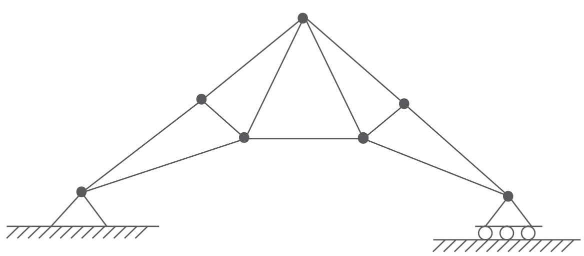
Chapter 1

Degree of Indeterminacy

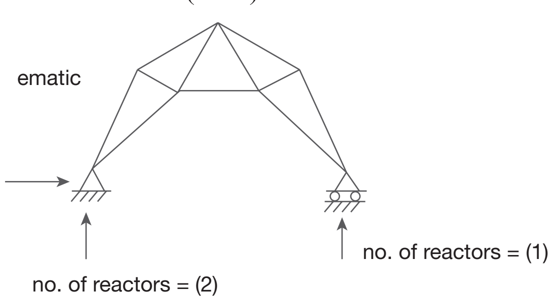
One-mark Questions

1. The kinematic indeterminacy of the plane truss shown in the figure is **[2016]**



(a) 11 (b) 8
(c) 3 (d) 0

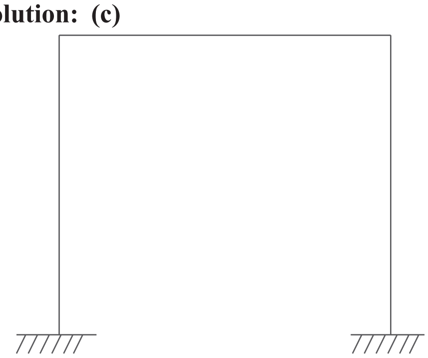
Solution: (a)
Number of joints (J) = 7
For rigid joint plan truss kinematic Indeterminacy = 2J – R = 2 × 7–(2 + 1) = 14 – 3 = 11



Hence, the correct option is (a).

2. The kinematic indeterminacy of single bay portal frame fixed at the base is **[1994]**
(a) 1 (b) 2
(c) 3 (d) 0

Solution: (c)



$$m = 3, \quad r = 6, \quad j = 4$$

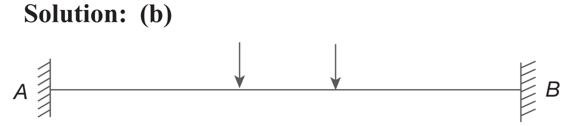
D_k: Kinematic indeterminacy

$$= 3j - r - m$$
$$= 3 \times 4 - 6 - 3 = 3$$

Hence, the correct option is (c).

3. A beam fixed at the ends and subjected to lateral loads only is statically indeterminate and the degree of indeterminacy is **[1994]**
(a) 1 (b) 2
(c) 3 (d) 0

Solution: (b)



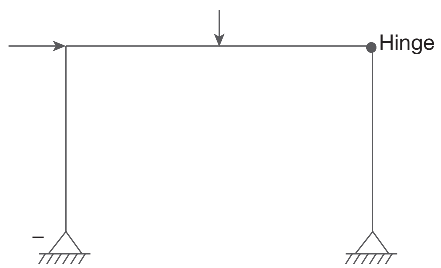
D_s: Degree of static indeterminacy = Number of reaction components – Number of equilibrium equations
Since the beam is subjected to only lateral loads, horizontal reaction at supports is equal to zero.
Number of available equilibrium equations = 2

Number of reaction components = 4

$$D_s = 4 - 2 = 2$$

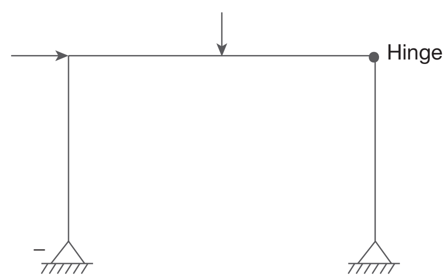
Hence, the correct option is (b).

4. The plane frame shown in figure is **[1993]**



(a) stable and statically determinate
(b) unstable and statically determinate
(c) stable and statically indeterminate
(d) unstable and statically indeterminate

Solution: (a)



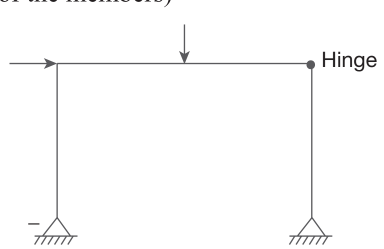
$$m = 3, \quad r = 4, \quad j = 4$$

D_s: Degree of static indeterminacy

$$= 3m + r - 3j - \Sigma(m' - 1)$$
$$= 3 \times 2 + 4 - 3 \times 4 - 1 = 0$$

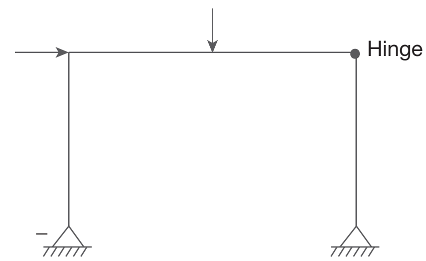
Therefore, the structure is stable and statically determinate.
Hence, the correct option is (a).

5. The kinematic indeterminacy of the plane frame shown in figure is (disregarding the axial deformation of the members) **[1993]**



(a) 7 (b) 5
(c) 6 (d) 4

Solution: (c)



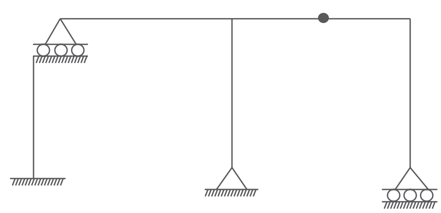
$$m = 3, \quad r = 4, \quad j = 4$$

D_k: Degree of kinematic indeterminacy

$$= 3j - r - m + \Sigma(m' - 1)$$
$$= 3 \times 4 - 4 - 3 + 1 = 6$$

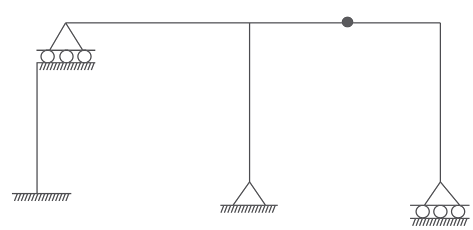
Hence, the correct option is (c).

6. A plane structure shown in the figure is **[1992]**



(a) stable and determinate
(b) stable and indeterminate
(c) unstable and determinated
(d) unstable and indeterminate

Solution: (a)



Consider the total structure into two plane structures.

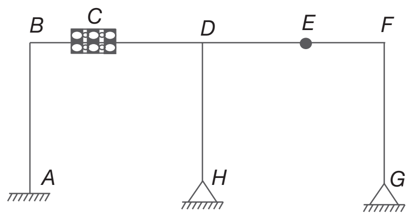
$$m = 5, \quad r = 4, \quad j = 6$$

D_s: Degree of static indeterminacy

$$= 3\,m + r - 3j - \Sigma(m' - 1)$$
$$= 3 \times 5 + 4 - 3 \times 6 - 1 = 0$$

For cantilever, $D_s = 0$
Total degree of static indeterminacy = 0
Therefore, the structure is stable and determinate.
Hence, the correct option is (a).

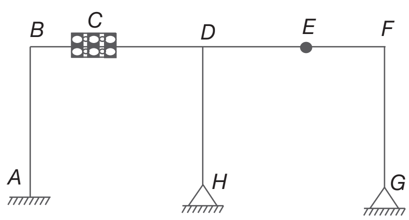
7. A plane frame *ABCDEFGH* shown in figure has clamp support at *A*, hinge supports at *G* and *H*, axial force release horizontal sleeve) at *C* and moment release (hinge) at *E*. The static (D_s) and kinematic (D_k) indeterminacies are **[1992]**



(a) $D_s = 4, D_k = 9$ (b) $D_s = 3, D_k = 11$

(c) $D_s = 2, D_k = 13$ (d) $D_s = 1, D_k = 14$

Solution: (c)



D_s: Degree of static indeterminacy

$$= 3m + r - 3j - \Sigma(m' - 1)$$

$$m = 7, \quad r = 7, \quad j = 8$$

$$D_s = 3\times 7 + 7 - 3\times 8 - 1 - 1 = 2$$

D_k: Degree of kinematic indeterminacy

$$= 3j - r - \Sigma m'$$

$$= 3\times 8 - 7 - 2 - 2 = 13$$

(or)
By considering *BD* and *DF* as individual members

$$m = 5, \quad r = 7, \quad j = 6$$

$$D_s = 3m + r - 3j - \Sigma(m' - 1)$$

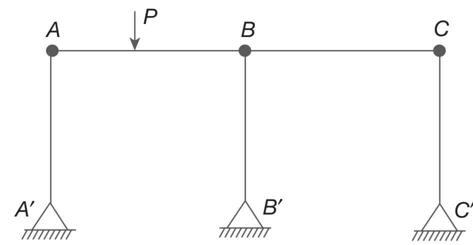
$$D_s = 3\times 5 + 7 - 3\times 6 - 1 - 1 = 2$$

$$D_k = 3j - r + \Sigma(m' - 1)$$

$$D_k = 3\times 6 - 7 + 1 + 1 = 13$$

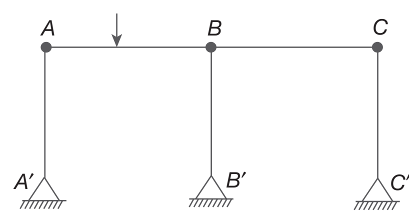
Hence, the correct option is (c).

8. The beam supported by 3 links and loaded as shown in the figure is **[1991]**



(a) stable and determinate
(b) unstable
(c) stable and indeterminate
(d) unstable but determinate

Solution: (b)



$M = 5, j = 6, R = 6$

Degree of static indeterminacy,

$$D_s = 3m + r - 3j - \Sigma(m' - 1)$$

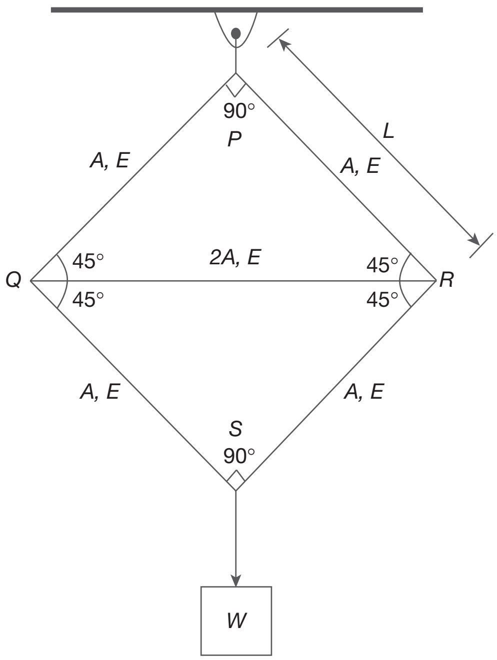
Number of members, $m = 5$
Number of reaction components, $r = 6$
Number of joints, $j = 6$
Number of members meeting at the internal hinge: m'

$$D_s = 3\times 5 + 6 - 3\times 6 - 1 - 2 - 1 = -1$$

Therefore, the given structure is unstable.
Hence, the correct option is (b).

Two-marks Questions

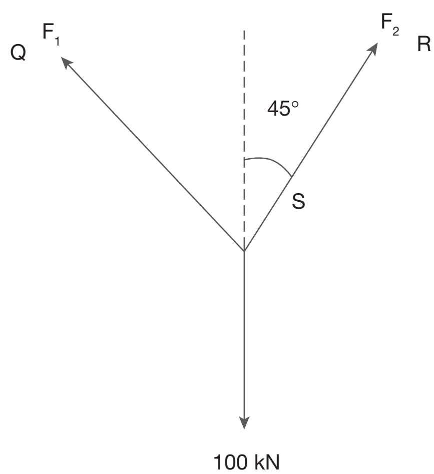
1. Consider the structural system shown in the figure under the action of weight (*W*). All the joints are hinged. The properties of the members in terms of length (*L*), area (*A*) and the modulus of elasticity (*E*) are also given in the figure. Let *L*, *A* and *E* be 1 m, 0.05 m^2 and 30 × 10^6 N/m^2, respectively, and *W* be 100 kN. **[2016]**

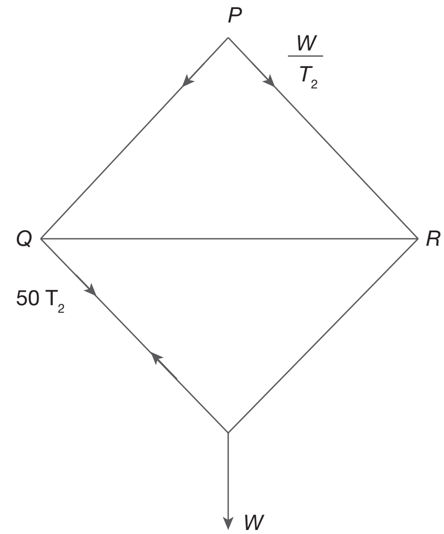


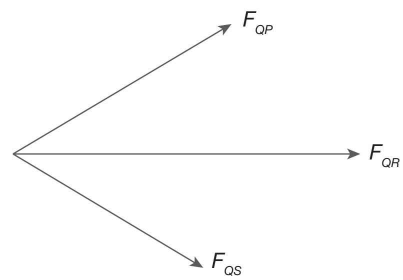
Which one of the following sets gives the correct values of the force, stress and change in length of the horizontal member QR?

