RL, FA, RE and RG

One-mark Questions

1. Which of the following is TRUE? **[2014]**

(a) The language $L = \{a^n b^n \mid n \geq 0\}$ is regular.

(b) The language $L = \{a^n \mid n \text{ is prime}\}$ is regular.

(c) The language $L = \{w|w$ has 3k + 1 b's for some $k \in N$ with $\Sigma = \{a, b\}$ } is regular.

(d) The language $L = \{ww \mid w \in \Sigma^*$ with $\Sigma = \{0, 1\}$ is regular.

Solution: (c)

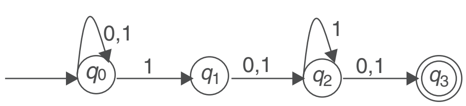
As per option (c), we get $L = \{w|$ w has $3k + 1$ b's for some $k \in N$ with $\Sigma = \{a, b\}$ } for every value of k, we get the language L with finite number of b's in a string of a's and b's. Let $n = 3k + b$.

Now, L is a language of all strings over the alphabet $\Sigma = \{a, b\}$ where every string contains exactly n b's. Then L is accepted by a finite autometa with $n + 2$ states.

So, L is a regular language.

Hence, the correct option is (c).

2. Consider the finite automaton of the following figure. **[2014]**

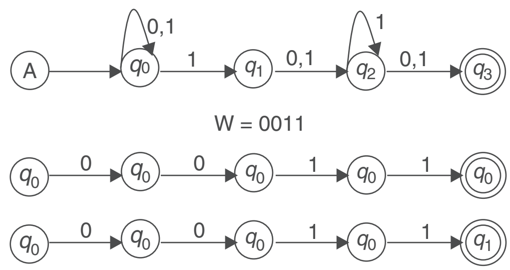


What is the set of reachable states?

(a) $\{q_0, q_1, q_2\}$ (b) $\{q_0, q_1\}$

(c) $\{q_0, q_1, q_2, q_3\}$ (d) $\{q_3\}$

Solution: (a)



So, by this analysis we can conclude that the states acceptable to the string w: $\{q_0, q_1, q_2\}$.

Hence, the correct option is (a).

3. If $L_1 = \{a^n | \ n \geq 0\}$ and $L_2 = \{b^n \mid n \geq 0\}$ **[2014]**

Consider I) $L_1.L_2$ is a regular language

II) $L_1.L_2 = a^n b^n \mid n \geq 0\}$

(a) Only I

(b) Only II

(c) Both I and II

(d) Neither I nor II

Solution: (a)

Because $L_1 = \{a^n | \ n \geq 0\}$, it is a regular language and $L_2 = \{b^n | \ n \geq 0\}$ is also a regular language.

So, it can be concluded that L_1. L_2 is also a regular language.

Hence, the correct option is (a).

4. The length of shortest string NOT in the language (over $\Sigma = \{a, b\}$) of the following regular expression is ______________. **[2014]**

$A^*b^*(ba)^*a^*$

Solution: $r = a^*b^*(ba)^*a^*$

'r' may generate all the string with length 0,1 and 2 but that does not guarantee all string will be of length 3.

5. Consider the language $L_1 = \Phi$ and $L_2 = \{a\}$. Which one of the following represents $L_1\ L_2\ U\ L_1^*$? **[2013]**

(a) $\{\varepsilon\}$ (b) ψ

(c) a^* (d) $\{\varepsilon, a\}$

Solution: (a)

$L_1 = \Phi$, i.e. $L_1^* = \{\varepsilon\}$

$L_2 = \{a\}\ L_2^* = a^*$

$L_1 \cdot L_2^* = \Phi$

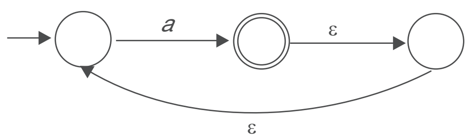
$L_1 \cdot L_2^* U\ L_1^*$ (putting $L_1 \cdot L_2^* = \Phi$)

$= \Phi\ U\ \{\varepsilon\} = \{\varepsilon\}$.

Hence, the correct option is (a).

6. What is the complement of the language accepted to be NFA shown below? **[2012]**

Assume $\Sigma = \{a\}$ and ε is the empty string.



(a) Φ (b) $\{\varepsilon\}$

(c) a^* (d) $\{a, \varepsilon\}$

Solution: (b)

By the definition of NFA, $L = a^+$ is accepted language and $\Sigma = \{a\}$

So L's complement $= \Sigma^* - a^+ = a^* - a^+ = \{\varepsilon\}$

Hence, the correct option is (b).

7. Let L_1 be recursive language. Let L_2 and L_3 be the languages that are recursively enumerable but not recursive. Which of the following statements is not necessarily true? **[2010]**

(a) $L_2 - L_1$ is recursively enumerable.

(b) $L_1 - L_3$ is recursively enumerable.

(c) $L_2 \cap L_1$ is recursively enumerable.

(d) $L_2 \cup L_1$ is recursively enumerable.

Solution: (b)

$L_2 - L_1 = L_2 \cap L_1$ is recursive enumerable.

$L_1 - L_3 = L_1 \cap L_3$ may not be a regular expression as L_3 need not RES.

Hence, the correct option is (b).

8. Which one of the following languages over the alphabet $\{0, 1\}$ is described by the regular expression $(0 + 1)^*0(0 + 1)^*0(0 + 1)^*$? **[2009]**

(a) The set of all strings containing the substring 00

(b) The set of all strings containing at most two 0's

(c) The set of all strings containing at least two 0's

(d) The set of all strings starting and ending with 0 or 1

Solution: (c)

$(0 + 1)^*0(0 + 1)^*0(0 + 1)^*$ will generate all the strings containing at least two zeros.

Hence, the correct option is (c).

9. Consider the set Σ^* of all strings over the alphabet $\Sigma = \{0, 1\}$. Σ^* with the concatenation operators for strings **[2003]**

(a) does not form a group.

(b) forms a non-commutative group.

(c) does not have a right identity element.

(d) forms a group if the empty string is removed from Σ^*.

Solution: (a)

This does not satisfy the inverse property.

Hence, the correct option is (a).

10. The regular expression $0^*(10^*)^*$ denotes the same set as **[2003]**

(a) $(1^*0)^*1^*$

(b) $0 + (0 + 10)^*$

(c) $(0 + 1)^*10(0 + 1)^*$

(d) none of these

Solution: (d)

Hence, the correct option is (d).

