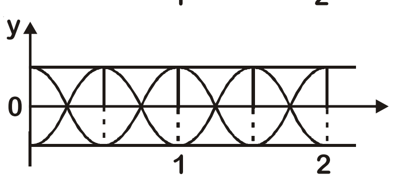

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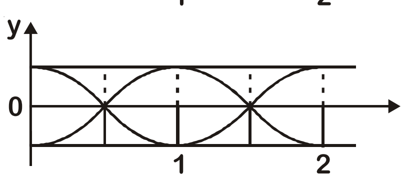


Answer (3)

Sol. Beat frequency = $|f_1 - f_2| = 11 - 9 = 2$ Hz

3. A spaceship orbits around a planet at a height of 20 km from its surface. Assuming that only gravitational field of the planet acts on the spaceship, what will be the number of complete revolutions made by the spaceship in 24 hours around the planet?

[Given: Mass of planet = 8×10^{22} kg,

Radius of planet = 2×10^6 m,

Gravitational constant G = 6.67×10^{-11} Nm^2/kg^2]

(1) 17 (2) 13

(3) 11 (4) 9

Answer (3)

Sol. $T = \frac{2\pi r}{v}, v = \sqrt{\frac{GM}{r}}$

$T = 2\pi r\sqrt{\frac{r}{GM}} = 2\pi\sqrt{\frac{r^3}{GM}}$

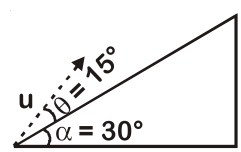
$T = 2\pi\sqrt{\frac{(202)^3 \times 10^{12}}{6.67\times10^{-11}\times8\times10^{22}}}$ sec

T = 7812.2 s

$T \simeq 2.17\text{hr} \Rightarrow 11\,\text{revolutions}$

4. A plane is inclined at an angle α = 30° with respect to the horizontal. A particle is projected with a speed u = 2 ms^{-1}, from the base of the plane, making an angle θ = 15° with respect to the plane as shown in the figure. The distance from the base, at which the particle hits the plane is close to :

(Take g = 10 ms^{-2})



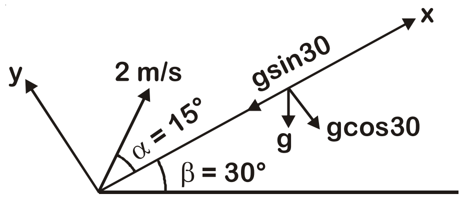
(1) 18 cm (2) 20 cm

(3) 14 cm (4) 26 cm

Answer (2)

Sol. Time of flight (T) $= \frac{2u\sin\alpha}{g\cos\beta}$

$T = \frac{(2)(2\sin15)}{g\cos30} = \frac{4}{10}\frac{\sin15}{\cos30}$



$$\text{Range (R)} = (2\cos 15)T - \frac{1}{2}g\sin 30(T)^2$$

$$= (2\cos 15)\frac{4}{10}\frac{\sin 15}{\cos 30} - \left(\frac{1}{3}\times 10\sin 30\right)\frac{16}{100}\frac{\sin^2 15}{\cos^2 30}$$

$$= \frac{16\sqrt{3}-16}{60} \simeq 0.1952\ \text{m} \simeq 20\ \text{cm}$$

5. A bullet of mass 20 g has an initial speed of 1 ms^{-1}, just before it starts penetrating a mud wall of thickness 20 cm. If the wall offers a mean resistance of 2.5×10^{-2} N, the speed of the bullet after emerging from the other side of the wall is close to :

(1) 0.4 ms^{-1} (2) 0.7 ms^{-1}

(3) 0.3 ms^{-1} (4) 0.1 ms^{-1}

Answer (2)

Sol. $v^2 = u^2 - 2aS$

$$v^2 = (1)^2 - (2)\left[\frac{2.5\times 10^{-2}}{20\times 10^{-3}}\right]\frac{20}{100}$$

$$v^2 = 1 - \frac{1}{2}$$

$$\Rightarrow \quad v = \frac{1}{\sqrt{2}}\ \text{m/s} = 0.7\ \text{m/s}$$

6. The time dependence of the position of a particle of mass m = 2 is given by $\vec{r}(t) = 2t\hat{i} - 3t^2\hat{j}$. Its angular momentum, with respect to the origin, at time t = 2 is :

(1) $-34\left(\hat{k}-\hat{i}\right)$ (2) $48\left(\hat{i}+\hat{j}\right)$

(3) $36\,\hat{k}$ (4) $-48\,\hat{k}$

Answer (4)

Sol. $\vec{r} = 2t\hat{i} - 3t^2\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = 2\hat{i} - 6t\hat{j}$$

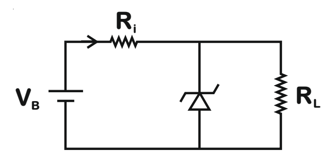
$$\vec{a} = \frac{d\overline{v}}{dt} = -6\hat{j}$$

$$\vec{F} = m\vec{a} = -12\hat{j}$$

$$\vec{r}(\text{at } t = 2) = 4\hat{i} - 12\hat{j}$$

$$\vec{L} = m(\vec{r}\times\vec{v}) = 2(4\hat{i} - 12\hat{j})\times(2\hat{i} - 12\hat{j}) = -48\hat{k}$$

7. The figure represents a voltage regulator circuit using a Zener diode. The breakdown voltage of the Zener diode is 6 V and the load resistance is, R_L = 4 kΩ. The series resistance of the circuit is R_i = 1 kΩ. If the battery voltage V_B varies from 8 V to 16 V, what are the minimum and maximum values of the current through Zener diode?

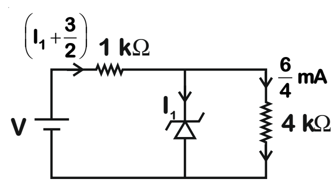


(1) 0.5 mA; 6 mA (2) 0.5 mA; 8.5 mA

(3) 1.5 mA; 8.5 mA (4) 1 mA; 8.5 mA

Answer (2)

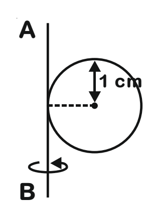
Sol.



$$I_1 = \left(8 - 6 - \frac{3}{2}\right) = \frac{1}{2} = 0.5\ \text{mA}$$

$$I_2 = \left(16 - 6 - \frac{3}{2}\right) = 8.5\ \text{mA}$$

8. A metal coin of mass 5 g and radius 1 cm is fixed to a thin stick AB of negligible mass as shown in the figure. The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5 s, is closed to :



(1) 4.0×10^{-6} Nm (2) 7.9×10^{-6} Nm

(3) 2.0×10^{-5} Nm (4) 1.6×10^{-5} Nm

Answer (3)

Sol. $\tau = I\alpha$

$\omega = \omega_0 + \alpha t$

$\Rightarrow \quad 25 \times 2\pi = (\alpha)5$

$\alpha = 10\pi$

$$\Rightarrow \quad \tau = \left(\frac{5}{4}mR^2\right)\alpha$$

$$\approx \left(\frac{5}{4}\right)\left(5\times10^{-3}\right)\left(10^{-4}\right)10\pi$$

$= 2.0 \times 10^{-5}$ Nm

9. In a Young's double slit experiment, the ratio of the slit's width is 4 : 1. The ratio of the intensity of maxima to minima, close to the central fringe on the screen, will be :

(1) $\left(\sqrt{3}+1\right)^4 : 16$ (2) 4 : 1

(3) 25 : 9 (4) 9 : 1

Answer (4)

Sol. $I_1 = 4\,I_0$

$I_2 = I_0$

$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2} = \left(\frac{9}{1}\right)$$

10. Two radioactive substances A and B have decay constants 5λ and λ respectively. At t = 0, a sample has the same number of the two nuclei. The time taken for the ratio of the number of nuclei to become $\left(\frac{1}{e}\right)^2$ will be :

(1) $\frac{1}{2\lambda}$ (2) $\frac{1}{4\lambda}$

(3) $\frac{1}{\lambda}$ (4) $\frac{2}{\lambda}$

Answer (1)

Sol. $N_x(\text{at } t) = N_0e^{-5\lambda t}$

$N_y(\text{at } t) = N_0e^{-\lambda t}$

$$\frac{N_x}{N_y} = \frac{1}{e^2} = e^{-4\lambda t}$$

$\Rightarrow \quad 4\lambda t = 2$

$$\Rightarrow \quad t = \frac{2}{4\lambda} = \left(\frac{1}{2\lambda}\right)$$

11. A cubical block of side 0.5 m floats on water with 30% of its volume under water. What is the maximum weight that can be put on the block without fully submerging it under water?

[Take, density of water = 10^3 kg/m^3]

(1) 30.1 kg (2) 46.3 kg

(3) 87.5 kg (4) 65.4 kg

Answer (3)

Sol. Given $(50)^3 \times \frac{30}{100} \times (1) \times g = M_{cube}g$ ...(i)

Let m mass should be placed

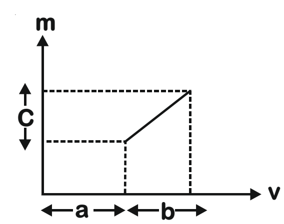
Hence $(50)^3 \times (1) \times g = \left(M_{cube} + m\right)g$ ...(ii)

equation (ii) – equation (i)

$\Rightarrow \quad mg = (50)^3 \times g(1 - 0.3) = 125 \times 0.7 \times 10^3\,g$

$\Rightarrow \quad m = 87.5$ kg

12. The graph shows how the magnification m produced by a thin lens varies with image distance v. What is the focal length of the lens used?



(1) $\frac{b^2}{ac}$ (2) $\frac{b^2c}{a}$

(3) $\frac{a}{c}$ (4) $\frac{b}{c}$

Answer (4)

Sol. As the graph between magnification (m) and image distance (v) varies linearly, then

$m = k_1v + k_2$

$$\Rightarrow \quad \frac{v}{u} = k_1v + k_2$$

$$\Rightarrow \quad \frac{1}{u} = k_1 + \frac{k_2}{v}$$

$$\Rightarrow \quad \frac{k_2}{v} - \frac{1}{u} = k_1$$

Clearly, $k_1 = \frac{1}{f}$ and $k_2 = 1$ here

$$\therefore \quad f = \frac{1}{\text{slope of } m - v \text{ graph}} = \frac{b}{c}$$

13. In Li^{++}, electron in first Bohr orbit is excited to a level by a radiation of wavelength λ. When the ion gets deexcited to the ground state in all possible ways (including intermediate emissions), a total of six spectral lines are observed. What is the value of λ?

