SECTION - A GENERAL APTITUDE

Q.1 If $\left(x-\frac{1}{2}\right)^2-\left(x-\frac{3}{2}\right)^2=x+2$, then the value of x is :

(a) 2
(b) 8
(c) 4
(d) 6

Ans. **(c)**

$$\left(x-\frac{1}{2}\right)^2-\left(x-\frac{3}{2}\right)^2 = x + 2$$

Using $a^2 - b^2 = (a + b)(a - b)$

$$\left(x-\frac{1}{2}+x-\frac{3}{2}\right)\left(x-\frac{1}{2}-x+\frac{3}{2}\right) = x + 2$$

$$(2x - 2) = x + 2$$

$$2x - x = 2 + 2$$

$$x = 4$$

End of Solution

Q.2 Listening to music during exercise improves exercise performance and reduces discomfort. Scientists researched whether listening to music while studying can help students learn better and the results were inconclusive. Students who needed external stimulation for studying fared worse while students who did not need any external stimulation benefited from music.

Which one of the following statements is the CORRECT inference of the above passage?

(a) Listening to music has a clear positive effect on learning in all students. Music has a positive effect only in some students who exercise.
(b) Listening to music has a clear positive effect on physical exercise. Music has a positive effect on learning only in some students.
(c) Listening to music has a clear positive effect both on physical exercise and on learning.
(d) Listening to music has no effect on learning and a positive effect on physical exercise.

Ans. **(b)**

"Only in some students" is the key in option (b) and that matches well with the given informations in the passage.

End of Solution

Q.3 Pen : Write : : Knife : _____

Which one of the following options maintains a similar logical relation in the above?

(a) Sharp (b) Cut

(c) Vegetables (d) Blunt

Ans. **(b)**

Pen is used to write and knife is used to cut.

End of Solution

Q.4 Gauri said that she can play keyboard _____ her sister.

(a) as worse as (b) as better as

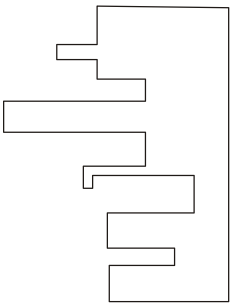
(c) as well as (d) as nicest as

Ans. **(c)**

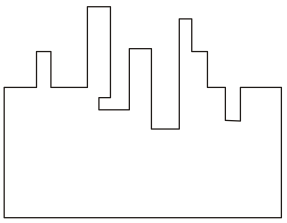
(as – as) and (so – as) are used in positive degree of comparison only. Worse, better are comparative degree and nicest is superlative degree, which are not fit between as–as and so–as.

End of Solution

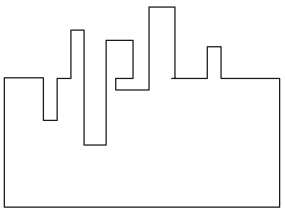
Q.5 What will be the remaining part to make it a rectangle?



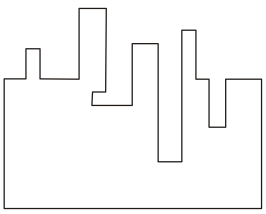
(a)



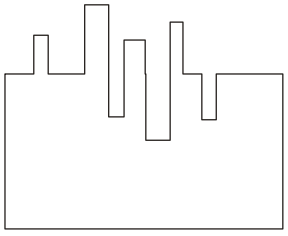
(b)



(c)



(d)



Ans. **(c)**

Option (c) is flipped.

End of Solution

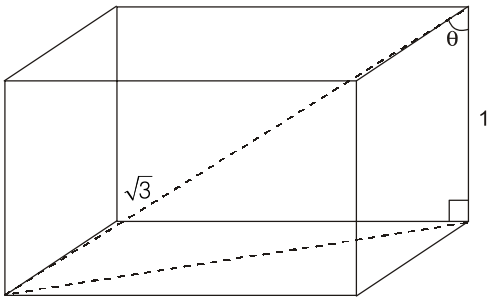
Q.6 If θ is the angle, in degrees, between the longest diagonal of the cube and any one of the edges of the cube, then $\cos\theta$ = ____.

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

Ans. **(a)**

Angle of longest diagonal of cube with an edge of cube.



$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{1}{\sqrt{3}}$$

End of Solution

Q.7 Six students P, Q, R, S, T and U, with distinct heights, compare their heights and make the following observations :

Observation I : S is taller than R.

Observation II : Q is the shortest of all.

Observation III : U is taller than only one student.

Observation IV : T is taller than S but is not the tallest.

The number of students that are taller than R is the same as the number of students shorter than _____.

(a) R (b) P

(c) S (d) T

Ans. **(c)**

$S > R$ ...(i)

Q is shortest and U is taller than only one. ...(ii)

$T > S$ ...(iii)

Hence, possible order is : $P > T > S > R > U > Q$.

$\because$ Number of students taller than R = 3.

$\therefore$ Number of students shorter than S = 3.

Hence, option (c) is the correct answer.

End of Solution

Q.8 The number of student in three classes is in the ratio 3 : 13 : 6. If 18 students are added to each class, the ratio changes to 15 : 35 : 21.
The total number of students in all the three classes in the beginning was:

(a) 88 (b) 110
(c) 66 (d) 22

Ans. **(a)**

3 : 13 : 6

Let $3k + 13k + 6k = n$

Now $+18 + 18 + 18$

$15 : 35 : 21$ ...(1)

$15y + 35y + 21y = 22k + 54$

$71y = 22k + 54$

Put value of k and satisfy

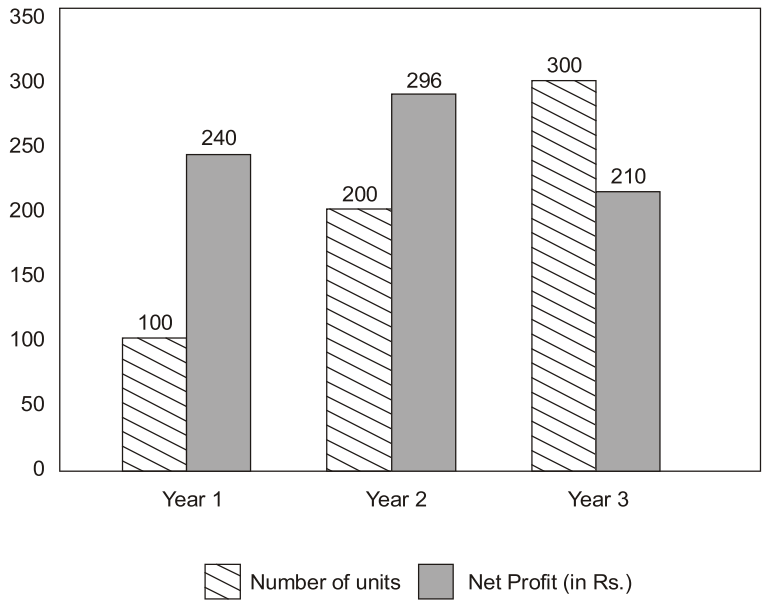
Here for $k = 4$

On putting $k = 4$ in equation (1)

$n = 88$

End of Solution

Q.9 The number of units of a product sold in three different years and the respective net profits are presented in the figure above. The cost/unit in year 3 was Rs. 1, which was half the cost/unit in year 2. The cost/unit in year 3 was one-third of the cost/unit in year 1. Taxes were paid on the selling price at 10%, 13% and 15% respectively for the three years. Net profit is calculated as the difference between the selling price and the sum of cost and taxes paid in that year.



The ratio of the selling price in Year 2 to the selling price in Year 3 is _____.

(a) 3 : 4 (b) 1 : 2
(c) 1 : 1 (d) 4 : 3

Ans. **(d)**

Cost/unit in year 3 = Rs. 1

Cost/unit in year 2 = Rs. 2

Cost/unit in year 1 = Rs. 3

Net Profit = S.P. – (Cost + Taxes)

In year 2, $296 = S.P. - (2 \times 200 + 0.13\ S.P.)$

$S.P. = 800$

Selling price in year 2 = Rs. 800

In year 3, $210 = S.P. - (300 \times 1 + 0.15\ S.P.)$

$210 = S.P. - 300 - 0.15\ S.P.$

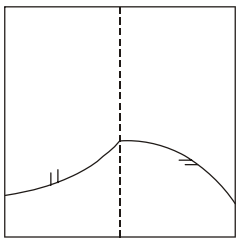
$$S.P. = \frac{210+300}{1-0.15}$$

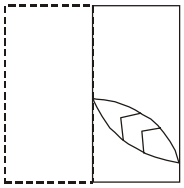
Selling price in year 3 = Rs. 600

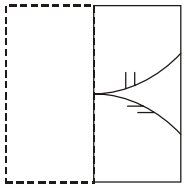
Hence, Required ratio = 800 : 600 = 4 : 3

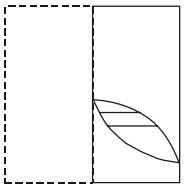
End of Solution

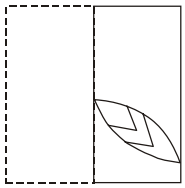
Q.10 A transparent square sheet shown above is folded along the dotted line. The folded sheet will look like _____.