11. Consider the following two statements **[2001]**

$S_1 : \{0^{2n} | n \geq 1\}$ is a regular language.

$S_2 : \{0^m10^n | m \geq 1 \text{ and } n \geq 1\}$ is a regular language.

(a) Only S_1 is correct

(b) Only S_2 is correct

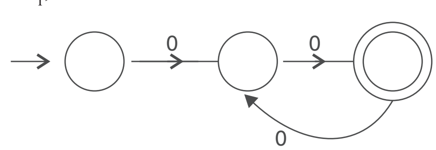
(c) Both S_1 and S_2 are correct

(d) None is correct

Solution: (a)

As for S_2 we cannot construct a FA.

For S_1, FA is



Hence, the correct option is (a).

12. Given an arbitrary non-deterministic automation with N states the maximum number of states in an equivalent minimized DFA is at least **[2001]**

(a) N^2 (b) 2^N

(c) $2N$ (d) $N!$

Solution: (b)

For $1 \leq M \leq 2^N$, number of states in the DFA is M.

Hence, the correct option is (b).

13. Let S & T be language over $\Sigma = \{a, b\}$ represented by regular expression $(a + b^*)^*$ and $(a + b)^*$, respectively, which of the following is true? **[2000]**

(a) S & 2 (b) $1 = \supset$

(c) S = T (d) $S \cap T = \Phi$

Solution: (c)

Same language is generated by S and T, as $(r_1 + r_2)^* = (r_1 + r_2^*)^*$.

Hence, the correct option is (c).

14. Let L denote the language generated by grammar $S \rightarrow 0S0|00$. Which one of the following is true? **[2000]**

(a) $L = 0+$

(b) L is regular but not $0+$

(c) L is context free but not regular

(d) L is not context free

Solution: (c)

By $S \rightarrow 0S0|00$, which is CNG, can generate a CFL but not a regular set.

Hence, the correct option is (c).

15. Consider the regular expression $(0 + 1)(0 + 1)\ldots$ n times. The minimum state finite automation that recognizes the language represented by this regular expression contains **[1999]**

(a) n states

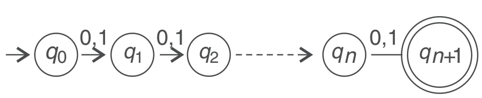
(b) $n + 1$ states

(c) $n + 2$ states

(d) none of the above

Solution: (c)

We know that $(0 + 1)(0 + 1) \ldots n$ times $= (0 + 1)^n$ and $(0 + 1)^n = \{w \varepsilon (0,1)^* |\, |w| = n\}$.



Here the string of length n can be defined by $n + 1$ states. Moreover, there is also a dead state.

So, total state $= (n + 1) + 1 = n + 2$.

Hence, the correct option is (c).

16. If the regular set A is represented by $A = (01 + 1)^*$ and the regular set B is represented by $B = ((01)^*1^*)^*$, which of the following is true? **[1998]**

(a) $A \subset B$

(b) $B \subset A$

(c) A and B are incomparable

(d) A = B

Solution: (d)

Applying rule $(r_1 + r_2)^* = (r_1^* + r_2^*)^*$,

Substituting $r_1 = (01)$ and $r_2 = 1$ in the above equation, we get

$[(01)^*1^*]^* = \{((01)^*)^* + (1^*)^*\}^*$.

As $(01)^*)^* = 01$ and $(1^*)^* = 1$, therefore $[(01)^*1^*]^* = \{ 01 + 1\}^*$.

Hence, the correct option is (d).

17. Which of the following sets can be recognized by a Deterministic Finite State Automation? **[1998]**

(a) the number 1, 2, 4, 8,…2^n, …. written in binary

(b) the number 1, 2, 4,... 2^n, …. written in unary

(c) the set of binary string in which the number of zeros is the same as the number of ones.

(d) the set {1, 101, 11011, 1110111, ….}.

Solution: (a)

1, 2, 4, 8… written in binary is deterministic autometa,

1, 2, 4, 8 … written in unary is not accepted by finite autometa

{1.101, 11011…} is not FA because for a set to be accepted by FA the count should be maintained between 0's and 1's.

Hence, the correct option is (a).

18. The string 1101 does not belong to the set represented by **[1998]**

(a) $110^*(0 + 1)$

(b) $1(0 + 1)^*101$

(c) $(10)^* (01)^*(00 + 11)^*$

(d) $(00 + (11)^*0)^*$

Solution: (c)

As (10)* (01)*(00 + 11)* is not having the string 1101.

Hence, the correct option is (c).

19. How many substrings of different length (non-zero) can be formed from a character string of length n? **[1998]**

(a) n (b) n^2

(c) 2^n (d) $\frac{n(n+1)}{2}$

Solution: (d)

Total sub-strings can be calculated as $\Sigma n = \frac{n(n+1)}{2}$.

Hence, the correct option is (d).

20. Given $\Sigma = \{a, b\}$, which one of the following sets is not countable. **[1997]**

(a) Set of all integers over Σ

(b) Set of all language over Σ

(c) Set of all regular languages over Σ

(d) Set of all language over Σ accepted by Turing Machines

Solution: (b)

Because of no one-to-one correspondence between Σ^* and N, set of all natural language is not countable.

Hence, the correct option is (b).

21. Let $L \subseteq \Sigma^*$, where $\Sigma = \{a, b\}$, which of the following is true? **[1996]**

(a) $L = \{ x | X$ has an equal number of a's and b's$\}$ is regular

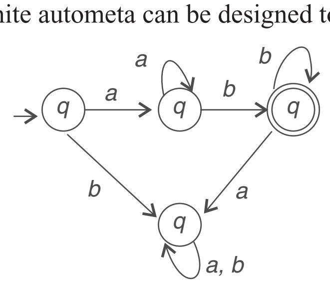
(b) $L = \{a^n b^n | n \geq 1\}$ is regular

(c) $L = \{x | x$ has more a's than b's$\}$ is regular

(d) $L = \{a^m b^n | m \geq 1, n \geq 1\}$ is regular

Solution: (d)

As a finite autometa can be designed to accept L.



Hence, the correct option is (d).