(Given: $h = 6.63 \times 10^{-34}$ Js; $c = 3 \times 10^8$ ms^{-1})

(1) 11.4 nm

(2) 12.3 nm

(3) 9.4 nm

(4) 10.8 nm

Answer (4)

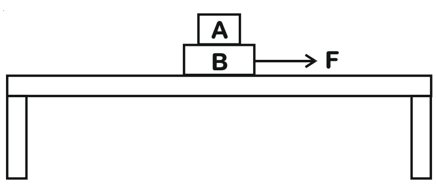
Sol. $\Delta E = \dfrac{hc}{\lambda}$

$$= (13.4)(3)^2\left[1-\frac{1}{16}\right]\text{eV}$$

$$\Rightarrow \quad \lambda = \frac{1242\times16}{(13.4)\times(9)(15)}\text{nm} \simeq 10.8\text{ nm}$$

14. Two blocks A and B of masses $m_A = 1$ kg and $m_B = 3$ kg are kept on the table as shown in figure. The coefficient of friction between A and B is 0.2 and between B and the surface of the table is also 0.2. The maximum force F that can be applied on B horizontally, so that the block A does not slide over the block B is:

[Take g = 10 m/s^2]



(1) 40 N

(2) 12 N

(3) 16 N

(4) 8 N

Answer (3)

Sol. $a_c = \left(\dfrac{F-f}{M+m}\right)$

$$a = \frac{F-(0.2)4\times10}{4} = \left(\frac{F-8}{4}\right)$$

We have $\dfrac{F-8}{4} \le (0.2)10$

$\Rightarrow \quad F - 8 \le 8$

$\Rightarrow \quad F \le 16$

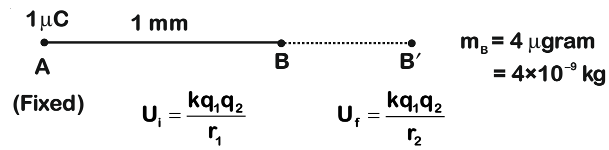
15. In free space, a particle A of charge 1 μC is held fixed at a point P. Another particle B of the same charge and mass 4 μg is kept at a distance of 1 mm from P. If B is released, then its velocity at a distance of 9 mm from P is :

$$\left[\text{Take } \frac{1}{4\pi\epsilon_0} = 9\times10^9\ \text{Nm}^2\text{C}^{-2}\right]$$

(1) 2.0×10^3 m/s (2) 3.0×10^4 m/s

(3) 1.5×10^2 m/s (4) 1.0 m/s

Answer (Bonus)



Conservation of energy

$$\frac{kq_1q_2}{r_1} = \frac{kq_1q_2}{r_2} + \frac{1}{2}mv^2$$

$$v^2 = \frac{2kq_1q_2}{m}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= \frac{2\times9\times10^9\times10^{-12}}{4\times10^{-9}\times10^{-3}}\left[1-\frac{1}{9}\right] = 4\times10^9$$

$$v = \sqrt{40}\times10^4\text{ m/s} = 6.32\times10^4\text{ m/s}$$

16. Water from a tap emerges vertically downwards with an initial speed of 1.0 ms^{-1}. The cross-sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water and that the flow is streamlined. The cross-sectional area of stream, 0.15 m below the tap would be:

(Take g = 10 ms^{-2})

(1) 1×10^{-5} m^2 (2) 5×10^{-5} m^2

(3) 5×10^{-4} m^2 (4) 2×10^{-5} m^2

Answer (2)

Sol. Using Bernoullie's equation $v_2 = \sqrt{v_1^2 + 2gh}$

Equation of continuity

$A_1V_1 = A_2V_2$

$$(1\text{ cm}^2)(1\text{ m/s}) = (A_2)\left(\sqrt{(1)^2 + 2\times10\times\frac{15}{100}}\right)$$

$$\Rightarrow \quad A_2\left(\text{In cm}^2\right) = \frac{1}{2}$$

$\Rightarrow \quad A_2 = 5 \times 10^{-5}$ m^2

17. When heat Q is supplied to a diatomic gas of rigid molecules, at constant volume its temperature increases by ΔT. The heat required to produce the same change in temperature, at a constant pressure is:

(1) $\frac{3}{2}Q$ (2) $\frac{2}{3}Q$

(3) $\frac{7}{5}Q$ (4) $\frac{5}{3}Q$

Answer (3)

Sol. Heat supplied at constant volume

$Q = nC_V\Delta T$

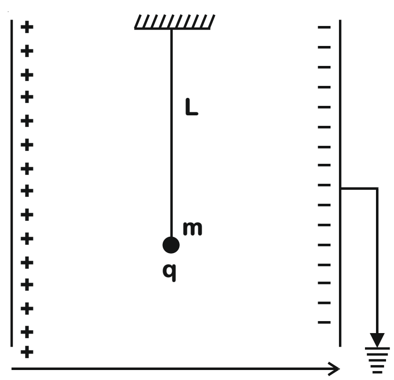
and heat supplied at constant pressure

$Q_1 = nC_p\Delta T$

$$\therefore \frac{Q_1}{Q} = \frac{C_p}{C_v}$$

$$\Rightarrow Q_1 = (Q)\left(\frac{7}{5}\right)$$

18. A simple pendulum of length L is placed between the plates of a parallel plate capacitor having electric field E, as shown in figure. Its bob has mass m and charge q. The time period of the pendulum is given by:



(1) $2\pi\sqrt{\frac{L}{\left(g-\frac{qE}{m}\right)}}$ (2) $2\pi\sqrt{\frac{L}{\left(g+\frac{qE}{m}\right)}}$

(3) $2\pi\sqrt{\frac{L}{\sqrt{g^2+\left(\frac{qE}{m}\right)^2}}}$ (4) $2\pi\sqrt{\frac{L}{\sqrt{g^2-\frac{q^2E^2}{m^2}}}}$

Answer (3)

Sol. $t = 2\pi\sqrt{\frac{L}{g_{eff}}}$

$$\Rightarrow g_{eff} = \sqrt{g^2+\left(\frac{gE}{m}\right)^2}$$

$$\Rightarrow t = 2\pi\sqrt{\frac{L}{\sqrt{g^2+\left(\frac{qE}{m}\right)^2}}}$$

19. A source of sound S is moving with a velocity of 50 m/s towards a stationary observer. The observer measures the frequency of the source as 1000 Hz. What will be the apparent frequency of the source when it is moving away from the observer after crosssing him? (Take velocity of sound in air is 350 m/s)

(1) 857 Hz (2) 1143 Hz

(3) 807 Hz (4) 750 Hz

Answer (4)

Sol. $f_{app} = f_{act}\left(\frac{V \pm V_0}{V \mp V_s}\right)$

$$1000 = f_{act}\left(\frac{350-0}{350+(-50)}\right) \text{ and } f' = f_{act}\left(\frac{350}{350+50}\right)$$

$$\Rightarrow f_{act} = \frac{1000\times 300}{400}$$

$f_{act} \approx 750\text{Hz}$

20. A 2 mW laser operates at a wavelength of 500 nm. The number of photons that will be emitted per second is:

[Given Planck's constant h = 6.6×10^{-34} Js, speed of light c = 3.0×10^8 m/s]

(1) 2×10^{16}

(2) 1.5×10^{16}

(3) 1×10^{16}

(4) 5×10^{15}

Answer (4)

Sol. $E = \frac{hc}{\lambda}$

Let no. of photons per sec. is N

$$\Rightarrow N\frac{hc}{\lambda} = 2\text{ mW}$$

$$\Rightarrow N = \frac{2\times\lambda}{hC} = \frac{2\times 5000\times 10^{-3}}{12420\times 1.6\times 10^{-19}}$$

N = 5×10^{15}

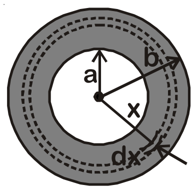
21. Space between two concentric conducting spheres of radii a and b (b > a) is filled with a medium of resistivity ρ. The resistance between the two spheres will be:

(1) $\frac{\rho}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$ (2) $\frac{\rho}{2\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$

(3) $\frac{\rho}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$ (4) $\frac{\rho}{4\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$

Answer (1)

Sol.



$$dR = \frac{(\rho)(dx)}{4\pi x^2}$$

$$R = \int dR$$

$$= \left(\frac{\rho}{4\pi}\right) = \int_a^b \frac{dx}{x^2}$$

$$= \left(\frac{\rho}{4\pi}\right)\cdot\left(\frac{1}{a}-\frac{1}{b}\right)$$

22. In an experiment, brass and steel wires of length 1 m each with areas of cross section 1 mm² are used. The wires are connected in series and one end of the combined wire is connected to a rigid support and other end is subjected to elongation. The stress required to produce a net elongation of 0.2 mm is,

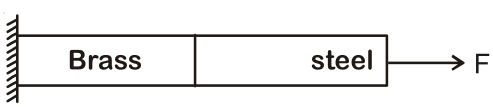
[Given, the Young's Modulus for steel and brass are, respectively, 120×10^9 N/m² and 60×10^9 N/m²]

(1) 1.8×10^6 N/m² (2) 1.2×10^6 N/m²

(3) 4.0×10^6 N/m² (4) 0.2×10^6 N/m²

Answer (Bonus)

Sol.