(a) 

(b) 

(c) 

(d) 

Ans. **(a)**

End of Solution

■■■■

SECTION - B TECHNICAL

Q.1 Consider a computer system with DMA support. The DMA module is transferring one 8-bit character in one CPU cycle from a device to memory through cycle stealing at regular intervals. Consider a 2 MHz processor. If 0.5% processor cycles are used for DMA, the data transfer rate of the device is _____ bits per second.

Ans. **(80000)**

y = 0.5 μsec (Transfer Time)

x = Preparation time

$$\% \text{ time CPU blocked} = \left(\frac{y}{x+y}\right)\times 100$$

$$0.5 = \left(\frac{0.5\ \mu\text{sec}}{x+0.5\ \mu\text{sec}}\right)\times 100$$

$$0.005x + 0.0025 = 0.5$$

$$0.005x = 0.4975$$

$$x = 99.5\ \mu\text{sec}$$

(99.5 μsec + 0.5 μsec) Total time ——————— 8 bit

1 sec ——————— ?

$$= \frac{8 \text{ bit}}{100\ \mu\text{sec}} = 80000 \text{ bits/sec}$$

End of Solution

Q.2 Let $L \subseteq \{0, 1\}^*$ be an arbitrary regular language accepted by a minimal DFA with k states. Which one of the following languages must necessarily be accepted by a minimal DFA with k states?

(a) $\{0, 1\}^* - L$ (b) $L \cup \{01\}$

(c) $L.L$ (d) $L - \{01\}$

Ans. **(a)**

If L is accepted by a min DFA with k states, by exchanging final and non-final states, we can make a minimal DFA with k states which accepts $\{0, 1\}^* - L = \overline{L}$.

So, option (a) is true.

End of Solution

Q.3. Consider a computer system with multiple shared resource types, with one instance per resource type. Each instance can be owned by only one process at a time. Owning and freeing of resources are done by holding a global lock (L). The following scheme is used to own a resource instance:

```
function OwnResource (Resource R)
    Acquire lock L // a global lock
    if R is available then
        Acquire R
        Release lock L
    else
        if R is owned by another process P then
            Terminate P, after releasing all resources owned by P
            Acquire R
            Restart P
            Release lock L
        end if
    end if
end function
```

Which of the following choice(s) about the above scheme is/are correct?

(a) The scheme violates the mutual exclusion property.
(b) The scheme ensures that deadlocks will not occur.
(c) The scheme may lead to live-lock.
(d) The scheme may lead to starvation.

Ans. **(b, c, d)**

- Mutual exclusion is not violated.
- Also, there will be no deadlock because of forceful preemption of resources.
- This may lead to starvation if the process is keeps on coming and preempting each other like P_1 is preempted by P_2 and P_2 is preempted by P_3.
- Live-lock is also possible due to continuous preemption of resources.

For option (c) consider two processes P_1 and P_2 now P_1 enter the code acquires lock and resource.

Now P_2 enters the else part kills P_1 and acquire R and restart P_1.

Now P_1 again acquire lock and kills the process P_2 this continues creating a live lock scenario but there is ambiguity in the code since "Release R" is not written any where so ambiguity is regarding how the process will release Resource R. According to the code, the only way to release the resource is by getting killed.

End of Solution

Q.4 Consider the three-way handshake mechanism followed during TCP connection establishment between hosts *P* and *Q*. Let *X* and *Y* be two random 32-bit starting sequence numbers chosen by *P* and *Q* respectively. Suppose *P* sends a TCP connection request message to *Q* with a TCP segment having SYN bit = 1. SEQ number = *X*, and ACK bit = 0. Suppose *Q* accepts the connection request. Which one of the following choices represents the information present in the TCP segment header that is sent by *Q* to *P*?

(a) SYN bit = 1, SEQ number = *Y*, ACK bit = 1, ACK number = *X* + 1, FIN bit = 0
(b) SYN bit = 1, SEQ number = *Y*, ACK bit = 1, ACK number = *X*, FIN bit = 0
(c) SYN bit = 0, SEQ number = *X* + 1, ACK bit = 0, ACK number = *Y*, FIN bit = 1
(d) SYN bit = 1, SEQ number = *X* + 1, ACK bit = 0, ACK number = *Y*, FIN bit = 0

Ans. **(a)**

Q will send the SYN bit = 1 to the connection establishment.
Q seq number will be Y different from X.
ACK bit = 1 because sending the ACK.
ACK number = X + 1 (Next seq number id)
FIN bit = 0 (Because establishing the connection)

End of Solution

Q.5 For a given biased coin, the probability that the outcome of a toss is a head is 0.4. This coin is tossed 1,000 times. Let *X* denote the random variable whose value is the number of times that head appeared in these 1,000 tosses. The standard deviation of *X* (rounded to 2 decimal places) is _____.

Ans. **(15.49)**

$$n = 1000,\ p = 0.4,\ q = 0.6$$

It is a binomially distributed random variable.

So, $$\text{S.D.} = \sqrt{npq} = \sqrt{1000 \times 0.4 \times 0.6} = \sqrt{240}$$
$$= 15.49$$

End of Solution

Q.6 Consider the following statements S_1 and S_2 about the relational data model :

S_1 : A relation scheme can have at most one foreign key.

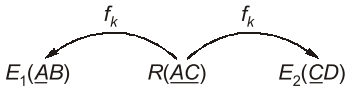
S_2 : A foreign key in a relation scheme R cannot be used to refer to tuples of R.

(a) Both S_1 and S_2 are false.
(b) S_1 is true and S_2 is false.
(c) S_1 is false and S_2 is true.
(d) Both S_1 and S_2 are true.

Ans. **(a)**

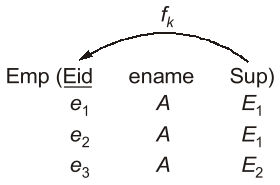
S_1 : A relation scheme can have at most one foreign key.

e.g.



More than 1 f_k also possible.

S_2 : A foreign key in a relational scheme R_1 cannot be used to refer to tuples of R.



e.g. Self referential relationship.

So, both S_1 and S_2 are false.

End of Solution

Q.7 The format of the single-precision floating-point representation of a real number as per the IEEE 754 standard is as follows:

sign	exponent	mantissa

Which one of the following choices is correct with respect to the smallest normalized positive number represented using the standard?

(a) exponent = 00000000 and maintissa = 00000000000000000000001
(b) exponent = 00000001 and maintissa = 00000000000000000000000
(c) exponent = 00000001 and maintissa = 00000000000000000000001
(d) exponent = 00000000 and maintissa = 00000000000000000000000

Ans. **(b)**

$$\left\{\begin{array}{c}\text{All 0's BE} \Rightarrow \text{Used for "0"} \\ \text{All 1's BE} \Rightarrow \text{Used for } (+\infty \text{ and } -\infty)\end{array}\right\}$$

End of Solution

Q.8 For a string w, we define w^R to be the reverse of w. For example, if w = 01101 then w^R = 10110. Which of the following languages is/are context-free?

(a) $\{wxw^RxR \mid w, x \in \{0, 1\}^*\}$ (b) $\{wxw^R \mid w, x \in \{0, 1\}^*\}$

(c) $\{^{ww^R}xx^R \mid w, x \in \{0, 1\}^*\}$ (d) $\{wxxw^Rw^R \mid w, x \in \{0, 1\}^*\}$

Ans. **(b, c, d)**

Option (a) : $\{wxw^Rx^R \mid w, x \in \{0, 1\}^*\}$

By putting w as "$\in$" we will get $\{xx^R \mid x \in \{0, 1\}^*\}$ which still has string matching. So, this will not be regular. Similarly, by putting x as $\in$ it will be $\{ww^R \mid w \in \{0, 1\}^*\}$ which still has string matching and will not become regular.

So, we need to do string matching but alternate order string matching is not possible in PDA. So, it is a CSL. Option (a) is a CSL but not CFL.

Option (b) : $L = \{wxw^R \mid w, x \in \{0, 1\}^*\}$

By putting w as "$\in$" we get $\{x \mid x \in \{0, 1\}^*\} = (0 + 1)^*$

Since a subset of L is $(0 + 1)^*$, L itself must be $(0 + 1)^*$ which is regular and hence CFL. Option (b) is a CFL.

Option (c) : $\{ww^Rxx^R \mid w, x \in \{0, 1\}^*\}$

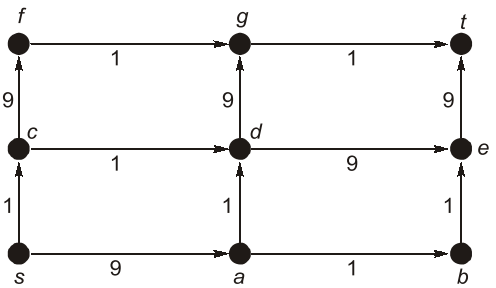
Here by putting x or w as $\in$, we cannot remove string matching. So, it is not regular. But it is CFL since in a NPDA we can push w, pop for w^R match it and then push x and pop for x^R and match it again and so this language is a CFL.

Option (d) : $\{wxx^Rw^R \mid w, x \in \{0, 1\}^*\}$

Here, also by putting w or x as $\in$, we cannot make it regular. NPDA can do this, push both w and x and then x^R pop and w^R pop and match. By push, push, pop, pop this can be accepted by NPDA. So, option (d) is CFL.