22. Which of the following four regular expressions are equivalent? **[1996]**

(i) $(00)^*(\varepsilon + 0)$ (ii) $(00)^*$

(iii) 0^* (iv) $0(00)^*$

(a) i & ii (b) ii & iii

(c) i & iii (d) iii & iv

Solution: (c)

$0^* = \{\varepsilon, 0, 00, 000\ldots\}$

$(00)^*(\varepsilon + 0) = (00)^* + (00)^*0 = \text{even} + \text{odd} = 0^*$

Or $0^* = (00)^*(0 + \varepsilon)$

$(00)^*$ is even number of 0's and $0(00)^*$ is odd number of 0's,

Hence, the correct option is (c).

23. State true or false with one line explanation: **[1994]**

A Finite State Machine (FSM) can be designed to add two integers of any arbitrary length (arbitrary number of digits).

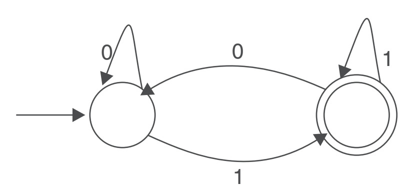
Solution: False

It cannot store carry to add arbitrary length because FSM is having finite memory.

Hence, the given statement is false.

Two-marks Questions

1. Which of the following regular expressions given below represent the following DFA? **[2014]**



(i) $0^*1\ (1 + 00^*1)^*$ (ii) $0^*1^*1 + 11^*0^*1$

(iii) $(0 + 1)^*1$

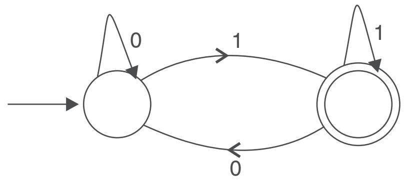
(a) i and ii only

(b) i and iii only

(c) ii and iii only

(d) i, ii and iii

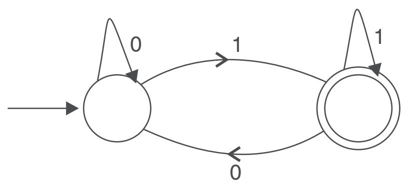
Solution: (b)



0*1 is accepted.

(1 + 00*1)* is accepted.

(i) 0*1(0 + 00*1)* is also accepted.



(0 + 11*0)*11* (0 + 1*0)

(ε + 1)0 = 1*0

(ii) (1*0)*1 + = (0 + 1)*

[0*1(1 + 00*1)*] and (0 + 1)*1 are equal and accepted by FA.

Hence, the correct option is (b).

2. Let L_1 = {w ∈ {0, 1}* | w have at least as many occurrences of (110)'s}. Let L_2 = {w ∈ {0, 1}* |w have at least as many occurrences of (000)'s as (111)'s}. Which one of the following is true? **[2014]**

(a) L_1 is regular but not L_2
(b) L_2 is regular but not L_1
(c) Both L_1 and L_2 are regular
(d) Neither L_1 and L_2 are regular

Solution: (a)

As L_1 is accepted by FA, L_1 = {w ∈ {0,1}* | w has at least as many occurrences of (110)'s}.

Let L_2 = {w ∈ {0, 1}* | w has at least as many occurrences of (000)'s as (111)'s}

L_2 requires to maintain a count. It is not accepted by FA.

L_2 is not regular.

Hence, the correct option is (a).

3. Consider the following languages

$L_1 = \{0^p\, 1^q\, 0^r | \, p, q, r \geq 0\}$

$L_2 = \{0^p\, 1^q\, 0^r | \, p, q, r \geq 0, p \neq r\}$

Which of the following statements is FALSE? **[2013]**

(a) L_2 is context free
(b) $L_1 \cap L_2$ is context free
(c) Complement of L_2 is recursive
(d) Complement of L_1 is context-free but not regular

Solution: (d)

L_1 = regular language.

L_2 = non-regular context-free language intersection of CFL with regular language $L_2 \cap L_1$ is CFL.

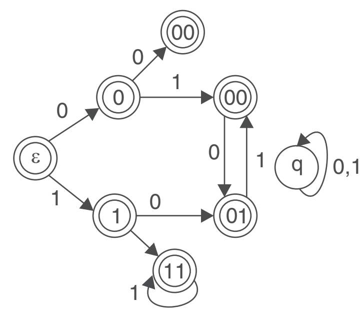
L_2 is DCFL and DCFLs complement. So, L_2 is DCFL.

L_2 is a recursive set and L_1 is a regular set.

Hence, the correct option is (d).

4. Consider the set of strings on {0, 1} in which every substring of symbol 3 has at most two zeros. For example, 1100001 and 011001 are in the language but 100010 are in the language, but 100010 is not. All strings of length less than 3 are also in the language. A partially completed DFA that accepts this language is shown below. **[2012]**

The missing arcs in DFA are



(a)

	00	01	10	11	q
00	1	0			
01				1	
10	0				
11			0		

(b)

	00	01	10	11	q
00		0			1
01		1		1	
10				0	
11		0			

(c)

	00	01	10	11	q
00		1			0
01		1			
10			0		
11		0			

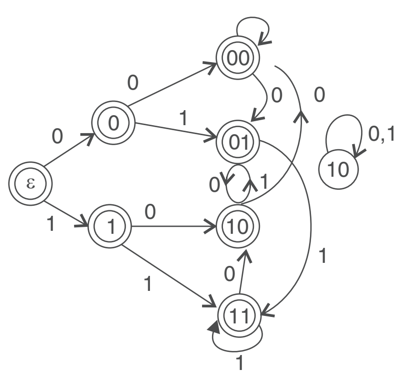
(d)

	00	01	10	11	q
00		1			0
01				1	
10	0				
11			0		

Solution: (d)

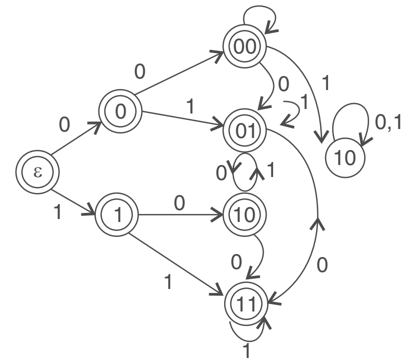
L = { All the string of 0's and 1's where substring of symbol s has at the most two zeros.