Corresponding to the stress (σ)

Total elongation $\Delta l_{net} = \frac{\sigma L_1}{Y_1} + \frac{\sigma L_2}{Y_2}$

$$\sigma = \Delta l\left(\frac{Y_1 Y_2}{Y_1 + Y_2}\right)$$

$$= 0.2 \times 10^{-3} \times \left(\frac{120 \times 60}{180}\right) \times 10^9$$

$$= 8 \times 10^6 \frac{N}{m^2}$$ (Answer is not matching)

23. A coil of self inductance 10 mH and resistance 0.1 Ω is connected through a switch to a battery of internal resistance 0.9 Ω. After the switch is closed, the time taken for the current to attain 80% of the saturation value is: [take ln5 = 1.6]

(1) 0.002 s (2) 0.324 s

(3) 0.103 s (4) 0.016 s

Answer (4)

Sol. $I = I_{sat}\left(1 - e^{-\frac{Rt}{L}}\right)$ Here $R = R_L + r = 1\Omega$

$$0.8 I_{sat} = I_{sat}\left(1 - e^{-\frac{t}{.01}}\right)$$

$$\Rightarrow \frac{4}{5} = 1 - e^{-100t}$$

$$\Rightarrow e^{-100t} = \left(\frac{1}{5}\right)$$

$\Rightarrow$ 100t = ln5

$$\Rightarrow t = \frac{1}{100}\ln 5$$

= 0.016 sec

24. The magnitude of the magnetic field at the center of an equilateral triangular loop of side 1 m which is carrying a current of 10 A is:

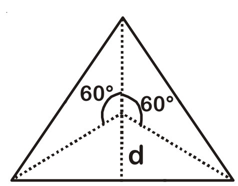
[Take $\mu_0 = 4\pi \times 10^{-7}$ NA⁻²]

(1) 18 μT (2) 1 μT

(3) 3 μT (4) 9 μT

Answer (1)

Sol.



$$d = \left(\frac{1}{3}\right)(a\sin 60)$$

$$d = \frac{a}{3} \times \frac{\sqrt{3}}{2} = \left(\frac{a}{2\sqrt{3}}\right)$$

$$B_0 = 3\left[\frac{\mu_0 I}{4\pi d}(\sin 60 + \sin 60)\right]$$

$$= \frac{3\mu_0 I}{4\pi\left(\frac{a}{2\sqrt{3}}\right)} = (2)\left(\frac{\sqrt{3}}{2}\right) \qquad d = \left(\frac{1}{3}\right)(a\sin 60)$$

$$= \frac{9}{2}\left(\frac{\mu_0 I}{\pi a}\right) \qquad d = \frac{a}{3} \times \frac{\sqrt{3}}{2}$$

$$= \frac{9 \times 2 \times 10^{-7} \times 10}{1} \qquad = \left(\frac{a}{2\sqrt{3}}\right)$$

$= 18\ \mu T$

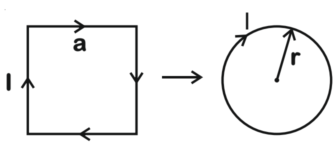
25. A square loop is carrying a steady current I and the magnitude of its magnetic dipole moment is m. If this square loop is changed to a circular loop and it carries the same current, the magnitude of the magnetic dipole moment of circular loop will be :

(1) $\frac{2m}{\pi}$ (2) $\frac{4m}{\pi}$

(3) $\frac{m}{\pi}$ (4) $\frac{3m}{\pi}$

Answer (2)

Sol.



$2\pi r = 4a \quad \Rightarrow \quad r = \left(\frac{2a}{\pi}\right)$

$m = (I)\ a^2$

$m_1 = (I)\ \pi r^2$

$$m_1 = (I)(\pi)\left(\frac{4a^2}{\pi^2}\right)$$

$$m_1 = \frac{4Ia^2}{\pi}$$

$$m_1 = \frac{4m}{\pi}$$

26. A submarine experiences a pressure of 5.05×10^6 Pa at a depth of d_1 in a sea. When it goes further to a depth of d_2, it experiences a pressure of 8.08×10^6 Pa. Then $d_2 - d_1$ is approximately (density of water = 10^3 kg/m^3 and acceleration due to gravity = 10 ms^{-2}) :

(1) 600 m (2) 500 m

(3) 300 m (4) 400 m

Answer (3)

Sol. $\Delta P = P_2 - P_1 = \rho g \Delta H$

$3.03 \times 10^6 = 10^3 \times 10 \times \Delta H$

$\Rightarrow \quad \Delta H \simeq 300$ m

27. The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod if it is to support a 400 N load without exceeding its elastic limit?

(1) 0.90 mm

(2) 1.16 mm

(3) 1.00 mm

(4) 1.36 mm

Answer (2)

Sol. Stress = $\frac{400}{\pi r^2} \le 379 \times 10^6\ \text{N/m}^2$

$$\Rightarrow \quad r^2 \ge \frac{400}{379 \times 10^6 \pi}$$

$2r \ge 1.15$ mm

28. Light is incident normally on a completely absorbing surface with an energy flux of 25 Wcm^{-2}. If the surface has an area of 25 cm^2, the momentum transferred to the surface in 40 min time duration will be :

(1) 3.5×10^{-6} Ns

(2) 6.3×10^{-4} Ns

(3) 5.0×10^{-3} Ns

(4) 1.4×10^{-6} Ns

Answer (3)

Sol. $P = \frac{\Delta E}{C}$

$$= \frac{(25 \times 25) \times 40 \times 60}{3 \times 10^8}\ \text{N-s}$$

$= 5 \times 10^{-3}$ N-s

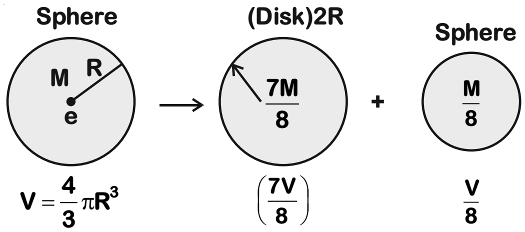
29. A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $\frac{7M}{8}$ and is converted into a uniform disc of radius 2R. The second part is converted into a uniform solid sphere. Let I_1 be the moment of inertia of the disc about its axis and I_2 be the moment of inertia of the new sphere about its axis. The ratio I_1/I_2 is given by :

(1) 140 (2) 185

(3) 285 (4) 65

Answer (1)

Sol.



$$I_1 = \left(\frac{7M}{8}\right)(2R)^2\frac{1}{2} = \frac{14}{8}MR^2$$

$$I_2 = \frac{2}{5}\left(\frac{M}{8}\right)r^2$$

$$\left[\begin{array}{l}\text{We have} \\ \frac{4}{3}\pi r^3\rho = \frac{M}{8} \\ \Rightarrow \ r = \left(\frac{R}{2}\right)\end{array}\right]$$

$$\Rightarrow \ I_2 = \frac{2}{5}\left(\frac{M}{8}\right)\left(\frac{R^2}{4}\right) = \frac{MR^2}{80}$$

$$\frac{I_1}{I_2} = \frac{14\times 80}{(8)} = 140$$

30. In the formula X = $5YZ^2$, X and Z have dimensions of capacitance and magnetic field, respectively. What are the dimensions of Y in SI units ?

(1) $[M^{-2}L^{-2}T^6A^3]$ (2) $[M^{-1}L^{-2}T^4A^2]$

(3) $[M^{-2}L^0T^{-4}A^{-2}]$ (4) $[M^{-3}L^{-2}T^8A^4]$

Answer (4)

Sol. X = $5YZ^2$

$$Y \propto \frac{X}{Z^2}$$

$$X = C = \frac{Q^2}{E} = \frac{[A^2T^2]}{[ML^2T^{-2}]}$$

$$X = [M^{-1}L^{-2}T^4A^2]$$

$$Z = B = \frac{F}{IL}$$

$$Z = [MT^{-2}A^{-1}]$$

$$Y = \frac{[M^{-1}L^{-2}T^4A^2]}{[MT^{-2}A^{-1}]^2}$$

$$Y = [M^{-3}L^{-2}T^8A^4]$$

PART–B : CHEMISTRY

1. The correct statement is :

(1) Zincite is a carbonate ore.

(2) Zone refining process is used for the refining of titanium.

(3) Aniline is a froth stabilizer.

(4) Sodium cyanide cannot be used in the metallurgy of silver.

Answer (3)

Sol. Ti is refined by Van Arkel method. Ag is leached by dilute solution of NaCN. Zincite is ZnO. Aniline is a froth stabilizer.

2. The pH of a 0.02 M NH_4Cl solution will be [given $K_b(NH_4OH) = 10^{-5}$ and log2 = 0.301]

(1) 2.65

(2) 5.35

(3) 4.35

(4) 4.65

Answer (2)

Sol. $NH_4^+ + H_2O \rightleftharpoons NH_4OH + H^+$

0.02 – x x x $K_h = \frac{10^{-14}}{10^{-5}} = 10^{-9}$

≈ 0.02

$$K_h = \frac{x^2}{0.02}$$

$10^{-9} \times 2 \times 10^{-2} = x^2$

$$x = \sqrt{20}\times 10^{-6}$$

$$pH = -\log\left(\sqrt{20}\times 10^{-6}\right)$$

pH = 5.35

3. The INCORRECT statement is :

(1) The spin-only magnetic moments of $[Fe(H_2O)_6]^{2+}$ and $[Cr(H_2O)_6]^{2+}$ are nearly similar.

(2) The gemstone, ruby, has Cr^{3+} ions occupying the octahedral sites of beryl.

(3) The spin-only magnetic moment of $[Ni(NH_3)_4(H_2O)_2]^{2+}$ is 2.83 BM.

(4) The color of $[CoCl(NH_3)_5]^{2+}$ is violet as it absorbs the yellow light.

Answer (2)

Sol. Ruby is aluminium oxide (Al_2O_3) containing about 0.5 – 1% Cr^{3+} ions which are randomly distributed in the position normally occupied by Al^{3+} ions.

4. **Number of stereo centers present in linear and cyclic structures of glucose are respectively :**

(1) 5 & 5

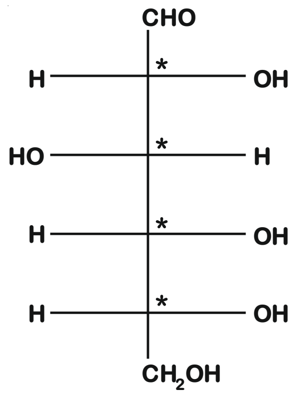
(2) 4 & 4

(3) 5 & 4

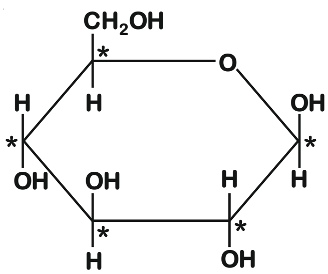
(4) 4 & 5

Answer (4)

Sol.