(a) Compressive force = 25 kN; Stress = 250 kN/m^2; Shortening = 0.0118 m
(b) Compressive force = 14.14 kN; Stress = 141.4 kN/m^2; Extension = 0.0118 m
(c) Compressive force = 100 kN; Stress = 1000 kN/m^2; Shortening = 0.0417 m
(d) Compressive force = 100 kN; Stress = 1000 kN/m^2; Extension = 0.0417 m

Solution: (c)







$$F_{SQ} = F_{SR}$$

$$\Rightarrow \quad 2F_{SQ}\cos 45° = w \quad \Rightarrow \quad F_{SQ} = \frac{w}{\sqrt{2}}$$

Similarly $F_{PQ} = F_{PR} = \dfrac{w}{\sqrt{2}}$

Now, Consider joint Q

$$\sum Fx = 0$$

$$\Rightarrow \quad F_{QP} \times \cos 45° + F_{QS}\cos 45° + F_{QR} = 0$$

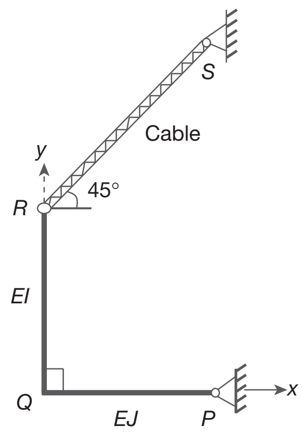
$$\Rightarrow \quad F_{QR} = w = 100 \text{ kN(Compressive)}$$

$$\Delta_{QR} = \frac{F_{QR} \times L}{2A_E} = \frac{100 \times \sqrt{2}L}{4 \times 0.05 \times 0.3 \times 106}$$

$$= 0.471 \text{(Shortening)}$$

Hence, the correct option is (c).

2. The degree of static indeterminacy of a rigid jointed frame PQR supported as shown in the figure is **[2014]**

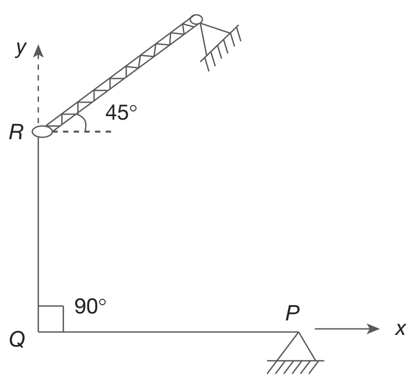


(a) 0 (b) 1
(c) 2 (d) unstable

Solution: (a)
Degree of static indeterminacy,

$$D_s = 3m + r - 3j - \Sigma(m' - 1)$$

Number of members, $m = 3$
Number of reaction components, $r = 4$
Number of joints, $j = 4$
Number of members meeting at hinge R, $m' = 2$



$D_s = 3 \times 3 + 4 - 3 \times 4 - 1 = 9 + 4 - 12 - 1 = 0$

The given rigid jointed frame is statically determinate.
(or)

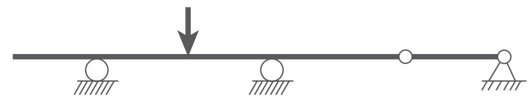
$$D_s = D_{se} + D_{si} = (r - 3) + 3c - \Sigma(m' - 1)$$

Number of closed loops, $c = 0$

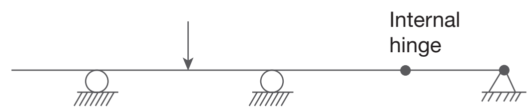
$$D_s = (4 - 3) + 3 \times 0 - (2 - 1) = 0$$

Hence, the correct option is (a).

3. The static indeterminacy of two span continuous beam with internal hinge, shown below, is _______. **[2014]**



Solution: 0



Degree of static indeterminacy,

$$D_s = 3m + r - 3j - \Sigma(m' - 1)$$

Number of members, $m = 4$
Number of reaction components, $r = 4$
Number of joints, $j = 5$
Number of members meeting at hinge, $m' = 2$

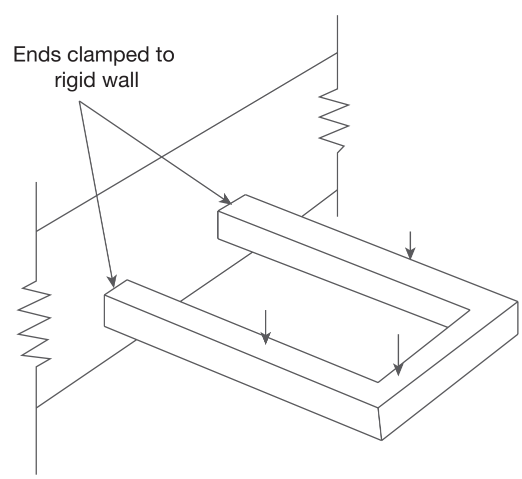
$$D_s = 3 \times 4 + 4 - 3 \times 5 - (2 - 1) = 12 + 4 - 15 - 1 = 0$$

(or)

$$\begin{aligned} D_s &= D_{se} + D_{si} \\ &= (r - 3) + 3c - \Sigma(m' - 1) \\ &= 4 - 3 + 0 - (2 - 1) = 1 - 1 = 0 \end{aligned}$$

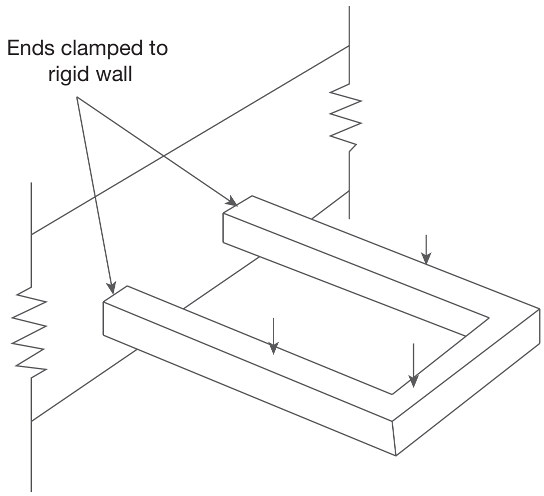
Hence, the answer is 0.

4. The degree of static indeterminacy of a rigidly jointed frame in a horizontal plane and subjected to vertical only, as shown in figure below, is **[2009]**



(a) 6 (b) 4
(c) 3 (d) 1

Solution: (c)



For a rigid jointed plane frame

$$D_s = (3m + r) - 3j \text{ Vertical loads only}$$

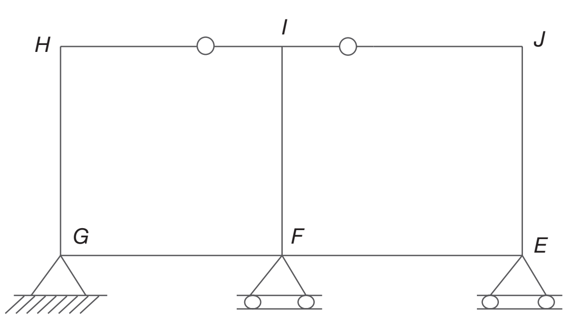
$$= (6m + r) - 6j$$

D_s: Degree of static indeterminacy
Number of members, $m = 3$
Number of external reactions, $r = 3 + 3 = 6$
Number of joints, $j = 4$

$$D_s = (3 \times 3 + 6) - 3 \times 4 = 3$$

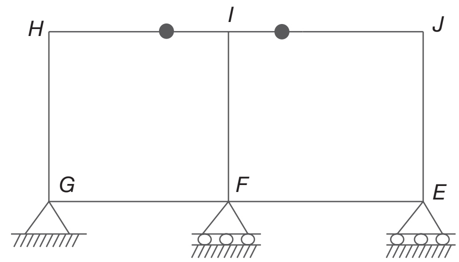
Hence, the correct option is (c).

5. The degree of static indeterminacy of the rigid fame having two internal hinges as shown in the figure below, is **[2008]**



(a) 8 (b) 7
(c) 6 (d) 5

Solution: (d)



Degree of static indeterminacy,

$$D_s = 3m + r - 3j - \Sigma(m' - 1)$$

Number of members, $m = 9$
Number of reaction components, $r = 2 + 1 + 1 = 4$
Number of rigid joints, $j = 6 + 2 = 8$

Number of members meeting at the internal hinge $= m'$

Total number of internal reaction components released,

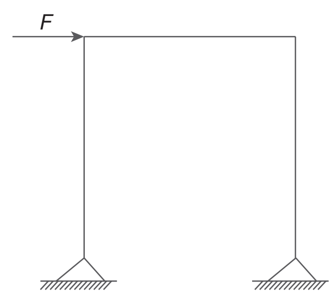
$$\Sigma(m' - 1) = (2 - 1) + (2 - 1) = 2$$

$$D_s = 3 \times 9 + 4 - 3 \times 8 - 2 = 27 + 4 - 24 - 2 = 5$$

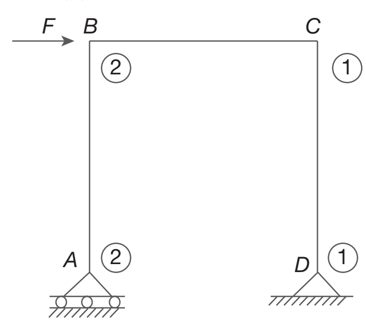
Hence, the correct option is (d).

6. Considering beam as axially rigid, the degree of freedom of a plane frame shown below is **[2005]**

(a) 9 (b) 8
(c) 7 (d) 6



Solution: (d)



D_k: Degree of freedom or degree of kinematic indeterminacy

$= 3j - r$ if axial deformation is considered

$= 3j - r - m$ if axial deformation is neglected

Since the frame is rigid, the axial deformation is neglected.

j: number of joints = 4

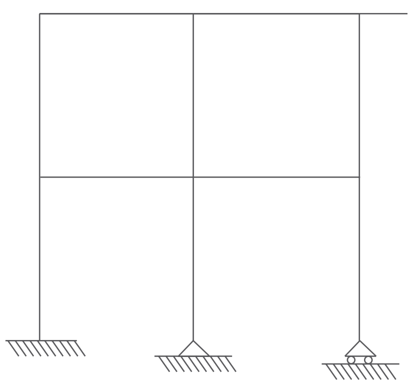
r: number of reaction components = 2 + 1 = 3

m: number of members = 4

$$D_k = 3 \times 4 - 3 - 3 = 6$$

Hence, the correct option is (d).

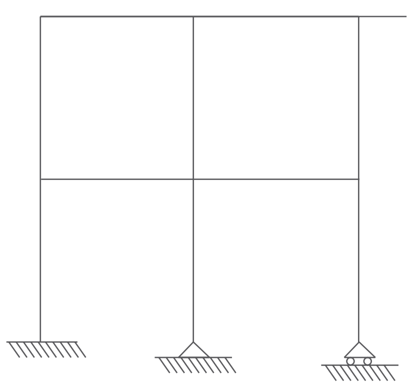
7. For the plane frame with an overhang as shown below, assuming negligible axial deformation, the degree of static indeterminacy, d, and the degree of kinematic indeterminacy, k, are **[2004]**



(a) $d = 3$ and $k = 10$ (b) $d = 3$ and $k = 13$

(c) $d = 9$ and $k = 10$ (d) $d = 9$ and $k = 13$

Solution: (d)



Degree of static indeterminacy, $D_s = 3m + r - 3j$

Number of members, $m = 11$

External reaction components, $r = 3 + 2 + 1 = 6$

Number of joints, $j = 10$

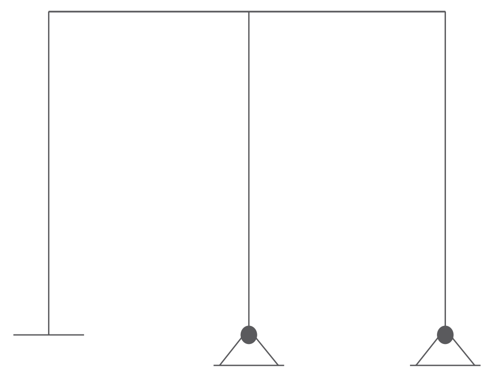
$$D_s = 3 \times 11 + 6 - 3 \times 10 = 33 + 6 - 30 = 9$$

D_k: Degree of kinematic indeterminacy = $(3j - r) - m$

$$= (3 \times 10 - 6) - 11 = 30 - 6 - 11 = 13$$

Hence, the correct option is (d).

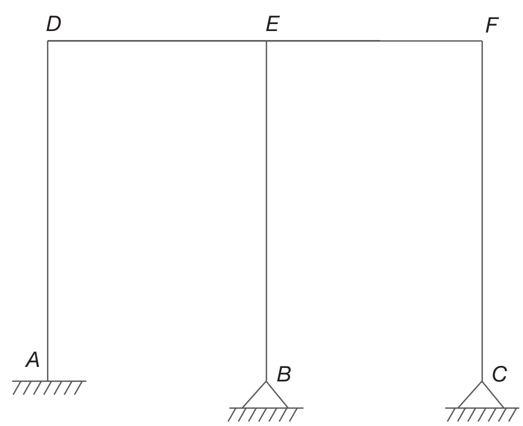
8. The degree of static indeterminacy, N_s, and the degree of kinematic indeterminacy, N_k, for the plane frame shown below, assuming axial deformations to be negligible, are given by **[2001]**



(a) $N_s = 6$ and $N_k = 11$ (b) $N_s = 6$ and $N_k = 6$

(c) $N_s = 4$ and $N_k = 6$ (d) $N_s = 4$ and $N_k = 4$

Solution: (c)



Degree of static indeterminacy,

$$D_s = 3 + 2 + 2 - 3 = 4$$

(or)

Degree of static indeterminacy, $D_s = 3m + r - 3j$

Number of members, $m = 5$

Number of reaction components, $r = 7$

Number of joints, $j = 6$

$$D_s = 3\times5+7-3\times6$$
$$=15+7-18=4$$

D_k: Degree of kinematic indeterminacy

$$=\delta_D,\theta_D,\theta_E,\theta_F,\theta_B,\theta_c$$
$$=6$$

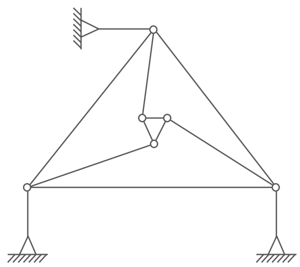
(or)

Degree of kinematic indeterminacy, $D_k = 3j - r - m$

$$D_k = 3\times6-7-5=18-7-5=6$$

Hence, the correct option is (c).