Case I: for option (a),

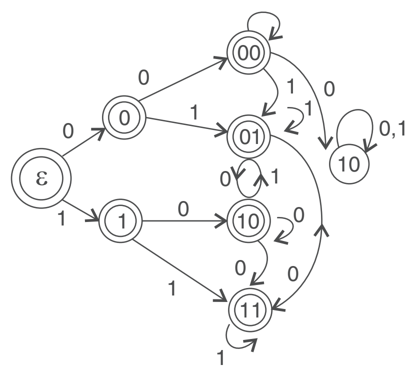


If we take substring 0000, it has a substring 000 containing more than two zeros. So, it is an invalid string but it is accepted by DFA. So, (a) is an invalid option.

Case III: for option (b),

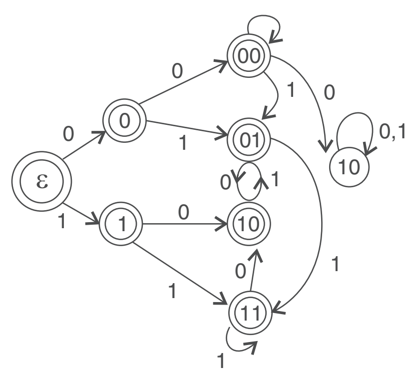


w = 000 has substring 0000, containing more than two zeros. So, it is an invalid string but it is accepted by DFA. So, (b) is an invalid option.



Case III: for option (b),

w = 001000 has substring 000, containing more than two zeros. So it is an invalid string but it is accepted by DFA. So, (c) is an invalid option.



Case IV: for option (d),

This substring accepts all substrings of three symbols has more than two zero's.

Hence, the correct option is (d).

5. Definition of the language L with alphabet $\{a\}$ is given as follows. $L = \{a^{nk} | \ k > 0, n$ is a positive integer constant$\}$. What is the minimum number of states needed in a DFA to recognize L? **[2011]**

(a) $k + 1$ (b) $n + 1$
(c) 2^{n+1} (d) 2^{k+1}

Solution: (b)

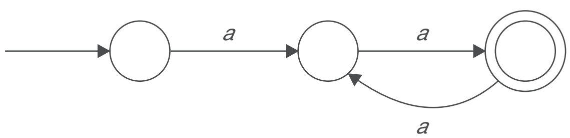
By using instantiation,

let $n = 2$.

$L = \{\in, a2, a4, a6, \ldots\} - \{\in\}$

= set of all even numbers of a's $-\{\in\}$.

That is accepted by minimal DFA of three states.



So, $n = 2$, and the answer = 3.

Hence, the correct option is (b).

6. Let $L = \{w \in (0 + 1)^* | \ w$ has even numbers of 1's$\}$, i.e. L is the set of all bit strings with even numbers of 1's. Which one of the regular expression below represents L? **[2010]**

(a) (0*10*1)
(b) 0*(10*10*)*
(c) 0*(10*1)*0*
(d) 0*1(10*1)*10*

Solution: (b)

Considering 111, option (d) is invalid.

Considering 1010101, option (c) is invalid.

Considering 01010, option (b) is invalid.

Hence, the correct option is (b).

7. Let w be any string of length n in $\{0, 1\}^*$. Let L be the set of all substrings of w. What is the minimum number of states in a non-deterministic finite automation that accepts L? **[2010]**

(a) $n - 1$ (b) n
(c) $n + 1$ (d) 2^{n-1}

Solution: (d)

Here, w is a string of length n. So, it requires minimum of $n + 1$ state to be accepted by NFA.

Hence, the correct option is (d).

8. $L = L_2 \cap L_1$ where language are defined as follows

$L_1 = [a^m \ b^m \ ca^n \ b^n | n, m \geq 0\}$

$L_2 = \{a^i \ b^j \ c^k \ | i, j, k \geq 0\}$ then **[2009]**

(a) Not recursive
(b) Regular
(c) Context free
(d) Recursively enumerable but not context free

Solution: (d)

$L = L_2 \cap L_1 = [a^m \ b^m \ c | \ m \geq 0\}$ which requires a stack to check the equality between a's and b's.

So, L is a context-free but not a regular language.

Hence, the correct option is (d).

9. The DFA accepts the set of all strings over $\{0, 1\}$ that **[2009]**

(a) begins either with 0 or 1
(b) ends with 0
(c) ends with 00
(d) contains the substring 00.

Solution: (c)

This DFA accepts all strings from x00 ending with 00.

Hence, the correct option is (c).

10. Given below are two finite state automata ($\rightarrow$ indicates the start state and F indicates the final state)

Y:

	a	b
$\rightarrow$ 1	1	2
2 (F)	2	1

Z:

	a	b
$\rightarrow$ 1	2	2
2 (F)	1	1

Which of the following represents the product automation ZY? **[2008]**

(a)

	a	b
$\rightarrow$ P	S	R
Q	R	S
R (F)	Q	P
S	Q	P

(b)

	a	b
→ P	S	Q
Q	R	S
R (F)	Q	P
S	Q	Q

(c)

	a	b
→ P	Q	S
Q	R	S
R (F)	Q	P
S	Q	P

(d)

	a	b
→ P	S	Q
Q	S	R
R (F)	Q	P
S	Q	P

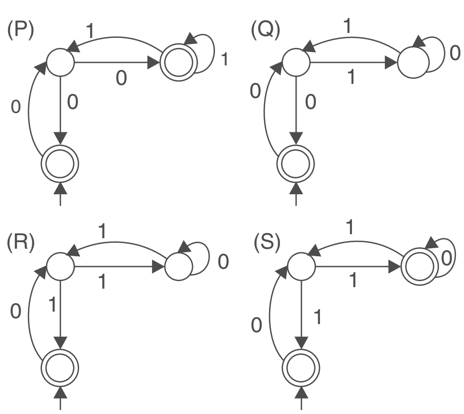
Solution: (a)

By trying input 'b', automation Z and Y go to finite state.

δ_0 $\delta[1Z, 1Y, b]$ final state. With output 'b' option *b, c, d* are not valid.

Hence, the correct option is (a).

11. Match the following NFAs with the regular expression they correspond to **[2008]**



(1) $\varepsilon + 0(01^*1 + 00)^*01^*$
(2) $\varepsilon + 0(10^*1 + 00)^*0$
(3) $\varepsilon + 0(10^*1 + 10)^*1$
(4) $\varepsilon + 0(10^*1 + 10)^*10^*$

(a) $P-2, Q-1, R-3, S-4$
(b) $P-1, Q-3, R-2, S-4$
(c) $P-1, Q-2, R-3, S-4$
(d) $P-3, Q-2, R-1, S-4$

Solution: (c)

P accepts 00. The invalid option is (d).