4 stereogenic centres



5 stereogenic centres

5. **The difference between ΔH and ΔU ($\Delta H - \Delta U$), when the combustion of one mole of heptane(l) is carried out at a temperature T, is equal to :**

(1) –3RT

(2) 4RT

(3) 3RT

(4) –4RT

Answer (4)

Sol. $\underset{(l)}{C_7H_{16}} + \underset{(g)}{11O_2} \xrightarrow{\Delta} \underset{(g)}{7CO_2} + \underset{(l)}{8H_2O}$

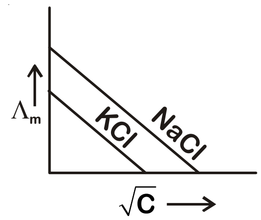
$\Delta H - \Delta U = \Delta n_g RT$

$\because\ \Delta n_g = -4$

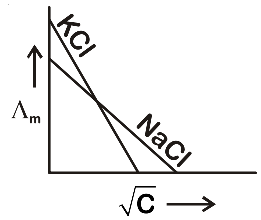
$\therefore\ \Delta H - \Delta U = -4RT$

6. **Which one of the following graphs between molar conductivity (Λ_m) versus $\sqrt{C}$ is correct?**

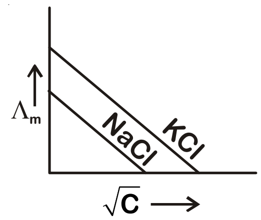
(1)



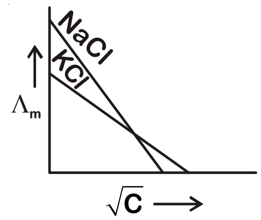
(2)



(3)

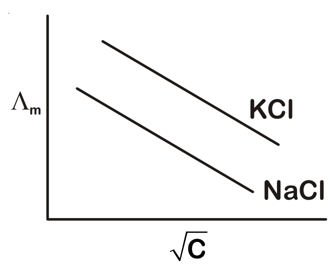


(4)



Answer (3)

Sol. KCl is more conducting than NaCl



7. **The crystal field stabilization energy (CFSE) of $[Fe(H_2O)_6]Cl_2$ and $K_2[NiCl_4]$, respectively, are :**

(1) $-2.4\Delta_o$ and $-1.2\Delta_t$

(2) $-0.6\Delta_o$ and $-0.8\Delta_t$

(3) $-0.4\Delta_o$ and $-0.8\Delta_t$

(4) $-0.4\Delta_o$ and $-1.2\Delta_t$

Answer (3)

Sol. $[Fe(H_2O)_6]^{2+}$ $\quad t_{2g}^4\ e_g^2$ $\quad$ CFSE = $-0.4\Delta_0$

$[NiCl_4]^{2-}$ $\quad e^4\ t_2^4$ $\quad$ CFSE = $-0.8\Delta_t$

8. For the reaction,

$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$,

$\Delta H = -57.2$ kJ mol^{-1} and $K_c = 1.7 \times 10^{16}$.

Which of the following statement is INCORRECT?

(1) The equilibrium constant is large suggestive of reaction going to completion and so no catalyst is required.

(2) The addition of inert gas at constant volume will not affect the equilibrium constant.

(3) The equilibrium will shift in forward direction as the pressure increases.

(4) The equilibrium constant decreases as the temperature increases.

Answer (1)

Sol. $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$

$K_c = 1.7 \times 10^{16}$ i.e. reaction goes to completion. Equilibrium constant has no relation with catalyst. Catalyst only affects the rate with which a reaction proceeds.

For the given reaction, catalyst V_2O_5 is used to speed up the reaction (Contact process).

9. A hydrated solid X on heating initially gives a monohydrated compound Y. Y upon heating above 373 K leads to an anhydrous white powder Z. X and Z, respectively are :

(1) Baking soda and dead burnt plaster.

(2) Baking soda and soda ash.

(3) Washing soda and soda ash.

(4) Washing soda and dead burnt plaster.

Answer (3)

Sol. $\underset{(X)}{Na_2CO_3 \cdot 10H_2O} \longrightarrow \underset{(Y)}{Na_2CO_3 \cdot H_2O} + 9H_2O$

$\underset{(Y)}{Na_2CO_3 \cdot H_2O} \longrightarrow \underset{(Z)}{Na_2CO_3} + H_2O$

X = Washing soda

Z = Soda ash

10. Which of these factors does not govern the stability of a conformation in acyclic compounds?

(1) Angle strain

(2) Steric interactions

(3) Electrostatic forces of interaction

(4) Torsional strain

Answer (1)

Sol. Angle strain is not present in acyclic compounds.

11. The highest possible oxidation states of uranium and plutonium, respectively, are :

(1) 6 and 7 (2) 7 and 6

(3) 6 and 4 (4) 4 and 6

Answer (1)

Sol. Maximum oxidation state shown by

Uranium = + 6

Plutonium = + 7

12. The correct option among the following is :

(1) Colloidal medicines are more effective because they have small surface area.

(2) Colloidal particles in lyophobic sols can be precipitated by electrophoresis.

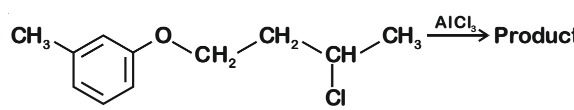
(3) Brownian motion in colloidal solution is faster if the viscosity of the solution is very high.

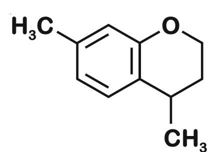
(4) Addition of alum to water makes it unfit for drinking.

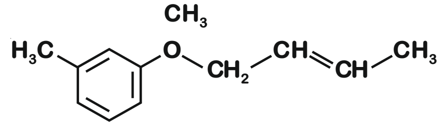
Answer (2)

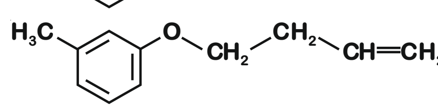
Sol. Electrophoresis is used to coagulate lyophobic colloids.

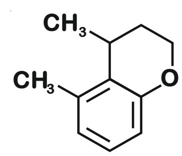
13. The major product obtained in the given reaction is :



(1) 

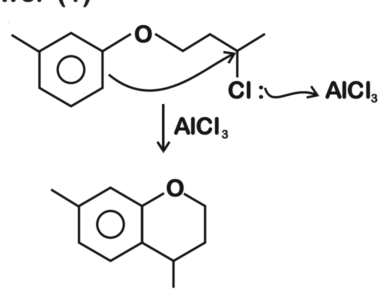
(2) 

(3) 

(4) 

Answer (1)

Sol.



14. The number of pentagons in C_{60} and trigons (triangles) in white phosphorus, respectively, are :
(1) 20 and 3
(2) 12 and 3
(3) 12 and 4
(4) 20 and 4

Answer (3)

Sol. Pentagons in C_{60} = 12
Triangles in P_4 = 4

15. For the reaction of H_2 with I_2, the rate constant is 2.5×10^{-4} dm^3 mol^{-1} s^{-1} at 327°C and 1.0 dm^3 mol^{-1} s^{-1} at 527°C. The activation energy for the reaction, in kJ mol^{-1} is :
($R = 8.314$ J K^{-1} mol^{-1})

(1) 150 (2) 59
(3) 72 (4) 166

Answer (4)

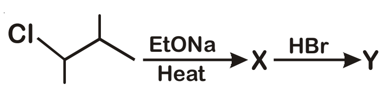
Sol. $\log \dfrac{K_2}{K_1} = \dfrac{E_a}{2.303R}\left(\dfrac{1}{T_1} - \dfrac{1}{T_2}\right)$

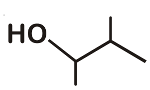
$$\log \frac{1}{2.5\times10^{-4}} = \frac{E_a}{8.314\times2.303}\left(\frac{1}{600}-\frac{1}{800}\right)$$

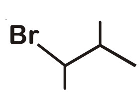
$$3.6 = \frac{E_a}{8.314\times2.303}\times\frac{200}{600\times800}$$

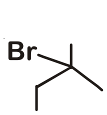
E_a = 165.4 kJ/mol
$\approx$ 166 kJ/mol

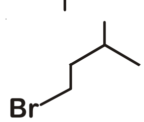
16. The major product 'Y' in the following reaction is :



(1) 

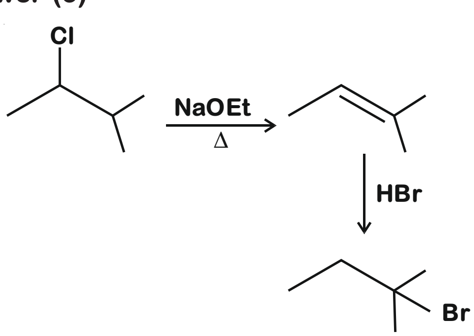
(2) 

(3) 

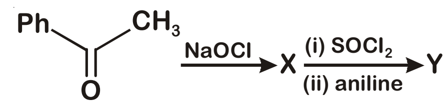
(4) 

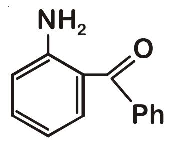
Answer (3)

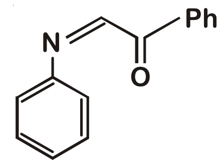
Sol.

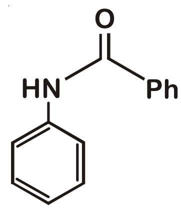


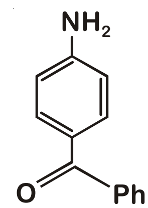
17. The major product 'Y' in the following reaction is :



(1) 

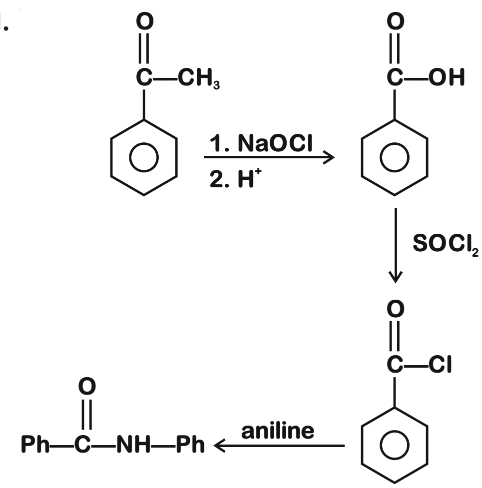
(2) 

(3) 

(4) 

Answer (3)

Sol.