9. The following two statements are made with reference to the planar truss shown below: **[2000]**



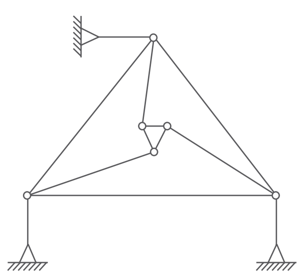
I. The truss is statically determinate

II. The truss is kinematically determinate

With reference to the above statements, which of the following applies?

(a) Both statements are true.
(b) Both statements are false.
(c) II is true but I false
(d) I is true but II false.

Solution: (d)



Degree of static indeterminacy, $D_s = m + r - 2j$
Number of members, $m = 12$
Number of reaction components, $r = 6$
Number of joints, $j = 9$

$$D_s = 12+6-2\times9=0$$

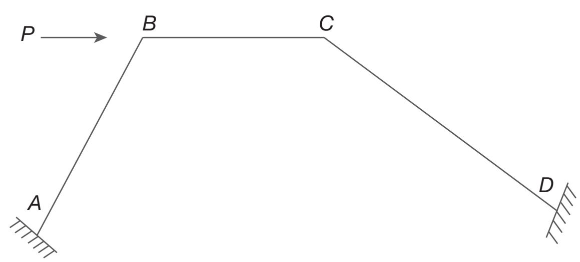
Therefore, the truss is statically determinate.
Degree of kinematic indeterminacy,

$$D_k = 2j - r$$

$$D_k = 2\times9-6=12$$

The truss is kinematically indeterminate.
Statement I is true but II is false.
Hence, the correct option is (d).

10. The degree of kinematic indeterminacy of the rigid frame with clamped ends at A and D shown in the figure is **[1997]**



(a) 4 (b) 3
(c) 2 (d) zero

Solution: (b)

Degree of kinematic indeterminacy $D_k = 3j - r$
Number of members, $m = 3$
Number of joints, $j = 4$,
Number of reaction components, $r = 6$,

$$D_k = 3\times4-6=6$$

$D_k = 3j - r - m$ If axial deformation is neglected

$$=3\times6-6-3=3$$

The displacements are: $\delta_B,\theta_B,\theta_c$

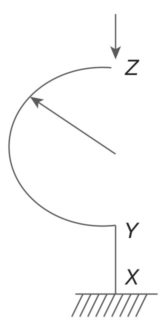
Hence, the correct option is (b).

Chapter 2

Analysis of Determinate Trusses and Frames

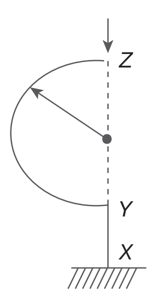
One-mark Questions

1. A curved member with a straight vertical leg is carrying a vertical load at Z, as shown in the figure. The stress resultants in the XY segments are **[2003]**



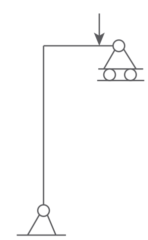
(a) bending moment, shear force and axial force
(b) bending moment and axial force only
(c) bending moment and shear force only
(d) axial force only

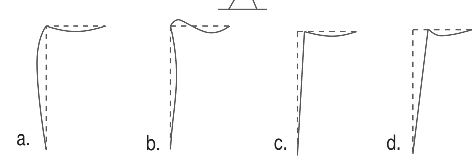
Solution: (d)



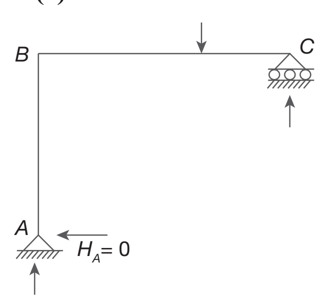
The line of action of load and the line segment XY coincides and hence no eccentricity. Therefore, the segment XY subjected to axial force only. But the segment YZ is subjected to axial force, shear force and bending moment.
Hence, the correct option is (d).

2. Identify the correct deflection diagram corresponding to the loading in the plane frame shown below: **[2001]**



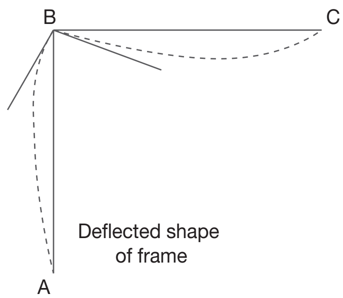


Solution: (a)



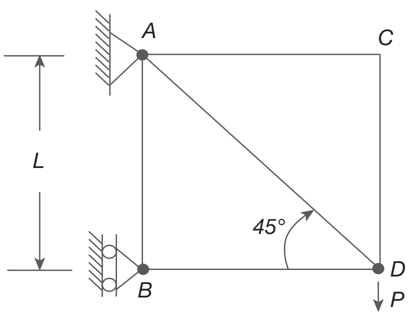
No horizontal reaction is induced at support A. The frame will not undergo any lateral displacement *ie.,* no sway.

The member BC deflects like sagging since the joint B is rigid, angle between the members BA and BC is same as that of frame without loading.



The slope at joint A is not equal to zero.
Hence, the correct option is (a).

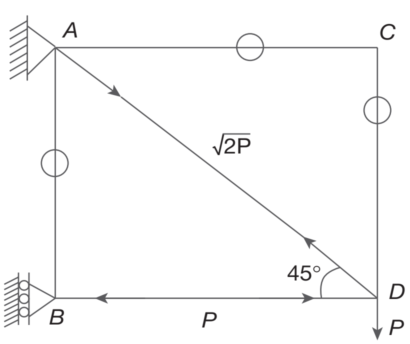
3. The strain energy stored in member AB of the pin-joined truss is shown in figure, when E and A are same for all members, is **[1998]**



(a) $\frac{2P^2L}{AE}$ (b) $\frac{P^2L}{AE}$

(c) $\frac{P^2L}{2AE}$ (d) zero

Solution: (d)
Forces in the members of the truss is shown in fig.

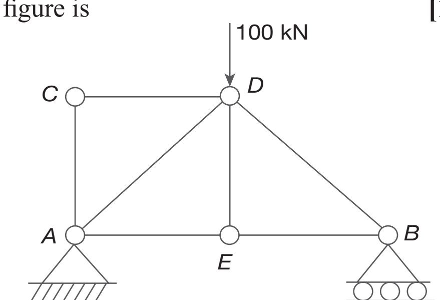


Force in the member AB, $F_{AB} = 0$
Strain energy stored in member AB,

$$U_{AB} = \left(\frac{F^2 l}{AE}\right)_{AB} = 0$$

Hence, the correct option is (d).

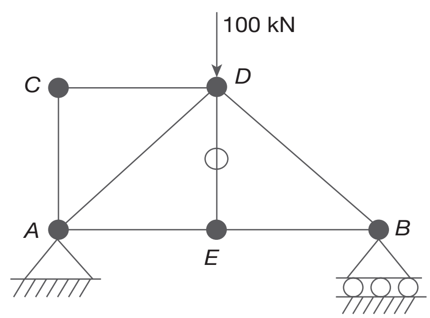
4. The force in the member DE of the truss shown in the figure is **[1997]**



(a) 100.0 kN (b) zero

(c) 35.5 kN (d) 25.0 kN

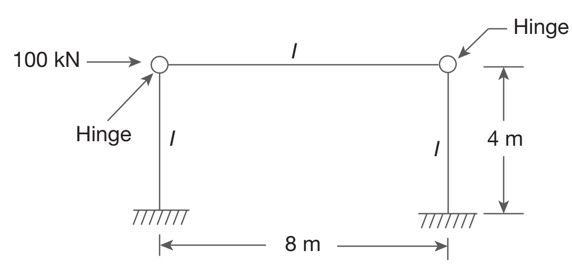
Solution: (b)



At joint E, out of three members, two members EA and EB are collinear and hence the force in third member ED is equal to zero.
At joint E, $\sum V = 0 \Rightarrow F_{ED} = 0$

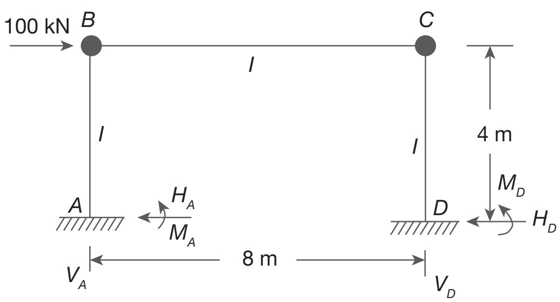
Hence, the correct option is (b).

5. For the frame shown in the figure, the maximum bending moment in the column is **[1997]**



(a) zero (b) 400 kNm
(c) 100 kNm (d) 200 kNm

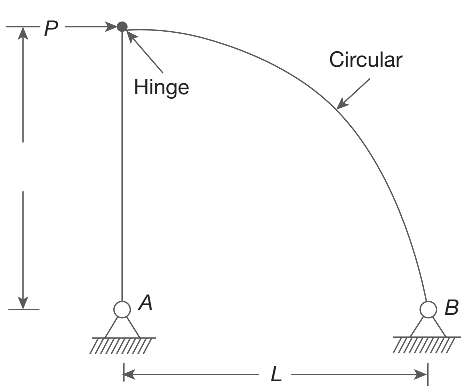
Solution: (d)



$$\sum V = 0 \Rightarrow V_A + V_D = 0$$
$$\sum H = 0 \Rightarrow H_A + H_D = 100$$
$$\sum M_B = 0 \Rightarrow H_A 4 - M_A = 0 \Rightarrow M_A = 4H_A$$
$$\sum M_C = 0 \Rightarrow H_D 4 - M_D = 0 \Rightarrow M_D = 4H_D$$
$$\sum M_B = 0 \Rightarrow H_D 4 - M_D - 8V_D = 0 \Rightarrow V_D = 0$$
$$\sum M_C = 0 \Rightarrow H_A 4 - M_A + 8V_A = 0 \Rightarrow V_A = 0$$
$$\sum M_D = V_A 8 + H_A 0 - M_A + 100 \times 4 = M_D$$
$$M_A + M_D = 400$$
$$H_A = H_D,\ M_A = M_D$$
$$H_A = H_D = 50\,\text{kN}, \quad M_A = M_D = 200\ \text{kN}$$

Max BM. in column = 200 kNm
Hence, the correct option is (d).

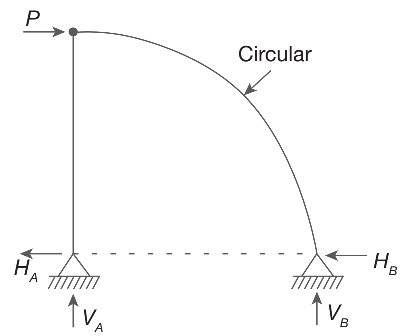
6. Vertical reaction at support B of the structure is **[1996]**



(a) P (b) $\sqrt{2}P$

(c) $\dfrac{P}{\sqrt{2}}$ (d) $\dfrac{P}{2}$

Solution: (a)

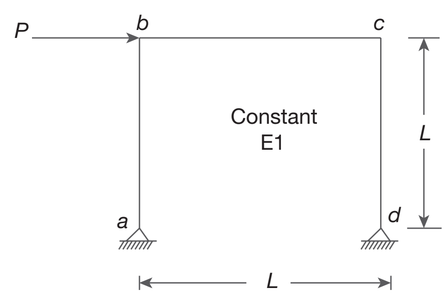


Taking moments of all forces about the hinge A,

$$\sum M_A = 0 \Rightarrow P \times L = V_B \times L = 0$$
$$V_B = P(\uparrow)$$

Hence, the correct option is (a).

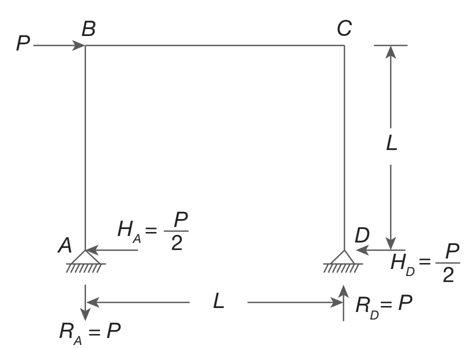
7. Bending moments at joint 'b' and 'c' of the portal frame are respectively **[1996]**



(a) $+\dfrac{PL}{2}, -\dfrac{PL}{2}$ (b) $+\dfrac{PL}{2}, +\dfrac{PL}{2}$

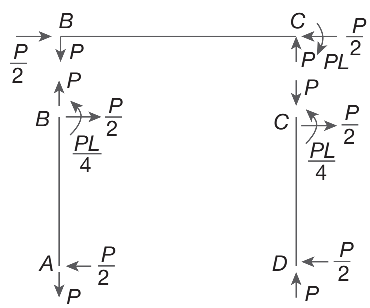
(c) $-\dfrac{PL}{4}, -\dfrac{PL}{4}$ (d) $+\dfrac{PL}{4}, +\dfrac{PL}{4}$

Solution: (a)



Bending moment at B, $BM_B = \dfrac{P}{2}\dfrac{L}{2} = \dfrac{PL}{4}$

Bending moment at C, $BM_C = \frac{P}{2}\frac{L}{2} = -\frac{PL}{4}$

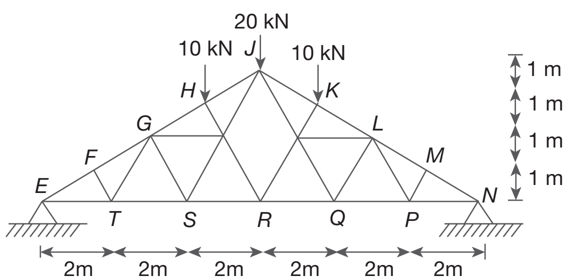


Free body diagrams of the members.

