## Detailed Analysis of GATE 2018 Paper

GATE ME Solved 2018 Paper (Set I) Detailed Analysis

| Subject | $\mathbf{1}$ Mark <br> Questions | $\mathbf{2 M a r k}$ <br> Questions | Total Marks |
| :--- | :---: | :---: | :---: |
| General Aptitude | 5 | 5 | 15 |
| Engineering Maths | 5 | 3 | 11 |
| Engineering Mechanics | 1 | 3 | 7 |
| Strength of Materials | 3 | 2 | 7 |
| Theory of Machines | 3 | 4 | 11 |
| Machine Design | 0 | 2 | 4 |
| Fluid Mechanics | 3 | 4 | 11 |
| Heat Transfer | 0 | 1 | 2 |
| Thermodynamics | 2 | 3 | 8 |
| Refrigeration and Air-Conditioning | 0 | 0 | 0 |
| Manufacturing and Industrial Engineering | 8 | 8 | 24 |
| Total Marks |  |  | 100 |

GATE ME Solved 2018 Paper (Set 2) Detailed Analysis

| Subject | 1 Mark <br> Questions | 2 Mark <br> Questions | Total Marks |
| :--- | :---: | :---: | :---: |$|$| General Aptitude | 5 | 5 |
| :--- | :---: | :---: |
| Engineering Maths | 5 | 4 |
| Engineering Mechanics | 1 | 1 |
| Strength of Materials | 2 | 2 |
| Theory of Machines | 2 | 5 |
| Machine Design | 2 | 0 |
| Fluid Mechanics | 1 | 3 |
| Heat Transfer | 1 | 2 |
| Thermodynamics | 3 | 2 |
| Refrigeration and Air-Conditioning | 0 | 2 |
| Manufacturing and Industrial Engineering | 8 | 9 |
| Total Marks |  |  |

# GATE 2018 Solved Paper ME: Mechanical Engineering Set - I 

Number of Questions: 65
Total Marks:100.0

Wrong answer for MCQ will result in negative marks, (-1/3) for 1 Mark Questions and (-2/3) for 2 Marks Questions.

## General Aptitude

## Number of Questions: 10

Section Marks: 15.0

## Q. 1 to Q. 5 carry 1 mark each and Q. 6 to Q. 10 carry 2 marks each.

## Question Number: 1 <br> Question Type: MCQ

A rectangle becomes a square when its length and breadth are reduced by 10 m and 5 m , respectively. During this process, the rectangle loses $650 \mathrm{~m}^{2}$ of area. What is the area of the original rectangle in square meters?
(A) 1125
(B) 2250
(C) 2924
(D) 4500

Solution: Consider the side of the final square be $x$. The dimensions of the rectangle will be $x+5, x+10$.
Reduction in area $=15 x+50=650$

$$
\begin{array}{rlrl} 
& & 15 x & =600 \\
\therefore & x & =40
\end{array}
$$

The area of the rectangle

$$
\begin{aligned}
& =(x+5)(x+10) \mathrm{m}^{2}=45(50) \mathrm{m}^{2} \\
& =2250 \mathrm{~m}^{2}
\end{aligned}
$$

Hence, the correct option is (B)
Question Number: 2
Question Type: MCQ
A number consists of two digits. The sum of the digits is 9 . If 45 is subtracted from the number, its digits are interchanged, What is the number?
(A) 63
(B) 72
(C) 81
(D) 90

Solution: If we assume the number to be ' $a b$ ', where $a$ is tens digit and $b$ is unit digit.
The value of the number of $10 a+b$

$$
\begin{equation*}
a+b=9 \tag{1}
\end{equation*}
$$

and

$$
\begin{array}{rlrl} 
& & (10+b)-45 & =(10 b+a) \\
\Rightarrow & 9(a-b) & =45 \\
\Rightarrow & a-b & =5 \tag{3}
\end{array}
$$

Solving (1), (3) we get

$$
a=7, b=2
$$

Thus the two digit number is 72 .
Hence, the correct option is (B)
Question Number: 3
Question Type: MCQ
"Going by the $\qquad$ that many hands make light work, the school $\qquad$ involved all the students in the task."
The words that best fill the blanks in the above sentence are
(A) principle, principal
(B) principal, principle
(C) principle, principle
(D) principal, principal

## Solution:

The correct option is (A)

## Question Number: 4

Question Type: MCQ
"Her $\qquad$ should not be confused with miserliness; she is ever willing to assist those in need."
(A) cleanliness
(B) punctuality
(C) frugality
(D) greatness

Solution: (C)
Question Number: 5
Question Type: MCQ
Seven machines take 7 minutes to make 7 identical toys. At the same rate, how many minutes would it take for 100 machines to make 100 toys?
(A) 1
(B) 7
(C) 100
(D) 700

Solution: 100 machines will also take seven minutes to make 100 identical toys.
Hence, the correct option is (B)
Question Number: 6
Question Type: MCQ
Which of the following functions describe the graph shown in the below figure?

(A) $y=||x|+1|-2$
(B) $y=||x|-1|-1$
(C) $y=||x|+1|-1$
(D) $y=||x-1|-1$

Solution: taking $x=0$ in the choices,
we get $-1,0,0,0$.
From the graph, $f(0)=0$
Thus option A is not correct.
We know that D can have only non negative values, while from the graph $y$ can be negative. Therefor option D is also not correct.
Setting $x=1$ in option C,
We get $-1,1$. The graph shows that

$$
f(1)=-1
$$

Therefore option C is also incorrect.
Thus we conclude that option B is correct.
Hence, the correct option is (B)

## Question Number: 7

Question Type: MCQ
Consider the following three statements:
(i) Some roses are red.
(ii) All red flowers fade quickly
(iii) Some roses fade quickly

Which of the following statements can be logically inferred from the above statements?
(A) If (i) is true and (ii) is false, then (iii) is false.
(B) If (i) is true and (ii) is false, then (iii) is true.
(C) If (i) and (ii) are true, then (iii) is true.
(D) If (i) and (ii) are false, then (iii) is false.

## Solution:

Hence, the correct option is (C)

Question Number: 8
Question Type: MCQ
For integers $a, b$ and $c$, what would be the minimum and maximum values respectively of $a+b+c$ if $\log |a|+\log$ $|b|+\log |c|=0$ ?
(A) -3 and 3
(B) -1 and 1
(C) -1 and 3
(D) 1 and 3

Solution: We know that

$$
\begin{aligned}
\log |a|+\log |b|+\log |c| & =0 \\
\log |a||b||c| & =0 \\
|a b c| & =1
\end{aligned}
$$

The maximum value of

$$
a+b+c=1+1+1=3
$$

The minimum value of

$$
a+b+c=(-1)+(-1)+(-1)=-3
$$

Hence, the correct option is (A)
Question Number: 9
Question Type: MCQ
Given that $a$ and $b$ are integers and $a+a^{2} b^{2}$ is odd, which one of the following statements is correct?
(A) $a$ and $b$ are both odd
(B) $a$ and $b$ are both even
(C) $a$ is even and $b$ is odd
(D) $a$ is odd and $b$ is even

Solution: As per question $a+a^{2} b^{2}$ is odd. Thus one of the terms is even and the other is odd. If a is even, both would be even. Hence, $b$ is even and $a$ is odd.
Hence, the correct option is (D)
Question Number: 10
Question Type: MCQ
From the time the front of a train enters a platform, it takes 25 seconds for the back of the train to leave the platform, while travelling at a constant speed of $54 \mathrm{~km} / \mathrm{h}$. At the same speed, it takes 14 seconds to pass a man running at $9 \mathrm{~km} / \mathrm{h}$ in the same direction as the train. What is the length of the train and that of the platform in meters, respectively?
(A) 210 and 140
(B) 162.5 and 187.5
(C) 245 and 130
(D) 175 and 200

Solution: Let length of the train be $L$, and length of the platform be $P$. Then we have

$$
\begin{aligned}
& \text { Speed }=54 \mathrm{~km} / \mathrm{hr}=15 \mathrm{~m} / \mathrm{s}, \\
& 9 \mathrm{~km} / \mathrm{hr}=2.5 \mathrm{~m} / \mathrm{s} \\
& \therefore \quad P+L=15(25) \mathrm{m}=375 \mathrm{~m} \\
& \text { and } \\
& L=(12.5)(14) \mathrm{m}=175 \mathrm{~m} \\
& \text { Hence } \quad P=200 \mathrm{~m} \\
& \text { Hence, the correct option is (D) }
\end{aligned}
$$

# Mechanical Engineering 

## Number of Questions: 55

Q. 11 to Q. 25 carry 1 mark each and Q. 26 to Q. 65 carry 2 marks each.

## Question Number: 11

Question Type: MCQ
In a linearly hardening plastic material, the true stress beyond initial yielding
(A) increases linearly with the true strain
(B) decreases linearly with the true strain
(C) first increases linearly and then decreases linearly with the true strain
(D) remains constant

Solution:


In stress strain curve given above for linearly hardening plastic material the true stress above initial yielding increases linearly with true strain.
Hence, the correct option is (A).
Question Number: 12
Question Type: MCQ
The type of weld represented by the shaded region in the figure is

(A) groove
(B) spot
(C) fillet
(D) plug

Solution:


The shaded region shown in above figure represents fillet. Hence, the correct option is (C)

## Section Marks: 85.0

Question Number: 13
Question Type: MCQ
Using the Taylor's tool life equation with exponent $n=0.5$, if the cutting speed is reduced by $50 \%$, the ration of new tool life to original tool life is
(A) 4
(B) 2
(C) 1
(D) 0.5

Solution: As we know that

$$
V_{1} T_{1}^{n}=V_{2} T_{2}^{n}
$$

Here $n=0.5$, then we have

$$
V_{1} T_{1}^{0.5}=V_{2} T_{2}^{0.5}
$$

Simplifying the above equation we get

$$
\begin{gathered}
\left(\frac{T_{2}}{T_{1}}\right)^{0.5}=\frac{V_{1}}{V_{2}}=\frac{V_{1}}{\frac{V_{1}}{2}}=2 \\
\frac{T_{2}}{T_{1}}=2^{1 / .05}=4
\end{gathered}
$$

Hence, the correct option is (A)
Question Number: 14

## Question Type: MCQ

A grinding ratio of 200 implies that the
(A) grinding wheel wears 200 times the volume of the material removed
(B) grinding wheel wears 0.005 times the volume of the material removed
(C) aspect ratio of abrasive particles used in the grinding wheel is 200
(D) ratio of volume of abrasive particle to that of grinding wheel is 200

Solution: Grinding ratio =
$\frac{\text { Volume of work material removed }\left(V_{\mathrm{m}}\right)}{\text { Volume of wheel wear }\left(V_{\mathrm{W}}\right)}$
$\frac{V_{\mathrm{m}}}{V_{\mathrm{m}}}=200$

$$
V_{\mathrm{w}}=\frac{V_{\mathrm{m}}}{200}=0.005 V_{\mathrm{m}}
$$

Hence, the correct option is (B)
Question Number: 15
Question Type: MCQ
Interpolator in CNC machine
(A) controls spindle speed
(B) coordinates axes movements
(C) operates tool changer
(D) commands canned cycle

Solution: Interpolar provides coordinates axes movements.
Hence, the correct option is (B).
Question Number: 16
Question Type: MCQ
The time series forecasting method that gives equal weightage to each of the most recent observations is
(A) Moving average method
(B) Exponential smoothing with linear trend
(C) Triple Exponential smoothing
(D) Kalman Filter

Solution: The time series forecasting method that gives equal weightage to each of the most recent observations is Moving average method.
Hence, the correct option is (A)
Question Number: 17
Question Type: MCQ
The number of atoms per unit cell and the number of slip systems, respectively, for a face centered cubic (FCC) crystal are
(A) 3,3
(B) 3,12
(C) 4,12
(D) 4,48

Solution: We know that for FCC structure, number of atoms per unit cell $=4$
Therefore, number of slip systems $=12$.
Hence, the correct option is (C)
Question Number: 18
Question Type: MCQ
A six-faced fair dice is rolled five times. The probability (in \%) of obtaining "ONE" at least four times is
(A) 33.3
(B) 3.33
(C) 0.33
(D) 0.0033

Solution:
Number of ways of getting "ONE" exactly four times when a fair dice is rolled five times

$$
={ }^{5} \mathrm{C}_{4} \times 5=5 \times 5=25
$$

Number of ways of getting "ONE" exactly five times $=1$ Number of ways of getting 'ONE' at least four times

$$
=25+1=26
$$

Probability (in \%) of obtaining 'ONE' at least four times

$$
=\frac{26}{6^{5}} \times 100=0.33
$$

Hence, the correct option is (C)
Question Number: 19
Question Type: NAT
A steel column of rectangular section ( $15 \mathrm{~mm} \times 10 \mathrm{~mm}$ ) and length 1.5 m is simply supported at both ends. Assuming modulus of elasticity, $E=200 \mathrm{GPa}$ for steel, the critical axial load (in kN ) is $\qquad$ (correct to two decimal places).

## Solution:

Length of steel column $L=1.5 \mathrm{~m}=1500 \mathrm{~mm}$
We know that for simply supported at both ends,

$$
\text { Load } P=\frac{\pi^{2} E I_{\min }}{L^{2}}
$$

Substituting the given values in above equation, we get

$$
\begin{aligned}
& P=\frac{\pi^{2} \times 200 \times 10^{3} \times \frac{15 \times 10^{3}}{12}}{1500^{2}} \\
& P=1096.62 \mathrm{~N}=1.096 \mathrm{kN}
\end{aligned}
$$

Hence, the correct answer is 1.096 .
Question Number: 20
Question Type: NAT
A four bar mechanism is made up of links of length 100, 200, 300 and 350 mm . If the 350 mm link is fixed, the number of links that can rotate fully is $\qquad$

## Solution:



$$
\begin{aligned}
& \text { Length } s=100 \mathrm{~mm} \\
& \text { Length } p=200 \mathrm{~mm} \\
& \text { Length } \ell=350 \mathrm{~mm}, \\
& \text { Length } q=300 \mathrm{~mm}
\end{aligned}
$$

We know that as per Grashoff's law

$$
\begin{aligned}
s+\ell & <p+q \\
100+450 & <200+300 \\
450 & <500
\end{aligned}
$$

Therefore, Grashoff's law is satisfied. The link adjacent to fixed link ( $s=100 \mathrm{~mm}$ ) is a crank which can rotate fully.
Hence, the correct answer is 1 .
Question Number: 21
Question Type: MCQ
Four red balls, four green balls and four blur balls are put in a box. Three balls are pulled out of the box at random one after another without replacement. The probability that all the three balls are red is
(A) $1 / 72$
(B) $1 / 55$
(C) $1 / 36$
(D) $1 / 27$

Solution: Probability

$$
\mathrm{red}=\frac{4 \times 3 \times 2}{12 \times 11 \times 10}=\frac{1}{55}
$$

Hence, the correct option is (B)
Question Number: 22
Question Type: MCQ
The rank of the matrix $\left[\begin{array}{ccc}-4 & 1 & -1 \\ -1 & -1 & -1 \\ 7 & -3 & 1\end{array}\right]$ is
(A) 1
(B) 2
(C) 3
(D) 4

Solution: Let $A=\left[\begin{array}{ccc}-4 & 1 & -1 \\ -1 & -1 & -1 \\ 7 & -3 & 1\end{array}\right]$

$$
\begin{aligned}
\operatorname{Det}(A) & =-4(-1-3)-1(-1+7)-1(3+7) \\
& =16-6-10 \\
& =0
\end{aligned}
$$

So, $\rho(A)<3$
And $\operatorname{Det}\left[\begin{array}{cc}-4 & 1 \\ -1 & -1\end{array}\right]=5 \neq 0$
$\therefore$ The rank of $A=2$.
Hence, the correct option is (B)

## Question Number: 23

Question Type: MCQ
According to the Mean Value Theorem, for a continuous function $f(x)$ in the interval $[a, b]$, there exists a value $\xi$ in this interval such that $\int_{a}^{b} f(x) d x=$ ?
(A) $f(\xi)(b-a)$
(B) $f(b)(\xi-a)$
(C) $f(a)(b-\xi)$
(D) 0

Solution: We know that as per mean value theorem,

$$
\int_{a}^{b} f(x) d x=f(\xi)(b-a)
$$

Hence, the correct option is (A)
Question Number: 24
Question Type: MCQ
$F(z)$ is a function of the complex variable $z=x+i y$ given by

$$
F(z)=i z+k \operatorname{Re}(z)+\operatorname{Im}(z)
$$

For what value of $k$ will $F(z)$ satisfy the Cauchy-Riemann equations?
(A) 0
(B) 1
(C) -1
(D) $y$

## Solution:

$$
\begin{aligned}
F(z) & =i z+k \operatorname{Re}(z)+i \operatorname{Im}(z) \\
& =i(x+i y)+k x+i y \\
& =i x-y+k x+i y \\
F(z) & =(k x-y)+i(x+y) \\
u & =k x-y \text { and } v=x+y \\
\frac{\partial u}{\partial x} & =k ; \frac{\partial u}{\partial y}=-1 ; \frac{\partial v}{\partial x}=1 \text { and } \frac{\partial v}{\partial y}=1
\end{aligned}
$$

As per Cauchy-Riemann equations,

$$
\Rightarrow \quad \begin{aligned}
\frac{\partial u}{\partial x} & =\frac{\partial v}{\partial y} \\
\Rightarrow & k
\end{aligned}
$$

Hence, the correct option is (B)
Question Number: 25
Question Type: MCQ
A bar of uniform cross section and weighing 100 N is held horizontally using two massless and inextensible strings $S_{1}$ and $S_{2}$ as shown in the figure.