18. 1 g of a non-volatile non-electrolyte solute is dissolved in 100 g of two different solvents A and B whose ebullioscopic constants are in the ratio of 1 : 5. The ratio of the elevation in their boiling points, $\dfrac{\Delta T_b(A)}{\Delta T_b(B)}$, is :

(1) 1 : 5 (2) 10 : 1
(3) 5 : 1 (4) 1 : 0.2

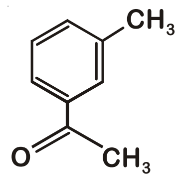
Answer (1)

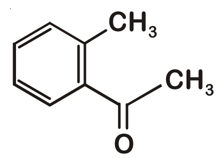
Sol. $\Delta T_b = k_b \times m$

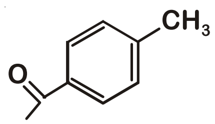
$$\frac{(k_b)_A}{(k_b)_B} = \frac{1}{5}$$

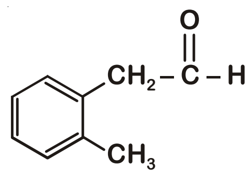
$$\therefore \quad \frac{(\Delta T_b)_A}{(\Delta T_b)_B} = \frac{(k_b)_A}{(k_b)_B} = \frac{1}{5}$$

19. Compound A($C_9H_{10}O$) shows positive iodoform test. Oxidation of A with $KMnO_4$/KOH gives acid B($C_8H_6O_4$). Anhydride of B is used for the preparation of phenolphthalein. Compound A is:

(1) 

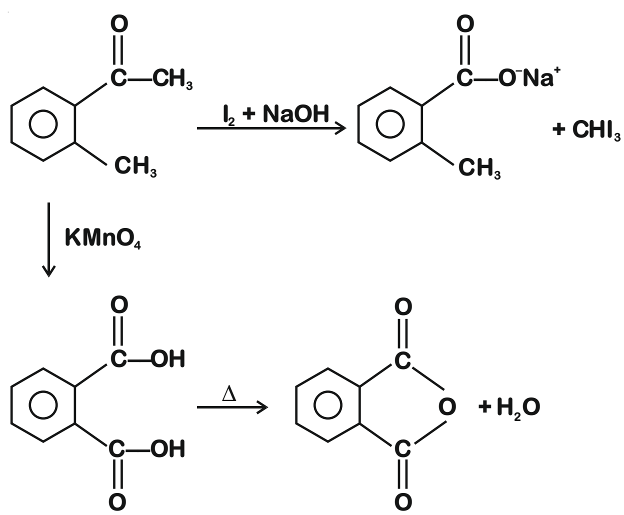
(2) 

(3) 

(4) 

Answer (2)

Sol.



$C_8H_6O_4$ anhydride

20. The ratio of the shortest wavelength of two spectral series of hydrogen spectrum is found to be about 9. The spectral series are :

(1) Paschen and Pfund

(2) Brackett and Pfund

(3) Lyman and Paschen

(4) Balmer and Brackett

Answer (3)

Sol. Shortest wavelength means $n_2 = \infty$

Lyman series $\bar{\nu}_L = \frac{1}{\lambda_L} = -1312 \times \frac{1}{1^2} \times 1^2$

Paschen series $\bar{\nu}_P = \frac{1}{\lambda_P} = -1312 \times \frac{1}{3^2} \times 1^2$

$\frac{\bar{\nu}_L}{\bar{\nu}_P} = \frac{\lambda_P}{\lambda_L} = 9$

21. The correct match between Item-I and Item-II is:

	Item – I		Item – II
(a)	High density polythene	(I)	Peroxide catalyst
(b)	Polyacrylonitrile	(II)	Condensation at high temperature and pressure
(c)	Novolac	(III)	Ziegler – Natta Catalyst
(d)	Nylon 6	(IV)	Acid or base catalyst

(1) (a) → (III), (b) → (I), (c) → (IV), (d) → (II)

(2) (a) → (III), (b) → (I), (c) → (II), (d) → (IV)

(3) (a) → (IV), (b) → (II), (c) → (I), (d) → (III)

(4) (a) → (II), (b) → (IV), (c) → (I), (d) → (III)

Answer (1)

Sol. a. HDPE – Ziegler-Natta Catalyst

b. Polyacrylonitrile Peroxide Catalyst

c. Novolac – Catalysed by acid or base

d. Nylon-6 – Condensation at High T and P

22. Air pollution that occurs in sunlight is :

(1) Fog (2) Oxidising smog

(3) Acid rain (4) Reducing smog

Answer (2)

Sol. Air pollution caused by sunlight is photochemical smog and it is oxidising.

23. The increasing order of nucleophilicity of the following nucleophiles is :

(a) $CH_3CO_2^{\ominus}$ (b) H_2O

(c) $CH_3SO_3^{\ominus}$ (d) $\overset{\ominus}{O}H$

(1) (d) < (a) < (c) < (b)

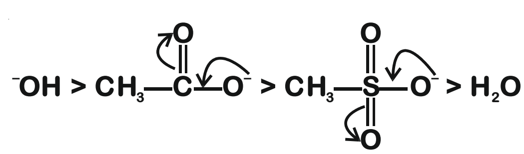
(2) (b) < (c) < (d) < (a)

(3) (a) < (d) < (c) < (b)

(4) (b) < (c) < (a) < (d)

Answer (4)

Sol. Greater the negative charge Present on a nucleophilic centre greater would be its nucleophilicity.



24. The correct order of the first ionization enthalpies is :

(1) Mn < Ti < Zn < Ni (2) Ti < Mn < Zn < Ni

(3) Ti < Mn < Ni < Zn (4) Zn < Ni < Mn < Ti

Answer (3)

Sol. Order for I.E. is

Ti < Mn < Ni < Zn

25. The noble gas that does NOT occur in the atmosphere is :

(1) Ne (2) Kr

(3) He (4) Ra

Answer (4)

Sol. Radon is not present in atmosphere.

26. Which of the following is NOT a correct method of the preparation of benzylamine from cyanobenzene?

(1) (i) $SnCl_2$ + HCl(gas) (ii) $NaBH_4$

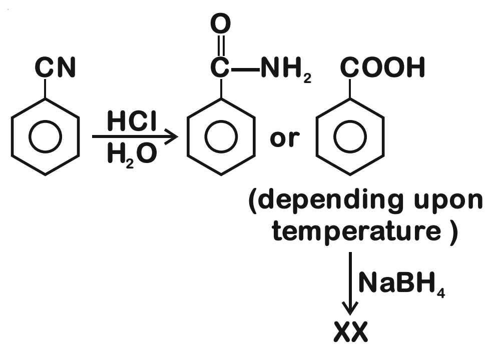
(2) H_2/Ni

(3) (i) $LiAlH_4$ (ii) H_3O^+

(4) (i) HCl/H_2O (ii) $NaBH_4$

Answer (4)

Sol.



27. The minimum amount of O_2(g) consumed per gram of reactant is for the reaction:

(Given atomic mass : Fe = 56, O = 16, Mg = 24, P = 31, C = 12, H = 1)

(1) $2Mg(s) + O_2(g) \rightarrow 2MgO(s)$

(2) $4Fe(s) + 3O_2(g) \rightarrow 2Fe_2O_3(s)$

(3) $C_3H_8(g) + 5O_2(g) \rightarrow 3CO_2(g) + 4H_2O(l)$

(4) $P_4(s) + 5O_2(g) \rightarrow P_4O_{10}(s)$

Answer (2)

Sol. (1) $2\ Mg + O_2 \longrightarrow 2\ MgO$

1 g requires $\frac{32}{48}$ g = 0.66 g of O_2

(2) $4Fe + 3O_2 \longrightarrow 2Fe_2O_3$

1 g Fe requires = 0.43 g of oxygen

(3) $C_3H_8 + 5O_2 \longrightarrow 3CO_2 + 4H_2O$

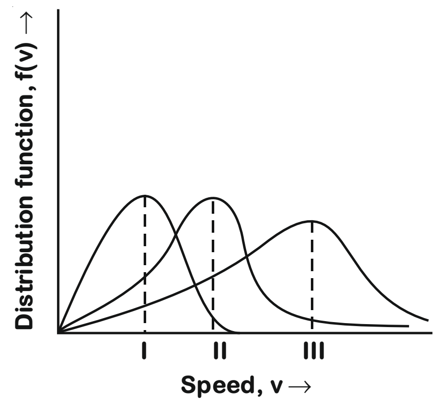
1 g of C_3H_8 requires = 3.6 g of O_2

(4) $P_4 + 5O_2 \longrightarrow P_4O_{10}$

1 g of P requires = 1.3 g of oxygen

28. Points I, II and III in the following plot respectively correspond to

(V_{mp} : most probable velocity)



(1) V_{mp} of N_2 (300 K); V_{mp} of O_2 (400 K); V_{mp} of H_2 (300 K)

(2) V_{mp} of H_2 (300 K); V_{mp} of N_2 (300 K); V_{mp} of O_2 (400 K)

(3) V_{mp} of N_2 (300 K); V_{mp} of H_2 (300 K); V_{mp} of O_2 (400 K)

(4) V_{mp} of O_2 (400 K); V_{mp} of N_2 (300 K); V_{mp} of H_2 (300 K)

Answer (1)

Sol. $V_{mp} = \sqrt{\frac{2RT}{M}}$

$\therefore$ as $\frac{T}{M}$ increases, V_{mp} increases

From curve

$(V_{mp})_I < (V_{mp})_{II} < (V_{mp})_{III}$

$(V_{mp})_{N_2} \propto \sqrt{\frac{300}{28}}, (V_{mp})_{O_2} \propto \sqrt{\frac{400}{32}}, (V_{mp})_{H_2} \propto \sqrt{\frac{300}{2}}$

$\therefore\ (V_{mp})_{N_2} < (V_{mp})_{O_2} < (V_{mp})_{H_2}$

(Under given Condition)

29. The correct statements among (a) to (d) are :

(a) Saline hydrides produce H_2 gas when reacted with H_2O.

(b) Reaction of $LiAlH_4$ with BF_3 leads to B_2H_6.

(c) PH_3 and CH_4 are electron - rich and electron - precise hydrides, respectively.

(d) HF and CH_4 are called as molecular hydrides.

(1) (a), (c) and (d) only.

(2) (c) and (d) only.

(3) (a), (b) and (c) only.

(4) (a), (b), (c) and (d).

Answer (4)

Sol. – With water saline hydrides produce H_2 gas

– $3LiAlH_4 + 4BF_3 \longrightarrow 2B_2H_6 + 3LiF + 3AlF_3$

– PH_3 is electron rich while CH_4 is electron precise hydride

– HF and CH_4 are molecular hydrides

30. In chromatography, which of the following statements is INCORRECT for R_f ?

(1) The value of R_f cannot be more than one.