P accepts 001. The invalid option is (a).

Q accepts 00. The invalid option is (b).

Hence, the correct option is (c).

12. Which of the following are regular sets? **[2008]**

I $\{a^n b^{2n} \mid n \geq 0, m \geq 0\}$
II $\{a^n b^n \mid n = 2m\}$
III $\{a^n b^n \mid n! = m\}$
IV $\{xcy \mid x, y \in \{a, b\}^*\}$

(a) I & IV only
(b) I & III only
(c) I only
(d) IV only.

Solution: (c)

Case-I, $L = a^*(b^2)^*$ and it is regular.

Case IV, $\{(x, y)x, y \in (a + b)^*$ and it is regular.

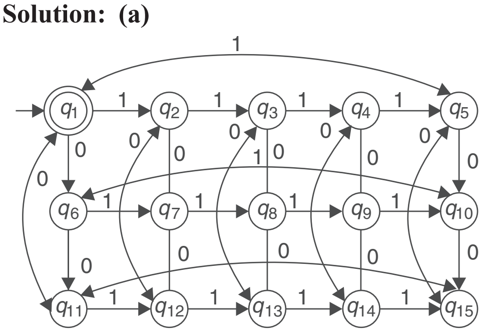
Hence, the correct option is (c).

13. A minimum state deterministic finite automation accepting the language **[2007]**

$L = \{w | w \in \{0, 1\}^*$ number of 0's and 1's in w are divisible by 3 and 5, respectively$\}$ has

(a) 15 states
(b) 11 states
(c) 10 states
(d) 9 states

Solution: (a)



Hence, the correct option is (a).

14. Which of following language is regular? **[2007]**

(a) $\{ww^r \mid w \in \{0, 1\}^+\}$
(b) $\{ww^r | x, w \in \{0, 1\}^+\}$
(c) $\{ww^r \mid x, w \in \{0, 1\}^+\}$
(d) $\{ww^r \mid x, w \in \{0, 1\}^+\}$

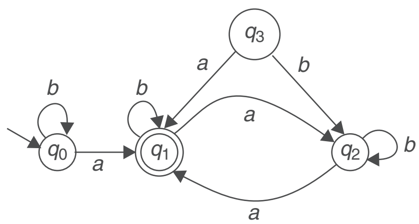
Solution: (c)

As it is generated by regular expression, it is a regular language.

$r = 0\,(0 + 1)^*0 + 1(0 + 1)^*1.$

Hence, the correct option is (c).

15. Consider the following finite state automaton. **[2007]**



The language accepted by this automation is given by the regular expression

(a) $b^*ab^*ab^*ab^*$ (b) $(a + b)^*$

(c) $b^*a(a + b)^*$ (d) $b^*ab^*ab^*$

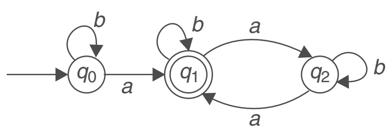
Solution: (c)

As the start state is not the final state $\in$ is not accepted by the DFA. So, (b) is an invalid option.

As *aaa* is accepted by FA, (d) is an invalid option as it does not accept *aaa*.

As the string must be accepted by finite automata, (*a*) is an invalid option.

Hence, the correct option is (c).



	a	b
$\rightarrow q_0$	$^*q_1$	q_0
$^*q_1$	q_2	$^*q_1$
q_2	$^*q_1$	q_2

16. The minimum state automation equiva lent to the above FSA has the following number of states:

(a) 1 (b) 2

(c) 3 (d) 4

[FPO]

Solution: (b)

See the construction of minimal DFA of the last solution.

Hence, the correct option is (b).

17. If *s* is a string over $(0 + 1)^*$ then let $n_0(s)$ denote the number of 0's in *s* and $n_1(s)$ the number of 1's in which one of the following language is not regular? **[2006]**

(a) $L = \{s \in \{0 + 1\}^* |\ n_0(s)$ is a 3-digit prime$\}$

(b) $L = \{s \in \{0 + 1\}^* |$ for every prefix of $s |\ n_0(s') - n_1(s') | \leq 2\}$

(c) $L = \{s \in \{0 + 1\}^* |\ n_0(s) - n_1(s) | \leq 4\}$

(d) $L = \{s \in \{0 + 1\}^* |\ n_0(s) \bmod 7 = n_0(s) \bmod 5 = 0\}$

Solution: (c)

Every finite language is regular and it is a finite language.

An infinite language, but by taking prefixes apart from comparing from 0's & 1's, DFA can be constructed. Hence, it is a regular language.

Is a language accepted by DFA with 35 state. So it is also regular.

Hence, the correct option is (c).

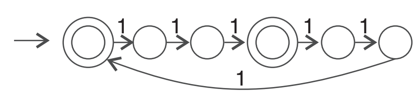
18. Consider the regular language $L = (111 + 11111)^*$. The minimum number of states in any DFA accepting this language is **[2006]**

(a) 3 (b) 5

(c) 8 (d) 9

Solution: (b)

Putting this pattern in figure, we will get



Hence, the correct option is (b).

19. What is the minimum number of ordered pairs of non-negative numbers that should be chosen to ensure that there are two pairs (a, b) and (c, d) in the chosen set such that $a \equiv c \bmod 3$ and $b \equiv d \bmod 5$. **[2005]**

(a) 4 (b) 6

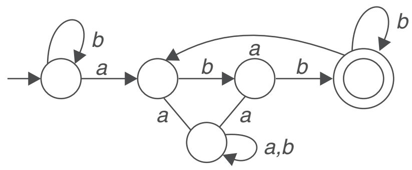
(c) 16 (d) 24

Solution: (c)

The required order pair will be $(3^*5 + 1) = 16$.

Hence, the correct answer is (c).

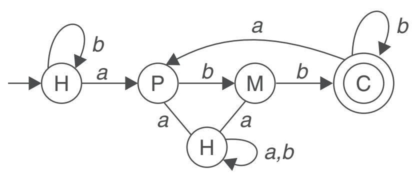
20. Consider the machine *M*. The language recognized by *M* is: **[2005]**



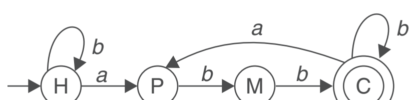
(a) $\{w \in \{a, b\}^*|$ every a in w is followed by exactly two b's$\}$
(b) $\{w \in \{a, b\}^*|$ every a in w is followed by at least two b's$\}$
(c) $\{w \in \{a, b\}^*|$ w contains the substring 'abb'$\}$
(d) $\{w \in \{a, b\}^*|$ w does not contain 'aa' as substring$\}$

Solution: (b)

The DFA state can be named



As *S* is a dead state or a trap state, that is why it can be deleted without affecting the set that is *w* accepted.