The tensions in the strings are
(A) $T_{1}=100 \mathrm{~N}$ and $T_{2}=0 \mathrm{~N}$
(B) $T_{1}=0 \mathrm{~N}$ and $T_{2}=100 \mathrm{~N}$
(C) $T_{1}=75 \mathrm{~N}$ and $T_{2}=25 \mathrm{~N}$
(D) $T_{1}=25 \mathrm{~N}$ and $T_{2}=75 \mathrm{~N}$

Sol. As per the given figure we get

$$
\begin{equation*}
T_{1}+T_{2}=100 \mathrm{~N} \tag{1}
\end{equation*}
$$

And

$$
T_{2} \times \frac{L}{2}=100 \times \frac{L}{2}
$$

Solving we get

$$
\begin{equation*}
T_{2}=100 \mathrm{~N} \tag{2}
\end{equation*}
$$

Substituting (2) in (1), we get

$$
T_{1}=0 \mathrm{~N} .
$$

Hence, the correct option is (A)
Question Number: 26
Question Type: MCQ
If $\sigma_{1}$ and $\sigma_{3}$ are the algebraically largest and smallest principal stresses respectively, the value of the maximum shear stress is
(A) $\frac{\sigma_{1}+\sigma_{3}}{2}$
(B) $\frac{\sigma_{1}-\sigma_{3}}{2}$
(C) $\sqrt{\frac{\sigma_{1}+\sigma_{3}}{2}}$
(D) $\sqrt{\frac{\sigma_{1}-\sigma_{3}}{2}}$

Solution: Choice (B)
Question Number: 27
Question Type: MCQ
The equation of motion for a spring-mass system excited by a harmonic force is

$$
M \ddot{x}+K x=F \cos (\omega t)
$$

where $M$ is the mass, $K$ is the spring stiffness, $F$ is the force amplitude and $\omega$ is the angular frequency of excitation. Resonance occurs when $\omega$ is equal to
(A) $\sqrt{\frac{M}{K}}$
(B) $\frac{1}{2 \pi} \sqrt{\frac{K}{M}}$
(C) $2 \pi \sqrt{\frac{K}{M}}$
(D) $\sqrt{\frac{K}{M}}$

## Solution:

We know that resonance will occur when $\omega=\omega_{n}=\sqrt{\frac{K}{M}}$ Hence, the correct option is (D)

Question Number: 28
Question Type: MCQ
For an Oldham coupling used between two shafts, which among the following statements are correct?
I. Torsional load is transferred along shaft axis.
II. A velocity ratio of $1: 2$ between shafts is obtained without using gears.
III. Bending load is transferred transverse to shaft axis.
IV. Rotation is transferred along shaft axis.
(A) I and III
(B) I and IV
(C) II and III
(D) II and IV

Solution: We know that in Oldham coupling, torsional load and rotation is transferred along shaft axis.
Hence, the correct option is (B)

## Question Number: 29

Question Type: MCQ
For a two-dimensional incompressible flow field given by $\vec{u}=A(x \hat{i}-y \hat{j})$, where $A>0$, which one of the following statements is FALSE?
(A) It satisfies continuity equation.
(B) It is unidirectional when $x \rightarrow 0$ and $y \rightarrow \infty$.
(C) Its streamlines are given by $x=y$.
(D) It is irrotational.

Sol. Flow field is given by

$$
\vec{u}=A x \hat{i}-A y \hat{j}
$$

Stream line equation is $\frac{d x}{u}=\frac{d y}{u}$

$$
\frac{d x}{A x}=\frac{d y}{-A y}
$$

Taking $\log$ on both sides of above equation, we get

$$
\begin{aligned}
\ln x & =-\ln y+\ln c \\
\ln x+\ln y & =\ln c \\
\ln x y & =\ln c \\
x y & =c
\end{aligned}
$$

Hence, the correct option is (C)
Question Number: 30
Question Type: MCQ
Which one of the following statements is correct for a superheated vapour?
(A) Its pressure is less than the saturation pressure at a given temperature.
(B) Its temperature is less than the saturation temperature at a given pressure.
(C) Its volume is less than the volume of the saturated vapour at a given temperature.
(D) Its enthalpy is less than the enthalpy of the saturated vapour at a given pressure.
Solution: For superheated vapour, the pressure is less than the saturated pressure at a given temperature.

$$
P<P_{\text {sat }}(\text { at a given } \mathrm{T})
$$

Hence, the correct option is (A)
Question Number: 31
Question Type: NAT
If the wire diameter of a compressive helical spring is increased by $2 \%$, the change in spring stiffness (in \%) is
$\qquad$ (correct to two decimal places).
Solution: If $d$ is the spring wire diameter, then the stiffness of helical spring can be expressed as

$$
K=\frac{G d^{4}}{64 R^{3} n}
$$

From above relation we conclude that

$$
K \propto d^{4}
$$

$$
\frac{K_{1}}{K_{2}}=\left(\frac{d_{1}}{d_{2}}\right)^{4}
$$

Since $d_{2}=1.02 d$ and $d_{1}=d$, then we have

$$
K_{2}=\left(\frac{1.02 d}{d}\right)^{4} \times K_{1}
$$

$$
K_{2}=1.08243 K_{1}
$$

\% increase in stiffness

$$
=\frac{K_{2}-K_{1}}{K} \times 100=8.243 \%
$$

Hence, the correct answer is 8.243 .

## Question Number: 32

Question Type: NAT
A flat plate of width $L=1 \mathrm{~m}$ is pushed down with a velocity $U=0.01 \mathrm{~m} / \mathrm{s}$ towards a wall resulting in the drainage of the fluid between the plate and the wall as shown in the figure. Assume two-dimensional incompressible flow and that the plate remains parallel to the wall. The average velocity, $U_{\text {avg }}$ of the fluid (in $\mathrm{m} / \mathrm{s}$ ) draining out at the instant shown in the figure is $\qquad$ (correct to three decimal places).


Solution: Width of flat plate $L=1 \mathrm{~m}$
Velocity of flat plate $U=0.01 \mathrm{~m} / \mathrm{s}$
Distance between flat plate and wall

$$
d=0.1 \mathrm{~m}
$$

If we assume the length of plate be $Z$
Rates of mass through plate $=$ Rate of mass displaced
between plates and wall.

$$
L Z \times d h=2 \times U_{\text {avg }} \times d \times Z d t
$$

Where $U_{\text {avg }}$ is the average velocity of the fluid (in $\mathrm{m} / \mathrm{s}$ ) draining out at the instant

$$
\begin{aligned}
L Z d h & =2 \times U_{\text {avg }} \times d \times z d t \\
\qquad U_{\text {avg }} & =\frac{L \times U}{2 d}=\frac{1 \times 0.01}{2 \times 0.1}=0.05 \mathrm{~m} / \mathrm{s} \quad\left[\because \frac{d h}{d t}=U\right]
\end{aligned}
$$

Hence, the correct answer is $0.05 \mathrm{~m} / \mathrm{s}$.

## Question Number: 33

Question Type: NAT
An ideal gas undergoes a process from state $1\left(T_{1}=300 \mathrm{~K}\right.$, $\left.p_{1}=100 \mathrm{kPa}\right)$ to state $2\left(T_{2}=600 \mathrm{~K}, p_{2}=500 \mathrm{kPa}\right)$. The specific heats of the ideal gas are: $C_{\mathrm{p}}=1 \mathrm{~kJ} / \mathrm{kg}-\mathrm{K}$ and $C_{\mathrm{v}}=$ $0.7 \mathrm{~kJ} / \mathrm{kg}-\mathrm{K}$. The change in specific entropy of the ideal gas from state 1 to state 2 (in $\mathrm{kJ} / \mathrm{kg}-\mathrm{K}$ ) is $\qquad$ (correct to two decimal places).

Solution: Intial Temperature $T_{1}=300 \mathrm{~K}$, Initial pressure $P_{1}=100 \mathrm{kPa}$
Final temperature $T_{2}=600 \mathrm{~K}$,
Final pressure $P_{2}=500 \mathrm{kPa}$

Specific heat at constant pressure $C_{\mathrm{p}}=1 \mathrm{~kJ} / \mathrm{kg}-\mathrm{K}$,
Specific heat at constant volume $C_{\mathrm{v}}=0.7 \mathrm{~kJ} / \mathrm{kg}-\mathrm{K}$
Change in specific entropy of the ideal gas from state 1 to state 2

$$
\begin{aligned}
\Delta s & =S_{2}-S_{1}=C_{\mathrm{p}} \ln \frac{T_{2}}{T_{1}}-R \ln \frac{P_{2}}{P_{1}} \\
S_{2}-S_{1} & =1 \times \ln \frac{600}{300}-0.3 \ln \frac{500}{100} \\
S_{2}-S_{1} & =0.21 \mathrm{~kJ} / \mathrm{kg}-\mathrm{K}
\end{aligned}
$$

Hence, the correct answer is 0.21 .

## Question Number: 34

## Question Type: NAT

For a Pelton wheel with a given water jet velocity, the maximum output power from the Pelton wheel is obtained when the ratio of the bucket speed to the water jet speed is
$\qquad$ (correct to two decimal places).

Solution: For maximum efficiency in pelton wheel, the ratio of the bucket speed to the water jet speed is $1 / 2=0.5$.
Hence, the correct answer is 0.5
Question Number: 35
Question Type: NAT
The height (in mm) fo a 125 mm sine bar to measure a taper of $27^{\prime} 32^{\prime}$ on a flat work piece is $\qquad$ (correct to three decimal places).

Solution: Taper $\theta=27^{\circ} 32^{1}$

$$
=27+\frac{32}{60}=27.533^{\circ}
$$

Height $L=125 \mathrm{~mm}$
Now we know that

$$
\begin{aligned}
\sin \theta & =\frac{H}{L} \\
\sin \theta & =\frac{H}{125} \\
H & =57.782 \mathrm{~mm} .
\end{aligned}
$$

Hence, the correct answer is 57.782 .
Question Number: 36
Question Type: MCQ
Let $X_{1}, X_{2}$ be two independent normal random variables with means $\mu_{1}, \mu_{2}$ and standard deviations $\sigma_{1}, \sigma_{2}$ respectively, consider $Y=X_{1}-X_{2}: \mu_{1}=\mu_{2}=1, \sigma_{1}=1, \sigma_{2}=2$. Then
(A) $Y$ is normally distributed with mean 0 and variance 1
(B) $Y$ is normally distributed with mean 0 and variance 5
(C) $Y$ has mean 0 and variance 5 . but is NOT normally distributed
(D) $Y$ has mean 0 and variance 1 . but is Not normally distributed

## Solution:

$Y$ is normally distributed

$$
\text { Mean of } \begin{aligned}
Y & =E(Y)=E\left(X_{1}-X_{2}\right) \\
& =E\left(X_{1}\right)-E\left(X_{2}\right)=\mu_{1}-\mu_{2}=0
\end{aligned}
$$

And variance of $Y=\operatorname{Var}(Y)$

$$
\begin{aligned}
& =\operatorname{Var}\left(X_{1}-X_{2}\right) \\
& =\operatorname{Var}\left(X_{1}\right)+\operatorname{Var}\left(X_{2}\right) \\
& \quad\left(\because X_{1} \text { and } X_{2} \text { are independent }\right) \\
& =\sigma_{1}^{2}+\sigma_{2}^{2}=1^{2}+2^{2}=5
\end{aligned}
$$

Hence, the correct option is (B)

## Question Number: 37

Question Type: MCQ
The value of the integral

$$
\oiint_{S} \vec{r} \cdot \vec{n} d s
$$

Over the closed surface $S$ bounding a volume $V$. where $\hat{r}=x \hat{i}+y \hat{j}+z \hat{k}$ is the position vector and $n$ is the normal to the surface $S$ is
(A) V
(B) 2 V
(C) 3 V
(D) 4 V

Solution: The given position vector is

$$
\bar{r}=x \bar{i}+y \bar{j}+z \bar{k}
$$

The divergence of given position vector will be

$$
\therefore \quad \operatorname{div} \bar{r}=3
$$

Applying Gauss' divergence theorem, we have

$$
\begin{aligned}
& \oiint_{S} \bar{r} \cdot \bar{n} d s=\iiint_{V} \operatorname{div} \bar{r} d v \\
& \iiint_{V} 3 d v=3 \mathrm{~V}
\end{aligned}
$$

Hence, the correct option is (C).

## Question Number: 38

Question Type: MCQ
A point mass is shot vertically up from ground level with a velocity of $4 \mathrm{~m} / \mathrm{s}$ at time $t=0$. It loses $20 \%$ of its impact velocity after each collision with the ground. Assuming that the acceleration due to gravity is $10 \mathrm{~m} / \mathrm{s}^{2}$ and that air resistance is negligible, the mass stops bouncing and comes to complete on the ground after a total time (in seconds) of
(A) 1
(B) 2
(C) 4
(D) $\infty$

Solution: Initial velocity $u=4 \mathrm{~m} / \mathrm{s}$,
Acceleration due to gravity $g=10 \mathrm{~m} / \mathrm{s}^{2}$
From first equation of motion, we get

$$
\begin{aligned}
& V=u+a t \\
& 0=4-10 t
\end{aligned}
$$

$$
\Rightarrow \quad t=0.4 \mathrm{~s}
$$

We know that after $1^{\text {st }}$ Collision,

$$
\begin{aligned}
& u_{1}=0.8 \times 4=3.2 \mathrm{~m} / \mathrm{s} \\
& v_{1}=u_{1}+a t_{1} \\
& t_{1}=\frac{3.2}{10}=0.32 \mathrm{~s}
\end{aligned}
$$

We know that after $2^{\text {nd }}$ Collision,

$$
\begin{aligned}
& u_{2}=0.8 \times 3.2=2.56 \mathrm{~m} / \mathrm{s} \\
& v_{2}=u_{2}+a t_{2} \\
& t_{2}=0.256 \mathrm{~s}
\end{aligned}
$$

Total time $=2\left[t+t_{1}+t_{2}+\ldots 0\right]$

$$
\begin{aligned}
& =2[0.4+0.32+0.256+\ldots] \\
& =2 \times \frac{0.4}{1-0.8}=4 \mathrm{~s}
\end{aligned}
$$

Hence, the correct option is (C)
Question Number: 39

## Question Type: MCQ

The state of stress oat a point, for a body in plane stress, is shown in the figure below. If the minimum principal stress is 10 kPa , then the normal stress $\sigma_{y}($ in kPa$)$ is

(A) 9.45
(B) 18.88
(C) 37.78
(D) 75.50

## Solution:

Minimum principal stress

$$
\begin{aligned}
& =\frac{\sigma_{x}+\sigma_{y}}{2}-\sqrt{\left(\frac{\sigma_{x}-\sigma_{y}}{2}\right)^{2}+\tau^{2} x y} \\
10 & =\frac{100+\sigma_{y}}{2}-\sqrt{\left(\frac{100-\sigma_{y}}{2}\right)^{2}+50^{2}} \\
40+\frac{\sigma_{y}}{2} & =\sqrt{\left(\frac{50-\sigma_{y}}{2}\right)^{2}+50^{2}}
\end{aligned}
$$

Squaring on both sides of above equation we get

$$
\begin{aligned}
1600+\frac{\sigma_{y}^{2}}{4}+40 \sigma_{y} & =2500+\frac{\sigma_{y}^{2}}{4}-50 \sigma_{y}+2500 \\
900 \sigma_{y} & =3400 \\
\sigma_{y} & =37.78 \mathrm{MPa}
\end{aligned}
$$

Hence, the correct option is (C)
Question Number: 40
Question Type: MCQ
An epicyclic gear train is shown in the figure below. The number of teeth on the gears $A, B$ and $D$ are 20,30 and 20 respectively. Gear $C$ has 80 teeth on the inner surface and 100 teeth on the outer surface. If the carrier $\operatorname{arm} A B$ is fixed and the sun gear $A$ rotates at 300 rpm in the clockwise direction then the rpm of $D$ in the clockwise direction is

(A) 240
(B) -240
(C) 375
(D) -375

## Solution:

Considering clockwise direction as positive,

$$
\begin{aligned}
& N_{\mathrm{A}}=+300 \\
& N_{\mathrm{B}}=\frac{-300 \times 20}{30}=-200 \\
& N_{\mathrm{C}}=-200 \times \frac{30}{80}=-75 \\
& N_{\mathrm{D}}=+75 \times \frac{100}{20}=+375
\end{aligned}
$$

Hence, the correct option is (C).

## Question Number: 41

Question Type: MCQ
A carpenter glues a pair of cylindrical wooden logs by bonding then end faces at an angle of $\theta=30^{\circ}$ as shown in the figure.