(2) Higher R_f value means higher adsorption.

(3) R_f value is dependent on the mobile phase.

(4) R_f value depends on the type of chromatography.

Answer (2)

Sol. R_f represents retardation factor in chromatography.

$$R_f = \frac{\text{Distance moved by the substance from base line}}{\text{Distance moved by the solvent from baseline}}$$

– Higher R_f value means lower adsorpation

PART–C : MATHEMATICS

1. Let $f(x) = \log_e(\sin x)$, $(0 < x < \pi)$ and $g(x) = \sin^{-1}(e^{-x})$, $(x \geq 0)$. If α is a positive real number such that $a = (fog)'(\alpha)$ and $b = (fog)(\alpha)$, then :

(1) $a\alpha^2 - b\alpha - a = 1$

(2) $a\alpha^2 + b\alpha + a = 0$

(3) $a\alpha^2 - b\alpha - a = 0$

(4) $a\alpha^2 + b\alpha - a = -2\alpha^2$

Answer (1)

Sol. $f(x) = \ln(\sin x)$, $g(x) = \sin^{-1}(e^{-x})$

$f(g(x)) = \ln(\sin(\sin^{-1} e^{-x}))$

$= -x$

$\Rightarrow -\alpha = b$

$f'(g(\alpha)) = a$

i.e., $a = -1$

$\therefore a\alpha^2 - b\alpha + 1 = -\alpha^2 + \alpha^2 + 1 = -a$

2. The angles A, B and C of a triangle ABC are in A.P. and $a : b = 1 : \sqrt{3}$. If $c = 4$ cm, then the area (in sq.cm) of this triangle is :

(1) $\frac{2}{\sqrt{3}}$

(2) $4\sqrt{3}$

(3) $2\sqrt{3}$

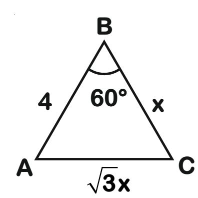
(4) $\frac{4}{\sqrt{3}}$

Answer (3)

Sol. $\because$ A, B, C, are in A.P

$\Rightarrow 2B = A + C$

$\Rightarrow B = \frac{\pi}{3}$



$\text{Area} = \frac{1}{2}(4x)\sin 60^\circ$

$= \sqrt{3}x$

Now $\cos 60^\circ = \frac{16 + x^2 - 3x^2}{8x}$

$\Rightarrow 4x = 16 - 2x^2$

$x = 2$ (as -4 is rejected)

Hence, area = $2\sqrt{3}$ sq. cm

3. The sum of the real roots of the equation

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0, \text{ is equal to :}$$

(1) 6

(2) 0

(3) -4

(4) 1

Answer (2)

Sol. $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$

$\Rightarrow$ $x(-3x^2 - 6x - 2x^2 + 6x) -6(-3x + 9 - 2x - 4)$

$-(4x - 9xA) = 0$

$\Rightarrow$ $x(-5x^2) -6(-5x + 5) - 4x + 9x = 0$

$\Rightarrow$ $x^3 - 7x + 6 = 0$

All the roots are real

$\therefore$ Sum of real roots $= \dfrac{0}{1} = 0$

4. If z and w are two complex numbers such that $|zw| = 1$ and $\arg(z) - \arg(w) = \dfrac{\pi}{2}$, then :

(1) $z\bar{w} = \dfrac{1-i}{\sqrt{2}}$ (2) $\bar{z}w = i$

(3) $z\bar{w} = \dfrac{-1+i}{\sqrt{2}}$ (4) $\bar{z}w = -i$

Answer (4)

Sol. $|zw| = 1$ ...(i)

$\arg\left(\dfrac{z}{w}\right) = \dfrac{\pi}{2}$ ...(ii)

$\therefore$ $\dfrac{z}{w} + \dfrac{\bar{z}}{\bar{w}} = 0$ $\Rightarrow z\bar{w} = -\bar{z}w$

from (i) $z\bar{z}w\bar{w} = 1$

$(\bar{z}w)^2 = -1$ $\Rightarrow \bar{z}w = \pm i$

from (ii) $-\arg(\bar{z}) - \arg w = \dfrac{\pi}{2}$

$\Rightarrow$ $\arg(\bar{z}w) = \dfrac{-\pi}{2}$

Hence, $\bar{z}w = -i$

5. The locus of the centres of the circles, which touch the circle, $x^2 + y^2 = 1$ externally, also touch the y-axis and lie in the first quadrant, is :

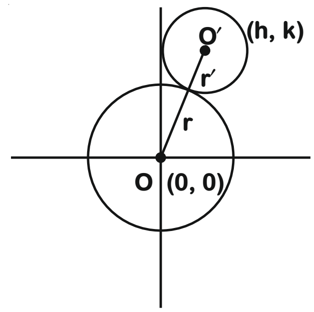
(1) $y = \sqrt{1+2x}, x \geq 0$ (2) $x = \sqrt{1+4y}, y \geq 0$

(3) $x = \sqrt{1+2y}, y \geq 0$ (4) $y = \sqrt{1+4x}, x \geq 0$

Answer (1)

Sol. Let centre of required circle is (h, k).

$\therefore$ $OO' = r + r'$



$\Rightarrow$ $\sqrt{h^2 + k^2} = 1 + h$

$h^2 + k^2 = 1 + h^2 + 2h$

$k^2 = 1 + 2h$

Locus is $y = \sqrt{1+2x}$

6. The sum $1 + \dfrac{1^3+2^3}{1+2} + \dfrac{1^3+2^3+3^3}{1+2+3} + \dots + \dfrac{1^3+2^3+3^3+\dots+15^3}{1+2+3+\dots+15} - \dfrac{1}{2}(1+2+3+\dots+15)$ is equal to :

(1) 1860 (2) 620

(3) 660 (4) 1240

Answer (2)

Sol. $S = 1 + \dfrac{1^3+2^3}{1+2} + \dfrac{1^3+2^3+3^3}{1+2+3} + \dots 15 \text{ terms}$

$$T_n = \frac{1^3+2^3+\dots n^3}{1+2+\dots n} = \frac{\dfrac{n^2(n+1)^2}{4}}{\dfrac{n(n+1)}{2}} = \frac{n(n+1)}{2}$$

$$S = \frac{1}{2}\left(\sum_{n=1}^{15} n^2 + \sum_{n=1}^{15} n\right) = \frac{1}{2}\left(\frac{15(16)(31)}{6} + \frac{15(16)}{2}\right)$$

$= 680$

$\Rightarrow$ $680 - \dfrac{1}{2}\dfrac{15(16)}{2} = 680 - 60 = 620$

7. The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2 + \dfrac{1}{x^3}\right)^n$ is ${}^nC_{23}$, is :

(1) 58 (2) 35

(3) 38 (4) 23

Answer (3)

Sol. $\left(x^2+\frac{1}{x^3}\right)^n$

General term $T_{r+1} = {}^nC_r\left(x^2\right)^{n-r}\left(\frac{1}{x^3}\right)^r$

${}^nC_r \cdot x^{2n-5r}$

for coefficient of x, $2n - 5r = 1$

Given ${}^nC_r = {}^nC_{23}$

$r = 23$ or $n - r = 23$

$\Rightarrow$ $n = 58$ or $n = 38$

Minimum value is n = 38

8. If both the mean and the standard deviation of 50 observations $x_1, x_2, \ldots x_{50}$ are equal to 16, then the mean of $(x_1 - 4)^2, (x_2 - 4)^2, \ldots (x_{50} - 4)^2$ is :

(1) 380 (2) 480

(3) 400 (4) 525

Answer (3)

Sol. $\frac{x_1+x_2+\ldots x_{50}}{50}=16$

$16^2=\frac{x_1^2+x_2^2\ldots x_{50}^2}{50}-16^2$

$2(16)^2\,50=x_1^2+x_2^2+\ldots x_{50}^2$

Required mean = $\frac{(x_1-4)^2+(x_2-4)^2+\ldots(x_{50}-4)^2}{50}$

$=\frac{16^2(100)+4^2(50)-8(16\times 50)}{50}$

$= 16^2(2) + 16 - 8(16) = 400$

9. If $\int x^5 e^{-x^2}dx = g(x)e^{-x^2}+c$, where c is a constant of integration, then g(–1) is equal to :

(1) –1 (2) $-\frac{1}{2}$

(3) 1 (4) $-\frac{5}{2}$

Answer (4)

Sol. $I=\int x^5\cdot e^{-x^2}dx$

Put $-x^2 = t$ $\Rightarrow -2xdx = dt$

$I=\int\frac{t^2\cdot e^t dt}{(-2)}=\frac{-1}{2}e^t\left(t^2-2t+2\right)+c$

$\therefore$ $g(x)=\frac{-1}{2}\left(x^4+2x^2+2\right)$

$g(-1)=\frac{-5}{2}$

10. Let a, b and c be in G.P. with common ratio r, where $a \neq 0$ and $0<r\le\frac{1}{2}$. If 3a, 7b and 15c are the first three terms of an A.P., then the 4th term of this A.P. is :

(1) $\frac{2}{3}a$ (2) a

(3) $\frac{7}{3}a$ (4) 5a

Answer (2)

Sol. Let $b = ar$, $c = ar^2$

AP : $3a, 7ar, 15ar^2$

$14ar = 3a + 15ar^2$

$\Rightarrow$ $15r^2 - 14r + 3 = 0$

$\Rightarrow$ $r=\frac{1}{3}\text{ or }\frac{3}{5}\text{(rejected)}$

Fourth term = $15ar^2 + 7ar - 3a$

$= a(15r^2 + 7r - 3)$

$= a\left(\frac{15}{9}+\frac{7}{3}-3\right)$

= a

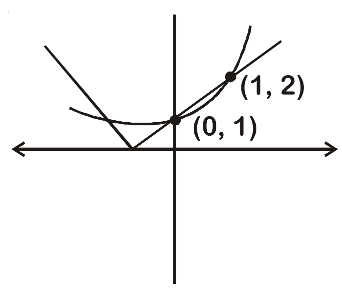
11. The area (in sq. units) of the region bounded by the curves $y = 2^x$ and $y = |x + 1|$, in the first quadrant is :

(1) $\frac{3}{2}-\frac{1}{\log_e 2}$ (2) $\frac{1}{2}$

(3) $\log_e 2+\frac{3}{2}$ (4) $\frac{3}{2}$

Answer (1)

Sol.