End of Solution

Q.9 In a directed acyclic graph with a source vertex s, the quality-score of a directed path is defined to be the product of the weights of the edges on the path. Further, for a vertex v other than s, the quality-score of v is defined to be the maximum among the quality-scores of all the paths from s to v. The quality-score of s is assumed to be 1.



The sum of the quality-scores of all the vertices in the graph shown above is _____.

Ans. **(929)**

Modify Dijkstra's Algo to get longest path in terms of quality scores (Use Max heap)

Algo:

1. Choose vertex u which is maximum quality score value.
   V is set of adjacency of u.
   for (each vertex (v))
   { if ($q[v] < q[u]$*cost (u, v)]
   {
   $q[v] = q[u] * \text{cost}(u, v)$
   prev[v] = 4
2. Repeat (1) for each vertex exactly once.

Quality score (q):

1	0	0	0	0	0	0	0	0
1	9	9	1	9	81	9	81	729

Initially, let $q[v] = 0$ or $-\infty$ (take smallest possible value)

Prev. :

–1	s	a	s	a	d	c	d	e

$S \Rightarrow \{a, c\}$ $q(a) = 9$, $q(c) = 1$

$a \Rightarrow \{d, b\}$ $q(d) = |ad|*q(a) = 1*9 = 9$, $q(b) = |ab|*q(a) = 1*9 = 9$

$b \Rightarrow \{e\}$ $q(e) = |be|*q(b) = 1*9 = 9$

$d \Rightarrow \{e, g\}$ $q(e) = |de|*q(d) = 9*9 = 81$, $q(g) = |dg|*q(d) = 9*9 = 81$

$e \Rightarrow \{t\}$ $q(t) = |et|*q(e) = 9*81 = 729$

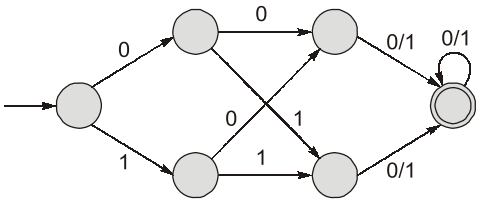
$g \Rightarrow \{t\}$ t already relaxed.

$c \Rightarrow \{f, d\}$ $q(f) = |cf|*q(c) = 9*1 = 9$

$f \Rightarrow \{g\}$ g already relaxed

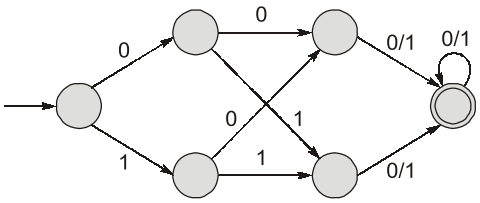
End of Solution

Q.10 Consider the following deterministic finite automation (DFA).



The number of strings of length 8 accepted by the above automation is _____.

Ans. (256)



The regular expression for $L(M)$ is $0(0 + 1)(0 + 1)(0 + 1)^* + 1(0 + 1)(0 + 1)(0 + 1)^*$

$= (0 + 1)(0 + 1)(0 + 1)(0 + 1)^*$

So, all strings of length ≥ 3 accepted.

Therefore number of strings of length 8 is $2^8 = 256$.

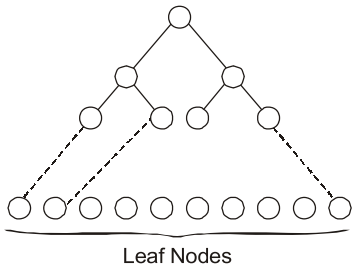
End of Solution

Q.11 Let H be a binary min-heap consisting of n elements implemented as an array. What is the worst case time complexity of an optimal algorithm to find the maximum element in H?

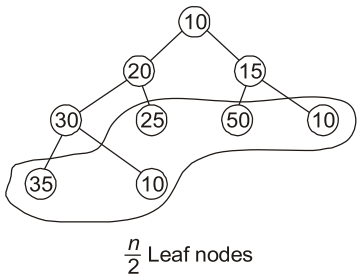
(a) $\Theta(\log n)$ (b) $\Theta(1)$

(c) $\Theta(n \log n)$ (d) $\Theta(n)$

Ans. (d)



OR



Maximum element is present somewhere in the leaf nodes.

$\therefore$ Find max in all leaf nodes from $\left(\left[\frac{n}{2}\right]+1\right).....a[n]$

$\Rightarrow$ Number of comparisons = $\frac{n}{2}-1 = \Theta(n)$

End of Solution

Q.12 In the context of compilers, which of the following is/are NOT an intermediate representation of the source program?

(a) Symbol table
(b) Three address code
(c) Control Flow Graph (CFG)
(d) Abstract Syntax Tree (AST)

Ans. **(a)**

Symbol table is a data structure which is used for storing the information about the variables. So, option (a) is correct.

There are three major categories of intermediate code representation.

Structural, linear and hybrid.

And CFG comes under the structural intermediate code representation.

End of Solution

Q.13 Consider the following two statements about regular languages:

S_1 : Every infinite regular language contains an undecidable language as a subset.
S_2 : Every finite language is regular.

Which one of the following choices is correct?

(a) Both S_1 and S_2 are true.
(b) Only S_2 is true.
(c) Only S_1 is true.
(d) Neither S_1 nor S_2 is true.

Ans. **(a)**

S_1 : Every infinite regular language contains an undecidable language as a subset.
S_2 : Every finite language is regular.

Clearly, S_2 is true, since for finite language, we can design FA by brute force, with a finite number of states.

Since, any language can be subset of an infinite language (No infinite language is closed under subset operation).

So, an infinite regular language can have any type of language as a subset including undecidable (non-REC) languages.

So, S_1 is also true. So, both S_1 and S_2 are true.

So, option (a) is correct.

End of Solution

Q.14 In an examination, a student can choose the order in which two questions (QuesA and QuesB) must be attempted.

- If the first question is answered wrong, the student gets zero marks.
- If the first questions is answered correctly and the second question is not answered correctly, the student gets the marks only for the first question.
- If both the questions are answered correctly, the student gets the sum of the marks of the two questions.

The following table shows the probability of correctly answering a question and the marks of the question respectively.

Question	Probability of answering correctly	Marks
QuesA	0.8	10
QuesB	0.5	20

Assuming that the student always wants to maximize her expected marks in the examination, in which order should she attempt the questions and what is the expected marks for that order (assume that the questions are independent)?

(a) First QuesA and then QuesB. Expected marks 14.
(b) First QuesB and then QuesA. Expected marks 22.
(c) First QuesB and then QuesA. Expected marks 14.
(d) First QuesA and then QuesB. Expected marks 16.

Ans. **(d)**

$X \rightarrow$ Random variable which represents total marks record.

$P(x) \rightarrow$ Probability of getting those marks.

$X \longrightarrow$	0	10	20	30
$P(x) \longrightarrow$	0.2×0.5	0.8×0.5	0.5×0.2	0.8×0.5
	11	11	11	11
	0.1	0.4	0.1	0.4
$\Sigma P(x) = 1$				

Now, if QuestionA is attempted first and it is correct.

Case-I:

$$E(x) = \Sigma(x) \cdot P(x)$$
$$= 0.4 \times 10 + 0.4 \times 30$$
$$= 4 + 12 = 16$$

Case-II:

If QuestionB is attempted first and is correct.

$$E(x) = \Sigma(x) \cdot P(x)$$
$$= 0.1\ (20) + 0.4\ (30)$$
$$= 2 + 12 = 14$$

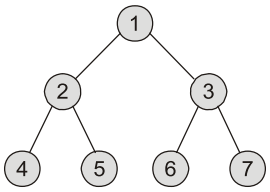
So case-I is giving maximum expected marks.

Hence option (d) is correct.

End of Solution

Q.15 Consider a complete binary tree with 7 nodes. Let A denote the set of first 3 elements obtained by performing Breadth-First Search (BFS) starting from the root. Let B denote the set of first 3 elements obtained by performing Depth-First Search (DFS) starting from the root. The value of $|A - B|$ is _____.

Ans. **(1)**



Using BFS, $A = \{1, 2, 3\}$

Using DFS, $B = \{1, 2, 4\}$

$|A - B|$ = Number of elements which are in A but not in B is only element {3}

So only 1 element present.

End of Solution

Q.16 A data file consisting of 1,50,000 student-records is stored on a hard disk with block size of 4096 bytes. The data file is sorted on the primary key RollNo. The size of a record pointer for this disk is 7 bytes. Each student-record has a candidate key attributed called ANum of size 12 bytes. Suppose an index file with records consisting of two fields, ANum value and the record pointer to the corresponding student record, is built and stored on the same disk. Assume that the records of data file and index file are not split across disk blocks. The number of blocks in the index file is _____.

Ans. **(698)**

Index entries = Number of DB file record ($\because$ Dense index)

Block factor for Index,

$$(\text{Block factor})_{\text{Index}} = \left\lfloor \frac{4096}{19} \right\rfloor \text{ entries/block} = 215$$

$$\therefore \text{Number of Index blocks} = \left\lceil \frac{1,50,000}{215} \right\rceil \text{ blocks}$$

$$= \lceil 697.67 \rceil = 698 \text{ index blocks}$$

End of Solution

Q.17 Consider the following sets, where $n \geq 2$:

S_1 : Set of all $n \times n$ matrices with entries from the set $\{a, b, c\}$

S_2 : Set of all functions from the set $\{0, 1, 2, ..., n^2 - 1\}$ to the set $\{0, 1, 2\}$

Which of the following choice(s) is/are correct?