The glue used at the interface fails if
Criterion 1: the maximum normal stress exceeds 2.5 MPa
Criterion 2: the maximum shear stress exceeds 1.5 MPa .
Assume that the interface fails before the logs fail. When a uniform tensile stress of 4 MPa is applied, the interface
(A) fails only because of criterion 1
(B) fails only because of criterion 2
(C) fails because of both criterion 1 and 2
(D) does not fail

Solution: Normal stress $\sigma=\sigma_{x} \cos ^{2} \theta$

$$
\begin{aligned}
\sigma & =4 \times \cos ^{2} 30=3 \mathrm{MPa} \\
\text { Shear stress } \tau & =\frac{\sigma_{\mathrm{x}}}{2} \sin 2 \theta \\
\tau & =2 \times \sin 60^{\circ}=1.73 \mathrm{MPa}
\end{aligned}
$$

Hence, the correct option is (C)
Question Number: 42
Question Type: MCQ
A Self-aligning ball bearing has a basic dynamic load rating ( $C_{10}$. For $10^{6}$ revolutions) of 35 kN . If the equivalent radial load on the bearing is 45 kN , the expected life (in $10^{6}$ revolutions) is
(A) below 0.5
(B) 0.5 to 0.8
(C) 0.8 to 1.0
(D) above 1.0

Solution: Basic dynamic load rating of ball $C=35 \mathrm{kN}$ Equivalent radial load on the bearing $P_{\mathrm{c}}=45 \mathrm{kN}$
Now we know that

$$
\begin{aligned}
& L_{90}=\left(\frac{\mathrm{C}}{P_{\mathrm{c}}}\right)^{3}=\left(\frac{35}{45}\right)^{3} \\
& L_{90}=0.4705
\end{aligned}
$$

Hence, the correct option is (A)
Question Number: 43
Question Type: MCQ
A tank open at the top with a water level of 1 cm , as shown in the figure, has a hole at a height of 0.5 m . A free jet leaves horizontally from the smooth hole. The distance $X$ (in m ) where the jet strikes the floor is

(A) 0.5
(B) 1.0
(C) 2.0
(D) 4.0

## Solution:

Height $h=0.5 \mathrm{~m}$
Acceleration due to gravity $g=9.81 \mathrm{~m} / \mathrm{s}^{2}$
velocity of free jet can be calculated using

$$
\begin{aligned}
& V=\sqrt{2 g h} \\
& V=\sqrt{2 \times 9.81 \times 05} \\
& V=3.3132 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$



If $t$ is the time taken for the liquid to travel from the opening to the ground, then we have

$$
\begin{aligned}
y & =\frac{1}{2} g t^{2} \\
y & =0.5=\frac{1}{2} g t^{2} \\
t^{2} & =\frac{2 \times 0.5}{9.81} \\
t & =0.319 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Distance travelled in time $t$ will be

$$
\begin{aligned}
& x=V t=3.3132 \times 0.319 \\
& x=1.05 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Hence, the correct option is (B)

## Question Number: 44

Question Type: MCQ
In a Lagrangian system, the position of a fluid particle in a flow is described as $x=x_{0} e^{-k t}$ and $y=y_{0} e^{k t}$ where $t$ is the time while $x_{\mathrm{o}}, y_{\mathrm{o}}$ and $k$ are constants. The flow is
(A) unsteady and one-dimensional
(B) steady and two-dimensional
(C) steady and one-dimensional
(D) unsteady and two-dimensional

Solution: Initial velocity $u=\frac{d x}{d t}=-k x_{0} e^{-\mathrm{kt}}$

$$
\text { Final velocity } V=\frac{d y}{d t}=-k y_{0} e^{k t}
$$

Now we know that in vector form

$$
\begin{gathered}
\vec{V}=u \hat{i}+v \hat{j} \\
=-k x_{0} e^{-\mathrm{kt}} \hat{i}+k y_{0} e^{\mathrm{kt}} \hat{j} \\
\frac{d u}{d x}+\frac{d v}{d y}=0 \\
\frac{d}{d x}\left(-k x_{0} e^{-\mathrm{kt}}\right)+\frac{d}{d y}\left(k y_{0} e^{\mathrm{kt}}\right)=0
\end{gathered}
$$

Since $x_{0}, y_{0}, k$ are constants continuity equation is satisfied. Hence, flow is $2-D$

$$
\begin{aligned}
& \frac{d u}{d t}=k^{2} x_{0} e^{-\mathrm{kt}} \neq 0 \\
& \frac{d v}{d t}=k^{2} x_{0} e^{\mathrm{kt}} \neq 0
\end{aligned}
$$

Hence, flow is unsteady.
Hence, the correct option is (D)
Question Number: 45
Question Type: MCQ
The maximum reduction in cross-sectional area per pass $(R)$ of a cold wire drawing process is

$$
R=1-e^{-(n+1)}
$$

Where $n$ represents the strain hardening coefficient. For the case of a perfectly plastic material. $R$ is
(A) 0.865
(B) 0.826
(C) 0.777
(D) 0.632

Solution: For perfectly plastic material strain hardening coefficient $n=0$

$$
\begin{aligned}
& R=1-e^{-(0+1)} \\
& R=1-e^{-1}=1-\frac{1}{e} \\
& R=0.632
\end{aligned}
$$

Hence, the correct option is (D)
Question Number: 46
Question Type: NAT
The percentage scrap in a sheet metal blanking operation of a continuous strip of sheet metal as shown in the figure is $\qquad$ (correct to two decimal places.)
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Solution:


Consider rectangle ABCD which repeats again on the strip of sheet,

$$
\begin{aligned}
A_{\mathrm{T}} & =\text { Total area } \\
& =\left(\frac{D}{5}+\frac{D}{2}+\frac{D}{2}+\frac{D}{5}\right)\left(\frac{D}{2}+\frac{D}{2}+\frac{D}{2}\right) \\
A_{\mathrm{T}} & =\frac{7}{5} D \times \frac{6}{5} D=\frac{42}{25} D^{2} \\
A_{\mathrm{B}} & =\text { Blanking area }=\frac{\pi}{4} D^{2} \\
\% \text { of scrap } & =\frac{A_{\mathrm{T}}-A_{\mathrm{B}}}{A_{\mathrm{T}}} \times 100 \% \\
& =\left[1-\frac{\left(\frac{\pi}{4}\right)}{\left(\frac{42}{25}\right)}\right] \times 100 \\
& =53.25 \%
\end{aligned}
$$

Hence, the correct answer is 53.25 .

## Question Number: 47

Question Type: NAT
An explicit forward Euler method is used to numerically integrated the differential equation

$$
\frac{d y}{d t}=y
$$

Using a time step of 0.1 . with the initial condition $y(0)=1$.
The value of $Y(1)$ denoted by this method is $\qquad$ (correct to two decimal places).

Solution: Given differential equation is

$$
\begin{aligned}
\frac{d y}{d x} & =y \\
& h=0.1, y(0)=1 \\
\therefore \quad & f(x, y) \\
\therefore & =y ; x_{0}=0 ; y_{0}=1
\end{aligned}
$$

By forward Ealer method,

$$
\begin{aligned}
y(0.1) & =y_{1} \\
& =y_{0}+h f\left(x_{0}, y_{0}\right) \\
& =y_{0}+h\left(y_{0}\right) \\
& =1+(0.1) 1 \\
& =1.1 \\
y(0.2) & =y_{2}=y_{1}+h f\left(x_{1}, y_{1}\right) \\
& =1.1+(0.1)(1.1) \\
& =1.21 \\
y(0.3) & =y_{3}=y_{2}+h f\left(x_{2}, y_{2}\right) \\
& =1.21+(0.1)(1.21) \\
& =1.331 \\
y(0.4) & =y_{4}=y_{3}+h f\left(x_{3}, y_{3}\right) \\
& =1.331+(0.1)(1.331) \\
& =1.4641 \\
y(0.5) & =y_{5}=y_{4}+h f\left(x_{4}, y_{4}\right) \\
& =1.4641+(0.1)(1.4641) \\
& =1.61051 \\
y(0.6) & =y_{6}=y_{5}+h f\left(x_{5}, y_{5}\right) \\
& =1.6105+(0.1)(1.6105) \\
& =1.77155 \\
y(0.7) & =y_{7}=y_{6}+h f\left(x_{6}, y_{6}\right) \\
& =1.7715+(0.1)(1.7715) \\
& =1.94865 \\
y(0.8) & =y_{8}=y_{7}+h f\left(x_{7}, y_{7}\right) \\
& =1.9486+(0.1)(1.9486) \\
& =2.14346 \\
y(0.9) & =y_{9}=y_{8}+h f\left(x_{8}, y_{8}\right) \\
& =2.1437+(0.1)(2.1437) \\
& =2.35807
\end{aligned}
$$

$$
\begin{aligned}
y(1) & =y_{10}=y_{9}+h f\left(x_{9}, y_{9}\right) \\
& =2.3581+(0.1)(2.3581) \\
& =2.59391 \\
\therefore \quad y(1) & =2.594 .
\end{aligned}
$$

Hence, the correct answer is 2.594 .

## Question Number: 48

Question Type: NAT
$F(s)$ is the Laplace transform of the function $f(t)=2 \mathrm{t}^{2} e^{-\mathrm{t}}$ $F(1)$ is $\qquad$ (correct to two decimal places).

Solution: The given function is

$$
f(t)=2 t^{2} e^{-t}
$$

Laplace transform of the function will be

$$
\left.\left.\begin{array}{rl}
\therefore \\
& =L[f(t)] \\
& =F(s) \\
& =2 \frac{d^{2}}{d s^{2}}\left(L\left[e^{-t}\right]\right) \\
& =2 \frac{d^{2}}{d s^{2}}\left(\frac{1}{s+1}\right) \\
& =2\left(\frac{2}{(s+1)^{3}}\right) \\
\therefore \quad \text { So, } \quad F(s) & =\frac{4}{(s+1)^{3}} \\
\therefore & F(1)
\end{array}\right)=\frac{4}{(1+1)^{3}}=\frac{4}{8}=\frac{1}{2}=0.5\right) ~ \$
$$

Hence, the correct answer is 0.5 .

## Question Number: 49

Question Type: NAT
A simple supported beam of width 100 mm . height 200 mm and length 4 m is carrying a uniformly distributed load of intensity $10 \mathrm{kN} / \mathrm{m}$. The maximum bending stress (in MPa) in the beam is $\qquad$ (correct to one decimal place).


Solution: Maximum bending moment,

$$
M=\frac{W L^{2}}{8}=\frac{10 \times 16}{8}=20 \mathrm{kNm}
$$

Maximum bending stress,

$$
\begin{aligned}
\sigma_{\max } & =\frac{m}{1} y_{\max }=\frac{20 \times 10^{3}}{\left(\frac{0.1 \times 0.2^{3}}{12}\right)} \times 0.1 \\
\sigma_{\max } & =30 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}=30 \mathrm{MPa}
\end{aligned}
$$

Hence, the correct answer is 30 .

## Question Number: 50

Question Type: NAT
A machine of mass $m=200 \mathrm{~kg}$ is supported on two mounts, each of stiffness $k=10 \mathrm{kN} / \mathrm{m}$. The machine is subjected to an external force (in N) $F(t)=50 \cos 5 t$. Assuming only vertical translator motion, the magnitude of the dynamic force (in N ) transmitted from each mount to the ground is
$\qquad$ (correct to two decimal places).


Solution: Mass $m=200 \mathrm{~kg}$,
stiffness $K=10 \mathrm{kN} / \mathrm{m}$

$$
=10000 \mathrm{~N} / \mathrm{m}
$$

Equivalent stiffness

$$
\begin{aligned}
K_{\mathrm{eq}} & =K+K=10000+10000 \\
& =20000 \mathrm{~N} / \mathrm{m}
\end{aligned}
$$

Now we have

$$
\begin{aligned}
F(t) & =50 \cos 5 t \\
F_{0} & =50 \mathrm{~N} \\
\omega & =5 \mathrm{rad} / \mathrm{s} \\
\omega_{n} & =\sqrt{\frac{K_{\mathrm{eq}}}{m}}=\sqrt{\frac{20000}{200}} \\
\omega_{n} & =\sqrt{100}=10 \mathrm{rad} / \mathrm{s} \\
\frac{\omega}{\omega_{\mathrm{n}}} & =\frac{5}{10}=\frac{1}{2} \\
\xi & =0
\end{aligned}
$$

[ $\because$ No damping]

Transmissibility

$$
\begin{aligned}
& T=\frac{\sqrt{1+\left(\frac{2 \xi \omega}{\omega_{\mathrm{n}}}\right)^{2}}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_{\mathrm{n}}}\right)^{2}\right]^{2}+\left[\frac{2 \xi \omega}{\omega_{\mathrm{n}}}\right]^{2}}} \\
& T=\frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_{\mathrm{n}}}\right)^{2}\right]^{2}}} \\
& T=\frac{1}{\sqrt{\left(1-\frac{1}{4}\right)^{2}}}=\frac{1}{1-\frac{1}{4}} \\
& T=\frac{4}{3} \\
& T=\frac{F_{\mathrm{t}}}{F_{0}} \\
& F_{t}=50 \times \frac{4}{3}=66.666 \mathrm{~N}
\end{aligned}
$$

Force from each mount

$$
=\frac{F_{\mathrm{t}}}{2}=\frac{66.666}{2}=33.33 \mathrm{~N}
$$

Hence, the correct answer is 33.33 .

## Question Number: 51

Question Type: NAT
A slider crank mechanism is shown in the figure. At some instant, the crank angle is $45^{\circ}$ and a force of 40 N is acting towards the left on the slider. The length of the crank is 30 mm and the connecting rod is 70 mm . Ignoring the effect of gravity, friction and inertial forces, the magnitude of the crankshaft torque (in Nm ) needed to keep the mechanism in equilibrium is $\qquad$ (correct to two decimal places).


Solution: Consider the figure given below


Length of connecting rod,$\ell=70 \mathrm{~mm}$
length of the crank, $r=30 \mathrm{~mm}$
crank angle $\theta=45^{\circ}$
We know that

$$
\begin{aligned}
r \sin \theta & =\ell \sin \beta \\
& =\frac{\sin 45^{\circ} \times 30}{70} \\
\tan \beta & =0.303, \beta=16.8^{\circ} \\
F_{\mathrm{CR}} \cos \beta & =F, F_{\mathrm{t}}=F_{\mathrm{CR}} \sin (\theta+\beta) \\
\text { Torque } & =F_{\mathrm{t}} \times r \\
& =\frac{F}{\cos \beta} \sin (\theta+\beta) \times r \\
& =\frac{40}{\cos 16.85} \sin (45+16.85) \times \frac{30}{1000} \\
\text { Torque } & =\frac{0.8817 \times 30 \times 40}{0.957 \times 1000} \\
T & =1.10 \mathrm{~N} \mathrm{~m} .
\end{aligned}
$$

Hence, the correct answer is 1.10 .

## Question Number: 52

Question Type: NAT
A sprinkler shown in the figure rotates about its hinge point in a horizontal plane due to water flow discharged through its two exit nozzles


The total flow rate $Q$ through the sprinkle is 1 litre $/ \mathrm{sec}$ and the cross-sectional area of each exit nozzle is $1 \mathrm{~cm}^{2}$. Assuming equal flow rate through both arms and a frictionless hinge, the steady state angular speed of rotation (in $\mathrm{rad} / \mathrm{s}$ ) of the sprinkler is $\qquad$ (correct to two decimal places).

Solution: Consider the figure given below


Total flow rate through the sprinkle is

$$
Q=1 \mathrm{lit} / \mathrm{sec}=10^{-3} \mathrm{~m}^{3} / \mathrm{sec}
$$

Area of each nozzle $\mathrm{A}_{1}=\mathrm{A}_{2}$

$$
A_{1}=A_{2}=1 \mathrm{~cm}^{2}=10^{-4} \mathrm{~m}^{2}
$$

$$
\therefore \quad V_{1}=V_{2}=\frac{Q / 2}{A}=\frac{10^{-3}}{2 \times 10^{-4}}=5 \mathrm{~m} / \mathrm{s}
$$

Let $\omega=$ angular speed in rad/sec
absolute velocity

$$
\begin{aligned}
V_{\mathrm{u} 1}= & V_{1}+r_{1} \omega=5+0.1 \omega \\
V_{\mathrm{u} 2}= & V_{2}-r_{2} \omega=5-0.2 \omega \\
\text { Torque }= & \rho Q\left(V_{\mathrm{u} 2} r_{2}\right)-\rho Q\left(V_{\mathrm{u} 1} r_{1}\right) \\
T= & \rho Q\left[\left(v_{2}-r_{2} \omega\right) r_{2}-\left(V_{1}+r \omega\right) r_{1}\right]=0 \\
& (5-0.2 \omega) 0.2-(5+0.1 \omega) 0.1=0 \\
1-0.04 \omega= & 0.5+0.01 \omega \\
0.5= & 0.05 \omega \\
\omega= & 10 \mathrm{rad} / \mathrm{sec}
\end{aligned}
$$

Hence, the correct answer is 10 .

## Question Number: 53

Question Type: NAT
A solid block of 2.0 kg mass slides steadily at a velocity $V$ along a vertical wall as shown in the figure below. A thin oil film of thickness $h=0.15 \mathrm{~mm}$ provides lubrication between the block and the wall. The surface area of the face of the block in contact with the oil film is $0.04 \mathrm{~m}^{2}$. The velocity distribution within the oil film gap is linear as shown in the figure. Take dynamic viscosity of oil as $7.10^{-3} \mathrm{~Pa}$-s and acceleration due to gravity as $10 \mathrm{~ms}^{2}$. Neglect weight of the oil. The terminal velocity $V(\mathrm{in} \mathrm{m} \mathrm{s}$ ) of the block is $\qquad$ (correct to one decimal place).


## Solution:

Mass $m=2 \mathrm{~kg}$
Acceleration $g=10 \mathrm{~m} / \mathrm{s}^{2}$
Height $h=15 \mathrm{~mm}$
Terminal velocity $V=$ ?
We know that terminal velocity is constant velocity, therefore net acceleration is zero.
Shear force due to oil film = weight of block

$$
\begin{aligned}
\tau A & =m g \\
\frac{\mu V}{h} \times A & =m g \\
V & =\frac{m g h}{\mu A}=\frac{2 \times 10 \times 0.15 \times 10^{-3}}{7 \times 10^{-3} \times 0.04} \\
V & =10.714 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Hence, the correct answer is 10.714 .

## Question Number: 54

Question Type: NAT
A tank of volume $0.05 \mathrm{M}^{3}$ contains a mixture of saturated water and saturated steam at $200^{\circ} \mathrm{C}$. The mass of the liquid present is 8 kg . The entropy (in $\mathrm{kJ} / \mathrm{kg} \mathrm{K}$ ) of the mixture is
$\qquad$ (correct to two decimal places).
Property data for saturated steam and water are:
At $200^{\circ} \mathrm{C} . \mathrm{P}_{\text {sat }}=1.5538 \mathrm{MPa}$

$$
\begin{aligned}
V_{f} & =0.001157 \mathrm{~m}^{3} / \mathrm{kg}, V_{t}=0.12736 \mathrm{~m}^{3} / \mathrm{kg} \\
S_{f g} & =4.1014 \mathrm{~kJ} / \mathrm{kg} \mathrm{~K}, S_{f}=2.33 .9 \mathrm{~kJ} / \mathrm{kg} \mathrm{~K}
\end{aligned}
$$

## Solution:

Volume of tank $V=0.05 \mathrm{~m}^{3}$
At $200^{\circ} \mathrm{C}$.

$$
\begin{aligned}
P_{\mathrm{sat}} & =1.5538 \mathrm{MPa} \\
V_{f} & =0.001157 \mathrm{~m}^{3} \mathrm{~kg}, \\
V_{t} & =0.12736 \mathrm{~m}^{3} / \mathrm{kg} \\
S_{f g} & =4.1014 \mathrm{~kJ} / \mathrm{kg} \mathrm{~K} . \\
S_{f} & =2.33 .9 \mathrm{~kJ} / \mathrm{kg} \mathrm{~K}
\end{aligned}
$$

mass of specific entropy of mixture

$$
\begin{aligned}
s & =s_{\mathrm{f}}+x s_{\mathrm{fg}} \\
x & =\frac{m_{\mathrm{V}}}{m_{\mathrm{V}}+m_{\mathrm{L}}} \\
V & =V_{\mathrm{L}}+V_{\mathrm{v}} \\
V & =m_{\mathrm{V}} \times \vartheta_{\mathrm{V}}+m_{\mathrm{L}} \times \vartheta_{\mathrm{L}} \\
0.05 & =8 \times 0.001157+m_{\mathrm{v}} \times \vartheta_{\mathrm{v}} \\
m_{\mathrm{v}} & =\frac{0.04074}{0.12736} \\
m_{\mathrm{v}} & =0.3198 \mathrm{~kg} \\
x & =\frac{0.3198}{0.3198+8} \\
x & =0.0384 \\
s & =2.3309+0.0384 \times 4.1054 \\
s & =2.488 \mathrm{~kJ} / \mathrm{kg} . \mathrm{K} .
\end{aligned}
$$

Hence, the correct answer is 2.488 .