Hence, the correct option is (a).

Two-marks Questions

1. A plane truss with applied loads is shown in the figure. **[2016]**



The members which do not carry any force are

(a) FT, TG, HU, MP, PL
(b) ET, GS, UR, VR, QL
(c) FT, GS, HU, MP, QL
(d) MP, PL, HU, FT, UR

Solution: (a)

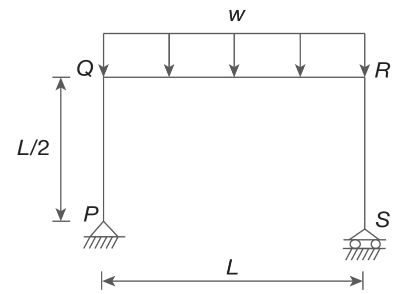
Conditions for zero force members are

i. The member meets at a joint and they are non-collinear and no external force acts at that joint. Both the members will be the zero force members.
ii. When the members meet at joint and two are collinear and no external force acts at the joint then third member will be zero force member.

According to the above statements, we can say that FT, TG, HU, MP and PL members are zero force members.

Hence, the correct option is (a).

2. The portal frame shown in the figure is subjected to a uniformly distributed vertical load w (per unit length). **[2016]**



The bending moment in the beam at the joint 'Q' is

(a) zero
(b) $\frac{wL^2}{24}$ (hogging)
(c) $\frac{wL^2}{12}$ (hogging)
(d) $\frac{wL^2}{8}$ (sagging)

Solution: (a)

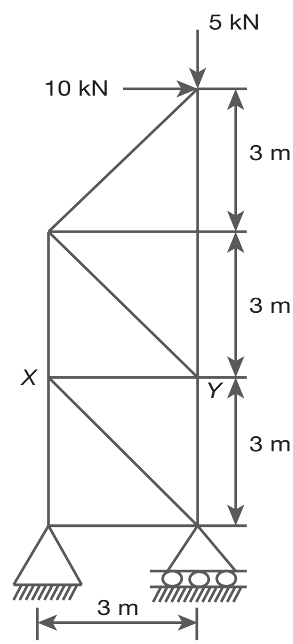
Since there is no external horizontal load.

So, $H_p = 0$

$\Rightarrow$ $M_\theta = 0$

Hence, the correct option is (a).

3. For the 2D truss with the applied loads shown below, the strain energy in the member XY is ________ kN-m. For member XY, assume AE = 30 kN, where A is cross-section area and E is the modulus of elasticity. **[2015]**

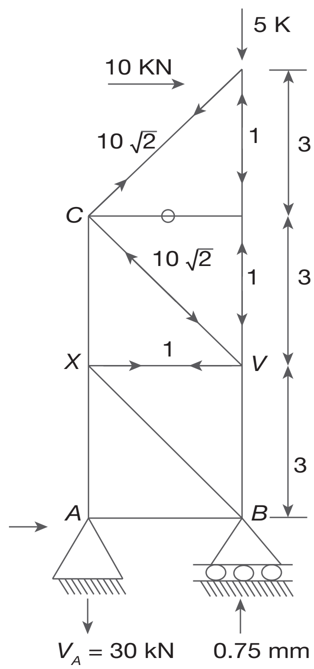


Solution: 5
Taking moments about A

$$R_B \times 3 = 10 \times 9 + 5 \times 3$$
$$R_B = 35 \text{ kN}$$

Taking moments about B

$$V_A \times 3 + 10 \times 9 = 0$$
$$V_A = -30 \text{ kN}$$



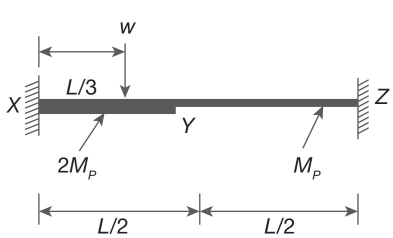
By method of joints,
Force in xy is calculated as

$$= 10 \text{ kN (compressive)}$$

Strain energy $U = \dfrac{P^2 L}{2AE} = \dfrac{10^2 \times 3}{2 \times 30} = 5$ kNm

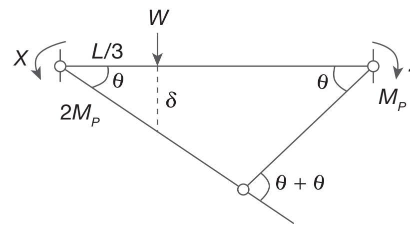
Hence, the answer is 5.

4. A fixed end beam is subjected to a load, W at 1/3rd span from the left support as shown in the figure. The collapse load of the beam is **[2015]**



(a) 16.5 M_p/L (b) 15.5 M_p/L
(c) 15.0 M_p/L (d) 16.0 M_p/L

Solution: (c)



Number of hinges required

$$= [4 - 2] + 1 = 3$$

[Only vertical reactions and moments are considered]
As no reaction in horizontal direction.
External work done = Internal work

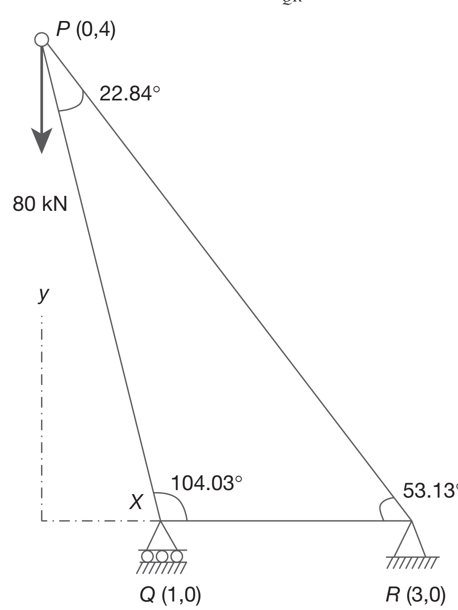
$$W \times \delta = 2M_P\theta + M_P(2\theta) + M_P\theta$$

$$W \times \frac{L}{3}\theta = 5M_p\theta$$

$$W = \frac{15M_p}{L}$$

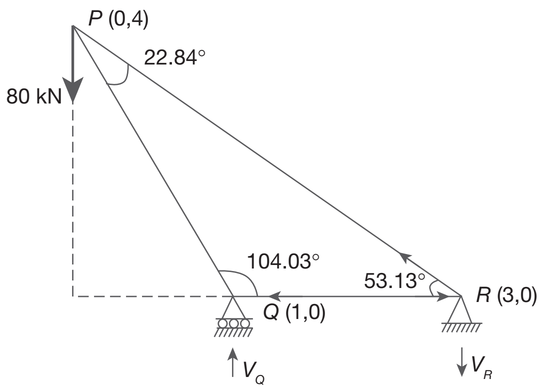
Hence, the correct option is (c).

5. Mathematical idealization of a crane has three bars with their vertices arranged as shown in the figure with a load of 80 kN hanging vertically. The coordinates of the vertices are given in parenthesis. The force in the member QR, F_{QR} will be **[2014]**



(a) 30 kN Compressive (b) 30 kN Tensile
(c) 50 kN Compressive (d) 50 kN Tensile

Solution: (a)



Taking moment of all forces about the joint Q,

$$\Sigma M_Q = 0 \Rightarrow -80 \times 1 + V_R \times 2 = 0$$

$$\Rightarrow \quad V_R = 40 \text{ kN}(\downarrow)$$

$$\Sigma V = 0 \Rightarrow V_Q - 40 = 80 \Rightarrow V_Q = 120 \text{ kN}(\uparrow)$$

Joint R:

$$\Sigma V = 0 \Rightarrow -40 + F_{PR} \sin 53.13° = 0$$

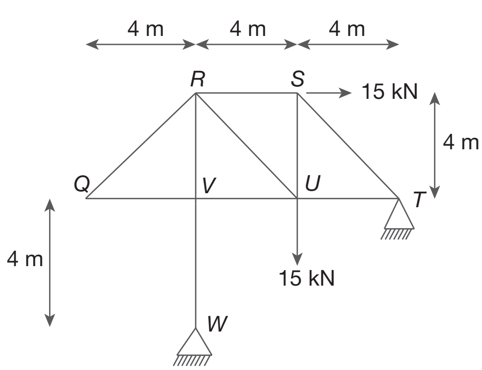
$$\Rightarrow \quad F_{PR} = 50 \text{ kN(T)}$$

$$\Sigma H = 0 \Rightarrow -50 \cos 53.13° + F_{QR} = 0$$

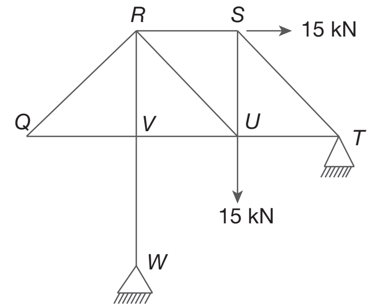
$$\Rightarrow \quad F_{QR} = 30 \text{ kN (C)}$$

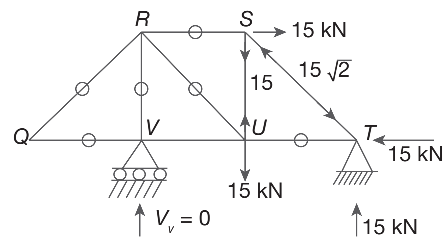
Hence, the correct option is (a).

6. The pin-jointed 2-D truss is loaded with a horizontal force of 15 kN at joint S and another 15 kN vertical force at joint U, as shown. Find the force in member RS (in kN) and report your answer taking tension as positive and compression as negative. ____________. **[2013]**



Solution: 0





Since the member VW is subjected to only vertical force, the member VW may be replaced by a vertical force at V.
Taking moments of all forces about joint T,

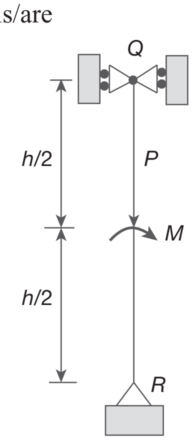
$$15 \times 4 - 15 \times 4 + V_V = 0 \Rightarrow V_V = 0$$

$$\sum V = 0 \Rightarrow V_T = 15 \text{ kN}(\uparrow)$$

$$\sum H = 0 \Rightarrow H_T = 15 \text{ kN}(\leftarrow)$$

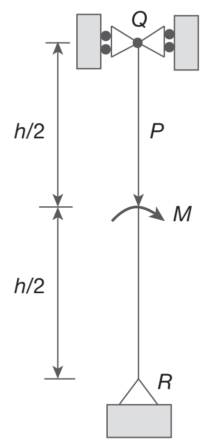
The forces in the members of the truss are shown in fig.
The force in member $RS = 0$
Hence, the answer is 0.

7. The sketch shows a column with a pin at the base and rollers at the top. It is subjected to an axial force P and a moment M at mid-height. The reaction (s) at R is/are **[2012]**



(a) a vertical force equal to P
(b) a vertical force equal to $P/2$
(c) a vertical force equal to P and a horizontal force equal to M/h
(d) a vertical force equal to $P/2$ and a horizontal force equal to M/h

Solution: (c)



$$\sum V = 0 \Rightarrow V_R = P(\uparrow)$$

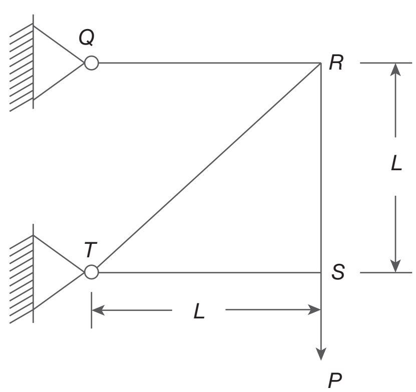
$$\sum M_0 = 0 \Rightarrow H_R h - M = 0,\ H_R = \frac{M}{h}$$

Vertical reaction at R, $V_R = P(\uparrow)$

Horizontal reaction at R, $H_R = \dfrac{M}{h}(\rightarrow)$

Hence, the correct option is (c).

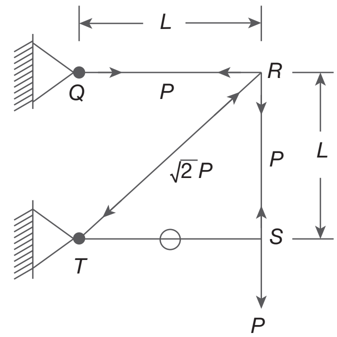
8. For the truss shown in the figure, the force in the member QR is **[2010]**



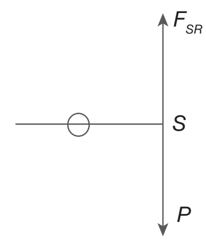
(a) Zero (b) $\dfrac{P}{\sqrt{2}}$

(c) P (d) $\sqrt{2}P$

Solution: (c)

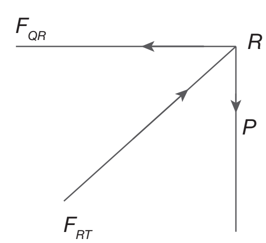


Joint S:



$$\sum V = 0 \Rightarrow P - F_{SR} = 0; \qquad F_{SR} = P(T)$$

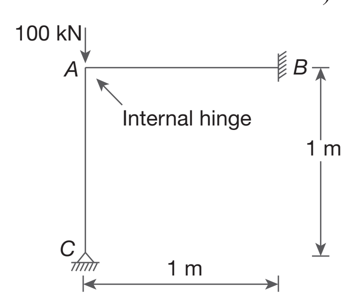
Joint R:



$$\sum V = 0 \Rightarrow P - F_{RT}\cos 45^\circ = 0\ ; \qquad F_{RT} = \sqrt{2}P(C)$$
$$\sum H = 0 \Rightarrow F_{RT}\sin 45^\circ - F_{QR} = 0\ ; \quad F_{QR} = P(T)$$

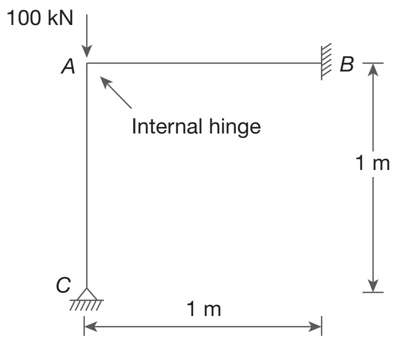
Hence, the correct option is (c).