(a) There exists a surjection from S_1 to S_2.

(b) There does not exist a bijection from S_1 to S_2.

(c) There does not exist an injection from S_1 to S_2.

(d) There exists a bijection from S_1 to S_2.

Ans. **(a, d)**

$$|S_1| = 3^{n^2}$$

Since each of the n^2 entries in $n \times n$ matrix can be fills in 3 ways.

$$|S_2| = 3^{n^2}$$

Since $|\{0, 1, 2\}| = 3$ and $|\{0, 1, 2, ... n^2 - 1\}| = n^2$)

Now the theorem says A bijection $f_{A \to B}$ exists iff $|A| = |B|$.

Here $|S_1| = |S_2|$

So, there has to be a bijection from S_1 to S_2. So, option (d) is correct.

If bijection exists surely surjection also exists. So, option (a) is also correct.

Option (b) and (c) are incorrect.

End of Solution

Q.18 Suppose the following functional dependencies hold on a relation U with attributes P, Q, R, S and T:

$$P \to QR$$
$$RS \to T$$

Which of the following functional dependencies can be inferred from the above functional dependencies?

(a) $PS \to T$ (b) $P \to R$

(c) $R \to T$ (d) $PS \to Q$

Ans. **(a, b, d)**

1. $PS \to T$ $(PS)^+ = \{P, S, Q, R, T\}$
2. $P \to R$ $(P)^+ = \{P, Q, R\}$
3. $R \to T$ $(R)^+ = \{R\}$
4. $PS \to Q$ $(PS)^+ = \{P, S, Q, R, T\}$

End of Solution

Q.19 Which of the following statement(s) is/are correct in the context of CPU scheduling?

(a) The goal is to only maximize CPU utilization and minimize throughput.

(b) Turnaround time includes waiting time.

(c) Round-robin policy can be used even when the CPU time required by each of the processes is not known apriori.

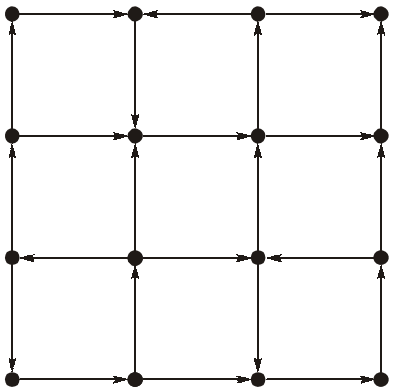
(d) Implementing preemptive scheduling needs hardware support.

Ans. **(b, c, d)**

- Goal is to maximize CPU utilization and maximize the throughput. So, statement (a) is false.
- Statement (b) is true, because turnaround time = completion time – arrival time and waiting time is included in this.
- Statement (c) is true because using time quantum, we can run the processes even if burst time is not known initially in round-robin.
- True for example, round robin scheduling requires hardware support which is timer.

End of Solution

Q.20 Consider the following directed graph:



Which of the following is/are correct about the graph?

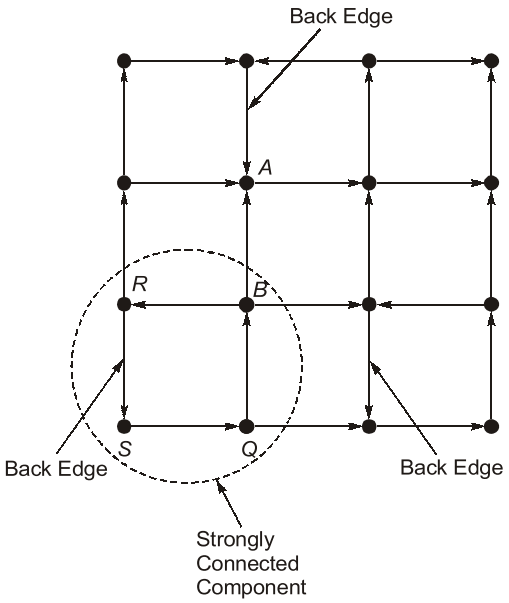
(a) The graph does not have a topological order.

(b) A depth-first traversal starting at vertex *S* classifies three directed edges as back edges.

(c) For each pair of vertices *u* and *v*, there is a directed path from *u* to *v*.

(d) The graph does not have a strongly connected component.

Ans. (a, b)



- As we can see there is cycle in given DAG. So, topological order is not possible.
- Statement (b) is also true.
- There is no path from A to B so statement (c) is false.
- In SQBR, it is strongly connected component. So, statement (d) is false.

End of Solution

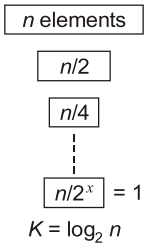
Q.21 What is the worst-case number of arithmetic operations performed by recursive binary search on a sorted array of size n?

(a) $\Theta(n)$ (b) $\Theta(n^2)$

(c) $\Theta\left(\sqrt{n}\right)$ (d) $\Theta(\log_2(n))$

Ans. (d)

Worst case is when the element not present in the sorted array.



Worst case occurrence relation is

$$T(n) = \begin{cases} T\left(\frac{n}{2}\right)+1, & n>1 \\ 1, & n\le 1 \end{cases}$$

$\therefore$ $\Theta(\log_2(n))$

End of Solution

Q.22 Consider a network using the pure ALOHA medium access control protocol, where each frame is of length 1,000 bits. The channel transmission rate is 1 Mbps (= 10^6 bits per second). The aggregate number of transmissions across all the nodes (including new frame transmissions and retransmitted frames due to collisions) is modelled as a Poisson process with a rate of 1,000 frames per second, Throughput is defined as the average number of frames successfully transmitted per second. The throughput of the network (rounded to the nearest integer) is _____.

Ans. **(135) [130 - 140]**

$$1 \text{ frames takes} = Tt = \frac{L}{B.W.}$$

$$\Rightarrow \quad \frac{1000}{10^6} = 1 \text{ millisec}$$

1000 frame Tt = 1000 × 1 millisec = 1 sec

In 1 sec, 1000 frames sends, which is 1 millisec per frame.

So, $G = 1$

Efficiency of Pure Aloha (η) = $G \times e^{-2G}$

where G = Number of requests per time slot willing to transmit

e = Mathematical constant approximately equal to 2.718

So, $\eta = 1 \times 2.718^{(-2 \times 1)} = 0.1353$

Therefore, in 1 sec 1000 frames = 0.1353 × 1000 = 135.3 (closest integer) ⇒ 135

Throughput ⇒ 135

End of Solution

Q.23 For a statement S in a program, in the context of liveness analysis, the following sets are defined :

$USE(S)$: the set of variables used in S

$IN(S)$: the set of variables that are live at the entry of S

$OUT(S)$: the set of variables that are live at the exit of S

Consider a basic block that consists of two statements, S_1 followed by S_2. Which one of the following statements is correct?

(a) $OUT(S_1) = USE(S_1) \cup IN(S_2)$ (b) $OUT(S_1) = IN(S_2) \cup OUT(S_2)$

(c) $OUT(S_1) = IN(S_2)$ (d) $OUT(S_1) = IN(S_1) \cup USE(S_1)$

Ans. **(c)**

In live variable analysis at any node, the set of variables live at just after the block are evaluated using the formula.

$$OUT = \cup \text{ IN (Successor nodes)}$$

So, the correct option is

$$OUT\ (S_1) = IN\ (S_2)$$

So, the correct option is (c).

End of Solution

Q.24 Let S be the following schedule of operations of three transactions T_1, T_2 and T_3 in a relational database system:

$$R_2(Y),\ R_1(X),\ R_3(Z),\ R_1(Y),\ W_1(X),\ R_2(Z),\ W_2(Y),\ R_3(X),\ W_3(Z)$$

Consider the statements P and Q below:

P : S is conflict-serializable.

Q : If T_3 commits before T_1 finishes, then S is recoverable.

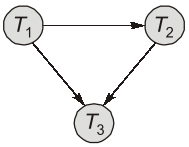
Which one of the following choices is correct?

(a) P is true and Q is false.
(b) Both P and Q are false.
(c) Both P and Q are true.
(d) P is false and Q is true.

Ans. **(a)**

No cycle in precedence graph.

$\therefore$ Conflict serializable.



T_1	T_2	T_3
⋮	⋮	⋮
$W_1(X)$		
	$R_2(Z)$	
	$W_2(Y)$	
		$R_3(X)$
		$W_3(Z)$

T_2 is doing dirty read of updated X by T_1.

So, recoverability only possible if T_3 commits after commit/RB of T_1.

So, option (a) is correct.

End of Solution

Q.25 Consider the following ANSI C program:

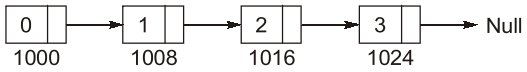
```
#include <stdio.h>
#include <stdlib.h>
struct Node{
        int value;
        struct Node *next;};
int main( ) {
    struct Node *boxE, *head, *boxN; int index = 0;
    boxE = head = (struct Node *) malloc(sizeof(struct Node));
    head → value = index;
    for (index = 1; index <= 3; index++) {
        boxN = (struct Node *) malloc(sizeof(struct Node));
        boxE → next = boxN;
        boxN → value = index;
        boxE = boxN; }
    for (index = 0; index <= 3; index++) {
        printf("Value at index %d is %d\n", index, head → value);
        head = head → next;
        printf("Value at index %d is %d\n", index+1, head → value); } }
```

Which one of the statements below is correct about the program?