## Question Number: 55

Question Type: NAT
Steam flows through a nozzle at a mass flow rate of in $=$ $0.1 \mathrm{~kg} / \mathrm{s}$ with a heat loss of 5 kW . The enthalpies at inlet and exit are $2500 \mathrm{~kJ} / \mathrm{kg}$ and $2350 \mathrm{~kJ} / \mathrm{kg}$ respectively. Assuming negligible velocity at inlet $\left(C_{1} \approx 0\right)$, the velocity $\left(C_{2}\right)$ of steam (in $\mathrm{m} / \mathrm{s}$ ) at the nozzle exit is $\qquad$ (correct to two decimal places).


## Solution:

Mass $m=0.1 \mathrm{~kg} / \mathrm{s}$
Heat loss $Q=5 \mathrm{~kW}$
enthalpies at inlet $h_{1}=2500 \mathrm{~kJ} / \mathrm{kg}$
velocity at inlet $c_{1}=0 \mathrm{~m} / \mathrm{s}$
enthalpies at outlet $h_{2}=2350 \mathrm{~kJ} / \mathrm{kg}$
According to SFEE, we get

$$
\begin{aligned}
& m\left(h_{1}+\frac{c_{1}^{2}}{2000}\right)+Q=m\left(h_{2}+\frac{c_{2}^{2}}{2000}\right) \\
& 0.1 \times 2500+0-5 \\
= & 0.1 \times 2350+\frac{c_{2}^{2}}{2000} \times 0.1 \\
c_{2}= & 447.21 \mathrm{~m} / \mathrm{s} .
\end{aligned}
$$

Hence, the correct answer is 447.2 .
Question Number: 56
Question Type: NAT
An engine working on air standard Otto cycle is supplied with air at 0.1 MPa and $35^{\circ} \mathrm{C}$. The compression ratio is 8 . The heat supplied is $500 \mathrm{~kJ} / \mathrm{kg}$. The maximum temperature (in K ) of the cycle is $\qquad$ (correct to one decimal place).

Solution: For air standard Otto cycle,
Pressure $P_{1}=0.1 \mathrm{MPa}$,
Temperature $T_{1}=35^{\circ} \mathrm{C}=308 \mathrm{~K}$

$$
r=8,
$$

heat supplied $q_{\mathrm{s}}=500 \mathrm{~kJ} / \mathrm{kg}$ specific heat at constant pressure $C_{\mathrm{p}}=1.005 \mathrm{~kJ} / \mathrm{kg} . \mathrm{K}$, specific heat at constant volume $C_{\mathrm{V}}=0.718 \mathrm{~kJ} / \mathrm{kg} . \mathrm{K}$ Gas constant $R=0.287 \mathrm{~kJ} / \mathrm{kg}$.K


From the above curve we get

$$
\begin{aligned}
\frac{T_{2}}{T_{1}} & =\left(\frac{V_{1}}{V_{2}}\right)^{\gamma-1} \\
\frac{T_{4}}{T_{3}} & =\left(\frac{V_{3}}{V_{4}}\right)^{\gamma-1} \\
\Rightarrow \quad \frac{T_{3}}{T_{4}} & =\left(\frac{V_{4}}{V_{4}}\right)^{\gamma-1} \\
\frac{T_{2}}{T_{1}} & =\frac{T_{3}}{T_{4}}(r)^{\gamma-1} \\
\gamma & =\frac{C_{\mathrm{p}}}{C_{\mathrm{v}}}=1.4 \\
T_{2} & =(8)^{1.4-1} \times 308 \\
T_{2} & =707.598 \mathrm{~K}
\end{aligned}
$$

Heat supplied $q_{\mathrm{s}}=C_{\mathrm{v}}\left(T_{3}-T_{2}\right)$

$$
\begin{aligned}
500 & =0.718\left(T_{3}-707.598\right) \\
0.718 T_{3} & =1008.055 \\
T_{3} & =1403.97^{\circ} \mathrm{C}
\end{aligned}
$$

Hence, the correct answer is 1403.97 .
Question Number: 57
Question Type: NAT
A plane slab of thickness $L$ and thermal conductivity $k$ is heated with a fluid on one side $(P)$, and the other side $(Q)$ is maintained at a constant temperature. $T_{\mathrm{Q}}$ of $25^{\circ} \mathrm{C}$, as shown in the figure. The fluid is at $45^{\circ} \mathrm{C}$ and the surface heat transfer coefficient, $h$, is $10 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$. The steady state temperature. $T_{\mathrm{P}},\left(\right.$ in $\left.{ }^{\circ} \mathrm{C}\right)$ of the side which is exposed to the fluid is $\qquad$ (correct to two decimal places).

## Solution:



Let heat transfer area is constant

$$
\begin{aligned}
& q=\frac{T_{\infty}-T_{Q}}{\frac{1}{h}+\frac{L}{k}}=\frac{45-25}{\frac{1}{10}+\frac{20 \times 10^{-2}}{2.5}} \\
& q=\frac{20}{0.1+0.06}=111.11 \mathrm{w} / \mathrm{m}^{2} \\
& q=\frac{T_{\infty}-T_{\mathrm{P}}}{\frac{1}{h}}=h\left(T_{\infty}-T_{\mathrm{P}}\right) \\
& T_{\mathrm{P}}=T_{\infty}-\frac{q}{h}=45-\frac{111.11}{10} \\
& T_{\mathrm{P}}=33.88^{\circ} \mathrm{C}
\end{aligned}
$$

Hence, the correct answer is $33.88^{\circ}$.

## Question Number: 58

Question Type: NAT
The true stress $(\sigma)$-true strain $(\varepsilon)$ diagram of a strain hardening material is shown in figure. First, there is loading up to point $A$, i.e., up to stress of 500 MPa and strain of 0.5 . Then from point $A$, there is unloading up to point $B$, i.e., to stress of 100 MPa . Given that the Young's modulus $E=200 \mathrm{GPa}$, the natural strain at point $b\left(\varepsilon_{\mathrm{B}}\right)$ is (correct to three decimal places).


Solution:


$$
E=\frac{\sigma}{E}=\tan \theta
$$

The slope of the stress - strain curve is $E$

$$
\therefore \begin{aligned}
\tan \theta & =\frac{(500-100)}{0.5-\epsilon_{\mathrm{B}}}=\tan \theta=E \\
\frac{400}{0.5-\epsilon_{\mathrm{B}}} & =2 \times 10^{5} \\
0.5-\epsilon_{\mathrm{B}} & =\frac{400}{2 \times 10^{5}} \\
\epsilon_{\mathrm{B}} & =0.498
\end{aligned}
$$

Hence, the correct answer is 0.498 .
Question Number: 59
Question Type: NAT
An orthogonal cutting operation is being carried out in which uncut thickness is 0.010 mm . cutting speed is 130 m min. rake angle is $15^{\circ}$ and width of cut is 6 mm . It is observed that the chip thickness is 0.015 mm . the cutting force is 60 N and the thrust force is 25 N . The ratio of friction energy to total energy is $\qquad$ (correct to two decimal places).

Solution: uncut thickness $t=0.01 \mathrm{~mm}$ cutting speed $V=130 \mathrm{~m} / \mathrm{min}$
minimum rake angle $\alpha=150^{\circ}$
width of cut $B=6 \mathrm{~mm}$
chip thickness $t_{\mathrm{c}}=0.015 \mathrm{~mm}$
cutting force $F_{\mathrm{c}}=60 \mathrm{~N}$
thrust force $F_{\mathrm{T}}=25 \mathrm{~N}$
Now using the relation given below

$$
\begin{aligned}
F & =F_{\mathrm{c}} \sin \alpha+F_{\mathrm{T}} \cos \alpha \\
& =60 \sin 15+25 \cos 15 \\
F & =39.672
\end{aligned}
$$

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Ration of frictional energy to total energy,

$$
\begin{aligned}
& =\frac{F}{F_{\mathrm{c}}} \times \frac{V_{\mathrm{c}}}{V}=\frac{F}{F_{\mathrm{c}}} \times \frac{t}{t_{\mathrm{c}}} \\
& =\frac{39.672}{60} \times \frac{0.01}{0.015} \\
& =0.4408
\end{aligned}
$$

Hence, the correct answer is 0.44 .

## Question Number: 60

Question Type: NAT
A bar is compressed to half of its original length. The magnitude of true strain produced in the deformed bar is
$\qquad$ (correct to two decimal places).

Solution: If $L_{1}$ and $L_{2}$ are initial and final lengths respectively

$$
\begin{aligned}
L_{2} & =\frac{L_{1}}{2} \\
\text { True strain } & =\epsilon_{\mathrm{T}}=\ln \frac{L_{1}}{L_{2}} \\
\epsilon_{\mathrm{T}} & =\ln \left[\frac{\frac{L_{1}}{2}}{L_{1}}\right]=\ln \frac{1}{2} \\
\epsilon_{\mathrm{T}} & =-0.693
\end{aligned}
$$

Magnitude of strain is 0.693 .
Hence, the correct answer is 0.693 .
Question Number: 61
Question Type: NAT
The minimum value of $3 x+5 y$ such that:

$$
\begin{gathered}
3 x+5 y \leq 15 \\
4 x+9 y \leq 8 \\
13 x+2 y \leq 2 \\
x \geq 0, y \geq 0
\end{gathered}
$$

is $\qquad$ -.

Solution: Consider
(i) $3 x+5 y \leq 15$
$3 x+5 y=15$
At, $x=0, y=3$
$x=5, y=0$


Consider:
(ii) $4 x+9 y \leq 8$
$4 x+9 y=8$
$x=0, y=\frac{8}{9}$
$x=2, y=0$
Consider:
(iii) $13 x+2 y \leq 2$
$13 x+2 y=2$
$x=0, y=1$

$$
x=\frac{2}{13}, y=0
$$

Comparing (ii) \& (iii),

$$
\begin{aligned}
4 x+9 y & =8 \\
13 x+2 y & =2 \\
x & =\frac{8-9 y}{4} \\
\frac{13(8-9 y)}{4}+2 y & =2 \\
104-117 y+8 y & =8 \\
109 y & =96 \\
y & =0.88, x=0.02 \\
z\left(0, \frac{8}{9}\right) & =3(0)+5\left(\frac{8}{9}\right)=4.44 \\
z(0.02,0.88) & =3(0.02)+5(0.88)=4.46 \\
z\left(\frac{2}{13}, 0\right) & =3\left(\frac{2}{13}\right)+5(0)=0.46
\end{aligned}
$$

$$
z(0,0)=0
$$

$\therefore z=0$ is minimum value
Hence, the correct answer is 0 .

## Question Number: 62

Question Type: NAT
Processing times (including setup times) and due dates for six jobs waiting to be processed at a work centre are given in the table. The average tardiness (in days) using shortest processing time rule is $\qquad$ (correct to two decimal places).

| Job | Processing Time (Days) | Due Date (Days) |
| :---: | :---: | :---: |
| A | 3 | 8 |
| B | 7 | 16 |
| C | 4 | 4 |
| D | 9 | 18 |
| E | 5 | 17 |
| F | 13 | 19 |

Solution: By SPT rule

| Job | Processing <br> Time | Due <br> Date | Flow <br> Time | Tardiness |
| :---: | :---: | :---: | :---: | :---: |
| A | 3 | 8 | $0+3=3$ | 0 |
| C | 4 | 8 | $3+4=7$ | 3 |
| E | 5 | 17 | $7+5=12$ | 0 |
| B | 7 | 16 | $12+7=19$ | 3 |
| D | 9 | 18 | $19+9=28$ | 10 |
| F | 13 | 19 | $28+13=41$ | 22 |
|  |  |  |  | 38 |

Total Tardiness $=38$

$$
\begin{aligned}
\text { Average tardiness per job } & =\frac{\text { Total tardiness }}{\text { Number of jobs }} \\
& =\frac{38}{6}=6.33 \text { days }
\end{aligned}
$$

Hence, the correct answer is 6.33 .

## Question Number: 63

Question Type: NAT
The schematic of an external drum rotating clockwise engaging with a short shoe is shown in the figure. The shoe is mounted at point $Y$ on a rigid lever $X Y Z$ hinged at point $X$. A force $F=100 N$ is applied at the free end of the lever shown. Given that the coefficient of friction between the shoe and the drum is 0.3 the braking torque (in Nm ) applied on the drum is $\qquad$ (correct to two decimal places).


Solution: Taking moments over point " $X$ "

$$
\begin{aligned}
F \times 300+f \times 300 & =R \times 200 \\
100 \times 300+\mu R \times 300 & =R \times 200 \\
100 \times 300+0.3 \times R \times 300 & =R \times 200 \\
1.1 R & =300 \\
R & =272.72 \mathrm{~N} \\
\text { Braking torque } & =\mu R \times r \\
& =0.3 \times 272.72 \times 0.1 \\
T & =8.18 \mathrm{Nm}
\end{aligned}
$$

Hence, the correct answer is 8.18 .
Question Number: 64
Question Type: NAT
Block $P$ of mass 2 kg slides down the surface and has a speed $20 \mathrm{~m} / \mathrm{s}$ at the lowest point. $Q$, where the local radius of curvature is 2 m as shown in the figure. Assuming $g=10 \mathrm{~m} / \mathrm{s}^{2}$. The normal force (in N ) at $Q$ is $\qquad$ (correct to two decimal places).


Solution: Mass of block $P, m=2 \mathrm{~kg}$, Acceleration $g=10 \mathrm{~m} / \mathrm{s}^{2}$
Now using the relation

$$
\begin{aligned}
N-m g & =\frac{m v^{2}}{R} \\
N & =m g+\frac{m v^{2}}{R}
\end{aligned}
$$

$$
N=2 \times 10+\frac{2 \times 20 \times 20}{2}=420 \mathrm{~N} .
$$

Hence, the correct answer is 420 N .
Question Number: 65
Question Type: NAT
An electrochemical machining (ECM) is to be used to cut a through hole into a 12 mm thick aluminum plate. The hole has rectangular cross-section $10 \mathrm{~mm} \times 30 \mathrm{~mm}$. The ECM operation will be accomplished in 2 minute, with efficiency of $90 \%$. Assuming specific removal rate for aluminum as $3.44 \times 10^{-2} \mathrm{~mm}^{3}(\mathrm{As})$, the current (in A ) required is (correct to two decimal places).

Solution: Volume of metal to be removed

$$
\begin{aligned}
& =10 \times 30 \times 12 \\
& =3600 \mathrm{~mm}^{3}
\end{aligned}
$$

Energy Required

$$
\begin{aligned}
& =\frac{\text { Volume of metal }}{\text { Specify removal rate } \times \text { efficiency }} \\
& =\frac{3600}{3.44 \times 10^{-2} \times 0.9} \\
& =\frac{104651.162}{0.9}
\end{aligned}
$$

Energy required $=116279.07 \mathrm{As}$
Given time $=2 \mathrm{~min}=120 \mathrm{~s}$
Current required

$$
\begin{aligned}
& I=\frac{116279.07}{120} \\
& I=968.992
\end{aligned}
$$

Hence, the correct answer is 968.99 .

# GATE 2018 Solved Paper ME: Mechanical Engineering Set - 2 

Number of Questions: 65
Total Marks:100.0

Wrong answer for MCQ will result in negative marks, (-1/3) for 1 Mark Questions and (-2/3) for 2 Marks Questions.

## General Aptitude

## Number of Questions: 10

## Q. 1 to Q. 5 carry 1 mark each and Q. 6 to Q. 10 carry 2 marks each.

## Question Number: 1 <br> Question Type: MCQ

The perimeters of a circle, a square and an equilateral triangle are equal. Which one of the following statements is true?
(A) The circle has the largest area.
(B) The square has the largest area.
(C) The equilateral triangle has the largest area.
(D) All the three shapes have same area.

Solution: The area would increase with the number of sides, if the perimeter of a regular polygon is constant. It would attain its maximum value when the polygon becomes a circle.
Hence, the correct option is (A)

## Question Number: 2

Question Type: MCQ
The value of the expression
$\frac{1}{1+\log _{u} v w}+\frac{1}{1+\log _{v} w u}+\frac{1}{1+\log _{w} u v}$ is $\qquad$ -.
(A) -1
(B) 0
(C) 1
(D) 3

## Solution:

$$
\begin{aligned}
& \frac{1}{1+\log _{u} v w}=\frac{1}{\log _{u} u+\log _{u} v w} \\
& =\frac{1}{\log _{u} u v w}=\log _{u v w} u
\end{aligned}
$$

$\therefore$ The given expression is $\log _{\mathrm{uvw}} u+\log _{\mathrm{uvw}} V+\log _{\mathrm{uvw}}=$ $\log _{\mathrm{uvw}} u \nu w=1$.
Hence, the correct option is (C)

## Question Number: 3

Question Type: MCQ
"The dress $\qquad$ her so well that they all immediately her on her appearance."
(A) complemented, complemented
(B) complimented, complemented
(C) complimented, complimented
(D) complemented, complimented

## Solution:

Hence, the correct option is (D)
Question Number: 4

## Question Type: MCQ

"The judge's standing in the legal community, though shaken by false allegations of wrongdoing, remained
$\qquad$ ."
(A) undiminished
(B) damaged
(C) illegal
(D) uncertain

## Solution:

Hence, the correct option is (A)
Question Number: 5
Question Type: MCQ
Find the missing group of letters in the following series:
BC, FGH, LMNO, $\qquad$
(A) UVWXY
(B) TUVWX
(C) STUVW
(D) RSTUV

Solution: Two letters B and C are written and after that D is omitted. In the next past three letters F, G and H are written and after that I, J and K are omitted. In the next part four letter are written. Following the same pattern, we have to skip four letters (P, Q, R and S). Hence T, U, V, W and X should be written.
Hence, the correct option is (B)
Question Number: 6
Question Type: MCQ
A contract is to be completed in 52 days and 125 identical robots were employed, each operational for 7 hours a day. After 39 days, five-sevenths of the work was completed. How many additional robots would be required to complete the work on time, if each robot is now operational for 8 hours a day?
(A) 50
(B) 89
(C) 146
(D) 175

Solution:
work done $=125(39)(7)$ robot-hours $(r h)$

The work is $5 / 7$ of the total work.
$\therefore$ Number of robots needed

$$
=\frac{50(39)(7)}{13(8)}=\frac{25(21)}{4}=\frac{525}{4}=131.25
$$

$\therefore 11.25$ additional robots are needed.