9. Vertical reaction developed at B in the frame below due to the applied load of 100 kN (with 150,00 mm^2 cross-sectional area and 3.125×10^9 mm^4 moment of inertia for both members) is **[2006]**



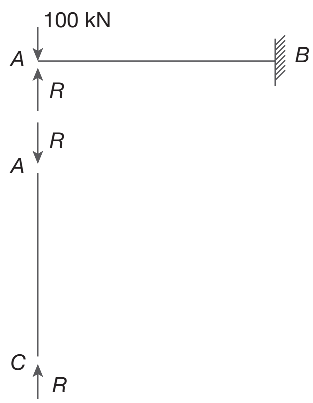
(a) 5.9 kN (b) 302 kN
(c) 66.3 kN (d) 94.1 kN

Solution: (a)



Cross sectional area, $A = 150,000 \text{ mm}^2$

Moment of Inertia of the section, $I = 3.125 \times 10^9 \text{ mm}^4$



At joint A, the deflection in beam AB is equal to the compression in column AC.

$$\frac{(100-R)l^3}{3EI} = \frac{RL}{AE} \; ; \; (100-R) = \frac{3R}{A}$$

$$100 - R = 3R \times \frac{3.125 \times 10^9 \times 10^{-12}}{150000 \times 10^{-6}} = 0.0625\, R$$

$$1.0625R = 100; \; R = 94.1 \text{ kN}$$

$$V_B = 100 - R = 100 - 94.1 = 5.9 \text{ kN}$$

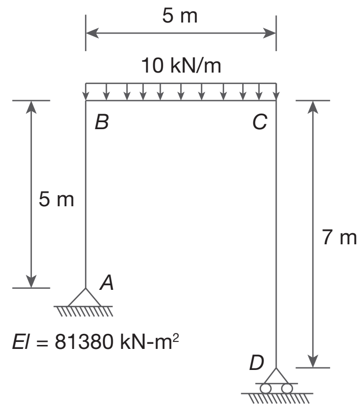
Hence, the correct option is (a).

10. The plane frame below is analyzed by neglecting axial deformations. Following statements are made with respect to the analysis.

I. Column AB carries axial force only

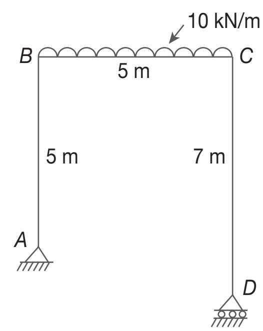
II. Vertical deflection at the center of beam BC is 1 mm

With reference to the above statements, which of the following applies? **[2004]**



(a) Both the statements are true
(b) Statement I is true but II is false
(c) Statement II is true but I is false
(d) Both the statements are false

Solution: (a)



Taking moments of all forces about the hinged support A,

$$\sum M_A = 0 \Rightarrow 10 \times 5 \times 2.5 - R_D 5 = 0$$

$$R_D = 25 \text{ kN}(\uparrow)$$

$$\sum V = 0 \Rightarrow R_A + R_D = 10 \times 5$$

$$R_A = 50 - 25 = 25 \text{ kN}(\uparrow)$$

Since the reactions at A and D are equal, beam BC will behave as simply supported beam. Therefore, the column AB carries axial force only.

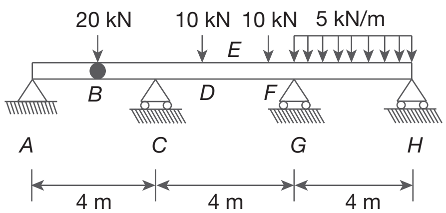
Deflection at the centre of beam BC, $\delta = \frac{5}{384}\frac{wl^4}{EI}$

$$\delta = \frac{5}{384}\frac{10(5)^4}{81380} \times 10^3 = 1 \text{ mm}$$

Hence, both the statements are true.
Hence, the correct option is (a).

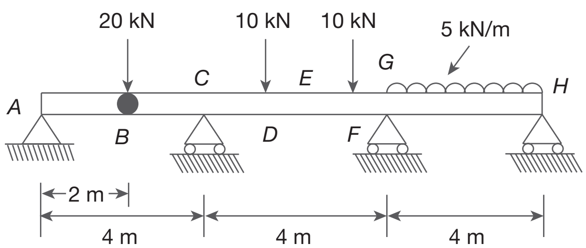
Common Data for Questions 11 and 12:
A three-span continuous beam has a internal hinge at B. Section B is at the mind-span of *AC*. Section *R* is at the mid-span of *CG*. The 20 kN load is applied at section B whereas 10 kN loads are applied at sections D and *F* as shown in the figure. Span *GH* is subjected to uniformly distributed load of magnitude 5 kN/m. For the loading shown, shear force immediate to the right of section E is 9.84 kN upwards and the sagging moment at section *E* is 10.3 kN-m.

AB = *BC* = 2 m
CD = *DE* = *EF* = *FG* = 1 m
GH = 4 m



11. The magnitude of the shear force immediate to the left and immediate to the right of section *B* are, respectively **[2004]**
(a) 0 and 20 kN
(b) 10 kN and 10 kN
(c) 20 kN and 0
(d) 9.84 kN and 10.16 kN

Solution: (a)



$$AB = BC = 2 \text{ m}$$

$$CD = DE = EF = FG = 1 \text{ m}; \; GH = 4 \text{ m}$$

SF to the right of $E = 9.84 \text{ kN}(\uparrow)$

BM at $E = 10.31 \text{ kNm}$ sagging

Taking moments of all forces about *B* from left,

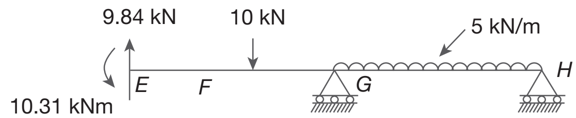
$$\sum M_B = 0 \Rightarrow R_A 2 = 0$$
$$R_A = 0$$

SF to the left of $B = 0$
SF to the right of $B = 20 \text{ kN}(\downarrow)$

Hence, the correct option is (a).

12. The vertical reaction at support H is **[2004]**
(a) 15 kN upward (b) 9.84 kN upward
(c) 15 kN downward (d) 9.84 kN downward

Solution: (b)

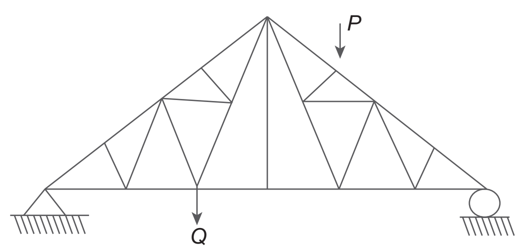


Taking moments of all forces about *G*,

$$R_H \times 4 - 5 \times 4 \times 2 + 10 \times 1 - 9.84 \times 2 + 10.31 = 0$$
$$4R_H = 39.37; \; R_H = 9.84 \text{ kN}(\uparrow)$$

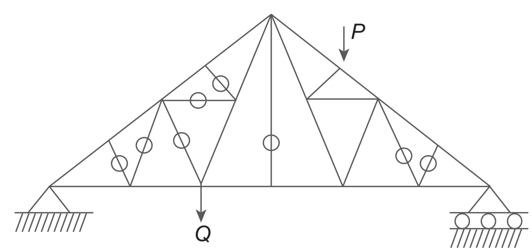
Hence, the correct option is (b).

13. For the plane truss shown in the figure, the number of zero force members for the given loading is **[2004]**



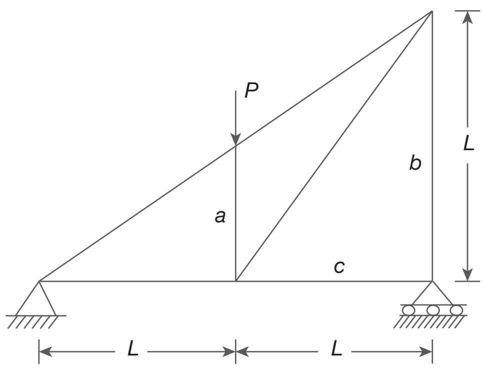
(a) 4 (b) 8
(c) 11 (d) 13

Solution: (b)



If three members meet at a joint and two of them are collinear, then the third member will carry zero force provided that there does not act any external load at the joint.
Number of members with zero forces = 8
Hence, the correct option is (b).

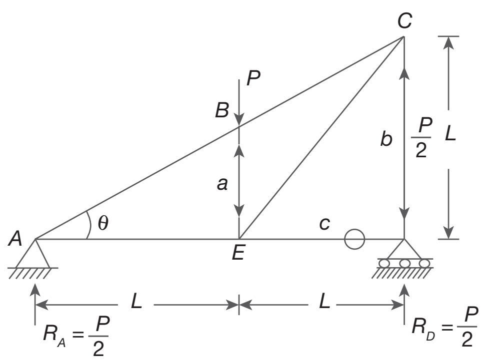
14. The forces in members 'a', 'b', 'c' in the truss shown are, respectively **[1995]**



(a) $P, \frac{P}{2}, 0$ (b) $\frac{P}{2}, P, 0$

(c) P, P, P (d) $\frac{P}{2}, \frac{P}{2}, 0$

Solution: (a)



Since the load is acting symmetrically,

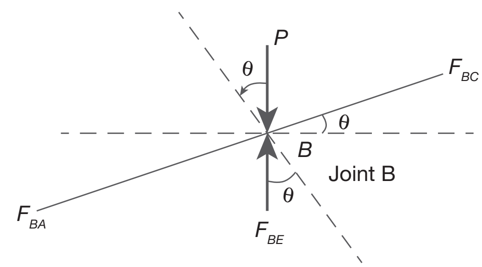
$$R_A = R_B = \frac{P}{2}$$

$$\text{Tan}\theta = \frac{L}{2L} = \frac{1}{2}, \theta = 26.56°$$

Resolving the forces at joint B in the direction perpendicular to AC,

$$P\cos\theta - P_{BE}\cos\theta = 0$$

$$P_{BE} = P$$

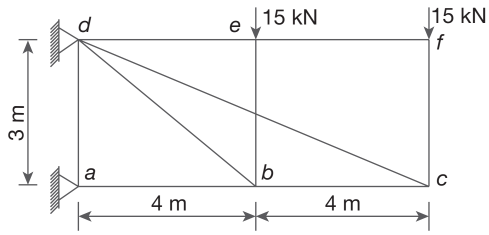


The forces in members 'a' 'b' and 'c' are: $P, \frac{P}{2}; 0$

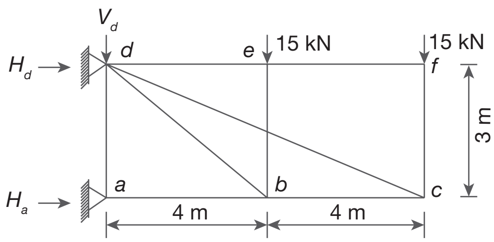
Hence, the correct option is (a).

Five-marks Questions

1. Compute the forces in members of the truss shown below in figure. **[1996]**



Solution:



$$D_{se} = r_e - 3 = 3 - 3 = 0$$

$$D_{si} = m - (2i - 3)$$

$$= 9 - (12 - 3) = 9 - 9 = 0$$

$$\therefore \quad D_s = D_{se} + D_{si} = 0 + 0 = 0$$

$$\sum F_x = 0$$

$$\Rightarrow \quad H_a + H_d = 0 \qquad (1)$$

$$\sum F_y = 0$$

$$\Rightarrow \quad V_a = V_d + 15 + 15$$

$$\Rightarrow \quad V_d = 30 \text{ kN}$$

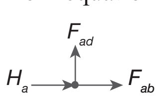
$$\sum M_d = 0$$

$$\Rightarrow \quad H_a \times 3 = 15 \times 4 + 15 \times 8$$

$$\Rightarrow \quad H_a = 60 \text{ kN}$$

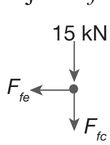
$$H_d = -60 \text{ kN}$$

[From equation (1)] Consider joint '*a*'



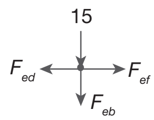
$\boxed{F_{ad} = 0}$ & $\boxed{F_{ab} = -H_a = -60 \text{ kN}}$

Consider joint '*f*'



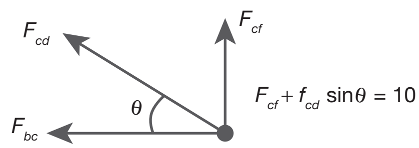
$\boxed{F_{fe} = 0 \text{ \& } F_{fc} = -15 \text{ kN}}$

Consider joint '*e*'



$\boxed{F_{fe} = F_{ed} = 0}$ $\boxed{F_{eb} = -15 \text{ kN}}$

Consider joint '*c*'



$$F_{cf} + f_{cd} \sin\theta = 0$$

$$\sin\theta = \frac{3}{8.54}$$

$$\cos\theta = \frac{3}{8.54}$$

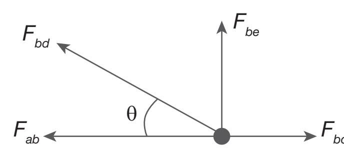
$$\Rightarrow \quad -15 + f_{cd} \times \frac{3}{8.54} = 0$$

$$\Rightarrow \quad f_{cd} = 42.7 \text{ kN and } f_{cd}\cos\theta + f_{bc} = 0$$

$$\Rightarrow \quad 42.7 \times \frac{8}{8.54} + f_{bc} = 0$$

$$\Rightarrow \quad f_{bc} = -40 \text{ kN}$$

Consider joint *b*



$$F_{bd} \sin\theta + F_{be} = 0$$

$$\Rightarrow \quad F_{bd} \times \frac{3}{8.54} = 15$$

$$\Rightarrow \quad F_{bd} = 42.7$$

Members	Force	Nature
ab	60	Compressive
ad	0	
bc	40	Compressive
bd	42.7	Tensile
be	15	Compressive
cf	15	Compressive
cd	42.7	Tensile
de	0	
ef	0	

Chapter 3

Propped Cantilevers and Fixed Beams

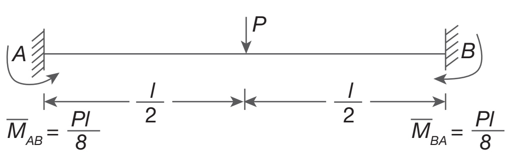
One-mark Questions

1. The fixed end moment of uniform beam of span l and fixed at the ends, subjected to a central point load P is **[1994]**

(a) $\dfrac{Pl}{2}$ (b) $\dfrac{Pl}{8}$

(c) $\dfrac{Pl}{12}$ (d) $\dfrac{Pl}{16}$

Solution: (b)



Fixed end moment at A, $\overline{M}_{AB} = \dfrac{Pl}{8}$ (anti-clockwise)

Fixed end moment at B, $\overline{M}_{BA} = \dfrac{Pl}{8}$ (clockwise)

Hence, the correct option is (b).