Area $=\int_0^1\left((x+1)-2^x\right)dx$

$=\left[\frac{x^2}{2}+x-\frac{2^x}{\ln 2}\right]_0^1$

$=\left(\frac{1}{2}+1-\frac{2}{\ln 2}\right)-\left(\frac{-1}{\ln 2}\right)$

$=\frac{3}{2}-\frac{1}{\ln 2}$

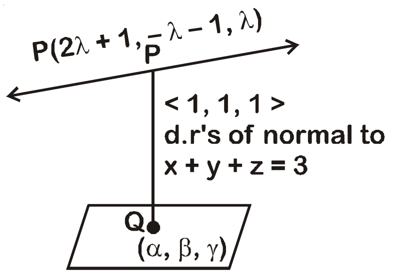
12. A perpendicular is drawn from a point on the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{1}$ to the plane x + y + z = 3 such that the foot of the perpendicular Q also lies on the plane x – y + z = 3. Then the co-ordinates of Q are :

(1) (1, 0, 2) (2) (2, 0, 1)

(3) (4, 0, –1) (4) (–1, 0, 4)

Answer (2)

Sol.



Let Q be (α, β, γ)

$\alpha + \beta + \gamma = 3$ ...(i)

$\alpha - \beta + \gamma = 3$ ...(ii)

$\therefore\ \alpha + \gamma = 3$ and $\beta = 0$

Equating DR's of PQ :

$$\frac{\alpha-2\lambda-1}{1}=\frac{\lambda+1}{1}=\frac{\gamma-\lambda}{1}$$

$\Rightarrow\ \alpha = 3\lambda + 2,\ \gamma = 2\lambda + 1$

Substituting in equation (i) we get

$\Rightarrow\ 5\lambda + 3 = 3$

$\lambda = 0$

Point is Q(2, 0, 1)

13. If the tangent to the curve $y=\frac{x}{x^2-3}$, $x\in R$, $(x\neq\pm\sqrt{3})$, at a point $(\alpha, \beta) \neq (0, 0)$ on it is parallel to the line 2x + 6y – 11 = 0, then :

(1) $|6\alpha + 2\beta| = 19$ (2) $|2\alpha + 6\beta| = 19$

(3) $|6\alpha + 2\beta| = 9$ (4) $|2\alpha + 6\beta| = 11$

Answer (1)

Sol. $y=\frac{x}{x^2-3}$

$$\frac{dy}{dx}=\frac{(x^2-3)-x(2x)}{(x^2-3)^2}=\frac{-x^2-3}{(x^2-3)^2}$$

$$\left.\frac{dy}{dx}\right|_{(\alpha,\beta)}=\frac{-\alpha^2-3}{(\alpha^2-3)^2}=-\frac{1}{3}$$

$3(\alpha^2 + 3) = (\alpha^2 - 3)^2$ ...(i)

i.e. $\alpha^2 = 9$

Also, $\beta=\frac{\alpha}{\alpha^2-3}\Rightarrow\alpha^2-3=\frac{\alpha}{\beta}\Rightarrow\frac{\alpha}{\beta}=6$

$\Rightarrow\ \alpha=\pm3,\ \beta=\pm\frac{1}{2}$

Which satisfies $|6\alpha + 2\beta| = 19$

14. Let y = y(x) be the solution of the differential equation, $\frac{dy}{dx}+y\tan x=2x+x^2\tan x, x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, such that y(0) = 1.Then :

(1) $y\left(\frac{\pi}{4}\right)+y\left(-\frac{\pi}{4}\right)=\frac{\pi^2}{2}+2$

(2) $y\left(\frac{\pi}{4}\right)-y\left(-\frac{\pi}{4}\right)=\sqrt{2}$

(3) $y'\left(\frac{\pi}{4}\right)+y'\left(-\frac{\pi}{4}\right)=-\sqrt{2}$

(4) $y'\left(\frac{\pi}{4}\right)-y'\left(-\frac{\pi}{4}\right)=\pi-\sqrt{2}$

Answer (4)

Sol. $\frac{dy}{dx}+y\tan x=2x+x^2\tan x$

P = tanx, Q = 2x + x^2tan x

$$\text{I.F.} = e^{\int\tan x\,dx}=e^{\ln|\sec x|}=|\sec x|$$

$$y(\sec x)=\int(2x+x^2\tan x)\sec x\,dx$$

$$=\int x^2\tan x\sec x\,dx+\int 2x\sec x\,dx$$

$$=x^2\sec x-\int 2x\sec x\,dx+\int 2x\sec x\,dx$$

$= x^2\sec x + c$

As y(0) = 1, c = 1

$\therefore\ y = x^2 + \cos x$

At $x=\frac{\pi}{4}$, $y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{16}+\frac{1}{\sqrt{2}}$

$$y\left(-\frac{\pi}{4}\right)=\frac{\pi^2}{16}+\frac{1}{\sqrt{2}}$$

$$y\left(\frac{\pi}{4}\right)-y\left(-\frac{\pi}{4}\right)=0$$

$$\frac{dy}{dx}=2x-\sin x$$

$$y'\left(\frac{\pi}{4}\right)=\frac{\pi}{2}-\frac{1}{\sqrt{2}},\ y'\left(-\frac{\pi}{4}\right)=-\frac{\pi}{2}+\frac{1}{\sqrt{2}}$$

$$y'\left(\frac{\pi}{4}\right)-y'\left(-\frac{\pi}{4}\right)=\pi-\sqrt{2}$$

15. Let a_1, a_2, a_3, ... be an A.P. with $a_6 = 2$. Then the common difference of this A.P., which maximises the product $a_1 a_4 a_5$, is :

(1) $\frac{2}{3}$ (2) $\frac{8}{5}$

(3) $\frac{3}{2}$ (4) $\frac{6}{5}$

Answer (1)

Sol. $a + 5d = 2$

Let $A = a_1 a_4 a_5 = a(a + 3d)(a + 4d)$

$= a(2 - 2d)(2 - d)$

$A = (2 - 5d)(4 - 6d + 2d^2)$

$\frac{dA}{dd} = 0$

$(2 - 5d)(-6 + 4d) + (4 - 6d + 2d^2)(-5) = 0$

$\Rightarrow 15d^2 - 34d + 16 = 0$

$d = \frac{8}{5}, \frac{2}{3}$

For $d = \frac{2}{3}, \frac{d^2A}{dd^2} < 0$

Hence $d = \frac{2}{3}$

16. If the plane $2x - y + 2z + 3 = 0$ has the distances $\frac{1}{3}$ and $\frac{2}{3}$ units from the planes $4x - 2y + 4z + \lambda = 0$ and $2x - y + 2z + \mu = 0$, respectively, then the maximum value of $\lambda + \mu$ is equal to :

(1) 13 (2) 15

(3) 5 (4) 9

Answer (1)

Sol. $P_1 : 2x - y + 2z + 3 = 0$

$P_2 : 2x - y + 2z + \frac{\lambda}{2} = 0$

$P_3 : 2x - y + 2z + \mu = 0$

Given $\frac{1}{3} = \frac{\left|3 - \frac{\lambda}{2}\right|}{\sqrt{9}} \Rightarrow \left|3 - \frac{\lambda}{2}\right| = 1$

$\lambda_{max} = 8$

Also, $\frac{2}{3} = \frac{|\mu - 3|}{\sqrt{9}} \Rightarrow \mu_{max} = 5$

$(\lambda + \mu)_{max} = 13$

17. If $\lim_{x\to 1} \frac{x^2 - ax + b}{x - 1} = 5$, then a + b is equal to:

(1) 5 (2) –4

(3) 1 (4) –7

Answer (4)

Sol. $\lim_{x\to 1} \frac{x^2 - ax + b}{x - 1} = 5$

As limit is finite, $1 - a + b = 0$

$\Rightarrow \lim_{x\to 1} \frac{2x - a}{1} = 5 \quad \left(\frac{0}{0} \text{ form}\right)$

i.e., $2 - a = 5$

or $a = -3$

$\therefore$ $b = -4$

$a + b = -3 - 4 = -7$

18. The number of real roots of the equation $5 + |2^x - 1| = 2^x(2^x - 2)$ is :

(1) 4 (2) 2

(3) 1 (4) 3

Answer (3)

Sol. Let $2^x - 1 = t$

$5 + |t| = (t + 1)(t - 1)$

$\Rightarrow |t| = t^2 - 6$

For $t > 0$, $t^2 - t - 6 = 0$

i.e., $t = 3$ or -2 (rejected)

For $t < 0$, $t^2 + t - 6 = 0$

i.e., $t = -3$ or 2 (both rejected)

$\therefore$ $2^x - 1 = 3$

$\Rightarrow x = 2$

19. Lines are drawn parallel to the line $4x - 3y + 2 = 0$, at a distance $\frac{3}{5}$ from the origin. Then which one of the following points lies on any of these lines?

(1) $\left(\frac{1}{4}, -\frac{1}{3}\right)$ (2) $\left(\frac{1}{4}, \frac{1}{3}\right)$

(3) $\left(-\frac{1}{4}, \frac{2}{3}\right)$ (4) $\left(-\frac{1}{4}, -\frac{2}{3}\right)$

Answer (3)

Sol. Let straight line be $4x - 3y + \alpha = 0$

Given $\frac{3}{5} = \left|\frac{\alpha}{5}\right|$

$\Rightarrow \alpha = \pm 3$

Line is $4x - 3y + 3 = 0$ or $4x - 3y - 3 = 0$

Clearly $\left(-\frac{1}{4}, \frac{2}{3}\right)$ satisfies $4x - 3y + 3 = 0$

20. Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number of beams is :

(1) 210 (2) 180

(3) 170 (4) 190

Answer (3)

Sol. Required number of beams = ${}^{20}C_2 - 20$

$= 190 - 20 = 170$

21. A spherical iron ball of radius 10 cm is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of the ice is 5 cm, then the rate at which the thickness (in cm/min) of the ice decreases, is :

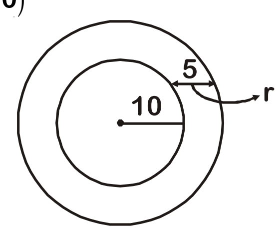
(1) $\frac{5}{6\pi}$ (2) $\frac{1}{36\pi}$

(3) $\frac{1}{9\pi}$ (4) $\frac{1}{18\pi}$

Answer (4)

Sol. $\frac{dV_{ice}}{dt} = 50$

$V_{ice} = \frac{4}{3}\pi(10+r)^3 - \frac{4}{3}\pi(10)^3$



$\frac{dV}{dt} = \frac{4}{3}\pi\, 3(10+r)^2 \frac{dr}{dt}$

$= 4\pi(10+r)^2 \frac{dr}{dt}$

At $r = 5, 50 = 4\pi(225)\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{50}{4\pi(225)}$

$= \frac{1}{18\pi}$ cm/min

22. Minimum number of times a fair coin must be tossed so that the probability of getting at least one head is more than 99% is :

(1) 8 (2) 6

(3) 5 (4) 7

Answer (4)

Sol. $1 - \left(\frac{1}{2}\right)^n > \frac{99}{100}$

$\left(\frac{1}{2}\right)^n < \frac{1}{100}$

$\therefore \quad n \geq 7$

Minimum value is 7.