## Question Number: 7

Question Type: MCQ
A house has a number which needs to be identified. The following three statements are given that can help in identifying the house number.
i. If the house number is a multiple of 3 , then it is a number from 50 to 59 .
ii. If the house number is NOT a multiple of 4 , then it is a number from 60 to 69 .
iii. If the house number is NOT a multiple of 6 , then it is a number from 70 to 79 .
What is the house number?
(A) 54
(B) 65
(C) 66
(D) 76

Solution:
Option (A) is not possible because this is not a multiple of 4 , (from ii) it must be a number from 60 to 69 . It is not.
Option (B) is not possible because this is not a multiple of 6, (from iii) it must be a number from 70 to 79 . It is not. The number cannot be 65 .
Option (C) is not possible because this is a multiple of 3 , (from i) it must be a number from 50 to 59. It is not. The number cannot be 66 .
Option (D) is not possible because this is not a multiple of 3 .
Hence, the correct option is (D)

## Question Number: 8

Question Type: MCQ
An unbiased coin is tossed six times in a row and four different such trials are conducted. One trial implies six tosses of the coin. If H stands for head and T stands for the tail, the following are the observations from the four trials:
(1) HTHTHT
(2) TTHHHT
(3) HTTHHT
(4) $\mathrm{HHHT}_{--}$.

Which statement describing the last two coin tosses of the fourth trial has the highest probability of being correct?
(A) Two $T$ will occur.
(B) One $H$ and one $T$ will occur.
(C) Two $H$ will occur.
(D) One $H$ will be followed by one T .

Solution: Observations in the 4 trials are when coin is tossed 6 times in each trail is given below
(1) $\mathrm{H} T H T H \mathrm{~T}$
(2) $\mathrm{T} T H \mathrm{H} H \mathrm{~T}$
(3) $\mathrm{H} T \mathrm{TH} H \mathrm{~T}$
(4) $\mathrm{HHH}_{--}$

The last two tosses in the fourth trial are independent of all preceding events. To decide which of the event (described in the 4 statements) in most likely, we can simply ignore all the given data.
Choice B in clearly the most likely. ( D is only half as probable as $B$. Also, each of $A$. C is half as probable as B. Also A, B, C are mutually exclusive and collectively exhaustive. Their probabilities are $\frac{1}{4}, \frac{1}{2} \frac{1}{4}$. D represents only half of B.)
Hence, the correct option is (B)
Question Number: 9
Question Type: MCQ
Forty students watched films A, B and C over a week. Each student watched either only one film or all three. Thirteen students watched film A, sixteen students watched film B and nineteen students watched film C. How many students watched all three films?
(A) 0
(B) 2
(C) 4
(D) 8

Solution: Consider the venn diagram given below


As all of them are watching one or three movies, then $b=$ $d=f=0$

$$
\text { Now, } \quad \begin{aligned}
& a+e=13 \\
& c+e=16 \\
& g+e=19
\end{aligned}
$$

Hence,

$$
\begin{equation*}
a+c+g+3 e=48 \tag{1}
\end{equation*}
$$

and

$$
\begin{equation*}
a+c+g+e=40 \tag{2}
\end{equation*}
$$

So, subtracting (1) and (2) we get $2 \mathrm{e}=8$
$\Rightarrow \quad e=4$
Therefore, the values of $a=9, c=12, g=15$.
So, 4 students watched all the three movies.
Hence, the correct option is (C)

## Question Number: 10

Question Type: MCQ
A wire would enclose an area of $1936 \mathrm{~m}^{2}$, if it is bent into a square. The wire is cut into two pieces. The longer piece is thrice as long as the shorter piece. The long and the short pieces are bent into a square and a circle, respectively. Which of the following choices is closest to the sum of the areas enclosed by the two pieces in square meters?
(A) 1096
(B) 1111
(C) 1243
(D) 2486

Solution: Area of the square is $1089 \mathrm{~m}^{2}$.
The smaller one is lent into a circle of radium, say $r$.

$$
2 \pi r=44 \Rightarrow r \approx 7
$$

The area $=\pi r^{2} \approx 22 / 7\left(7^{2}\right) \mathrm{m}^{2}=154 \mathrm{~m}^{2}$.
The sum of the areas $\approx(1089+154) \mathrm{m}^{2}=1243 \mathrm{~m}^{2}$
Hence, the correct option is (C).

## Mechanical Engineering

## Number of Questions: 55

Q. 11 to Q. 25 carry 1 mark each and Q. 26 to Q. 65 carry 2 marks each.

## Question Number: 11

Question Type: MCQ
Select the correct statement for $50 \%$ reaction stage in a steam turbine.
(A) The rotor blade is symmetric.
(B) The stator blade is symmetric.
(C) The absolute inlet flow angle is equal to absolute exit flow angle.
(D) The absolute exit flow angle is equal to inlet angle of rotor blade.

## Solution:

Hence, the correct option is (D)
Question Number: 12
Question Type: MCQ
Denoting $L$ as liquid and $M$ as solid in a phase-diagram with the subscripts representing different phases, a eutectoid reaction is described by
(A) $M_{1} \rightarrow M_{2}+M_{3}$
(B) $L_{1} \rightarrow M_{1}+M_{2}$
(C) $L_{1}+M_{1} \rightarrow M_{2}$
(D) $M_{1}+M_{2} \rightarrow M_{3}^{2}$

Solution: We know that Eutectoid reaction is expressed as

$$
M_{1} \rightarrow M_{2}+M_{3}
$$

Hence, the correct option is (A)
Question Number: 13
Question Type: MCQ
During solidification of pure molten metal, the grains in the casting near the mould wall are
(A) coarse and randomly oriented
(B) fine and randomly oriented
(C) fine and ordered
(D) coarse and ordered

Solution: During solidification of pure molten metal, the grains in the casting near the mould wall are fine and randomly oriented because rate of solidification is high at surface of mould during solidification of pure molten metal. Hence, the correct option is (B)

Section marks: $\mathbf{8 5 . 0}$

## Question Number: 14

Question Type: MCQ
Match the following products with the suitable manufacturing process

|  | Product | Manufacturing process |  |
| :--- | :--- | :--- | :--- |
| P. | Toothpaste tube | 1. | Centrifugal casting |
| Q. | Metallic pipes | 2. | Blow moulding |
| R. | Plastic bottles | 3. | Rolling |
| S. | Threaded bottles | 4. | Impact extrusion |

(A) P-4, Q-3, R-1, S-2
(B) P-2, Q-1, R-3, S-4
(C) P-4, Q-1, R-2, S-3
(D) P-1, Q-3, R-4, S-2

## Solution:

Hence, the correct option is (C)
Question Number: 15
Question Type: MCQ
Feed rate in slab milling operation is equal to
(A) rotation per minute (rpm)
(B) product of rpm and number of teeth in the cutter
(C) product of rpm, feed per tooth and number of teeth in the cutter
(D) product of rpm, feed per tooth and number of teeth in contact

Solution: If $f_{\mathrm{t}}$ is the feed per tooth, $z$ is the number of teeth in cutter and $N$ is the revolution per minute then feed rate can be expressed as

$$
f=f_{\mathrm{t}} \times N \times Z
$$

Hence, the correct option is (C)
Question Number: 16
Question Type: MCQ
Metal removal in electric discharge machining takes place through
(A) ion displacement
(B) melting and vaporization
(C) corrosive reaction
(D) plastic shear

## Solution:

Hence, the correct option is (B)
Question Number: 17
Question Type: MCQ
The preferred option for holding an odd-shaped workpiece in a centre lathe is
(A) live and dead centres
(B) three jaw chuck
(C) lathe dog
(D) four jaw chuck

## Solution:

Hence, the correct option is (D)
Question Number: 18
Question Type: MCQ
A local tyre distributor expects to sell approximately 9600 steel belted radial tyres next year. Annual carrying cost is $₹ 16$ per tyre and ordering cost is ₹ 75 . The economic order quantity of the tyres is
(A) 64
(B) 212
(C) 300
(D) 1200

Solution: Number of tyres sold $D=9600$
Annual carrying cost $C_{\mathrm{n}}=₹$ 16/year
Annual ordering cost $C_{0}=₹ 75 /$ order
We know that economic order quantity can be expressed as

$$
\begin{aligned}
\mathrm{EOQ} & =\sqrt{\frac{2 D C}{C_{h}}}=\sqrt{\frac{2 \times 9600 \times 75}{16}} \\
& =\sqrt{1200 \times 75}=300
\end{aligned}
$$

Hence, the correct option is (C)
Question Number: 19
Question Type: NAT
If $A=\left[\begin{array}{lll}1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 1\end{array}\right]$ then $\operatorname{det}\left(A^{-1}\right)$ is $\qquad$ (correct to two
decimal places).
Solution: Given $A=\left[\begin{array}{lll}1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 1\end{array}\right]$

$$
\operatorname{det}\left(A^{-1}\right)=\frac{1}{\operatorname{det}(A)}=\frac{1}{1 \times 4 \times 1}=\frac{1}{4}=0.25
$$

Hence, the correct answer is 0.25 .
Question Number: 20
Question Type: NAT
A hollow circular shaft inner radius 10 mm , outer radius 20 mm and length 1 m is to be used as a torsional spring. If the shear modulus of the material of the shaft is 150 GPa , the torsional stiffness of the shaft (in $\mathrm{kN}-\mathrm{m} / \mathrm{rad}$ ) is $\qquad$ (correct to two decimal places).

Solution: We know that torsional stiffness of the shaft (in $\mathrm{kN}-\mathrm{m} / \mathrm{rad}$ ) can be expressed as

$$
\begin{aligned}
\text { Torsional stiffness } & =\frac{G I}{L} \\
& =\frac{150 \times 10^{9} \times \frac{\pi}{32}\left[0.04^{4}-0.02^{4}\right]}{1} \\
& =35.342 \mathrm{kNm} / \mathrm{rad}
\end{aligned}
$$

Hence, the correct answer is 35.342 .
Question Number: 21
Question Type: MCQ
The Fourier cosine series for an even function $f(x)$ is given by

$$
f(x)=a_{0}+\sum_{n=1}^{\infty} a_{\mathrm{n}} \cos (n x) .
$$

The value of the coefficient $a_{2}$ for the function $f(x)=\cos ^{2}(x)$ in $[0, \pi]$ is
(A) -0.5
(B) 0.0
(C) 0.5
(D) 1.0

Solution: The given function is $f(x)=\cos ^{2}(x)$
Fourier cosine series for an even function $f(x)$ will be

$$
\begin{aligned}
a_{2} & =\frac{2}{\pi} \int_{0}^{\pi} f(x) \cos 2 x d x \\
& =\frac{2}{\pi} \int_{0}^{\pi} \cos ^{2}(x) \cdot \cos 2 x d x \\
& =\frac{2}{\pi} \int_{0}^{\pi}\left[\frac{1+\cos 2 x}{2}\right] \cos 2 x d x \\
& =\frac{1}{\pi} \int_{0}^{\pi}\left[\cos 2 x+\cos ^{2}(2 x)\right] d x \\
& =\frac{1}{\pi} \int_{0}^{\pi}\left[\cos 2 x+\left(\frac{1+\cos 4 x}{2}\right)\right] d x \\
& =\frac{1}{\pi} \int_{0}^{\pi}\left[\cos 2 x+\frac{1}{2}+\frac{1}{2} \cos 4 x\right] d x \\
& =\frac{1}{\pi}\left[\frac{\sin 2 x}{2}+\frac{x}{2}+\frac{1}{2}\left(\frac{\sin 4 x}{4}\right)\right]_{0}^{\pi} \\
& =\frac{1}{\pi} \times \frac{\pi}{2}=\frac{1}{2}=0.5
\end{aligned}
$$

Hence, the correct option is (C)

Question Number: 22
Question Type: MCQ
The divergence of the vector field $\vec{u}=e^{x}(\cos y \hat{i}+\sin y \hat{j})$ is
(A) 0
(B) $e^{x} \cos y+e^{x} \sin y$
(C) $2 e^{x} \cos y$
(D) $2 e^{x} \sin y$

Solution: The divergence of the vector field is given as

$$
\begin{aligned}
\vec{u} & =e^{x}(\cos y \hat{i}+\sin y \hat{j}) \\
& =e^{x} \cos y \hat{i}+e^{x} \sin y \hat{j} \\
\therefore \quad \operatorname{div}(\vec{u}) & =\frac{\partial}{\partial x}\left(e^{x} \cos y\right)+\frac{\partial}{\partial y}\left(e^{x} \sin y\right) \\
& =e^{x} \cos y+e^{x} \cos y \\
& =2 e^{x} \cos y
\end{aligned}
$$

Hence, the correct option is (C)
Question Number: 23
Question Type: MCQ
Consider a function $u$ which depends on position $x$ and time $t$. The partial differential equation $\frac{\partial u}{\partial t}=\frac{\partial^{2} u}{\partial x^{3}}$ is known as the
(A) Wave equation
(B) Heat equation
(C) Laplace's equation
(D) Elasticity equation

## Solution:

Hence, the correct option is (B)
Question Number: 24
Question Type: MCQ
If $y$ is the solution of the differential equation $y^{3} \frac{d y}{d x}+$ $x^{3}=0, y(0)=1$, the value of $y(-1)$ is
(A) -2
(B) -1
(C) 0
(D) 1

Solution: Given differential equation is

$$
\begin{equation*}
y^{3} \frac{d y}{d x}+x^{3}=0 \tag{1}
\end{equation*}
$$

and

$$
\begin{array}{rlrl}
y(0) & =1 \\
& & y^{3} \frac{d y}{d x}+x^{3} & =0 \\
\Rightarrow & y^{3} \frac{d y}{d x} & =-x^{3} \\
\Rightarrow & y^{3} d y & =-x^{3} d x \\
\Rightarrow & \int y^{3} d y & =-\int x^{3} d x \\
\Rightarrow & \frac{y^{4}}{4} & =-\frac{x^{4}}{4}+C
\end{array}
$$

$\Rightarrow \quad x^{4}+y^{4}=4 C$
From (2), $\quad 0^{4}+1^{4}=4 C$
$\Rightarrow \quad 4 C=1$
$\therefore$ (3) becomes,

$$
\begin{array}{rlrl} 
& x^{4}+y^{4} & =1 \\
\text { At } x=-1, & (-1)^{4}+y^{4}=1 & \Rightarrow 1+y^{4}=1 \\
\Rightarrow & y & =0 & \Rightarrow y(-1)=0
\end{array}
$$

Hence, the correct option is (C)
Question Number: 25
Question Type: MCQ
The minimum axial compressive load, $P$, required to initiate buckling for a pinned-pinned slender column with bending stiffness EI and length $L$ is
(A) $P=\frac{\pi^{2} E I}{4 L^{2}}$
(B) $P=\frac{\pi^{2} E I}{L^{2}}$
(C) $P=\frac{3 \pi^{2} E I}{4 L^{2}}$
(D) $P=\frac{4 \pi^{2} E I}{L^{2}}$

## Solution:

Hence, the correct option is (B).
Question Number: 26
Question Type: MCQ
A frictionless gear train is shown in the figure. The leftmost 12 -teeth gear is given a torque of $100 \mathrm{~N}-\mathrm{m}$. The output torque from the 60 -teeth gear on the right in $\mathrm{N}-\mathrm{m}$ is

(A) 5
(B) 20
(C) 500
(D) 2000

Solution: Torque of 12-teeth gear $T=100 \mathrm{~N} \mathrm{~m}$
Let the output torque from the 60 -teeth gear on the right be $T_{4}=$ ?
Now we know that

$$
N_{2}=N_{1} \times \frac{T_{1}}{T_{2}}=N_{1} \times \frac{12}{48}=\frac{N_{1}}{4}
$$

$N_{3}$ and $N_{2}$ rotates at same speed.

$$
\begin{aligned}
N_{4} & =N_{3} \times \frac{T_{3}}{T_{4}}=\frac{N_{1}}{4} \times \frac{12}{60}=\frac{N_{1}}{20} \\
T_{1} \times N_{1} & =T_{4} \times N_{4}
\end{aligned}
$$

$$
\begin{aligned}
& T_{4}=\frac{T_{1} \times N_{1}}{\frac{N_{1}}{20}} \\
& T_{4}=20 \mathrm{~T}_{1}=2000 \mathrm{~N}-\mathrm{m}
\end{aligned}
$$

Hence, the correct option is (D)
Question Number: 27
Question Type: MCQ
In a single degree of freedom underdamped spring-massdamper system as shown in the figure, an additional damper is added in parallel such that the system still remains underdamped. Which one of the following statements is ALWAYS true?

(A) Transmissibility will increase
(B) Transmissibility will decrease
(C) Time period of free oscillations will increase.
(D) Time period of free oscillations will decrease.