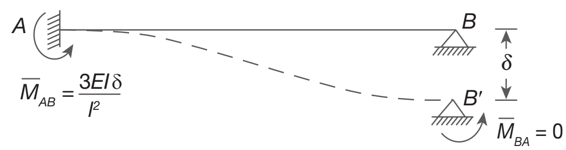
2. The moments at the ends 'A' and 'B' of a beam 'AB' where end 'A' is fixed and 'B' is hinged, when the end 'B' sinks by an amount Δ, are given as **[1991]**

(a) $\dfrac{6EI\Delta}{l^2}, \dfrac{6EI\Delta}{l^2}$ (b) $\dfrac{6EI\Delta}{l^2}, 0$

(c) $\dfrac{3EI\Delta}{l^2}, \dfrac{3EI\Delta}{l^2}$ (d) $\dfrac{3EI\Delta}{l^2}, 0$

Solution: (d)

The deflected shape of the beam is shown in fig.



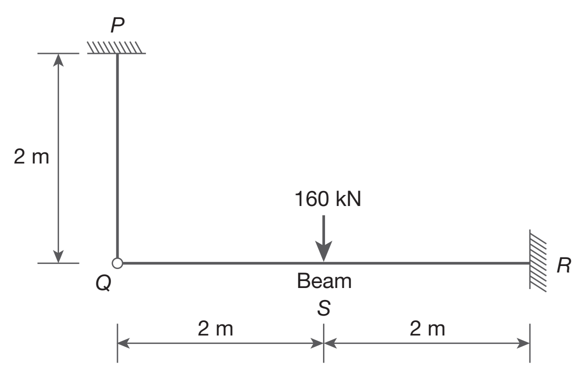
BM at end A, $\overline{M}_{AB} = -\dfrac{3EI\delta}{l^2}$

BM at end B, $\overline{M}_{BA} = 0$

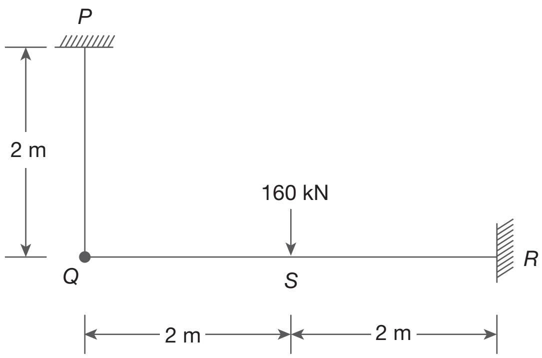
Hence, the correct option is (d).

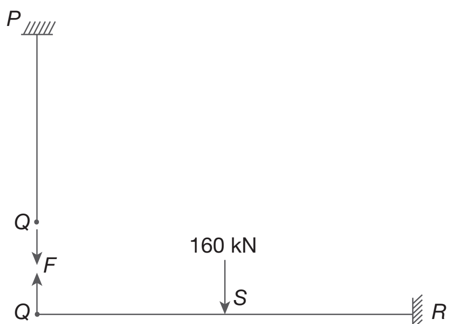
Two-marks Questions

1. The axial load (in kN) in the member PQ for the assembly/arrangement shown in the figure given below is ________. **[2014]**



Solution: 50





Free body diagrams

Since the joint Q is hinged, vertically downward force in member PQ is equal to the upward force at Q in beam QR. Let F be the tensile force in the member PQ. Elongation of member PQ = Downward deflection at Q in beam QR

$$\frac{F\times 2}{AE}=\frac{160\times 2^2}{3EI}+\frac{160\times 2^2}{2EI}\times 2-\frac{F\times 4^3}{3EI}$$

Deflection due to axial forces will be very small as compared to bending, and hence neglected.

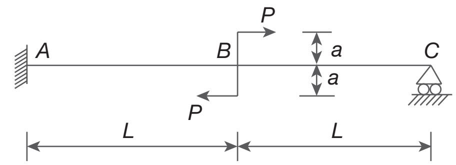
$$\frac{160\times 2^3}{3EI}+\frac{160\times 2^2}{2EI}\times 2-\frac{F\times 4^3}{3EI}=0$$

$$426.67+640-21.33F=0$$

$$F=50\,\text{kN}$$

Hence, the answer is 50.

Statement for Linked Questions 2 and 3:
Consider a propped cantilever beam ABC under two loads of magnitude P each as shown in the figure blow. Flexural rigidity of the beam is EI. **[2006]**

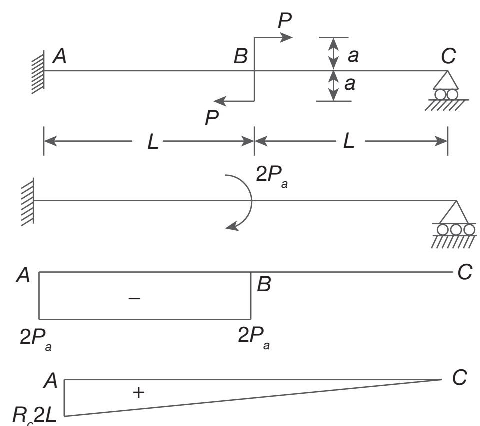


2. The reaction at C is

(a) $\frac{9Pa}{16L}$ (upwards) (b) $\frac{9Pa}{16L}$ (downwards)

(c) $\frac{9Pa}{8L}$ (upwards) (d) $\frac{9Pa}{8L}$ (downwards)

Solution: (c)



Deflection at C due to moment,

$$\delta_{C1}=\frac{2PaL}{EI}\left(\frac{L}{2}+L\right)=\frac{3PaL^2}{EI}(\downarrow)$$

Deflection due to reaction at C,

$$=\delta_{C2}=\frac{R_c(2L)^3}{3EI}=\frac{8R_cL^3}{3EI}(\uparrow)$$

$$\frac{3PaL^2}{EI}=\frac{8R_cL^3}{3EI};R_c=\frac{9Pa}{8L}(\uparrow)$$

Hence, the correct option is (c).

3. The rotation at B is

(a) $\frac{5PLa}{16EI}$ (clockwise)

(b) $\frac{5PLa}{16EI}$ (anti-clockwise)

(c) $\frac{59PLa}{16EI}$ (clockwise)

(d) $\frac{59PLa}{16EI}$ (anti-clockwise)

Solution: (a)

Rotation at B due to moment, $\theta_{B1} = \dfrac{2PaL}{EI}(\uparrow)$

Rotation at B due to reaction

$$R_c, \theta_{B2} = \frac{RL^2}{2EI} + \frac{RL^2}{EI} = \frac{3RL^2}{2EI} = \frac{27}{16}\frac{PaL}{EI}(\uparrow)$$

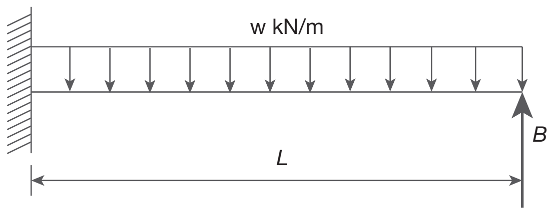
Rotation at B, $\theta_B = \theta_{B1} - \theta_{B2}$

$$= \frac{2PaL}{EI} - \frac{27}{16}\frac{PaL}{EI} = \frac{5}{16}\frac{PaL}{EI}(\downarrow)$$

$$R_B = \frac{3}{8}wL$$

Hence, the correct option is (a).

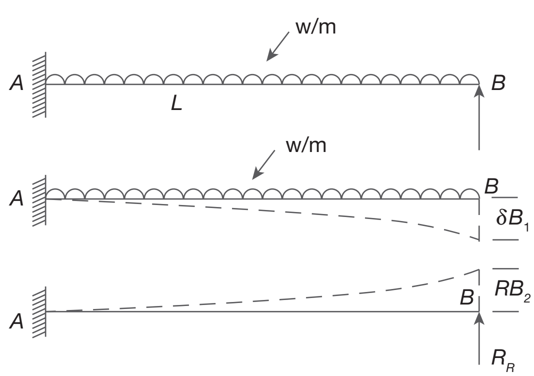
4. In the propped cantilever beam carrying a uniformly distributed load of w N/m shown in the following figure, the reaction at the support B is **[2002]**



(a) $\dfrac{5}{8}wL$ (b) $\dfrac{3}{8}wL$

(c) $\dfrac{1}{2}wL$ (d) $\dfrac{3}{4}wL$

Solution: (b)



The given propped cantilever is equivalent to the sum of cantilever subjected to udl and cantilever subjected to reaction at B.

Deflection at the support B, $\delta_B = 0$

$$\delta_{B1} - \delta_{B2} = 0$$

$$\frac{wL^4}{8EI} = \frac{R_B L^3}{3EI} \Rightarrow R_B = \frac{3}{8}wL$$

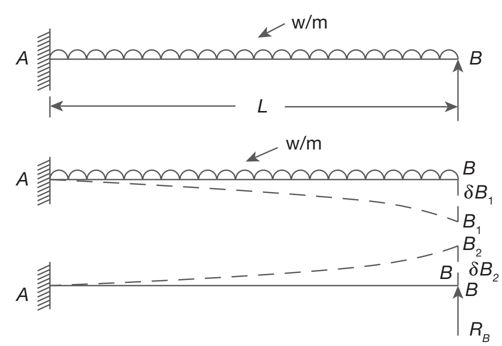
Hence, the correct option is (b).

5. A propped cantilever beam of span L, is loaded with uniformly distributed load of intensity w/unit length, all through the span. Bending moment at the fixed end is **[1997]**

(a) $\dfrac{wL^2}{8}$ (b) $\dfrac{wL^2}{2}$

(c) $\dfrac{wL^2}{12}$ (d) $\dfrac{wL^2}{24}$

Solution: (a)



Downward deflection due to udl, $\delta_{B_1} = \dfrac{wL^4}{8EI}$

Upward deflection due to reaction R_B, $\delta_{B_2} = \dfrac{R_B L^3}{3EI}$

At the prop support, $\delta_B = 0$

$$\delta_{B1} = \delta_{B2}$$

$$\frac{wL^4}{8EI} = \frac{R_B L^3}{3EI} \Rightarrow R_B = \frac{3}{8}wL$$

BM at fixed end,

$$BM_A = \frac{3}{8}wLL - wL\frac{L}{2} = \frac{3}{8}wL^2 - \frac{wL^2}{2} = -\frac{wL^2}{8}$$

$$= \frac{wL^2}{8}\text{ (hogging)}$$

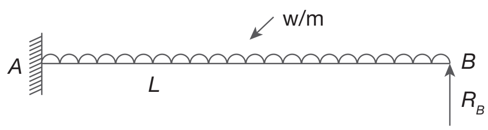
Hence, the correct option is (a).

6. A cantilever beam of span l subjected to uniformly distributed load w per unit length resting on a rigid prop at the tip of the cantilever. The magnitude of the reaction at the prop is **[1994]**

(a) $\frac{1}{8}wl$ (b) $\frac{2}{8}wl$

(c) $\frac{3}{8}wl$ (d) $\frac{4}{8}wl$

Solution: (c)



Let R_B: Reaction of the prop at B

Downward deflection at B due to udl, $\delta_{B1} = \frac{wL^4}{8EI}$

Upward deflection at B due to prop reaction,

$$\delta_{B2} = \frac{R_B L^3}{3EI}$$

Since the prop is rigid, $\delta_B = 0$

$$\delta_{B1} - \delta_{B2} = 0 \Rightarrow \delta_{B1} = \delta_{B2}$$

$$\frac{wL^4}{8EI} = \frac{R_B L^3}{3EI}$$

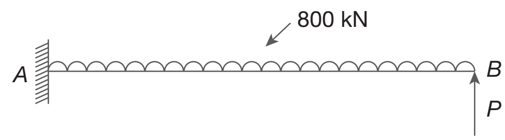
Hence, the correct option is (c).

7. A cantilever beam of span L is subjected to a downward load of 800 kN uniformly distributed over its length and a concentrated upward load P, at its free end. For vertical displacement to be zero at the free end, the value of P is **[1992]**

(a) 300 kN (b) 500 kN

(c) 800 kN (d) 1000 kN

Solution: (a)



Downward deflection at B due to $800\,\text{kN}$ load,

$$\delta_{B1} = \frac{wL^4}{8EI} = \frac{WL^3}{8EI} = \frac{800L^3}{8EI} = \frac{100L^3}{EI}$$

Upward deflection at B due to P, $\delta_{B2} = \frac{PL^3}{3EI}$

Given $\delta_B = 0 \Rightarrow \delta_{B1} - \delta_{B2} = 0$

$$\frac{100L^3}{EI} = \frac{PL^3}{3EI}$$

$$P = 300\text{ kN}$$

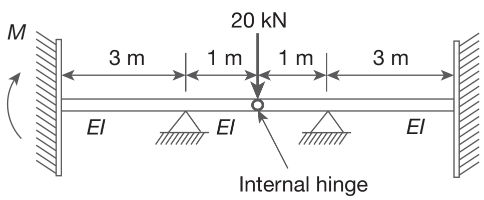
Hence, the correct option is (a).