(a) Upon execution, the program creates a linked-list of five nodes.
(b) It has a missing return which will be reported as an error by the compiler.
(c) It dereferences an uninitialized pointer that may result in a run-time error.
(d) Upon execution, the program goes into an infinite loop.

Ans. **(c)**

As we can see in the loop, i runs from 1 to 3. So, four nodes will be created because one node is already created with value 0.



When index = 3, then head value = 3

Head = Head → Next **(Now head will point to NULL)**

Head → Value [which is in print. So it will generate Run time Error].

So, option (c) is correct.

End of Solution

Q.26 If x and y are two decimal digits and $(0.1101)_2 = (0.8xy5)_{10}$, the decimal value of $x + y$ is _____.

Ans. **(3)**

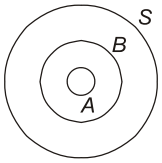
$$(0.1101)_2 = 1 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4}$$
$$= 0.5 + 0.25 + 0 + 0.0625 = (0.8125)_{10}$$
$$(0.8125)_{10} = (0.8xy5)_{10}$$
$$\therefore \quad x = 1,\ y = 2,\ x + y = 1 + 2 = 3$$

End of Solution

Q.27 Let S be a set consisting of 10 elements. The number of tuples of the form (A, B) such that A and B are subsets of S and $A \subseteq B$ is _____.

Ans. **(59049)**

The Venn diagram for this is



Now every element x in S has only 3 options. It can be $x \in A$ or $x \in B - A$ or $x \in S - B$. So the number of ways to choose A and B such that $A \subseteq B \subseteq S$ is 3^{10}.

End of Solution

Q.28 Consider the following ANSI C program:

```
int main( ) {
    Integer x;
    return 0;
}
```

Which one of the following phases in a seven-phase C compiler will throw an error?

(a) Syntax analyzer
(b) Machine dependent optimizer
(c) Semantic analyzer
(d) Lexical analyzer

Ans. **(*)**

[Note: Official answer key given by **IIT-Bombay** is option (c)]

End of Solution

Q.29 Assume a two-level inclusive cache hierarchy. L1 and L2, where L2 is the larger of the two. Consider the following statements.

S_1 : Read misses in a write through L1 cache do not result in writebacks of dirty lines to the L2.

S_2 : Write allocate policy must be used in conjunction with write through caches and no-write allocate policy is "used with writeback caches.

Which of the following statements is correct?

(a) S_1 is true and S_2 is true.
(b) S_1 is true and S_2 is false.
(c) S_1 is false and S_2 is true.
(d) S_1 is false and S_2 is false.

Ans. **(b)**

End of Solution

Q.30 A bag has r red balls and b black balls. All balls are identical except for their colours. In a trial, a ball is randomly drawn from the bag, its colour is noted and the ball is placed back into the bag along with another ball of the same colour. Note that the number of balls in the bag will increase by one, after the trial. A sequence of four such trials is conducted. Which one of the following choices gives the probability of drawing a red ball in the fourth trial?

(a) $\frac{r}{r+b}$

(b) $\frac{r}{r+b+3}$

(c) $\left(\frac{r}{r+b}\right)\left(\frac{r+1}{r+b+1}\right)\left(\frac{r+2}{r+b+2}\right)\left(\frac{r+3}{r+b+3}\right)$

(d) $\frac{r+3}{r+b+3}$

Ans. **(a)**

There are 10 favourable ways to calculate the probability of red ball in 4^{th} trial.
(RFR)R ≈ R or (BRR)R ≈ 1 way or (RRR)R ≈ 3 ways or (BBR)R ≈ 3 ways

$$P(RRRR) = \frac{r}{r+b}\times\frac{r+1}{r+1+b}\times\frac{r+2}{r+2+b}\times\frac{+2}{r+3+b} \quad ...(1)$$

$$P(BBBR) = \frac{b}{r+b}\times\frac{b+1}{r+b+1}\times\frac{b+2}{r+b+2}\times\frac{r}{r+b+3} \quad ...(2)$$

$$P(RRBR) = \frac{3!}{2!}\times\frac{r}{r+b}\times\frac{r+1}{r+b+1}\times\frac{b}{r+b+2}\times\frac{r+2}{r+b+3} \quad ...(3)$$

$$P(BBRR) = \frac{3!}{2!}\times\frac{b}{r+b}\times\frac{b+1}{r+b+1}\times\frac{r}{r+b+2}\times\frac{r+1}{r+b+3} \quad ...(4)$$

Required probability = (1) + (2) + (3) + (4)

$$= \frac{r(r+1)(r+2)(r+3)+b(b+1)(b+2)r+3r(r+1)b(r+2)+3b(b+1)r(r+1)}{(r+b)(r+b+1)(r+b+2)(r+b+3)}$$

On solving it we get,

$$= \frac{r(r+1+b)}{(r+b)(r+b+1)} = \frac{r}{r+b}$$

End of Solution

Q.31 Consider the following multi-threaded code segment (in a mix of *C* and pseudo-code), invoked by two processes P_1 and P_2, and each of the processes spawns two threads T_1 and T_2:

```
int x = 0;      // global
Lock L1;        // global
main( ) {
   create a thread to execute foo( );   // Thread T1
   create a thread to execute foo( );   // Thread T2
   wait for the two threads to finish execution;
   print (x);}
foo( ) {
   int y = 0;
   Acquire L1;
   x = x + 1;
   y = y + 1;
   Release L1;
   print (y); }
```

Which of the following statement(s) is/are correct?

(a) At least one of P_1 and P_2 will print the value of x as 4,

(b) Both P_1 and P_2 will print the value of x as 2.

(c) Both T_1 and T_2, in both the processes, will print the value of y as 1.

(d) At least one of the threads will print the value of y as 2.

Ans. **(b, c)**

P_1 and P_2 can spawn two threads T_1 and T_2.

```
int x = 0; //global
Lock L1; //global
main( ) {
    foo( ); //Thread T1
    foo( ); //Thread T2
    print(x);
}
foo( ) {
a.  int y = 0;
b.  Acquire L1;
c.  x = x + 1;
d.  y = y + 1;
    Release L1;
    print(y);
}
```

- Let P_1 executed T_1 and in foo( ).

(a) $P_1 - T_1$ X

(b) $P_1 - T_1$ ✓ 1

(c) $x = x + 1$ $(P_1 - T_1)$

.... Preempt T_1 of P_1.

Similarly, perform thread T_2 of P_1 then $x = 2$.

Now, if we similarly perform both threads of P_2 then x will be maximum 2.

Note: It is not mentioned about IPC or concurrent mechanism so the process will not shared the global variable. Each process will have its on own address space so the maximum value of x by each process can be 2 only.

Note: But as we know every foo call will have its own copy of variable y so y can not be more than 1 in any case.

End of Solution

Q.32 Suppose we want to design a synchronous circuit that processes a string of 0's and 1's. Given a string, it produces another string by replacing the first 1 in any subsequence of consecutive 1's by a 0. Consider the following example.

Input sequence : 00100011000011100

Output sequence : 00000001000001100

A Mealy Machine is a state machine where both the next state and the output are functions of the present state and the current input.

The above mentioned circuit can be designed as a two-state Mealy machine. The states in the Mealy machine can be represented using Boolean values 0 and 1. We denote the current state, the next state, the next incoming bit, and the output bit of the Mealy machine by the variables s, t, b and y respectively.

Assume the initial state of the Mealy machine is 0.

What are the Boolean expressions corresponding to t and y in terms of s and b?

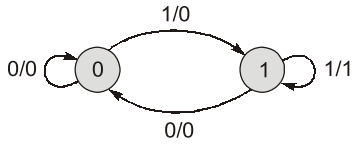
(a) $t = b$
$y = s\bar{b}$

(b) $t = s + b$
$y = sb$

(c) $t = s + b$
$y = s\bar{b}$

(d) $t = b$
$y = sb$

Ans. **(d)**



PS s	Next State t, $b = 0$	O/P y, $b = 1$
0	0,0	1,0
1	0,0	1,1

$t = \bar{s}b + sb = b$

$y = sb$

End of Solution

Q.33 Consider the following ANSI C code segment:

```
z = x + 3 + y → f1 + y → f2;
for (i = 0; i < 200; i = i + 2) {
    if (z > i) {
        p = p + x + 3;
        q = q + y → f1;
    } else {
        p = p + y → f2;
        q = q + x + 3;
    }
}
```

Assume that the variable *y* points to a struct (allocated on the heap) containing two fields *f*1 and *f*2, and the local variables x, y, z, p, q, and i are allotted registers. Common sub-expression elimination (CSE) optimization is applied on the code. The number of addition and dereference operations (of the form $y \rightarrow f1$ or $y \rightarrow f2$) in the optimized code, respectively, are :

(a) 303 and 2 (b) 403 and 102

(c) 203 and 2 (d) 303 and 102

Ans. **(a)**

$t_1 = y \rightarrow f_1$ (1 dereference)

$t_2 = y \rightarrow f_2$ (1 dereference)

$t_3 = x + 3$ (1 add)

$z = t_3 + t_1 + t_2$ (2 additions)

```
For (i = 0; i < 200; i+ = 2)
{
    if (z > i)
    {
        p = p + t3
        q = q + t1 (2 add)
    }
    else
    {
        p = p + p2
        q = q + t3 (2 add)
    }
}
```

If else condition ⇒

Either if is executed (or) else is executed.