Solution: We know that transmissibility, can be expressed as

$$
\epsilon=\frac{\sqrt{1+\left(\frac{2 \xi \omega}{\omega_{n}}\right)}}{\sqrt{\left[1-\left[\frac{\omega}{\omega_{n}}\right]^{2}\right]^{2}+\left[\frac{2 \xi \omega}{\omega_{n}}\right]^{2}}}
$$

When the additional damper is added parallelly, then the damping will increase and hence the transmissibility $\xi$ will increase.

$$
\omega_{\mathrm{d}}=\sqrt{1-\xi^{2}} \times \omega_{n}
$$

$\omega_{\mathrm{d}}$ will decrease because $\xi^{2}$ will increase.

$$
T=\frac{2 \pi}{\omega_{d}}
$$

$\therefore$ Time period will increase.
Hence, the correct option is (C)
Question Number: 28
Question Type: MCQ
Pre-tensioning of a bolted joint is used to
(A) strain harden the bolt head
(B) decrease stiffness of the bolted joint
(C) increase stiffness of the bolted joint
(D) prevent yielding of the thread root

Solution: Pre-tensioning of bolt increases stiffness of the bolted joint.
Hence, the correct option is (C)
Question Number: 29
Question Type: MCQ
The peak wavelength of radiation emitted by a black body at a temperature of 2000 K is $1.45 \mu \mathrm{~m}$. If the peak wavelength of emitted radiation changes to $2.90 \mu \mathrm{~m}$, then the temperature (in K ) of the black body is
(A) 500
(B) 1000
(C) 4000
(D) 8000

## Solution:

Initial peak Wavelength $\lambda_{1}=1.45 \mu \mathrm{~m}$
Initial Temperature $T_{1}=2000 \mathrm{~K}$
Final peak wavelength $\lambda_{2}=2.90 \mu \mathrm{~m}$
Let the final temperature be $T_{2}=$ ?
Now using the relation given below

$$
\begin{aligned}
\lambda_{1} T_{1} & =\lambda_{2} T_{2} \\
1.45 \times 2000 & =T_{2} \times 2.90 \\
T_{2} & =\frac{2000}{2}=1000 \mathrm{~K}
\end{aligned}
$$

Hence, the correct option is (B)
Question Number: 30
Question Type: MCQ
For an ideal gas with constant properties undergoing a quasi-static process, which one of the following represents the change of entropy $(\Delta s)$ from state 1 to 27
(A) $\Delta s=C_{\mathrm{p}} \ln \left(\frac{T_{2}}{T_{1}}\right)-R \ln \left(\frac{P_{2}}{P_{1}}\right)$
(B) $\Delta s=C_{\mathrm{v}} \ln \left(\frac{T_{2}}{T_{1}}\right)-C_{\mathrm{p}} \ln \left(\frac{V_{2}}{V_{1}}\right)$
(C) $\Delta s=C_{p} \ln \left(\frac{T_{2}}{T_{1}}\right)-C_{\mathrm{v}} \ln \left(\frac{P_{2}}{P_{1}}\right)$
(D) $\Delta s=C_{v} \ln \left(\frac{T_{2}}{T_{1}}\right)+R \ln \left(\frac{V_{1}}{V_{2}}\right)$

## Solution:

Hence, the correct option is (A)
Question Number: 31
Question Type: NAT
Fatigue life of a material for a fully reversed loading condition is estimated from $\sigma_{a}=1100 \mathrm{~N}^{-0.15}$, where $\sigma_{\mathrm{a}}$ is the stress amplitude in MPa and $N$ is the failure life in cycles. The maximum allowable stress amplitude (in MPa ) for a life of $1 \times 10^{5}$ cycles under the same loading condition is
$\qquad$ (correct to two decimal places).

## Solution:

The maximum allowable stress amplitude (in MPa) can be expressed as

$$
\begin{aligned}
\sigma_{\max } & =1100 \mathrm{~N}^{-0.15} \\
& =1100 \times\left(10^{5}\right)^{-0.15} \\
& =\frac{1100}{5.62} \\
& =195.61 \mathrm{MPa}
\end{aligned}
$$

Hence, the correct answer is 195.61 .
Question Number: 32
Question Type: NAT
The viscous laminar flow of air over a flat plate results in the formation of a boundary layer. The boundary layer thickness at the end of the plate of length $L$ is $\delta_{\mathrm{L}}$. When the plate length is increased to twice its original length, the percentage change in laminar boundary layer thickness at the end of the plate (with respect to $\delta_{\mathrm{L}}$ ) is $\qquad$ (correct to two decimal places).

Solution: Consider the figure given below


As per problem $L_{2}=2 L$
We know that for laminar flow,

$$
\begin{aligned}
\delta & =\frac{5 x}{\sqrt{R e_{x}}} \\
\delta_{1} & \propto x^{1 / 2} \\
\frac{\delta_{1}}{\sqrt{x_{1}}} & =\frac{\delta_{2}}{\sqrt{x_{2}}} \\
\delta_{2} & =\sqrt{\frac{x_{2}}{x_{1}}} \times \delta_{1} \\
\delta_{2} & =\sqrt{\frac{2 L}{L}} \times \delta_{1} \\
\delta_{2} & =\sqrt{2} \times \delta_{1} \\
\frac{\delta_{2}}{\delta_{1}} & =1.414 \\
\% \text { change } & =\frac{1.414-1}{1} \times 100 \\
& =41.4 \%
\end{aligned}
$$

Hence, the correct answer is 41.4.

Question Number: 33
Question Type: NAT
An engine operates on the reversible cycle as shown in figure. The work output from the engine (in $\mathrm{kJ} / \mathrm{cycle}$ ) is
$\qquad$ (correct to two decimal places)


Solution: Consider the figure given below


From the above given figure required work can be calculated as

$$
\begin{aligned}
\text { Work output } & =\frac{1}{2} \times A B \times A C \\
& =\frac{1}{2} \times(650-400) \times(2.5-2) \\
& =62.5 \mathrm{~kJ}
\end{aligned}
$$

Hence, the correct answer is 62.5 .
Question Number: 34
Question Type: NAT
The arrival of customers over fixed time intervals in a bank follow a Poisson distribution with an average of 30 customers/hour. The probability that the time between successive customer arrival is between 1 and 3 minutes is $\qquad$ (correct to two decimal places).

Solution: Arrival rate,

$$
\begin{aligned}
& \lambda=30 / \text { hour } \\
& \lambda=\frac{1}{2} \mathrm{~min}
\end{aligned}
$$

The probability can be expressed as

$$
P=1-e^{-\lambda t}
$$

probability for time $t=1$ minute will be

$$
P(1)=1-e^{-\frac{1}{2}}=0.393
$$

probability for time $t=3$ minute will be

$$
P(3)=1-e^{-\frac{3}{2}}=0.7768
$$

The probability that the time between successive customer arrival is between 1 and 3 minutes will be

$$
\begin{aligned}
P(1 \leq T \leq 3 \mathrm{~min}) & =0.7768-0.393 \\
& =0.383
\end{aligned}
$$

Hence, the correct answer is 0.383 .
Question Number: 35
Question Type: NAT
A ball is dropped from rest from a height of 1 m in a frictionless tube as shown in the figure. If the tube profile is approximated by two straight lines (ignoring the curved position), the total distance travelled (in m ) by the ball is
$\qquad$ (correct to two decimal places).


Solution: Consider the figure given below


From the above figure we conclude that

$$
\begin{aligned}
& \frac{B C}{A B}=\sin 45^{\circ} \\
& A B=\frac{B C}{\sin 45^{\circ}}=\frac{1}{\sin 45^{\circ}}=1.4142 \mathrm{~m}
\end{aligned}
$$

Total distance travelled by ball will be

$$
\begin{aligned}
D & =O A+A B \\
& =1+1.4142=2.414 \mathrm{~m}
\end{aligned}
$$

Hence, the correct answer is 2.414 .

## Question Number: 36

## Question Type: MCQ

Let $z$ be a complex variable. For a counter-clockwise integration around a unit circle $C$, centered at origin.

$$
\oint_{\mathrm{C}} \frac{1}{5 z-4} d z=A \pi i
$$

the value of $A$ is
(A) $2 / 5$
(B) $1 / 2$
(C) 2
(D) $4 / 5$

Solution: Let $\oint_{\mathrm{C}} \frac{1}{5 z-4} d z$
$z=\frac{4}{5}$ is a singularity of $\frac{1}{5 z-4}$ and it lies inside $C$.


$$
\begin{aligned}
\therefore \quad I= & \oint_{\mathrm{C}} \frac{1}{5 z-4} d z \\
& \oint_{\mathrm{C}} \frac{1}{5\left(z-\frac{4}{5}\right)} d z \\
& =\frac{1}{5} \oint_{\mathrm{C}} \frac{1}{5 z-\frac{4}{5}} d z=\frac{1}{5} \times 2 \pi i
\end{aligned}
$$

(By Cauchy's integral formula)
$\therefore \quad I=\oint_{\mathrm{C}} \frac{1}{5 z-4} d z=\frac{2}{5} \pi i$
Given

$$
\oint_{\mathrm{C}} \frac{1}{5 z-4} d z=A \pi i
$$

$\Rightarrow \quad \frac{2}{5} \pi i=A \pi i$
$\Rightarrow \quad A=\frac{2}{5}$
Hence, the correct option is (A)
Question Number: 37
Question Type: MCQ
Let $X_{1}$ and $X_{2}$ be two independent exponentially distributed random variables with means 0.5 and 0.25 , respectively.
Then $Y=\min \left(X_{1}, X_{2}\right)$ is
(A) exponentially distributed with mean $1 / 6$
(B) exponentially distributed with mean 2
(C) normally distributed with means $3 / 4$
(D) normally distributed with mean $1 / 6$

Solution: As $X_{1}$ and $X_{2}$ are two independent exponentially distributed random variables, $Y=\min \left(X_{1}, X_{2}\right)$ is also exponentially distributed
If $\theta_{1}$ and $\theta_{2}$ be the parameters of $X_{1}$ and $X_{2}$ respectively
We know that mean of $X_{1}=E\left(X_{1}\right)=0.5$

$$
\begin{array}{ll}
\text { i.e., } & \frac{1}{\theta_{1}}=0.5 \\
\Rightarrow & \theta_{1}=\frac{1}{0.5}=2
\end{array}
$$

Similarly mean of $X_{2}=E\left(X_{2}\right)=0.25$

$$
\begin{array}{ll}
\text { i.e., } & \frac{1}{\theta_{2}}=0.25 \\
\Rightarrow & \theta_{2}=\frac{1}{0.25}=4
\end{array}
$$

$\therefore$ The parameter of $Y=\min \left(X_{1}, X_{2}\right)$

$$
\begin{array}{cc}
\text { is } \theta_{1}+\theta_{2}=2+4=6 \\
\therefore \quad & \text { Mean of } Y=\frac{1}{\theta_{1}+\theta_{2}}=\frac{1}{6}
\end{array}
$$

Hence, the correct option is (A)
Question Number: 38
Question Type: MCQ
For a position vector $\bar{r}=x \bar{i}+y \bar{j}+z \bar{k}$ the norm of the vector can be defined as $|\vec{r}|=\sqrt{x^{2}+y^{2}+z^{2}}$. Given a function $\phi=\ln |\vec{r}|$, its gradient $\phi$ is:
(A) $\vec{r}$
(B) $\frac{\vec{r}}{|\vec{r}|}$
(C) $\frac{\vec{r}}{\vec{r} \cdot \vec{r}}$
(D) $\frac{\vec{r}}{|\vec{r}|^{3}}$

## Solution:

The position vector is $\bar{r}=x \bar{i}+y \bar{j}+z \bar{k}$
The given function is expressed as

$$
\begin{aligned}
\phi & =\ln |\bar{r}|=\ln \left(\sqrt{x^{2}+y^{2}+z^{2}}\right) \\
& =\ln \left(x^{2}+y^{2}+z^{2}\right)^{1 / 2} \\
& =\frac{1}{2} \ln \left(x^{2}+y^{2}+z^{2}\right)
\end{aligned}
$$

$$
\begin{align*}
\therefore \quad \nabla \phi= & \bar{i} \frac{\partial}{\partial x}\left[\frac{1}{2} \ln \left(x^{2}+y^{2}+z^{2}\right)\right] \\
& +\bar{j} \frac{\partial}{\partial y}\left(\frac{1}{2} \ln \left(x^{2}+y^{2}+z^{2}\right)\right) \\
& +\bar{k} \frac{\partial}{\partial z}\left(\frac{1}{2} \ln \left(x^{2}+y^{2}+z^{2}\right)\right) \tag{1}
\end{align*}
$$

Consider $\frac{\partial}{\partial x}\left(\frac{1}{2} \ln \left(x^{2}+y^{2}+z^{2}\right)\right)$

$$
=\frac{1}{2}, \frac{2 x}{x^{2}+y^{2}+z^{2}}=\frac{x}{x^{2}+y^{2}+z^{2}}
$$

As per symmetry,

$$
\begin{equation*}
\frac{\partial}{\partial y}\left(\frac{1}{2} \ln \left(x^{2}+y^{2}+z^{2}\right)\right)=\frac{y}{x^{2}+y^{2}+z^{2}} \tag{2}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{\partial}{\partial z}\left(\frac{1}{2} \ln \left(x^{2}+y^{2}+z^{2}\right)\right)=\frac{z}{x^{2}+y^{2}+z^{2}} \tag{3}
\end{equation*}
$$

Substituting (2) and (3) in equation (1), we get

$$
\begin{aligned}
\nabla \phi= & \bar{i} \frac{x}{x^{2}+y^{2}+z^{2}}+\bar{j} \frac{y}{x^{2}+y^{2}+z^{2}} \\
& +\bar{k} \frac{z}{x^{2}+y^{2}+z^{2}} \\
& =\frac{x \bar{i}+y \bar{i}+z \bar{k}}{x^{2}+y^{2}+z^{2}}=\frac{\bar{r}}{|\bar{r}|^{2}}=\frac{\bar{r}}{\bar{r} \cdot \bar{r}}
\end{aligned}
$$

Hence, the correct option is (C)
Question Number: 39
Question Type: MCQ
In a rigid body in plane motion the point $R$ is acclerating with respect to point $P$ at $10 \angle 180^{\circ} \mathrm{m} / \mathrm{s}^{2}$. If the instantaneous acceleration of point $Q$ is zero, the acceleration (in $\mathrm{m} / \mathrm{s}^{2}$ ) of point $R$ is

(A) $8 \angle 233^{\circ}$
(B) $10 \angle 225^{\circ}$
(C) $10 \angle 217^{\circ}$
(D) $8 \angle 217^{\circ}$

Solution: Consider the figure given below


We know that

$$
\begin{aligned}
\tan \theta & =\frac{16}{12}=\frac{4}{3} \\
\theta & =53.13^{\circ},
\end{aligned}
$$

Therefore $\alpha=90+\theta=36.86^{\circ}$ centripetal acceleration can be expressed as

$$
\begin{aligned}
\alpha_{\mathrm{RP}} & =r \omega^{2} \\
10 & =20 \omega^{2} \\
\omega^{2} & =0.5 \\
a_{\mathrm{R}} & =(Q R) \omega^{2}=16 \times 0.5 \\
a_{\mathrm{R}} & =8 \mathrm{~m} / \mathrm{s}^{2} \\
\text { angle } & =180+\alpha=180+36.86 \\
& =217^{\circ} \\
a_{\mathrm{R}} & =8 \angle 217^{\circ}
\end{aligned}
$$

Hence, the correct option is (D)

## Question Number: 40

## Question Type: MCQ

A rigid rod of lengh 1 m is resting at an angle $\theta=45^{\circ}$ as shown in the figure. The end $P$ is dragged with a velocity of $U=5 \mathrm{~m} / \mathrm{s}$ to the right. At the instant shown, the magnitude of the velocity $V$ (in $\mathrm{m} / \mathrm{s}$ ) of point $Q$ as it moves along the wall without losing contact is

(A) 5
(B) 6
(C) 8
(D) 10

Solution: Consider the figure given below


In the above figure $I$ is Instantaneous centre, therefore

$$
\begin{aligned}
& V=I Q \times \omega \\
& U=I P \times \omega
\end{aligned}
$$

Dividing (1) and (2) we get

$$
\begin{aligned}
\frac{V}{U} & =\frac{I Q}{I P}=\frac{L \cos 45^{\circ}}{L \sin 45^{\circ}} \\
V & =U=5 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Hence, the correct option is (A)
Question Number: 41
Question Type: MCQ
A bar of circular cross section is clamped ends $P$ and $Q$ as shown in the figure. A torsional moment $T=150 \mathrm{Nm}$ is applied at a distance of 100 mm from end P . The torsional reactions $\left(T_{\mathrm{P}}, T_{\mathrm{Q}}\right)$ in Nm at the ends $P$ and $Q$ respectively are

(A) $(50,100)$
(B) $(75,75)$
(C) $(100,50)$
(D) $(120,30)$

## Solution:

torsional moment $T=150 \mathrm{Nm}$
Distance $=100 \mathrm{~mm}$
Angular twist $q$ is equal in both sections.

$$
\begin{aligned}
\theta_{1} & =\theta_{2} \\
\frac{T_{1} L_{1}}{G J} & =\frac{T_{2} L_{2}}{G J} \\
T_{1} \times 100 & =T_{2} \times 200 \\
T_{1} & =2 T_{2}
\end{aligned}
$$

Torques in both sections, $T_{1}+T_{2}=T$

$$
\begin{aligned}
T_{1}+T_{2} & =150 \\
3 T_{2} & =150 \\
T_{2} & =50 \mathrm{Nm} \\
T_{1} & =2 \times 50 \mathrm{Nm}=100 \mathrm{Nm}
\end{aligned}
$$

Hence, the correct option is (C)
Question Number: 42
Question Type: MCQ
In a cam-follower, the follower rises by $h$ as the cam rotates by $\delta$ (radians) at constant angular velocity $w$ (radians). The follower is uniformly accelerating during the first half of the rise period and it is uniformly decelerating in the latter half of the rise period. Assuming that the magnitude of the acceleration and deceleration are same, the maximum velocity of the follower is
(A) $\frac{4 h \omega}{\delta}$
(B) $h \omega$
(C) $\frac{2 h \omega}{\delta}$
(D) $2 h \omega$

Solution: For constant acceleration motion, according to second equation of motion

$$
\begin{aligned}
& S=u t+\frac{1}{2} a t^{2} \\
& S=\frac{1}{2} a t^{2} \\
& a=\frac{2 S}{t^{2}}
\end{aligned}
$$

where $S=\frac{h}{2}$,
also

$$
\begin{aligned}
\theta & =\omega t \\
\frac{\delta}{2} & =\omega t \\
t & =\frac{\delta}{2 \omega} \\
a & =\frac{2\left(\frac{h}{2}\right)}{\left(\frac{\delta}{2 \omega}\right)^{2}} \\
a & =\frac{4 h \omega^{2}}{\delta^{2}}
\end{aligned}
$$

we know that when the acceleration is constant, then velocity is linear

$$
U=\frac{d s}{d t}=a t
$$

$$
\begin{aligned}
U & =\frac{4 h \omega^{2}}{\delta^{2}} \times \frac{\theta}{\omega} \\
U & =\frac{4 h \omega \theta}{\delta^{2}} \\
\theta & =0, U=0 \\
\theta & =\frac{\delta}{2}, U=U_{\max }=\frac{4 h \omega}{\delta^{2}} \times \frac{\delta}{2} \\
U_{\max } & =\frac{2 h \omega}{\delta}
\end{aligned}
$$