Chapter 4

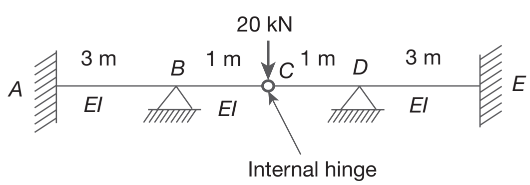
Analysis of Indeterminate Structures

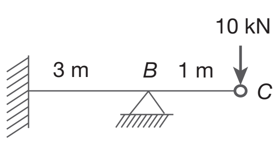
ONE-MARK QUESTIONS

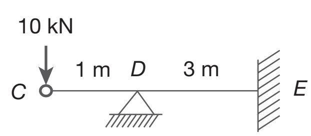
1. For the beam shown below, the value of the support moment M is _______ kN-m. **[2015]**



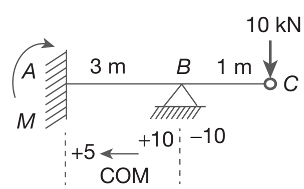
Solution: 5







The given beam symmetrical.
Hence, considering F.B.D. of laft side half
Adopting Moment distribution method
M = 5 kN-m



Hence, the answer is 5.

2. Identify the FALSE statement from the following, pertaining to the methods of structural analysis. **[2001]**

(a) Influence lines for stress resultants in beams can be drawn using Muller Breslau's Principle.
(b) The Moment Distribution Method is a force method of analysis, not a displacement method.
(c) The Principle of Virtual Displacements can be used to establish a condition of equilibrium.
(d) The Substitute Frame Method is not applicable to frames subjects to significant sides sway.

Solution: (b)

Influence lines for stress resultant can be drawn using Muller Breslau Principle. True.

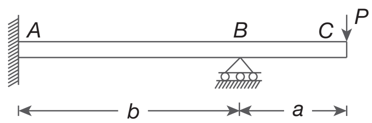
Displacement methods: Slope deflection method, Moment distribution method, Stiffness methods.

The principle of virtual work is used to establish a condition of equilibrium. True

Substitute frame method is used for analysis the frames subjected to only gravity loads, *ie.,* not applicable for lateral loads. True.

Hence, the correct option is (b).

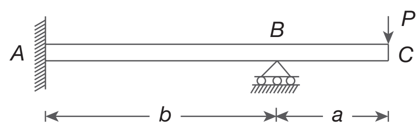
3. The magnitude of the bending moment at the fixed support of the beam is equal to **[1995]**



(a) Pa (b) $\frac{Pa}{2}$

(c) Pb (d) $P(a + b)$

Solution: (b)



Distribution factors at joint B:

$$(DF)_{BA} : (DF)_{BC} = 1 : 0$$

Joint	A	B	
DF	–	1	0
Member	AB	BA	BC
FEM's			$-Pa$
Balancing		Pa	
Carry over moment	$\frac{Pa}{2}$		
Final moments	$\frac{Pa}{2}$	Pa	$-Pa$

Bending moment at A, $M_A = \frac{Pa}{2}$ (clockwise)

Hence, the correct option is (b).

4. The number of simultaneous equations to be solved in the slope deflection method is equal to **[1995]**

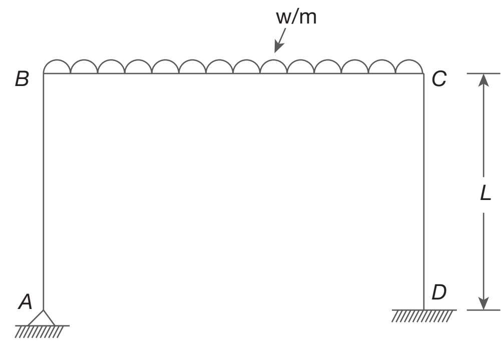
(a) the degree of static indeterminacy
(b) the degree of kinematic indeterminacy
(c) the number of joints in the structure
(d) None of the above

Solution: (a)
The number of simultaneous equations to be solved in the slope deflection method is equal to the degree of static indeterminacy.
Hence, the correct option is (a).

5. A single bay single storey portal frame has hinged left support and a fixed right support. It is loaded with uniformly distributed load on the beam. Which one of the following statements is true with regard to the deformation of the frame? **[1995]**

(a) It would sway to the left side
(b) It would sway to the right side
(c) It would not sway at all
(d) None of the above

Solution: (a)



Stiffness of member BA, $K_{BA} = \frac{3EI}{L}$

Stiffness of member CD, $K_{CD} = \frac{4EI}{L}$

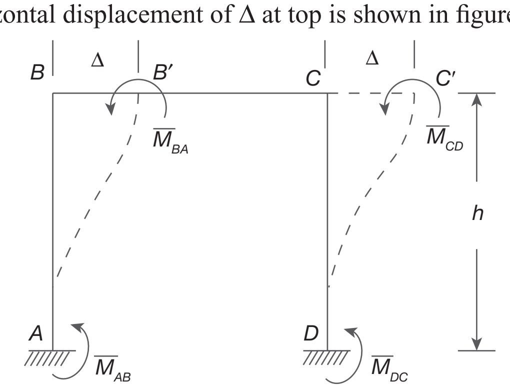
The frame will sway towards the column of low stiffness. Therefore, the frame would sway towards left side.
Hence, the correct option is (a).

6. A signal bay portal frame of height h fixed at the base is subjected to horizontal displacement Δ at the top. The base moments developed is proportional to **[1994]**

(a) $\frac{1}{h}$ (b) $\frac{1}{h^2}$

(c) $\frac{1}{h^3}$ (d) None of these

Solution: (b)
The deflected shape of the portal frame for horizontal displacement of Δ at top is shown in figure.



$$\bar{M}_{AB} = \bar{M}_{BC} = \bar{M}_{CD} = \bar{M}_{DC} = -\frac{6EI\Delta}{h^2}$$

Therefore, the moment at base is proportional to $\frac{1}{h^2}$

Hence, the correct option is (b).

7. Match the following methods with appropriate analysis **[1994]**

a. Strain energy method	I. Influence line for redundant structures
b. Complementary energy method	II. Deflection of linear structures
c. Muller Breslau Principle	III. Deflection of non-linear structures
d. kani's method of analysis	IV. Analysis of multistoreyed frames

Solution:
A: II; B: III; C: I; D: IV

Method	Use
Strain energy method	Deflection of linear structures
Complementary energy method	Deflection of non linear structures
Muller Breslau principle	Influence lines for redundant structures
Kani's method of analysis	Analysis of multistoreyed frames.

8. Methods of indeterminate structural analysis may be grouped under either force method or displacement method. Which of the groupings given below is correct? **[1993]**

Group-I (Force method)	Group-II (Displacement method)
a. Moment distribution method Consistent deformation method	1. Method of three moments Slope deflection method
b. Method of three methods Consistent deformation method	2. Moment distribution method Slope deflection method
c. Slope deflection method Consistent deformation method	3. Moment distribution method Method of three moment
d. Moment distribution method Method of three moments	4. Slope deflection method Consistent deformation method

Solution: (b)
Force method of analysis
1. Clayperson's theorem of three moments
2. Castigliano's theorem
3. Consistency deformation method.
4. Unit load method
5. Virtual work method
6. Minimum potential energy method
7. Column analogy method
8. Flexibility method.

Displacement methods of analysis
1. Slope deflection method
2. Moment distribution method
3. Kani's method
4. Stiffness method.

Hence, the correct option is (b).

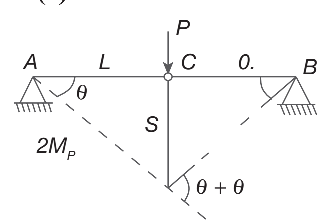
TWO-MARKS QUESTIONS

1. In a system, two connected rigid bars AC and BC are of identical length, L with pin supports at A and B. The bars are interconnected at C by a frictionless hinge. The rotation of the hinge is restrained by a rotational spring of stiffness, k. The system initially assumes a straight line configuration, ACB. Assuming both the bars as weightless, the rotation at supports, A and B, due to a transverse load, P applied at C is: **[2005]**

(a) $\frac{PL}{4\,k}$ (b) $\frac{PL}{2\,k}$

(c) $\frac{P}{4\,k}$ (d) $\frac{Pk}{4\,L}$

Solution: (a)

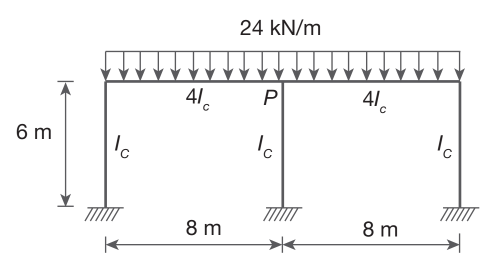


Overturning Moment = PL
Resisting Moment = k 4θ

$$PL = k4\theta;\ \theta = \frac{PL}{4\,k}$$

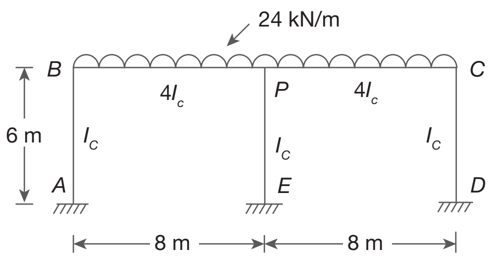
Hence, the correct option is (a).

2. Considering the symmetry of a rigid frame as shown, the magnitude of the bending moment (in kNm) at P (Preferably using the moment distribution method) is **[2014]**



(a) 170 (b) 172
(c) 176 (d) 178

Solution: (c)



Distribution factors:

Joint	Member	Relative stiffness, k	Total relative stiffness, $\sum k$	$DF = \frac{k}{\sum k}$
B	BA	$\frac{I}{6}$	$\frac{I}{6}+\frac{I}{2}=\frac{2}{3}I$	$\frac{I}{6}/\frac{2}{3}I=\frac{1}{4}$
	BP	$4\frac{I}{8}/\frac{I}{2}$		$\frac{I}{2}/\frac{2}{3}I=\frac{3}{4}$
P	PB	$\frac{4I}{8}=\frac{I}{2}$	$\frac{I}{2}+\frac{I}{6}+\frac{I}{2}=\frac{7}{6}I$	$\frac{I}{2}/\frac{I}{6}I=\frac{3}{7}$
	PE	$\frac{I}{6}$		$\frac{I}{6}/\frac{7I}{6}=\frac{1}{7}$
	PC	$\frac{4I}{8}=\frac{I}{2}$		$\frac{I}{2}/\frac{7I}{6}=\frac{3}{7}$
C	CP	$\frac{4I}{8}=\frac{I}{2}$	$\frac{I}{2}+\frac{I}{6}=\frac{2}{3}I$	$\frac{I}{2}/\frac{2}{3}I=\frac{3}{4}$
	CD	$\frac{I}{6}$		$\frac{I}{6}/\frac{2}{3}I=\frac{1}{4}$

Fixed end moments:

$$\bar{M}_{AB}=\bar{M}_{BA}=\bar{M}_{PE}=\bar{M}_{EP}=\bar{M}_{CD}=\bar{M}_{DC}=0$$

$$\bar{M}_{BP}=-\frac{24\times 8^2}{12}=-128 \text{ kNm}$$

$$\bar{M}_{PB}=128 \text{ kNm}$$

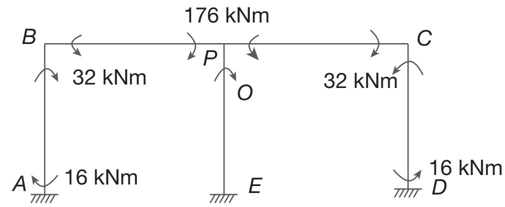
$$\bar{M}_{PC}=-128 \text{ kNm}$$

$$\bar{M}_{CP}=128 \text{ kNm}$$

Moment distribution table:

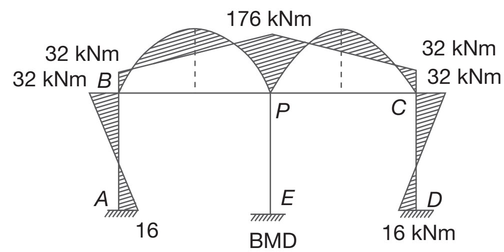
A	B		P	C		D		Joint
	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{7}$	$\frac{3}{7}$	$\frac{3}{4}$	$\frac{1}{4}$		DF
0	0	-128	128	-128	128	-128	0	FEM's
	32	96	-	-	-96	-32		Balancing
16			48	-48			-16	COM
16	32	-32	176	-176	32	-32	-16	Final moments

Bending moment at $P = 176$ kNM



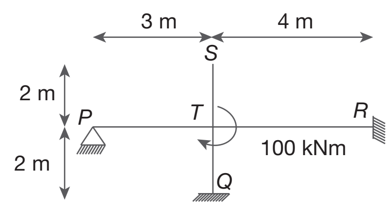
Simply supported bending moment at mid span

$$=\frac{24\times 8^2}{8}=192 \text{ kNm}$$

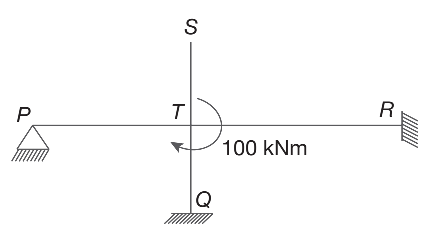


Hence, the correct option is (c).