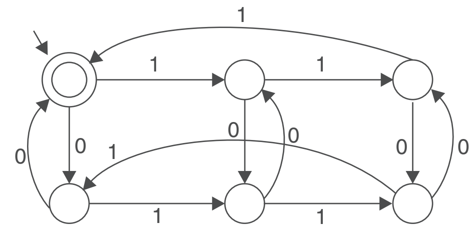
Option (d) the string $\in$ is in $\{a, b\}$ and does not contain '*aa*' as a substring. As the start state is not the final state $\in$ is not accepted by the DFA. So, it is an invalid option.

Option (a) considers the string *abbb*. It is accepted by the DFA. So, it is a valid option.

Option (c) $\in$ is accepted by the DFA. So, it is a valid option.

Hence, the correct option is (b).

21. The following finite state machine accept all those binary strings in which the number of 1's and 0's are, respectively **[2004]**



(a) Divisible by 3 and 2
(b) Odd and even
(c) Even and odd
(d) Divisible by 2 and 3

Solution: (a)

By the method of elimination:

Option (b) considers the string 100. The number of 1's is odd and the number of 0's is even.

The string is not accepted by DFA, so this option is not valid.

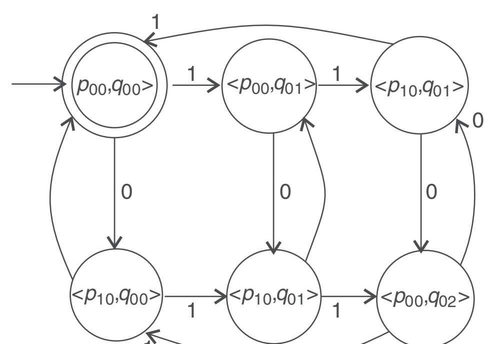
Option (c) considers string 11000. The number of 1's is even and the number of 0's is odd.

As this is not accepted by DFA, so this option is not valid.

Option (d) consider s string 11000. The number of 1's is divisible by 2 and the number of 0's is divisible by 3.

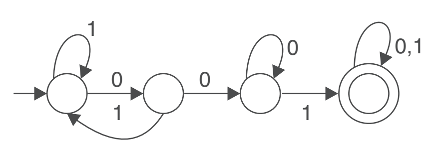
As this is not accepted by DFA, so this option is not valid.

In Option (a) the given DFA will be



Hence, the correct answer is (a).

22. Consider the following deterministic finite state automation *M*. **[2003]**



Let S denote the set of seven bit binary strings in which the first, the forth, and the last bits are 1. The number of strings in S accepted by M is

(a) 1 (b) 5
(c) 7 (d) 8

Solution: (c)

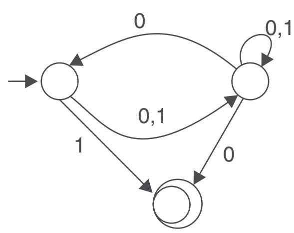
The pattern is represented as 1- -1- -1 . Now it will need $2^4 = 16$ ways to fill the blank spaces.

So, for this pattern we will have 16 patterns and all strings will not be acceptable but only seven.

1001 001 is accepted
1001 011 is accepted
1001 101 is accepted
1001 111 is accepted
1011 001 is accepted
1101 001 is accepted
1111 001 is accepted

Hence, the correct option is (c).

23. Consider NFA M shown below:

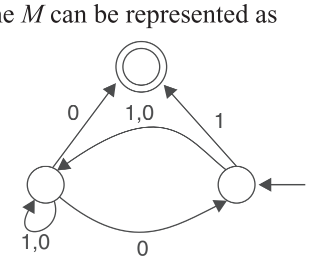


Let the language accepted by M be L. let L_1 be the language accepted by the NFA, M_1 obtained by changing the accepting state of M to a non-accepting state and by changing the non-accepting state of M to accepting states. Which of the following statement is true? **[2003]**

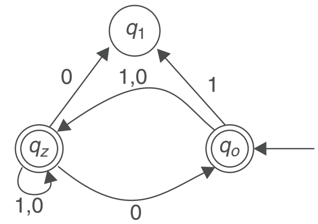
(a) $L_1 = \{0, 1\}^* - L$
(b) $L_1 = \{0, 1\}^*$
(c) $L_1 \subseteq L$
(d) $L_1 = L$

Solution: (b)

Machine M can be represented as



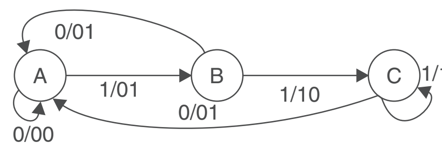
When we will change final and non-final state, w will get



Final NFA will be the language $L_1 = \{ 0, 1\}^*$.

Hence, the correct option is (b).

24. The finite state machine is described by the following state diagram with A as starting state, where an arc label is x/y and x stands for 1-bit input and y stands for 2-bit output. **[2002]**



(a) Outputs the same of the present and previous bits of the input
(b) Outputs 01 whenever the input sequence contains 11
(c) Outputs 00 whenever the input sequence contains 10
(d) None of the above

Solution: (a)

If input is 10, output is 01 = 1 + 0 = 01

If input is 110, output is 01 = 1 + 0

If input is 111, output is 10 = 1 + 1

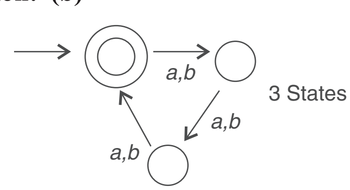
Hence, the correct option is (a).

25. The smallest finite which accepts the language **[2002]**

$L = \{x|$ length of x is divisible by $3\}$

(a) 2 states (b) 3 states
(c) 4 states (d) 5 states

Solution: (b)



depicts the three states.

Hence, the correct option is (b).