Hence, the correct option is (C)

## Question Number: 43

## Question Type: MCQ

A bimetallic cylindrical bar of cross sectional area $1 \mathrm{~m}^{2}$ is made by bonding Steel (Young's modulus $=210 \mathrm{GPa}$ ) and Aluminium (Young's modulus $=70 \mathrm{GPa}$ ) as shown in the figure. To maintain axial strain of magnitude $10^{-6}$ in Steel bar and compressive axial strain of magnitude $10^{-6}$ in Aluminium bar, the magnitude of the required force $P$ (in kN ) along the indicated direction is

(A) 70
(B) 140
(C) 210
(D) 280

## Solution:



Young's modulus of steel $=210 \mathrm{GPa}$
Young's modulus of alunimium $=70 \mathrm{GPa}$
The magnitude of required force will be

$$
\begin{aligned}
& P_{\mathrm{s}}+P_{\mathrm{Al}}=P \\
& \Delta_{\mathrm{S}}+\Delta_{\mathrm{Al}}=0 \\
& \frac{P_{\mathrm{s}} \times \frac{L}{2}}{A E_{\mathrm{s}}}+\frac{\left(P_{\mathrm{s}}-P\right) \frac{L}{2}}{A E_{\mathrm{AI}}}=0 \\
& \frac{P_{\mathrm{s}}}{210}+\frac{P_{\mathrm{s}}-P}{70}=0 \\
& P_{\mathrm{s}}=\frac{3 \mathrm{P}}{4}
\end{aligned}
$$

$$
\begin{aligned}
\Delta_{\mathrm{s}} & =\frac{P_{\mathrm{s}} \times \frac{L}{2}}{A E_{\mathrm{s}}} \\
\epsilon_{\mathrm{s}} & =\frac{P_{\mathrm{s}}}{A E_{\mathrm{s}}}=\frac{3 P}{4 A E_{\mathrm{s}}} \\
P & =\frac{E_{s} \times 4 \times A \times \epsilon_{s}}{3} \\
& =\frac{10^{-6} \times 4 \times 1 \times 2.10 \times 10^{9}}{3} \\
P & =280 \mathrm{kN}
\end{aligned}
$$

Hence, the correct option is (D)

## Question Number: 44

Question Type: MCQ
Air flows at the rate of $1.5 \mathrm{~m}^{3} / \mathrm{s}$ through a horizontal pipe with a gradually reducing cross-section as shown in the figure. The two cross-sections of the pipe have diameters of 400 mm and 200 mm . Take the air density as $1.2 \mathrm{~kg} / \mathrm{m}^{3}$ and assume inviscid incompressible flow. The change in pressure $\left(p_{2}-p_{1}\right)$ (in kPa ) between section 1 and 2 is

(A) -1.28
(B) 2.56
(C) -2.13
(D) 1.28

Solution: We know that

$$
\begin{gathered}
\frac{P_{1}}{\rho g}+\frac{V_{1}^{2}}{2 g}+Z_{1}=\frac{P_{2}}{\rho g}+\frac{V_{2}^{2}}{2 g}+Z_{2} \\
\because \\
Z_{1}=Z_{2} \\
\frac{P_{1}}{\rho g}+\frac{V_{1}^{2}}{2 g}=\frac{P_{2}}{\rho g}+\frac{V_{2}^{2}}{2 g} \\
A_{1}=\frac{\pi}{4} \times 0.4^{2}=0.1256 \mathrm{~m}^{2} \\
A_{2}=\frac{\pi}{4} \times 0.2^{2}=0.0314 \mathrm{~m}^{2} \\
\frac{P_{2}-P_{1}}{\rho g}+\frac{V_{1}^{2}-V_{2}^{2}}{2 g} \\
Q= \\
\frac{A_{1} V_{1}=A_{2} V_{2} \quad(\text { continuity equation })}{2}=\frac{\left(\frac{Q}{A_{1}}\right)^{2}-\left(\frac{Q}{A_{2}}\right)^{2}}{2}
\end{gathered}
$$

$$
\begin{aligned}
& P_{2}-P_{1}=\frac{1.2}{2}\left[\left(\frac{1.5}{0.1256}\right)^{2}-\left(\frac{1.5}{0.0314}\right)^{2}\right] \\
& P_{2}-P_{1}=-1.28 \mathrm{kPa}
\end{aligned}
$$

Hence, the correct option is (A)
Question Number: 45
Question Type: MCQ
The problem of maximizing $z=x_{1}-x_{2}$ subject to constraints $x_{1}+x_{2} \leq 10, x_{1} \geq 0, x_{2} \geq 0$ and $x_{2} \leq 5$ has
(A) no solution
(B) one solution
(C) two solutions
(D) more than two solutions

Solution: As per problem,

$$
Z=X_{1}-X_{2}
$$

Consider, the conditions

$$
\begin{aligned}
X_{1}+X_{2} & =\leq 10 \\
X_{1}+X_{2} & =10 \\
X_{1} & =0, X_{2}=10 \\
X_{1} & =10, X_{2}=0 \\
X_{1} & =5, X_{2}=5
\end{aligned}
$$



$$
\begin{aligned}
Z(0,5) & =0-5=-5 \\
Z(5,5) & =5-5=0 \\
Z(10,0) & =10-0=10 \\
Z_{\max } & =10 \text { at }(10,10)
\end{aligned}
$$

Hence, the correct option is (B)
Question Number: 46
Question Type: NAT
Given the ordinary differential equation $\frac{d^{2} y}{d x^{3}}+\frac{d y}{d x}-6 y=0$ with $y(0)=0$ and $\frac{d y}{d x}(0)=1$, the value of $y(1)$ is $\qquad$ (correct to two decimal places).
Solution: The ordinary differential equation is given as

$$
\begin{equation*}
\frac{d^{2} y}{d x^{2}}+\frac{d y}{d x}-6 y=0 \tag{1}
\end{equation*}
$$

where $\quad y(0)=0$ and $\frac{d y}{d x}(0)=1$
Applying Laplace transform on both sides of Equation (1), we get

$$
\begin{array}{ll} 
& L\left[\frac{d^{2} y}{d x^{2}}+\frac{d y}{d x}-6 y\right]=L[0] \\
\Rightarrow & L\left[\frac{d^{2} y}{d x^{2}}\right]+L\left[\frac{d y}{d x}\right]-6 L[y]=0 \\
\Rightarrow & {\left[s^{2} \bar{y}-\operatorname{sy}(0)-\frac{d y}{d x}(0)\right]+[s \bar{y}-y(0)]-6 \bar{y}=0}
\end{array}
$$

where $\bar{y}=L[y]$

$$
\begin{array}{cc}
\Rightarrow & s^{2} \bar{y}-s \times 0-1+s \bar{y}-0-6 \bar{y}=0 \\
\Rightarrow & \begin{array}{c}
\left(s^{2}+s-6\right) \bar{y}=1 \\
\Rightarrow
\end{array} \\
& =\frac{1}{5}=\frac{1}{s^{2}+s-6}\left[\frac{1}{(s-2)}-\frac{1}{(s+3)}\right] \\
\therefore & \begin{aligned}
&(s+3)(s-2) \\
& \therefore=L^{-1}[\bar{y}]=L^{-1}\left[\frac{1}{5}\left(\frac{1}{s-2}-\frac{1}{s+3}\right)\right] \\
&=\frac{1}{5}\left(L^{-1}\left[\frac{1}{s-2}\right]-L^{-1}\left[\frac{1}{s+3}\right]\right) \\
& \therefore y
\end{aligned} \\
& =\frac{1}{5}\left[e^{2 x}-e^{-3 x}\right]
\end{array}
$$

Now $y(1)=y_{\mathrm{a} t \mathrm{x}=1}$

$$
=\frac{1}{5}\left[e^{2}-e^{-3}\right]=1.4679
$$

Hence, the correct answer is 1.468 .
Question Number: 47

## Question Type: NAT

A thin-walled cylindrical can with rigid end caps has a mean radius $R=100 \mathrm{~mm}$ and a wall thickness of $t=5$ mm . The can is pressurized and an additional tensile stress of 50 MPa is imposed along the axial direction as shown in the figure. Assume that the state of stress in the wall is uniform along its length. If the magnitudes of axial and circumferential components of stress in the can are equal, the pressure (in MPa) inside the can is $\qquad$ (correct to two decimal places).


## Solution:

mean radius $R=100 \mathrm{~mm}$
wall thickness of $t=5 \mathrm{~mm}$
Additional tensile stress $=50 \mathrm{MPa}$
Now we know that

$$
\begin{aligned}
\sigma_{\mathrm{h}} & =\frac{P R}{t} \\
\sigma_{1} & =\frac{P R+50}{2 t} \\
\sigma_{\mathrm{h}} & =\sigma_{1} \\
\frac{P R}{t} & =\frac{P R+50}{2 t} \mathrm{MPa} \\
\frac{P R}{2 t} & =50 \\
P & =\frac{50 \times 2 \times 5}{100}=5 \mathrm{MPa}
\end{aligned}
$$

Hence, the correct answer is 5 .

## Question Number: 48

Question Type: NAT
A bar is subjected to a combination of a steady of 60 kN and a load fluctuating between -10 kN and 90 kN . The corrected endurance limit of the bar is 150 MPa , the yield strength of the material is 480 MPa and the ultimate strength of the material is 600 MPa . the bar cross-section is square with side $a$. If the factor of safety is 2 , the value of a (in mm ), according to the modification Goodman's criterion, is $\qquad$ (correct to two decimal places).

Solution: yield strength of the material $\sigma_{\mathrm{yt}}=480 \mathrm{MPa}$ the ultimate strength of the material $\sigma_{u t}=600 \mathrm{MPa}$ The corrected endurance limit of the bar $\sigma_{\mathrm{e}}=150 \mathrm{MPa}$ We know that

$$
P_{\mathrm{m}}=60+\frac{90-10}{2}=100 \mathrm{KN}
$$

$$
P_{\mathrm{v}}=\frac{90-(-10)}{2}=50 \mathrm{KN}
$$

We know that as Good man's criterion,

$$
\begin{aligned}
& \frac{\sigma_{m}}{\sigma_{u t}}+\frac{\sigma_{v}}{\sigma_{e}}=\frac{1}{2} \\
& \frac{100 \times 10^{3}}{a^{2} \times 600}+\frac{50 \times 10^{3}}{a^{2} \times 150}=\frac{1}{2} \\
& a^{2}=2\left[\frac{100 \times 10^{3}}{600}+\frac{50 \times 10^{3}}{150}\right] \\
& a^{2}=1000 \mathrm{~mm} \\
& a=31.62 \mathrm{~mm}
\end{aligned}
$$

Hence, the correct answer is 31.62 .

## Question Number: 49

Question Type: NAT
A force of 100 N is applied to the centre of a circular disc, of mass 10 kg and radius 1 m , resting on a floor as shown in figure. If the disc rolls without slipping on the floor, the linear acceleration (in $\mathrm{m} / \mathrm{s}^{2}$ ) of the centre of the disc is
$\qquad$ (correct to two decimal places)


Solution: Force $F=100 \mathrm{~N}$,
Mass $m=10 \mathrm{~kg}$
Radius $r=1 \mathrm{~m}$
Let the linear acceleration (in $\mathrm{m} / \mathrm{s}^{2}$ ) of the centre of the disc be $a=$ ?


From the above figure we get

$$
\begin{align*}
& 100-f_{\mathrm{s}}=m \times a \\
& 100-f_{s}=10 \times a \tag{i}
\end{align*}
$$

Since disk is rolling,

$$
\begin{aligned}
T & =I \alpha \\
f_{\mathrm{s}} \times r & =\frac{m r^{2}}{2} x \alpha \\
f_{\mathrm{s}} & =\frac{m r \times \alpha}{2}=\frac{m a}{2}
\end{aligned}
$$

substituting in (i),

$$
\begin{aligned}
100-\frac{m a}{2} & =10 \times a \\
100-\frac{10 a}{2} & =10 a \\
15 a & =100 \\
a & =6.6 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

Hence, the correct answer is 6.66 .

## Question Number: 50

Question Type: NAT
A frictionless circular piston of area $10^{-2} \mathrm{~m}^{2}$ and mass 100 kg sinks into a cylindrical container of the same area filled with water of density $1000 \mathrm{~kg} / \mathrm{m}^{3}$ as shown in the
figure. The container has a hole of area $10^{-3} \mathrm{~m}^{2}$ at the bottom that is open to the atmosphere. Assuming there is no leakage from the edges of the piston and considering water to be incompressible, the magnitude of the piston velocity (in $\mathrm{m} / \mathrm{s}$ ) at the instant shown is $\qquad$ (correct to two decimal places).


## Solution:

According ton equation of contuinity

$$
\begin{aligned}
& A_{1} V_{1}=A_{2} V_{2} \\
& V_{2}=\left(\frac{A_{1}}{A_{2}}\right) V_{1} \\
& \frac{P_{1}}{\rho g}+\frac{V_{1}^{2}}{2 g}+z_{1}=\frac{P_{2}}{\rho g}+\frac{V_{2}^{2}}{2 g}+z_{2} \\
& {\left[\frac{P_{\text {atm }}+\frac{100 \times 10}{10^{-2}}}{\rho g}\right]+\frac{V_{1}^{2}}{2 g}+0.5} \\
& =\frac{P_{\mathrm{atm}}}{\rho g}+\frac{A_{1}^{2}}{A_{2}^{2}} \times \frac{V_{1}^{2}}{2 g} \\
& \frac{1000}{10^{-2} \times 100 \times 10}+0.5=\left(\frac{A_{1}^{2}}{A_{2}^{2}}-1\right) \cdot \frac{V_{1}^{2}}{2 g} \\
& \frac{V_{1}^{2}}{2 g}\left(10^{2}-1\right)=10.5 \\
& V_{1}^{2}=\frac{10.5 \times 2 \times 10}{99}=2.12 \\
& V_{1}=1.456 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Hence, the correct answer is 1.456 .

## Question Number: 51

## Question Type: NAT

A 0.2 m thick black plate having a thermal conductivity of $3.96 \mathrm{~W} / \mathrm{m}-\mathrm{K}$ is exposed to two infinite black surfaces at 300 K and 400 K as shown in the figure. At steady state, the surface temperature of the plate facing the cold side is 350 K . The value of Stefan-Boltzmann constant, $\sigma$, is 5.67 $\times 10^{-8} \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}^{4}$. Assuming 1-D heat conduction, the magnitude of heat flux through the plate (in $\mathrm{W} / \mathrm{m}^{2}$ ) is $\qquad$ (correct to two decimal places).


Solution: Consider the figure given below

$$
\begin{aligned}
300 \mathrm{~K}
\end{aligned} \begin{aligned}
q_{\text {plate - cold surface }} & =\left(T_{2}^{4}-T_{0}^{4}\right) \sigma \\
& =5.67 \times\left[\left(\frac{T_{2}}{100}\right)^{4}-\left(\frac{T_{0}}{100}\right)^{4}\right] \\
& =5.67 \times\left[\left(\frac{350}{100}\right)^{4}-\left(\frac{300}{100}\right)^{4}\right] \\
& =391.58 \mathrm{~W} / \mathrm{m}^{2}
\end{aligned}
$$

Hence, the correct answer is 391.58 .

## Question Number: 52

Question Type: NAT
Air is held inside a non-insulated cylinder using a piston (mass $M=25 \mathrm{~kg}$ and area $A=100 \mathrm{~cm}^{2}$ ) and stoppers (of negligible area), as shown in the figure. The initial pressure $P_{1}$ and temperature $T_{1}$ of air inside the cylinder are 200 kPa and $400^{\circ} \mathrm{C}$, respectively. The ambient pressure $P_{\infty}$ and temperature $T_{\infty}$ are 100 kPa and $27^{\circ} \mathrm{C}$, respectively. The temperature of the air inside the cylinder $\left({ }^{\circ} \mathrm{C}\right)$ at which the piston will begin to move is $\qquad$ (correct to two decimal places).


Solution: Pressure of air inside the cylinder when piston will begin to move is equal to

$$
\begin{aligned}
P_{0} & =P_{\infty}+\text { Pressure due to piston weight } \\
P_{0} & =100+\frac{25 \times 10}{100 \times 10^{-4} \times 10^{3}} \\
& =100+25=125 \mathrm{kPa}
\end{aligned}
$$

If we assume $T_{0}$ to be the corresponding temperature and the process should be constant volume,

$$
\begin{aligned}
\frac{P_{0}}{T_{0}} & =\frac{P_{\mathrm{i}}}{T_{\mathrm{i}}} \\
\therefore \quad T_{0} & =\frac{P_{0}}{P_{\mathrm{i}}} \times T_{\mathrm{i}} \\
T_{0} & =\frac{125}{200} \times 673=420.625 \mathrm{~K} \\
T_{0} & =147.625^{\circ} \mathrm{C}
\end{aligned}
$$

Hence, the correct answer is 147.62 .
Question Number: 53
Question Type: NAT
A standard vapor compression refirgeration cycle operating with a condensing temperature of $35^{\circ} \mathrm{C}$ and an evaporating temperature of $-10^{\circ} \mathrm{C}$ develops 15 kW of cooling. The p-h diagram shows the enthalpies of various states. If the isentropic efficiency of the compressor is 0.75 the magnitude of compressor power (in kW ) is $\qquad$ (correct to two decimal places).