23. If $5x + 9 = 0$ is the directrix of the hyperbola $16x^2 - 9y^2 = 144$, then its corresponding focus is :

(1) $\left(\frac{5}{3}, 0\right)$ (2) $\left(-\frac{5}{3}, 0\right)$

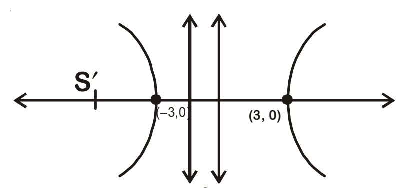
(3) $(-5, 0)$ (4) $(5, 0)$

Answer (3)

Sol. $16x^2 - 9y^2 = 144$

i. e. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

Focus $S'(-ae, 0)$



$x = \frac{-9}{5}$

$a = 3, b = 4$

$e^2 = 1 + \frac{16}{9} = \frac{25}{9}$

$S' \equiv \left(-3 \times \frac{5}{3}, 0\right) \equiv (-5, 0)$

24. Let λ be a real number for which the system of linear equations

$x + y + z = 6$

$4x + \lambda y - \lambda z = \lambda - 2$

$3x + 2y - 4z = -5$

has infinitely many solutions. Then λ is a root of the quadratic equation :

(1) $\lambda^2 + 3\lambda - 4 = 0$ (2) $\lambda^2 - \lambda - 6 = 0$

(3) $\lambda^2 + \lambda - 6 = 0$ (4) $\lambda^2 - 3\lambda - 4 = 0$

Answer (2)

Sol. $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 4 & \lambda & -\lambda \\ 3 & 2 & -4 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ 4-\lambda & 2\lambda & -\lambda \\ 1 & 6 & -4 \end{vmatrix} = 0 \quad \Rightarrow \quad \lambda = 3$$

$$\Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ \lambda-2 & \lambda & -\lambda \\ -5 & 2 & -4 \end{vmatrix} = 0 \quad \text{for } \lambda = 3$$

$$\Delta_2 = \begin{vmatrix} 1 & 6 & 1 \\ 4 & \lambda-2 & -\lambda \\ 3 & -5 & -4 \end{vmatrix} = 0 \quad \text{for } \lambda = 3$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 4 & \lambda & \lambda-2 \\ 3 & 2 & -5 \end{vmatrix} = 0 \quad \text{for } \lambda = 3$$

$\therefore$ For $\lambda = 3$, infinitely many solutions is obtained.

25. The integral $\int_{\pi/6}^{\pi/3} \sec^{2/3} x \operatorname{cosec}^{4/3} x \, dx$ is equal to :

(1) $3^{7/6} - 3^{5/6}$ (2) $3^{5/3} - 3^{1/3}$

(3) $3^{5/6} - 3^{2/3}$ (4) $3^{4/3} - 3^{1/3}$

Answer (1)

Sol. $I = \int_{\pi/6}^{\pi/3} \sec^{2/3} x . \operatorname{cosec}^{4/3} x \, dx$

$$= \int_{\pi/6}^{\pi/3} \frac{1.dx}{\cos^{2/3} x . \sin^{4/3} x}$$

$$= \int_{\pi/6}^{\pi/3} \frac{1 \quad dx}{\cos^2 x . \tan^{4/3} x} = \int_{\pi/6}^{\pi/3} \frac{\sec^2 x dx}{\tan^{4/3} x}$$

Let tan x = t

$$I = \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} t^{-4/3} dt = \frac{3\left[t^{-1/3}\right]_{1/\sqrt{3}}^{\sqrt{3}}}{-1}$$

$$= -3\left[3^{-1/6} - \frac{1}{3^{-1/6}}\right]$$

$$= -3(3^{-1/6} - 3^{-1/6})$$

$$= 3(3^{1/6} - 3^{-1/6})$$

$$= 3^{7/6} - 3^{5/6}$$

26. If the line ax + y = c, touches both the curves $x^2 + y^2 = 1$ and $y^2 = 4\sqrt{2}x$, then $|c|$ is equal to :

(1) $\frac{1}{\sqrt{2}}$

(2) $\frac{1}{2}$

(3) 2

(4) $\sqrt{2}$

Answer (4)

Sol. Tangent on $y^2 = 4\sqrt{2}x$ is $yt = x + \sqrt{2}t^2$

As it is tangent on circle also,

$$\left|\frac{\sqrt{2}t^2}{\sqrt{1+t^2}}\right| = 1$$

$2t^4 = 1 + t^2$ i.e. $t^2 = 1$

Equation is $\pm y = x + \sqrt{2}$

Hence $|c| = \sqrt{2}$

27. The tangent and normal to the ellipse $3x^2 + 5y^2 = 32$ at the point P(2, 2) meet the x-axis at Q and R, respectively. Then the area (in sq. units) of the triangle PQR is :

(1) $\frac{16}{3}$

(2) $\frac{14}{3}$

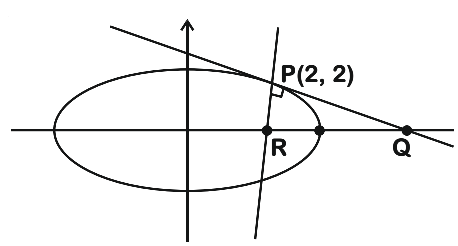
(3) $\frac{34}{15}$

(4) $\frac{68}{15}$

Answer (4)

Sol. For $\frac{3x^2}{32} + \frac{5y^2}{32} = 1$

Tangent at P is



$$\frac{3(2)x}{32} + \frac{5(2)y}{32} = 1$$

$$\frac{3x}{16}+\frac{5y}{16}=1$$

$$Q\equiv\left(\frac{16}{3},0\right)$$

Normal at P is $\frac{32x}{3(2)}-\frac{32y}{5(2)}=\frac{32}{3}-\frac{32}{5}$

$$R\equiv\left(\frac{4}{5},0\right)$$

area of ΔPQR = $\frac{1}{2}$ (PQ) (PR) = $\frac{1}{2}\sqrt{\frac{136}{3}}.\sqrt{\frac{136}{5}}$

$= \frac{68}{15}$

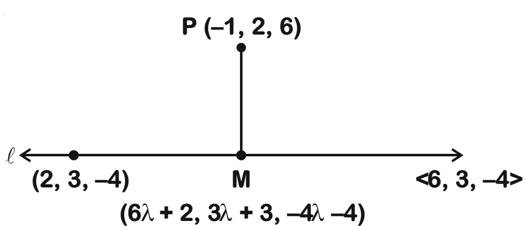
28. The distance of the point having position vector $-\hat{i}+2\hat{j}+6\hat{k}$ from the straight line passing through the point (2, 3, –4) and parallel to the vector, $6\hat{i}+3\hat{j}-4\hat{k}$ is :

(1) 7 (2) $4\sqrt{3}$

(3) 6 (4) $2\sqrt{13}$

Answer (1)

Sol. Equation of l is $\frac{x-2}{6}=\frac{y-3}{3}=\frac{z+4}{-4}$



Let M $(6\lambda + 2, 3\lambda + 3, -4\lambda - 4)$

DR's of PM is $< 6\lambda + 3, 3\lambda + 1, -4\lambda - 10 >$

$\Rightarrow$ $(6\lambda + 3)(6) + (3\lambda + 1)(3) + (-4\lambda - 10)(-4) = 0$

$\Rightarrow$ $\lambda = -1$

i.e. $M \equiv (-4, 0, 0)$

$\therefore$ PM = $\sqrt{9+4+36}=7$

29. If $\cos^{-1}x-\cos^{-1}\frac{y}{2}=\alpha$, where $-1\le x\le 1$, $-2\le y\le 2$, $x\le\frac{y}{2}$, then for all x, y, $4x^2-4xy\cos\alpha+y^2$ is equal to :

(1) $2\sin^2\alpha$

(2) $4\sin^2\alpha - 2x^2y^2$

(3) $4\cos^2\alpha + 2x^2y^2$

(4) $4\sin^2\alpha$

Answer (4)

Sol. $\cos^{-1}x - \cos^{-1}\frac{y}{2}=\alpha$

$$\Rightarrow \cos^{-1}\left(\frac{xy}{2}+\sqrt{1-x^2}.\sqrt{1-\frac{y^2}{4}}\right)=\alpha$$

$$\Rightarrow \frac{xy}{2}+\frac{\sqrt{1-x^2}\sqrt{4-y^2}}{2}=\cos\alpha$$

$$\Rightarrow xy + \sqrt{1-x^2}\sqrt{4-y^2}=2\cos\alpha$$

$(xy - 2\cos\alpha)^2 = (1 - x^2)(4 - y^2)$

$x^2y^2 + 4\cos^2\alpha - 4xy\cos\alpha = 4 - y^2 - 4x^2 + x^2y^2$

$4x^2 - 4xy\cos\alpha + y^2 = 4\sin^2\alpha$

30. The negation of the Boolean expression $\sim s \vee (\sim r \wedge s)$ is equivalent to :

(1) $s \wedge r$

(2) r

(3) $\sim s \wedge \sim r$

(4) $s \vee r$

Answer (1)

Sol. $\sim s \vee (\sim r \wedge s)$

$\equiv (\sim s \vee \sim r) \wedge (\sim s \vee s)$

$\equiv (\sim s \vee \sim r)$ ($\because$ $(\sim s \vee s)$ is tautology)

$\equiv \sim(s \wedge r)$

Hence its negation is $s \wedge r$

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