26. Consider DFA over $\Sigma = \{a, b\}$ accepting all strings which have number of a's divisible by 6 and number of b's divisible by 8. What is the minimum number of states that the DFA will have? **[2001]**

(a) 8 (b) 14
(c) 15 (d) 48

Solution: (d)

The 48 state DFA follows.

	a	b
$\rightarrow q_{00}$	q_{10}	q_{01}

Input of length 1

	a	b
q_{10}	q_{20}	q_{11}
q_{01}	q_{11}	q_{02}

Input of length 2

	a	b
q_{20}	q_{30}	q_{21}
q_{11}	q_{21}	q_{12}
q_{02}	q_{12}	q_{03}

Input of length 3

	a	b
q_{30}	q_{40}	q_{31}
q_{21}	q_{31}	q_{22}
q_{12}	q_{22}	q_{13}
q_{03}	q_{13}	q_{04}

Input of length 4

	a	b
q_{40}	q_{50}	q_{41}
q_{31}	q_{41}	q_{32}
q_{22}	q_{32}	q_{23}
q_{13}	q_{23}	q_{14}
q_{04}	q_{14}	q_{05}

Input of length 5

	a	b
q_{50}	q_{50}	q_{41}
q_{41}	q_{51}	q_{32}
q_{32}	q_{42}	q_{33}
q_{23}	q_{33}	q_{24}
q_{14}	q_{24}	q_{15}
q_{05}	q_{15}	q_{15}

Input of length 6

	a	b
q_{00}	q_{10}	q_{01}
q_{51}	q_{61}	q_{52}
q_{42}	q_{52}	q_{43}
q_{33}	q_{43}	q_{34}
q_{24}	q_{34}	q_{25}
q_{15}	q_{25}	q_{16}
q_{00}	q_{16}	q_{07}

Input of length 7

	a	b
q_{10}	q_{20}	q_{11}
q_{01}	q_{11}	q_{02}
q_{52}	q_{62}	q_{53}
q_{43}	q_{53}	q_{44}
q_{34}	q_{44}	q_{35}
q_{25}	q_{35}	q_{26}
q_{16}	q_{26}	q_{17}
q_{06}	q_{17}	q_{00}

Input of length 8

Hence, the correct option is (d).

27. Consider the following languages:

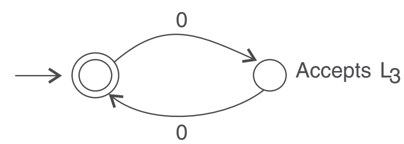
(a) $L_1 = \{w\, w \mid w \in \{a, b\}^*\}$
(b) $L_2 = \{w\, w^r \mid w \in \{a, b\}^*$, w^r is the reverse of $w\}$
(c) $L_3 = \{0^{2i} \mid I$ is an integer$\}$
(d) $L_4 = \{0^{i2} \mid I$ is an integer$\}$

Which of the following languages are regular? **[2001]**

(a) Only L_1 L_2
(b) Only L_2, L_3, L_4
(c) Only L_3, L_4
(d) Only L_3

Solution: (d)

For the expression $L_3 = \{0^{2i} \mid I$ is an integer$\}$



accepts L_3

L_3 is regular.

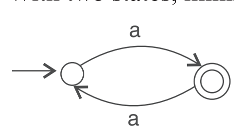
Hence, the correct option is (d).

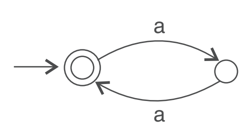
28. What can be said about the regular language L over $\{a\}$ whose minimal finite state automation has two states? **[2000]**

(a) must be $\{a^n \mid n$ is odd$\}$
(b) must be $\{a^n \mid n$ is even$\}$
(c) must be $\{a^n \mid n \geq 0\}$
(d) Either L must be $\{a^n \mid n$ is odd$\}$ or L must be $\{a^n \mid n$ is even$\}$

Solution: (d)

With two states, minimal finite automation.



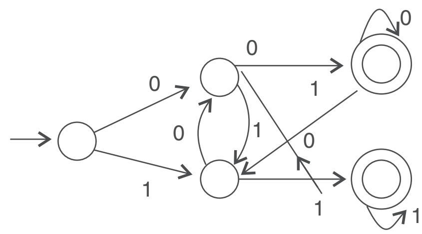


Hence, the correct option is (d).

29. Let L be the set of all binary strings whose last two symbols are the same. The numbers of states in the minimum state deterministic finite-state automation accepting L is **[1998]**

(a) 2 (b) 5
(c) 8 (d) 3

Solution: (b)



Hence, the correct option is (b).

30. Which of the following regular expression over {0, 1} denotes the set of all strings not containing 100 as substring? **[1997]**

(a) 0*(1 + 0)* (b) 0*1010*
(c) 0*1*01* (d) 0*(10 + 1)*

Solution: (d)

0*1*01* and 0*(10 + 1)* both generate the string that doesn't contain 100 as a substring, and it can also be said that (c) does not guarantee ε but the option (d) guarantees ε.

Hence, the correct option is (d).

31. Which of the following definitions given below generate the same language? **[1995]**

Where $L = \{x^n y^n \mid n \geq 1\}$

(i) $E \to xEy \mid xy$ (ii) $xy \mid (x^+ xyy^+)$
(iii) $x^+ y^+$

$L = \{x^n\, y^n \mid n \geq 1\}$ generates string with equal number of x and equal number of y's.

$E \to xBy \mid xy$ abo generators tips same.

(a) i only (b) i & ii
(c) ii & iii (d) ii only

Solution: (a)

In expression $L = \{x^n y^n \mid n \geq 1\}$, the strings are generated with equal number of x and y.

$E \to xEy \mid xy$ also generates the same string as of L.

Hence, the correct option is (a).

32. A finite state machine with the following state table has a single input X and a single output Z.

[1995]

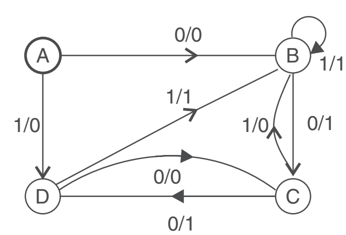
Present state	Next state, Z	
	X = 1	X = 0
A	D, 0	B, 0
B	B, 1	C, 1
C	B, 0	D, 1
D	B, 1	C, 0

If the initial state is unknown, then the shortest input sequence to reach the final state C is here, since initial make unknown m 10 input we can each final state C with shortest path.

(a) 01
(b) 10
(c) 101
(d) 110

Solution: (b)

Here, the initial state is unknown, so on 10 input we can reach the final state C in shortest path.



Hence, the correct option is (b).