Solution: Refrigerating capacity $=15 \mathrm{~kW}$

$$
\begin{aligned}
m\left(h_{1}-h_{4}\right) & =15 \mathrm{~kW} \\
m(400-250) & =15 \\
m & =0.1 \mathrm{~kg} / \mathrm{sec} \\
W_{\text {isentropic, compressor }} & =h_{1}-h_{2} \\
W_{\text {isen, comp }} & =4+5-40=75 \mathrm{~kJ} / \mathrm{kg} \\
\eta_{\text {compressor }} & =\frac{W_{\text {isentropic,comp }}}{W_{\text {actual }}} \\
W_{\text {actual }} & =\frac{75}{0.75}=100 \mathrm{~kJ} / \mathrm{kg}
\end{aligned}
$$

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$$
W_{\text {actual }}=\mathrm{m} \times W_{\text {actual }}=0.1 \times 100
$$

Compressor power $P=10 \mathrm{kw}$
Hence, the correct answer is 10 .
Question Number: 54
Question Type: NAT
Ambient air is at a pressure of 100 kPa , dry bulb temperature of $30^{\circ} \mathrm{C}$ and $60 \%$ relative humidity. The saturation pressure of water at $30^{\circ} \mathrm{C}$ is 4.24 kPa . The specific humidity of air (in $\mathrm{g} / \mathrm{kg}$ of dry air) is $\qquad$ (correct to two decimal places).

## Solution:

dry bulb temperature $\mathrm{DBT}=30^{\circ} \mathrm{C}$
saturation pressure of water at $30^{\circ} \mathrm{C}$

$$
P_{\mathrm{vs}}=4.24 \mathrm{kPa}
$$

Now we know that

$$
\begin{aligned}
\phi & =\frac{P_{\mathrm{v}}}{P_{\mathrm{vs}}} \\
P_{\mathrm{r}} & =0.6 \times 4.24 \\
P_{\mathrm{r}} & =2.544 \mathrm{kPa}
\end{aligned}
$$

Specific humidity can be calculated as

$$
\begin{aligned}
& \omega=0.622 \times \frac{P_{\mathrm{v}}}{P_{\mathrm{atm}}-P_{\mathrm{v}}} \\
& \omega=0.622 \times \frac{2.544}{100-2.544} \\
& \omega=16.236 \text { gram } / \mathrm{kg} \text { of dry air }
\end{aligned}
$$

Hence, the correct answer is 16.23 .
Question Number: 55
Question Type: NAT
A test is conducted on a one-fifth scale model of a Francis turbine under a head of 2 m and volumetric flow rate of $1 \mathrm{~m}^{3} / \mathrm{s}$ at 450 rpm . Take the water density and the acceleration due to gravity as $10^{3} \mathrm{~kg} / \mathrm{m}^{3}$ and $10 \mathrm{~m} / \mathrm{s}^{2}$, respectively. Assume no losses both in model and prototype turbines. The power (in MW) of a full sized turbine while working under a head of 30 m is $\qquad$ (correct to two decimal places).

Solution: Scale ratio can be expressed as

$$
C_{\mathrm{r}}=\frac{1}{5}=\frac{D_{\mathrm{m}}}{D_{\mathrm{p}}}
$$

Head of Francis turbine $H=2 \mathrm{~m}$
Discharge $Q=1 \mathrm{~m}^{3} / \mathrm{sec}$
Speed $N_{\mathrm{m}}=450 \mathrm{rpm}$
We know that the Power of model can be expressed as

$$
P_{\mathrm{m}}=\rho g Q H=10^{3} \times 10 \times 1 \times 2=0.02 \mathrm{MW}
$$

$$
\begin{aligned}
\frac{\sqrt{H}}{N D} & =\text { Constant } \\
\left(\frac{H}{D^{2} N^{2}}\right)_{\mathrm{m}} & =\left(\frac{H}{D^{2} N^{2}}\right)_{\mathrm{p}} \\
N_{\mathrm{p}}^{2} & =\frac{H_{\mathrm{p}}}{H_{\mathrm{m}}} \times\left(\frac{D_{\mathrm{m}}}{D_{\mathrm{p}}}\right)^{2} \times N_{\mathrm{m}}^{2} \\
& =\frac{30}{2} \times\left(\frac{1}{5}\right)^{2} \times(450)^{2}
\end{aligned}
$$

Speed of Prototype $=348.56 \mathrm{rpm}$
Also, $\frac{P}{D^{5} N^{3}}=$ Constant

$$
\begin{aligned}
\left(\frac{P}{D^{5} N^{3}}\right)_{\mathrm{m}} & =\left(\frac{P}{D^{5} N^{3}}\right)_{\mathrm{p}} \\
P_{\mathrm{p}} & =\left(\frac{D_{\mathrm{p}}}{D_{\mathrm{m}}}\right)^{5} \times\left(\frac{N_{\mathrm{p}}}{N_{\mathrm{m}}}\right)^{2} \times P_{\mathrm{m}} \\
P_{\mathrm{p}} & =5^{2} \times\left(\frac{348.56}{450}\right)^{3} \times 0.02 \\
P_{\mathrm{p}} & =29.047 \mathrm{MW}
\end{aligned}
$$

Hence, the correct answer is 29.05 .
Question Number: 56
Question Type: NAT
The true stress (in MPa) versus true strain relationship for a metal is given by

$$
\sigma=1020 \varepsilon^{0.4}
$$

The cross-sectional area at the start of a test (when the stress and strain values are equal to zero) is $100 \mathrm{~mm}^{2}$. The cross-sectional area at the time of necking (in $\mathrm{mm}^{2}$ ) is
$\qquad$ (correct to two decimal places).

## Solution:

True stress (in MPa) versus true strain relationship is given as

$$
\sigma=1020 \in^{0.4}
$$

We know that necking begins when true strain is equal to strain hardening exponent.

$$
\begin{aligned}
\epsilon & =n=0.4 \\
\ln \frac{A_{0}}{A_{\mathrm{f}}} & =\epsilon \\
\ln \frac{100}{A_{\mathrm{f}}} & =0.4 \\
A_{\mathrm{f}} & =\frac{100}{e^{0.4}}=67.03 \mathrm{~mm}^{2}
\end{aligned}
$$

Hence, the correct answer is 67.03 .

## Question Number: 57

Question Type: NAT
A steel wire is drawn from an initial diameter $\left(d_{\mathrm{i}}\right)$ of 10 mm to a final diameter $\left(d_{\mathrm{f}}\right)$ of 7.5 mm . The half cone angle $(\alpha)$ of the die is $5^{\circ}$ and the coefficient of friction $(\mu)$ between the die and the wire is 0.1 . The average of the initial and final yield stress $\left[\left(\sigma_{\mathrm{y}}\right)_{\text {avg }}\right]$ is 350 MPa . The equation for drawing stress $\sigma_{\mathrm{f}},($ in MPa$)$ is given as

$$
\sigma_{\mathrm{f}}=\left(\sigma_{\mathrm{y}}\right)_{\mathrm{avg}}\left\{1+\frac{1}{\mu \cot \alpha}\right\}\left[1-\left(\frac{d_{\mathrm{f}}}{d_{\mathrm{i}}}\right)^{2 \mu \cot \alpha}\right]
$$

The drawing stress (in MPa ) required to carry out this equation is $\qquad$ (correct to two decimal places).
Solution: initial diameter of steel wire $d_{\mathrm{i}}=10 \mathrm{~mm}$
Final diameter of steel wire $d_{\mathrm{f}}=7.5 \mathrm{~mm}$
Half cone angle $(\alpha)$ of the die $a=5^{\circ}$
Average of the initial and final yield stress is $\left(\sigma_{y}\right)_{\text {avg }}=350 \mathrm{mPa}$ The equation for drawing stress $\sigma_{\mathrm{f}}$, (in MPa) is given as

$$
\begin{aligned}
\sigma_{\mathrm{f}} & =\left(\sigma_{\mathrm{y}}\right)_{\text {avg }}\left\{1+\frac{1}{\mu \cot \alpha}\right\}\left[1-\left(\frac{d_{f}}{d_{i}}\right)^{2 \mu \cot \alpha}\right] \\
\sigma_{\mathrm{f}} & =350\left[1+\frac{1}{0.1 \cot 5}\right]\left[1-\left(\frac{7.5}{10}\right)^{2 \times 0.1 \cot 5}\right] \\
& =316.28 \mathrm{MPa}
\end{aligned}
$$

Hence, the correct answer is 316.28 MPa .

## Question Number: 58

Question Type: NAT
Following data correspond to an orthogonal turning of a 100 mm diameter rod on a lathe. Rake angle: $+15^{\circ}$; Uncut chip thickness: 0.5 mm ; nominal chip thickness after the cut: 1.25 mm . The shear angle (in degrees) for this process is $\qquad$ (correct to two decimal places).

Solution: Diameter of $\operatorname{rod} D=100 \mathrm{~mm}$
Rake angle $a=15^{\circ}$
Uncut chip thickness $t=0.5 \mathrm{~mm}$
nominal chip thickness after the cut $t_{\mathrm{c}}=1.25 \mathrm{~mm}$
Now using the relation given below

$$
\begin{aligned}
\tan \phi & =\frac{r \cos \alpha}{1-r \sin \alpha} \\
r & =\frac{t}{t_{\mathrm{c}}}=0.4 \\
\tan \phi & =\frac{0.4 \cos 15^{\circ}}{1-0.4 \sin 15^{\circ}} \\
\phi & =\tan ^{-1}\left(\frac{0.4 \cos 15}{1-0.4 \sin 15}\right)
\end{aligned}
$$

$$
\phi=23.31^{\circ}
$$

Hence, the correct answer is $23.31^{\circ}$.
Question Number: 59
Question Type: NAT
Taylor's tool life equation is used to estimate the life of a batch of identical HSS twist drills by drilling through holes at constant feed in 20 mm thick mild steel plates, In test 1 , a drill lasted 300 holes at 150 rpm while in test 2 , another drill lasted 200 holes at 300 rpm . The maximum number of holes that can be made by another drill from the above batch at 200 rpm is $\qquad$ (correct to two decimal places).
Solution: We know that Taylors tool life equation is $V T^{n}$ = constant
Revolution per minute $V_{1}=150 \mathrm{rpm}$,
Number of holes $T_{1}=300$ holes
Revolution per minute $V_{2}=300 \mathrm{rpm}$,
Number of holes $T_{2}=200$ holes
Revolution per minute $V_{3}=200 \mathrm{rpm}$
Number of holes $T_{3}=$ ?
Now using the relation

$$
\begin{aligned}
V_{1} T_{1}^{n} & =V_{2} T_{2}^{n} \\
150 \times 300^{n} & =300 \times 200^{n} \\
1.5^{n} & =2 \\
n & =\frac{\ln 2}{\ln 1.5}=1.7
\end{aligned}
$$

Now using relation

$$
\begin{aligned}
V_{3} T_{3}^{n} & =V_{1} T_{1}{ }^{n} \\
200 \times T_{3}^{n} & =150 \times 300^{n} \\
\left(\frac{T_{3}}{300}\right)^{n} & =\frac{150}{200}=\frac{3}{4} \\
T_{3} & =300 \times\left(\frac{3}{4}\right)^{\frac{1}{1.7}} \\
T_{3} & =253.29 \text { holes }
\end{aligned}
$$

Hence, the correct answer is 253.29 .
Question Number: 60
Question Type: NAT
For sand-casting a steel rectangular plate with dimensions $80 \mathrm{~mm} \times 120 \mathrm{~mm} \times 20 \mathrm{~mm}$, a cylindrical riser has to be designed. The height of the riser is equal to its diameter. The total solidification time for the casting is 2 minutes. In Chvorinov's law for the estimation of the total solidification time, exponent is to be taken as 2 . For a solidification time of 3 minutes in the riser, the diameter (in mm ) of the riser is $\qquad$ (correct to two decimal places).

Solution: We know that

$$
\begin{align*}
\frac{\tau_{\mathrm{R}}}{\tau_{\mathrm{C}}} & =\left(\frac{m_{\mathrm{R}}}{m_{\mathrm{C}}}\right)^{2} \\
m_{\mathrm{C}} & =\frac{80 \times 120 \times 20}{2[(80 \times 120)+(120 \times 20)+(80 \times 20)]} \\
& =7.05 \tag{1}
\end{align*}
$$

We know that for side riser,

$$
\begin{equation*}
m_{\mathrm{R}}=\frac{d}{6} \tag{2}
\end{equation*}
$$

From (1) and (2), we get

$$
\begin{aligned}
\frac{m_{\mathrm{R}}}{m_{\mathrm{C}}} & =\sqrt{1.5} \\
\frac{d}{6 \times 7.05} & =\sqrt{1.5} \\
d & =51.8
\end{aligned}
$$

Hence, the correct answer is 51.8.

## Question Number: 61 <br> Question Type: NAT

The arc lengths of a directed graph of a project are as shown in the figure. The shortest path length from node 1 to node is $\qquad$ .


Solution: From the given figure shortest path is

$$
1-2-5-4-6
$$

and shortest path length is 7 .
Hence, the correct answer is 7.
Question Number: 62
Question Type: NAT
A circular hole of 25 mm diameter and depth of 20 mm is machined by EDM process. The material removal rate (in $\left.\mathrm{mm}^{3} / \mathrm{mm}\right)$ is expressed as $4 \times 10^{4} \mathrm{IT}^{-1.23}$
where $I=300 \mathrm{~A}$ and the melting point of the material, $T=$ $1600^{\circ} \mathrm{C}$. The time (in minutes) for machining this hole is
$\qquad$ (correct to two decimal places)

Solution: $\mathrm{MRR}=4 \times 10^{4} \times \mathrm{IT}^{-1.23}$

$$
\begin{aligned}
\mathrm{MRR} & =4 \times 10^{4} \times 300(1600)^{-1.23} \\
& =1374.40 \mathrm{~mm}^{3} / \mathrm{min}
\end{aligned}
$$

Volume required to remove $=\frac{\pi}{4} D^{2} L$

$$
\begin{aligned}
& =\frac{\pi}{4} \times 25^{2} \times 20 \\
& =9817.477 \mathrm{~mm}^{3}
\end{aligned}
$$

Time required $=\frac{9817.477}{1374.4}=7.1431 \mathrm{~min}$
Hence, the correct answer is 7.1431 .

## Question Number: 63

Question Type: NAT
A welding opreation is being performed with voltage $=$ 30 V and current $=100 \mathrm{~A}$. The cross-sectional area of the weld bead is $20 \mathrm{~mm}^{2}$. The work-piece and filler are of titanium for which the specific energy of melting is $14 \mathrm{~J} / \mathrm{mm}^{3}$. Assuming a thermal efficiency of the welding process 70\%, the welding speed (in $\mathrm{mm} / \mathrm{s}$ ) is $\qquad$ (correct to two decimal places).

Solution: Give values are
Voltage $=30 \mathrm{~V}$
Current $=100 \mathrm{~A}$
Cross-sectional area $A=20 \mathrm{~mm}^{2}$
Specific energy of melting $=14 \mathrm{~J} / \mathrm{mm}^{3}$

$$
\eta_{\text {thermal }}=70 \%
$$

We know that

$$
\begin{aligned}
\text { Power } & =\eta_{\mathrm{th}} \times V I \\
& =0.7 \times 30 \times 100 \\
& =2100 \mathrm{~J} / \mathrm{S} \\
\text { Specific energy } & =\frac{\text { Power }}{A \times V}
\end{aligned}
$$

The welding speed $(V)$ will be

$$
\begin{aligned}
V & =\frac{\text { Power }}{\text { Specific energy } \times A} \\
& =\frac{2100}{14 \times 20} \\
V & =7.5 \mathrm{~mm} / \mathrm{s}
\end{aligned}
$$

Hence, the correct answer is 7.5 V .
Question Number: 64
Question Type: NAT
Steam in the condenser of a thermal power plant is to be condensed at a temperature of $30^{\circ} \mathrm{C}$ with cooling water which enters the tubes of the condenser at $14^{\circ} \mathrm{C}$ and exits at $22^{\circ} \mathrm{C}$. The total surface area of the tubes is $50 \mathrm{~m}^{2}$, and the overall heat transfer coefficient is $2000 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$. The
heat transfer (in MW) to the condenser is $\qquad$ (correct to two decimal places).

## Solution:


$T_{\mathrm{hi}}=30^{\circ} \mathrm{C}, T_{\mathrm{ci}}=14^{\circ}$
$T_{\text {he }}=30^{\circ} \mathrm{C}, T_{\mathrm{ce}}=22^{\circ}$
$\theta_{1}=T_{\mathrm{hi}}-T_{\mathrm{ci}}=16^{\circ} \mathrm{C}$
$\theta_{2}=T_{\mathrm{he}}-T_{\mathrm{ce}}=8^{\circ} \mathrm{C}$
LMTD $=\frac{\theta_{1}-\theta_{2}}{\ln \left(\frac{\theta_{1}}{\theta_{2}}\right)}=\frac{16-8}{\ln \left(\frac{16}{8}\right)}$
LMTD $=11.54^{\circ} \mathrm{C}$
Heat transfer $\theta=U A$ LMTD

$$
\begin{aligned}
\theta & =2000 \times 50 \times 11.54 \\
& =1.154 \mathrm{MW}
\end{aligned}
$$

Hence, the correct answer is 1.154 .
Question Number: 65
Question Type: NAT
A vehicle powered by a spark ignition engine follows air standard Otto cycle $(\gamma=1.4)$. The engine generates 70 kW
while consuming $10.3 \mathrm{~kg} / \mathrm{hr}$ of fuel. The calorific value of fuel is $44.000 \mathrm{~kJ} / \mathrm{kg}$. the compression ratio is $\qquad$ (correct to two decimal places).

Solution: $\gamma=1.4$
Power $P=70 \mathrm{~kW}$
Fuel consumption $m_{\mathrm{f}}=10.3 \mathrm{~kg} / \mathrm{hr}$

$$
=\frac{10.3}{3600} \mathrm{~kg} / \mathrm{sec}
$$

Calorific value $C_{\mathrm{v}}=44000 \mathrm{~kJ} / \mathrm{kg}$
Now using the relation

$$
\begin{aligned}
\eta_{\text {cycle }} & =\frac{70}{\frac{10.3}{3600} \times 44000} \\
\eta_{\text {cycle }} & =0.55 \\
1-\frac{1}{r^{\gamma-1}} & =0.55 \\
\frac{1}{r^{\gamma-1}} & =0.44556 \\
r^{\gamma-1} & =\frac{1}{0.44556} \\
r & =\left(\frac{1}{0.44556}\right)^{\frac{1}{1.4-1}} \\
r & =7.6
\end{aligned}
$$

Hence, the correct answer is 7.6.