⇒ At any iteration 2 addition operations will be executed.

So, in loop the iterations are $\left(\frac{200}{2}\right)$ = 100 times.

∴ In loop the number of additions = 100 × 2 = 200 additions

$\therefore$ Total additions = 200 + 1 + 2 + 100 loop additions (inside for loop)
= 303

and 2 dereferences.

$\therefore$ Correct answer is 303 and 2 which is option (a).

End of Solution

Q.34 For two n-dimensional real vectors P and Q, the operation $s(P, Q)$ is defined as follows:

$$s(P, Q) = \sum_{i=1}^{n} (P[i].Q[i])$$

Let $\mathcal{L}$ be a set of 10-dimensional non-zero real vectors such that for every pair of distinct vectors $P, Q \in \mathcal{L}$, $s(P, Q) = 0$. What is the maximum cardinality possible for the set $\mathcal{L}$?

(a) 10 (b) 9

(c) 100 (d) 11

Ans. **(a)**

$\mathcal{L}$ is the set of 10-dimensional orthogonal vectors. So cardinality of $\mathcal{L} \leq 10$.

i.e., Maximum cardinality of $\mathcal{L} = 10$.

So, option (a) is correct.

End of Solution

Q.35 The relation scheme given below is used to store information about the employees of a company, where empId lis the key and deptId indicates the department to which the employee is assigned. Each employee is assigned to exactly one department.

emp(empId, name, gender, salary, deptId)

Consider the following SQL query:

```
select deptId, count(*)
from emp
where gender = "female" and salary > (select avg(salary) from emp)
group by deptId;
```

The above query gives, for each department in the company, the number of female employees whose salary is greater than the average salary of

(a) female employees in the department.

(b) employees in the department.

(c) female employees in the company.

(d) employees in the company.

Ans. (d)

emp	empId	Name	Gender	Salary	deptid
21K>20K	e_1	*X*	Female	210K	CS
	e_2	*Y*	Male	19K	CS
25K>20K	e_3	*XZ*	Female	25K	EC
	e_4	*YZ*	Male	14K	EC
21K>20K	e_5	*a*	Female	21K	CS

Average salary of all employees in the company.

O/P

deptid	count(*)
CS	2
EC	1

For each department, number of female employees whose salary is greater than average salary of employees in the company.

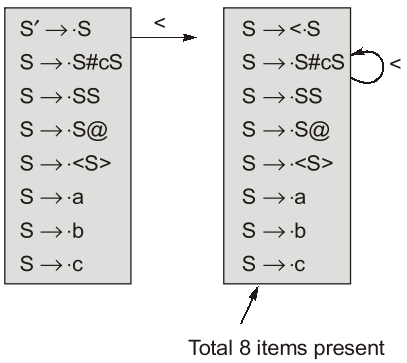
End of Solution

Q.36 Consider the following augmented grammar with {#, @, <, >, a, b, c} as the set of terminals.

$S' \rightarrow S$
$S \rightarrow S \# cS$
$S \rightarrow SS$
$S \rightarrow S@$
$S \rightarrow <S>$
$S \rightarrow a$
$S \rightarrow b$
$S \rightarrow c$

Let I_0 = CLOSURE($S' \rightarrow \bullet S$}). The number of items in the set GOTO(GOTO(I_0, <), <) is _____.

Ans. (8)



End of Solution

Q.37 Consider the following ANSI C function:

```
int SomeFunction(int x, int y)
{
    if ((x == 1)||(y == 1)) return 1;
    if (x == y) return x;
    if (x > y) return SomeFunction(x – y, y);
    if (y > x) return SomeFunction(x, y – x);
}
```

The value returned by SomeFunction(15, 255) is _____.

Ans. **(15)**

This function will keep on subtracting till both x and y becomes equal that is 15.

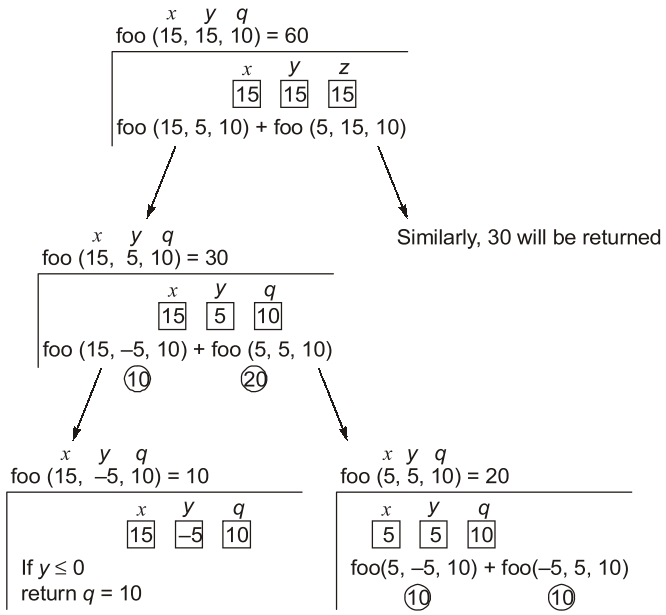
End of Solution

Q.38 Consider the following ANSI C program:

```
#include <stdio.h>
int foo(int x, int y, int q)
{
    if ((x <= 0) || (y <= 0))
        return q;
    if (x <= 0)
        return foo(x, y – q, q)
    if (y <= 0)
        return foo(x – q, y, q);
    return foo(x, y – q, q) + foo(x – q, y, q);
}
int main( )
{
    int r = foo(15, 15, 10);
    printf("%d", r);
    return 0;
}
```

The output of the program upon execution is _____.

Ans. (60)



End of Solution

Q.39 Consider a Boolean function $f(w, x, y, z)$ such that

$$f(w, 0, 0, z) = 1$$
$$f(1, x, 1, z) = x + z$$
$$f(w, 1, y, z) = wz + y$$

The number of literals in the minimal sum-of-products expression of f is _____.

Ans. (6)

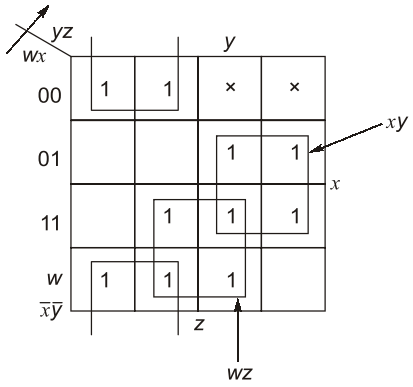
$f(w, 0\ 0, z) = 1$ ...(1)

$f(1, x, 1, z) = x + z$ ...(2)

$f(w, 1, y, z) = wz + y$ ...(3)

	$wxyz$	Eqn. 1	Eqn. 2	Eqn. 3	f
0	0000	1			1
1	0001	1			1
2	0010				×
3	0011				×
4	0100			0	0
5	0101			0	0
6	0110			1	1
7	0111			1	1
8	1000	1			1
9	1001	1			1
10	1010		0		0
11	1011		1		1
12	1100			0	0
13	1101			1	1
14	1110		1	1	1
15	1111		1	1	1

$f(w, x, y, z) = \Sigma m(0, 1, 6, 7, 8, 9, 11, 13, 14, 15) + d(2, 3)$



$\therefore \quad f = \bar{x}\bar{y} + xy + wz$

The number of literals in the minimal SOP expression is 6.

End of Solution

Q.40 Consider the cyclic redundancy check (CRC) based error detecting scheme having the generator polynomial $X^3 + X + 1$. Suppose the message $m_4m_3m_2m_1m_0 = 11000$ is to be transmitted. Check bits $c_2c_1c_0$ are appended at the end of the message by the transmitter using the above CRC scheme, The transmitted bit string is denoted by $m_4m_3m_2m_1m_0c_2c_1c_0$. The value of the checkbit sequence $c_2c_1c_0$ is

(a) 110 (b) 111

(c) 100 (d) 101

Ans. **(c)**

$x^3 + x + 1 = 1011$

$$\begin{array}{r|l} 1011 & 11000000 \\ & 1011 \\ \hline & 1110 \\ & 1011 \\ \hline & 1010 \\ & 1011 \\ \hline & 100 \end{array}$$

End of Solution

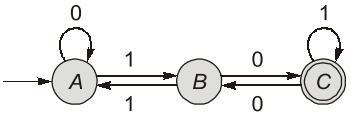
Q.41 Which of the following regular expressions represent(s) the set of all binary numbers that are divisible by three? Assume that the string e is divisible by three.

(a) (0*(1(01*0)*1)*)* (b) (0 +1(01*0)* 1)*

(c) (0 + 11 + 10(1 + 00)*01)* (d) (0 + 11 + 11(1 + 00)*00)*

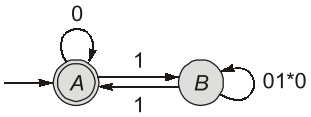
Ans. **(a, b, c)**

The DFA for accepting all binary strings divisible by 3 is given below:



where A is residue 0 state, B is residue 1 state and C is residue 2 state.