33. The number of substring (of all length inclusive) that can be formed from a character string of length n is **[1994]**

(a) n (b) n^2

(c) $\dfrac{n\,(n-1)}{2}$ (d) $\dfrac{n\,(n+1)}{2}$

Solution: (d)

Let s be the string

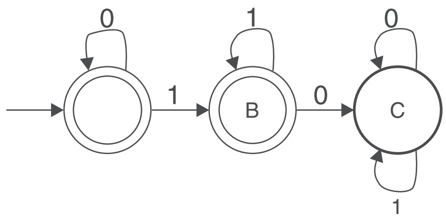
Let the length of s be 'n'.

No. of substrings = $\Sigma n + 1 = \dfrac{n(n+1)}{2} + 1$.

Hence, the correct option is (d)

34. The regular expression for the language recognized by the finite state automation of is_______

[1994]



Solution:

By the given diagram the expression 0* + 0*11* defines it properly.

35. Which of the following regular expression identities are true? **[1992]**

(a) $R(*) = r^*$

(b) $(r^*s^*)^* = (r + s)^*$

(c) $(r + s)^* = r^* + s^*$

(d) $r^*s^* = r^* + s^*$

Solution: (b)

As r and s are regular expressions, the expression $(r + s)^* = (r^* + s^*)^*$ will generate the same language.

Hence, the correct option is (b).

36. If G is a context-free grammar and w is a string of length n in $L(G)$, how long is derivation of w in G, if G is in Chomsky normal form? **[1992]**

(a) $2n$ (b) $2n + 1$

(c) $2n - 1$ (d) n

Solution: (c)

The length of derivation tree = $2n - 1$.

As $S \to AB$

$A \to BC|a$

$B \to CC|b|$

Derivation: $w = ab$

$S \to AB$

$S \to aB$

$S \to ab$

Let the string $w = ab$ and $|w| = 2(n)$

Number of product in derivation = $2n - 1 = 4 - 1 = 3$.

Hence, the correct option is (c).

37. Let $r = 1(1 + 0)^*$, $s = 11^*0$ and $t = 1^*0$ be three regular expressions. Which one of the following is true? **[1991]**

(a) $L(s) \subseteq L(r)$ & $L(s) \subseteq L(t)$

(b) $L(t) \subseteq L(s)$ & $L(s) \subseteq L(t)$

(c) $L(s) \subseteq L(t)$ & $L(s) \subseteq L(r)$

(d) $L(t) \subseteq L(s)$ & $L(s) \subseteq L(r)$

Solution: (a)

Expanding $r = 1\,(1 + 0)^*$ = all strings start with 1.

Expanding $s = 11^*0$ = 10, 110, 1110,……

Expanding $t = 1^*0$ = 0, 10, 110, 1110…..

So, it can be concluded that $L(s) \subseteq L(r)$ & $L(s) \subseteq L(t)$

Hence, the correct option is (a).

38. Let R_1 and R_2 be regular sets defined over the alphabet Σ then: **[1990]**

(a) $R_1 \cap R_2$ is not regular

(b) $R_1 \cup R_2$ is regular

(c) $\Sigma^* - R_1$ is regular

(d) R_1^* is not regular

Solution: (c)

Let Σ be any alphabet. Σ^* is a universal language which is accepted by a finite automata.

Also R_1 is a regular language and complement of $R_1 = \Sigma^* - R_1$ that is also a finite automata.

Finite automata for R_1c can be obtained by interchanging final and non-final states from FA of R_1.

Hence, if R_1 is a regular language, then $\Sigma^* - R_1$ is also regular.

Hence, the correct option is (c).

39. How many substrings (of all lengths inclusive) can be formed from a character string of length n? Assume all characters to be distinct. Prove your answer. **[1989]**

Solution:

Total no of substrings = $\Sigma\, n + 1$

$$\frac{n(n+1)}{2} + 1$$

For example, let s = Pearson be the string then $|s| = 7$

s has substring of length l = 0,1, 2, …7

substring of length 0, $\in$

substring of length 1 p, e, a, r, s, o, n

substring of length 2, pe, pa, pr…..

substring of length 3, pea, par,….

Similarly,

substring of length 7, pearson.

So, total no of substring is $\Sigma 7 + 1$.

so, total no. of substring = $\dfrac{n(n+1)}{2} + 1$.

Five-marks Questions

1. Given that language L_1 is regular and that the language $L_1 \cap L_2$ is regular. Is the language L_2 always regular? **[1994]**

Solution: L_2 need to be always regular, since

$L_1 = \{a^m b^n \mid m, n \geq 0\}$ is regular.

$L_2 = \{a^m b^n \mid m = n\}$ is non-regular.

Now L_2 is a subset of L_1.

Then $L_1 \cap L_2 = L_1$ is regular.

2. Is the class of regular sets closed under infinite union? Explain. **[1989]**

Solution: Infinite union of regular sets need not be regular.

Ex: $L_0 = \{\varepsilon\}$

$L_1 = \{a, b\}$

$L_2 = \{a^2b^2\}$….

$L_n = \{a^m b^n\}$ and so on, where each L_n is regular.

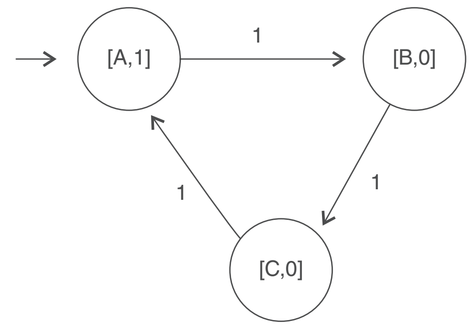
Then L = n = $\{a^i b^i \mid i \geq 0$ is not regular$\}$

3. Give minimum DFA that performs as MOD-3 counter, i.e. outputs a 1 each time the number of 1's in the input sequence is a sequence is a multiple of 3. **[1987]**

Solution: $\Sigma = \{1\}$ $\Delta = \{0,1\}$

$Q = \{A, B, C\}$. 1 for accept

Number of 1's $\cong$ 0 mod 3. 0 for reject



4. Give the regular expression over {0,1} to denote the set of proper non-null substring of the string 0110. **[1987]**

Solution: $\Sigma = \{0, 1\}$

W = 0110

Proper substring of 0110 : {0, 1, 01, 11, 10, 011, 110, 0110}.

The regular expression is

0 + 1 + 01 + 11 + 10 + 011 + 110 + 0110.