From this we get by deleting (c)



We get option (b) is correct (0 + 1(01*0)*1)*

Now option (a) = option (b)

because $(r^*s^*)^* = (r + s)^*$

So, option (a) is also correct.

Option (c) can be obtained by resolving the loop between B and C on "C" instead of on "B".

Option (c) is also correct.

Also note that whatever string option (b) can derive, option (c) also can derive. No counter example possible.

So, option (c) is also correct.

Option (d) (0 + 11 + 11(1 + 00)*00)* cannot derive "1001" which is accepted by machine.
So, option (d) is incorrect.
So, options (a), (b) and (c) are correct.

End of Solution

Q.42 Consider the following ANSI C program:

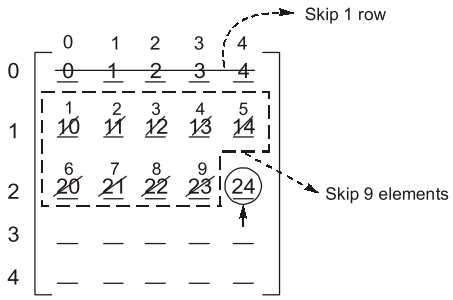
```
#include <stdio.h>
int main( ) {
    int arr[4] [5] ;
    int i, j;
    for (i = 0; i < 4; i++) {
        for (j = 0; j < 5; j++) {
            arr[i] [j] = 10 * i + j;
        }
    }
    printf("%d", * (arr[i] + 9));
    return 0;
}
```

What is the output of the above program?

(a) 24 (b) 20

(c) 14 (d) 30

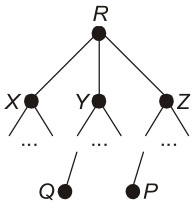
Ans. **(a)**



int *a*[4][5]
[*(*a* + 1) + 9]
It means skip one row and then skip 9 elements.
So, the resultant value is 24.

End of Solution

Q.43 Consider a computer network using the distance vector routing algorithm in its network layer. The partial topology of the network is as shown below.



The objective is to find the shortest-cost path from the router *R* to routers *P* and *Q*. Assume that *R* does not initially know the shortest routes to *P* and *Q*. Assume that *R* has three neighbouring routers denoted as *X*, *Y* and *Z*. During one iteration, *R* measures its distance to its neighbours *X*, *Y* and *Z* as 3, 2 and 5, respectively. Router *R* gets routing vectors from its neighbours that indicate that the distance to router *P* from routers *X*, *Y* and *Z* are 7, 6 and 5, respectively. The routing vector also indicates that the distance to router *Q* from routers *X*, *Y* and *Z* are 4, 6 and 8, respectively. Which of the following statement(s) is/are correct with respect to the new routing table of *R*, after updation during this iteration?

(a) The distance from *R* to *Q* will be stored as 7.
(b) The distance from *R* to *P* will be stored as 10.
(c) The next hop router for a packet from *R* to *Q* is *Z*.
(d) The next hop router for a packet from *R* to *P* is *Y*.

Ans. **(a, d)**

Given *R* gets the distance vector (3, 2, 5)
After the one iteration distance vector from *X* to *P*, *Y* to *P*, and *Z* to *P* is (7, 6, 5) respectively.
The distance vector from *R* to *P* via *XYZ* is (3 + 7, 2 + 6, 5 + 5) = (10, 8, 10)
So, take minimum distance from *R* to *P* which is 8 via *Y*.
After the iteration distance vector from *X* to *Q*, *Y* to *Q*, *Z* to *Q* is (4, 6, 8) respectively.
The distance vector from *R* to *Q* via *XYZ* is (3 + 4, 2 + 6, 5 + 8) = (7, 8, 13).
So, take minimum distance from *R* to *Q* which is 7 via *X*.

End of Solution

Q.44 Let L_1 be a regular language and L_2 be a context-free language. Which of the following languages is/are context-free?

(a) $L_1 \cap \overline{L_2}$

(b) $\overline{\overline{L_1} \cup \overline{L_2}}$

(c) $L_1 \cup (L_2 \cup \overline{L_2})$

(d) $(L_1 \cap L_2) \cup (\overline{L_1} \cap L_2)$

Ans. **(b, c, d)**

$L_1 \rightarrow$ Regular

$L_2 \rightarrow$ CFL

1. $L_1 \cap \overline{L_2} = \text{Reg} \cap \overline{\text{CFL}} = \text{Reg} \cap \overline{\text{CSL}}$
   $= \text{Reg} \cap \text{CSL} = \text{CSL}$ (need not be CFL)
2. $\overline{\overline{L_1} \cup \overline{L_2}} = L_1 \cap L_2 = \text{Reg} \cap \text{CFL} = \text{CFL}$
3. $L_1 \cup (L_2 \cup \overline{L_2}) = L_1 \cup \Sigma^* = \Sigma^* = \text{Regular and hence CFL}$
4. $(L_1 \cap L_2) \cup (\overline{L_1} \cap L_2) = (L_1 \cup \overline{L_1}) \cap L_2$
   $= \Sigma^* \cap L_2$
   $= L_2 = \text{CFL}$

End of Solution

Q.45 For constants $a \geq 1$ and $b > 1$, consider the following recurrence defined on the non-negative integers:

$$T(n) = a \cdot T\left(\frac{n}{b}\right) + f(n)$$

Which one of the following options is correct about the recurrence $T(n)$?

(a) If $f(n)$ is $\Theta(n^{\log_b(a)})$, then $T(n)$ is $\Theta(n^{\log_b(a)})$.

(b) If $f(n)$ is $O(n^{\log_b(a) - e})$ for some $e > 0$, then $T(n)$ is $\Theta(n^{\log_b(a)})$.

(c) If $f(n)$ is $\dfrac{n}{\log_2(n)}$, then $T(n)$ is $\Theta(\log_2(n))$.

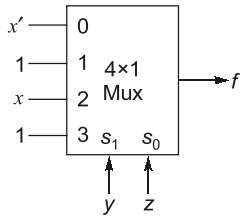
(d) If $f(n)$ is $n \log_2(n)$, then $T(n)$ is $\Theta(n \log_2(n))$.

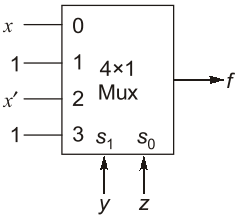
Ans. **(b)**

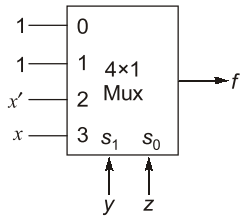
According to Standard Master Theorem, only option (b) is correct.

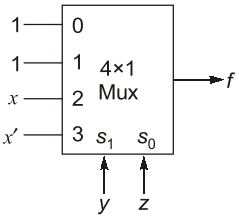
End of Solution

Q.46 Which one of the following circuits implements the Boolean function given below?
$f(x, y, z) = m_0 + m_1 + m_3 + m_4 + m_5 + m_6$, where m_i is the i^{th} minterm.

(a)


(b)


(c)


(d)


Ans. **(d)**

$$\Sigma(x, y, z) = m_0 + m_1 + m_3 + m_4 + m_5 + m_6$$
$$= \Sigma m(0, 1, 3, 4, 5, 6)$$

As per the given options, variable y is connected to multiplexer select input S_1 and z is connected to select input S_0.

		S_1 y 00	S_0 z 01	I_2 10	I_3 11
		I_0	I_1		
x	0	⓪	①	2	③
x	1	④	⑤	⑥	7
		1	1	x	$\bar{x}$

So, the correct option is (d).

End of Solution

Q.47 Consider the string abbccddeee. Each letter in the string must be assigned a binary code satisfying the following properties:

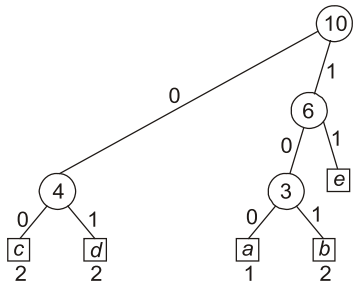
1. For any two letters, the code assigned to one letter must not be a prefix of the code assigned to the other letter.
2. For any two letters of the same frequency, the letter which occurs earlier in the dictionary order is assigned a code whose length is at most the length of the code assigned to the other letter.

Among the set of all binary code assignments which satisfy the above two properties, what is the minimum length of the encoded string?

(a) 23 (b) 25
(c) 21 (d) 30

Ans. (a)

Letters used in string :	*a*	*b*	*c*	*d*	*e*
Frequencies	1	2	2	2	3



Letters used in string:

∴ Minimum length of encoded string

= Sum frequency count using Huffman coding

= 1 * 3 + 2 * 3 + 2 * 2 + 2 * 2 + 3 * 2

= 3 + 6 + 4 + 4 + 6 = 23

End of Solution

Q.48 If the numerical value of a 2-byte unsigned integer on a little endian computer is 255 more than that on a big endian computer, which of the following choices represent(s) the unsigned integer on a little endian computer?

(a) 0x6665 (b) 0x0001

(c) 0x0100 (d) 0x4243

Ans. (a, c)

Option (a): In little endian 0x6665 on converting it to decimal = 26213

In big endian it will be 6566 on converting it to decimal = 25958

Now little endian – big endian = 26231 – 25958 = 255 which is correct.

Option (b): In little endian 0x0001 which is 1 in decimal in big endian 0x0100 which is greater than little endian in decimal. So this is incorrect option.

Option (c): Little endian = 0x0100 which is 256 in decimal and big endian = 0x0001 which is 1 in decimal an difference will be 255 hence this is also correct option.

Option (d): Little endian = 0x4243 and big endian = 0x4342, big endian value is greater than little endian so this is incorrect.

End of Solution

Q.49 Consider a pipelined processor with 5 stages. Instruction Fetch (IF), Instruction Decode (ID), Execute (EX), Memory Access (MEM), and Write Back (WB). Each stage of the pipeline, except the EX stage, takes one cycle. Assume that the ID stage merely decodes the instruction and the register read is performed in the EX stage. The EX stage takes one cycle for ADD instruction and two cycles for MUL instruction. Ignore pipeline register latencies. Consider the following sequence of 8 instructions:

ADD, MUL, ADD, MUL, ADD, MUL, ADD, MUL

Assume that every MUL instruction is data-dependent on the ADD instruction just before it and every ADD instruction (except the first ADD) is data-dependent on the MUL instruction just before it. The Speedup is defined as follows:

$$\text{Speedup} = \frac{\text{Execution time without operand forwarding}}{\text{Execution time with operand forwarding}}$$

The Speedup achieved in executing the given instruction sequence on the pipelined processor (rounded to 2 decimal places) is _____.

Ans. **(1.87) (1.87 - 1.88)**

With operand forwarding:

8 Instructions + 4 MUL Instruction × 1 Extra Cycle in Ex-stage

$\Rightarrow$ $n = 12$ (finite)

$\Rightarrow$ $K = 5$

$ET_{Pipe} = (K + n - 1)$ Cycles

$= (5 + 12 - 1)$

$= 16$ cycles

Without operand forwarding :

- 8 Instructions + 4 MUL Instruction × 2 Stalls at ID stage for ADD O/P
  + 3 ADD Instruction × 3 Stalls at ID stage for MUL O/P
  + 1 MUL Instruction × 1 Extra Cycle in Ex-Stage (Last Instruction)

$\Rightarrow$ $n = 26$

$\Rightarrow$ $K = 5$

$ET_{Pipe} = (K + n - 1)$ Cycles

$= (5 + 26 - 1)$ Cycles

$= 30$ Cycles

$\therefore$ $S = \frac{30}{16} = 1.875$

End of Solution

Q.50 Choose the correct choice(s) regarding the following propositional logic assertion S:

$$S : ((P \wedge Q) \rightarrow R) \rightarrow ((P \wedge Q) \rightarrow (Q \rightarrow R))$$

(a) S is a tautology.
(b) S is neither a tautology nor a contradiction.
(c) The antecedent of S is logically equivalent to the consequent of S.
(d) S is a contradiction.

Ans. **(a, c)**

$$\begin{aligned} S : ((P \wedge Q) \rightarrow R) \rightarrow ((P \wedge Q) &\rightarrow (Q \rightarrow R)) \\ &\equiv (pq \rightarrow r) \rightarrow (pq \rightarrow (q \rightarrow r)) \\ &\equiv [(pq)' + r] \rightarrow [(pq)' + (q' + r)] \\ &\equiv [(pq)' + r]' + [(pq)' + q' + r] \\ &\equiv [pq \cdot r'] + [p' + q' + q' + r] \\ &\equiv pqr' + p' + q' + r \\ &\equiv (p + p')(qr' + p') + q' + r \\ &\equiv qr' + p' + q' + r \\ &\equiv (q + q')(r' + q') + p' + r \\ &\equiv r' + q' + p' + r \equiv r' + r + q' + p' \\ &\equiv 1 + q' + p' \equiv 1 \text{ (Tautology)} \end{aligned}$$

So, S is a tautology.
So, option (a) is true.
Option (b) and (d) are false.
Option (c) antecedent of S is

$$\begin{aligned} pq \rightarrow r &\equiv (pq)' + r \\ &\equiv p' + q' + r \end{aligned}$$

The consequent of S is $pq \rightarrow (q \rightarrow r)$

$$\begin{aligned} &\equiv (pq)' + q' + r \\ &\equiv p' + q' + q' + r \\ &\equiv p' + q' + r \end{aligned}$$

So, Antecedent of $S \equiv$ Consequent of S
So, option (c) is also true.

End of Solution

Q.51 Suppose that P is a 4×5 matrix such that every solution of the equation $Px = 0$ is a scalar multiple of $[2 \quad 5 \quad 4 \quad 3 \quad 1]^T$. The rank of P is _____.

Ans. **(4)**

$P_{4 \times 5} \Rightarrow$ Number of unknowns $(n) = 5$ in $PX = 0$ ...(1)
Also it is given that Nullity $PX = 0$ is one, i.e., $N(P) = 1$.
Hence, Nullity = Number of unknowns – Rank

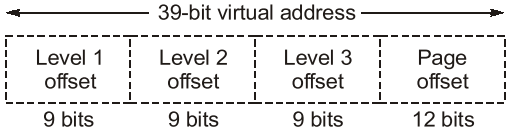
$$1 = 5 - \rho(P)$$

or

$$\rho(P) = 5 - 1 = 4$$

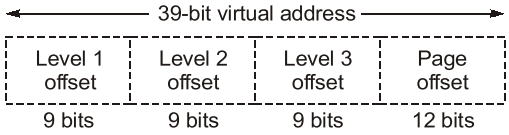
End of Solution

Q.52 Consider a three-level page table to translate a 39-bit virtual address to a physical address as shown below:



The page size is 4 KB (1 KB = 2^{10} bytes) and page table entry size at every level is 8 bytes. A process *P* is currently using 2 GB (1 GB = 2^{30} bytes) virtual memory which is mapped to 2 GB of physical memory. The minimum amount of memory required for the page table of P across all levels is _____ KB.

Ans. **(4108)**



Since process P is using 2 GB physical memory and page size 2^{12} bytes.

So, Number of pages = $\frac{2^{31}}{2^{12}} = 2^{19}$

There are 2^9 entries per page table in 3rd level.

So, we need = $\frac{2^{19}}{2^9}$ = 2^{10} page tables in 3rd level

Now, it means 2^{10} entries in 2nd level so number of page tables in 2nd level

$$= \frac{2^{10}}{2^9} = 2 \text{ page table}$$

Now we have 2 entries in 1st level hence we need only 1 page table in 1st level.
Total number of page table = 2^{10} + 2 + 1 = 1027 per table
Each page table has 2^9 entries and each entry size is 8 bytes.
So, total size of page table in bytes = 1027 × 2^9 × 8 bytes = 4108 KB

End of Solution

Q.53 Suppose that $f: R \to R$ is a continuous function on the interval [–3, 3] and a differentiable function in the interval (–3, 3) such that for every x in the interval. $f'(x) \le 2$. If $f(-3) = 7$, then then $f(3)$ is at most _____.

Ans. **(19)**

$$f'(x) \le 2,\ f(-3) = 7$$

$$f'(x) = \frac{f(3)-7}{3-(-3)} \qquad \text{[Using Lagrange]}$$

$$f'(x) = \frac{f(3)-7}{6}$$

$$6f'(x) = f(3) - 7$$

$$f(3) = 6f'(x) + 7$$

Given max value of $f'(x)$ is 2

So, $f(3) = 6 \times 2 + 7$

$f(3) = 19$

End of Solution

Q.54 Let G be a connected undirected weighted graph. Consider the following two statements.

S_1 : There exists a minimum weight edge in G which is present in every minimum spanning tree of G.

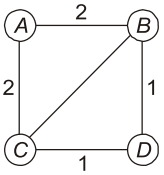
S_2 : If every edge in G has distinct weight, then G has a unique minimum spanning tree.

Which one of the following options is correct?

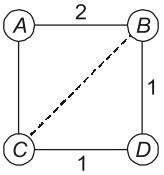
(a) S_1 is false and S_2 is true.
(b) Both S_1 and S_2 are false.
(c) S_1 is true and S_2 is false.
(d) Both S_1 and and S_2 are true.

Ans. **(a)**

S_1 : Consider the graph:



One of the possible MSTS can be:



But the minimum weight edge BC in G is not present. So, the S_1 is a false statement.

S_2 : In any undirected graph G, distinct edge weights means Unique MST.

So, the S_1 is false and S_2 is true.

Option (a) is correct.

End of Solution

Q.55 Consider a set-associative cache of size 2 KB (1 KB = 2^{10} bytes) with cache block size of 64 bytes. Assume that the cache is byte-addressable and a 32-bit address is used for accessing the cache. If the width of the tag field is 22 bits, the associativity of the ache is _____.

Ans. **(2)**

Set Associative Map

CM Site = 2 KB

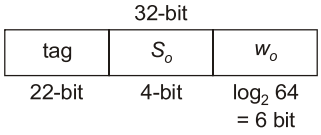
Block Size = 64 B

$$\text{Number of lines} = \frac{2^{11}}{2^6} \Rightarrow 2^5(32)$$

MM Adder = 32 bit

Tag field size = 22 bits

Set associative CM adder format



Set offset (S_o) = 4 bit

$\therefore$ Number of sets (S) = $2^4(16)$

$$\text{Number of Sets } (S) = \frac{N}{P\text{-way}}$$

$$16 = \frac{32}{P\text{-way}}$$

$$\text{P-way} = \frac{32}{16} = 2$$

End of Solution

■■■■