3. All members in the rigid-jointed frame shown are prismatic and have the same flexural stiffness EI. Find the magnitude of the bending moment at Q (in kNm) due to the given loading. _______. **[2013]**



Solution: 25



The distribution factors at joint T are computed as

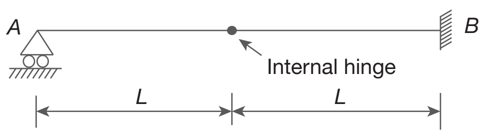
Joint	Member	Relative stiffness	Total relative stiffness	DF
T	TP	$\frac{I}{3}\cdot\frac{3}{4}=\frac{I}{4}$		$\frac{I}{4}/I=0.25$
	TQ	$\frac{I}{2}$	$\frac{I}{4}+\frac{I}{2}+\frac{I}{4}=I$	$\frac{I}{2}/I=0.50$
	TR	$\frac{I}{4}$		$\frac{I}{4}/I=0.25$
	TS	0		

$$M_{TQ} = 0.5 \times 100 = 50\,\text{kNm}$$

$$M_{QT} = \frac{1}{2} \times 50 = 25\,\text{kNm}$$

Hence, the answer is 25.

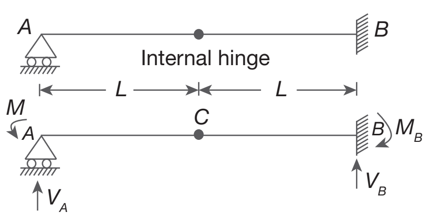
4. Carry-over factor C_{AB} for the beam shown in the figure below is **[2006]**



(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) 1

Solution: (d)



Applying moment M at A,

Taking moments of all forces to the left of C about C,

$$\sum M_c = 0 \Rightarrow -M + V_A L = 0;\ V_A = \frac{M}{L}(\uparrow)$$

$$\sum V = 0 \Rightarrow V_A + V_B = 0;\quad V_B = \frac{-M}{L} = \frac{M}{L}(\downarrow)$$

Taking moments of all forces to the right of C about C,

$$M_B + \frac{M}{L}L = 0;\ M_B = -M = M(\uparrow)$$

$$\text{Carry Over Factor, } C_{AB} = \frac{\text{Carry over moment to } B}{\text{Applied moment at } A}$$

$$= \frac{M}{M} = 1$$

Hence, the correct option is (d).

5. Match the following: **[2005]**

Group 1	Group 2
P. Slope deflection method	1. Force method
Q. Moment distribution method	2. Displacement method
R. Method of three moments	
S. Castigliano's second theorem	

(a) P: 1; Q: 2; R: 1; S: 2
(b) P: 1; Q: 1; R: 2; S: 2
(c) P: 2; Q: 2; R: 1; S: 1
(d) P: 2; Q: 1; R: 2; S: 1

Solution: (c)

Force methods are

i. Method of three moments
ii. Castigliano's second theorem (Energy methods)
iii. Column analogy method
iv. Flexibility method

Displacement methods are

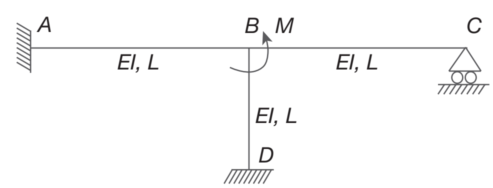
i. Slope deflection method
ii. Moment distribution method
iii. Kani's method
iv. Stiffness method

Hence, the correct option is (c).

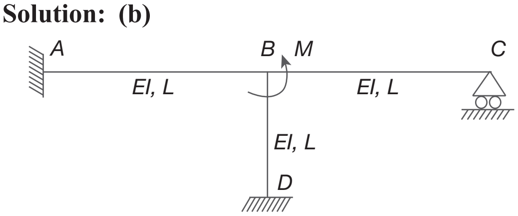
6. All members of the frame shown below have the same flexural rigidity EI and length L. if a moment M is applied at joint B, the rotation of the joint is **[2005]**

(a) $\frac{ML}{12EI}$ (b) $\frac{ML}{11EI}$

(c) $\frac{ML}{8EI}$ (d) $\frac{ML}{7EI}$



Solution: (b)



The moment M is distributed to three member meeting at joint B according to flexural rigidity, length and end condition.

The rotation 'θ' of all members meeting at joint B is same.

$$M = \frac{4EI}{L}\theta + \frac{4EI}{L}\theta + \frac{3EI}{L}\theta = \frac{11EI}{L}\theta$$

$$\text{Therefore, } \theta = \frac{ML}{11EI}$$

Hence, the correct option is (b).

7. For a linear elastic structural system, minimization of potential energy yields **[2004]**

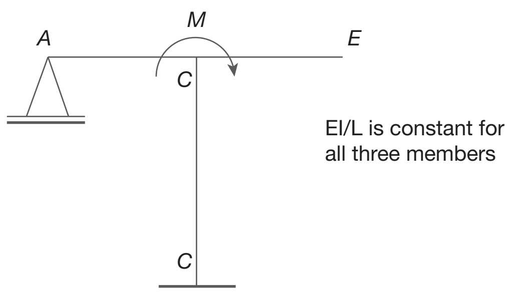
(a) compatibility conditions
(b) constitutive relations
(c) equilibrium equations
(d) strain-displacement relations

Solution: (a)

Compatibility conditions deals with balancing of displacements and minimization of potential energy.

Hence, the correct option is (a).

8. The frame below shows three beam elements *OA*, *OB* and *OC*, with identical length *L* and flexural rigidly *EI*, subject to an external moment *M* applied at the rigid joint *O*. The correct set of bending moments [M_{OA}, M_{OB}, M_{OC}] that develop at *O* in the three beam elements *OA*, *OB* and *OC* respectively, is **[2001]**



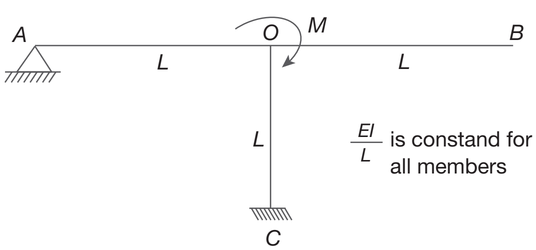
(a) $[3M/8, M/8, 4M/8]$

(b) $[3M/11, 4M/11, 4M/11]$

(c) $[M/3, M/3, M/3]$

(d) $[3M/7, 0, 4M/7]$

Solution: (d)



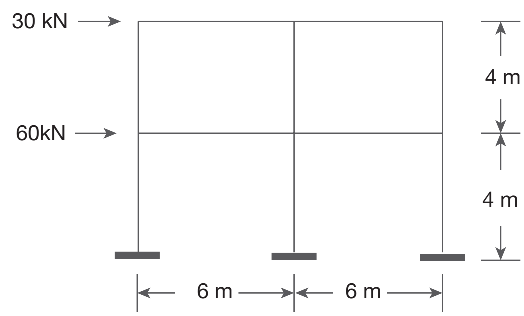
The joint moment is distributed among the members meeting at that joint is proportional to their distribution factors.

Member	Relative stiffness	Total relative stiffness	Distribution factor (D.F)
OA	$\frac{3}{4}\frac{EI}{L}$	$\frac{7}{4}\frac{EI}{L}$	$\left(\frac{3}{4}\right)/\left(\frac{7}{4}\right)=\left(\frac{3}{7}\right)$
OB	0		0
OC	$\frac{EI}{L}$		$1/\left(\frac{7}{4}\right)=\frac{4}{7}$

$$M_{OA}=\frac{3}{7}M\ ;\ M_{OB}=0\ ;\ M_{OC}=\frac{4}{7}M$$

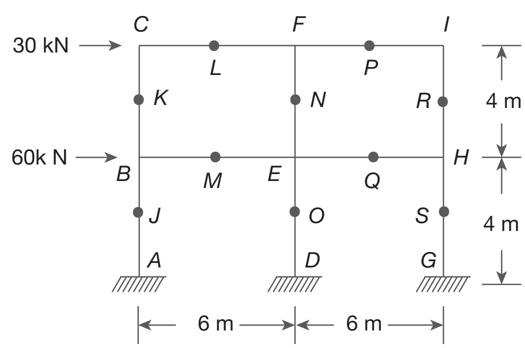
Hence, the correct option is (d).

9. The end moment (in kNm units) developed in the roof level beams in the laterally loaded frame shown below (with all columns having identical cross-sections), according to the Cantilever Method of simplified analysis, is **[2001]**



(a) 7.5
(b) 15
(c) 20
(d) 30

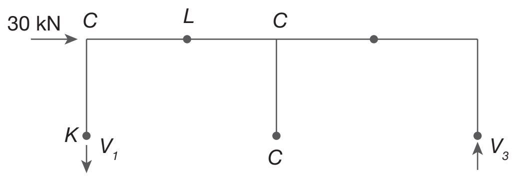
Solution: (b)



According to cantilever method, the point of contraflexure in each member lies at its mid span or mid height.

The axial stresses in the columns are directly proportional to their distance from the centroidal vertical axis of the frame.

Let V_1, V_2 and V_3 be the axial forces in columns 1, 2 and 3 respectively.



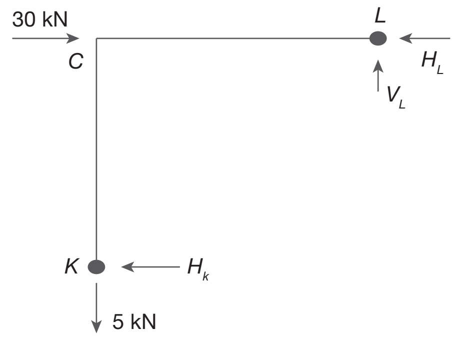
Since the frame is symmetrical, centre of gravity of column areas pass through column 2.

$$\frac{V_1}{6} = \frac{V_2}{0} = \frac{V_3}{6} \Rightarrow V_3 = V_1,\ V_2 = 0$$

Taking moments of all forces about hinge K,

$$30 \times 2 - V_3 \times 12 = 0 \Rightarrow V_3 = 5\text{ kN}(\uparrow)$$

$$V_1 = 5\text{ kN}(\downarrow)$$



$$\sum M_L = 0 \Rightarrow -5 \times 3 + H_K \times 2 = 0 \Rightarrow H_K = 70\text{ kN}$$

$$\sum H = 0 \Rightarrow H_L = 30 - 7.5 = 22.5\text{ kN}$$

$$\sum V = 0 \Rightarrow V_L = 5\text{ kN}(\uparrow)$$

BM at $C = 5 \times 3 = 15$ kNm

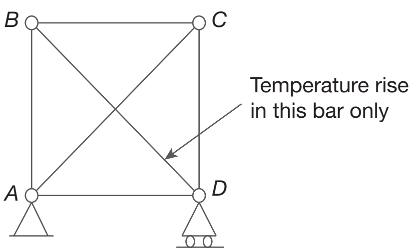
Hence, the correct option is (b).

Chapter 5

Energy Principles

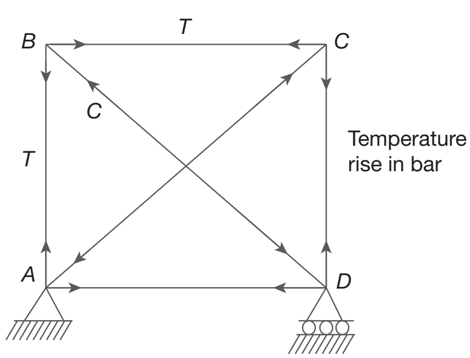
ONE-MARK QUESTIONS

1. Identify that FALSE statement from the following, pertaining to the effects due to a temperature rise ΔT in the bar BD alone in the plane truss shown below: **[2001]**



(a) No reactions develop at supports A and D.
(b) The bar BD will be subject to a tensile force.
(c) The bar AC will be subject to a compressive force.
(d) The bar BC will be subject to a tensile force.

Solution: (b)



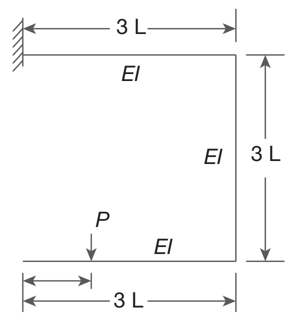
Due to rise of temperature in bar BD, the bar will tend to elongate. But the joints B and D will offer resistance in preventing the expansion of bar BD. Therefore, the bar BD is subjected to compressive force.

The forces in the members of the truss are as shown in the figure.

No reactions induced at supports A and D of the truss. Bar AC is subjected to a compressive force and bar BC is subjected to tensile force.

Hence, the correct option is (b).

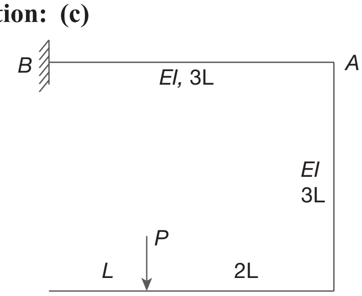
2. For the structure shown below, the vertical deflection at point A is given by **[2000]**



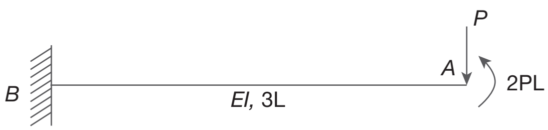
(a) $\dfrac{PL^3}{81EI}$ (b) $\dfrac{2PL^3}{81EI}$

(c) Zero (d) $\dfrac{PL^3}{72EI}$

Solution: (c)



Consider the free body diagram for member *AB*,



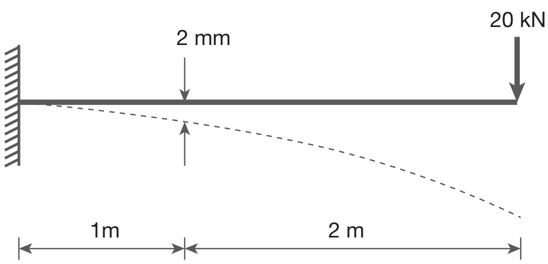
Deflection at *A*,

$$\delta_A = \frac{P(3L)^3}{3EI} - \frac{2PL.(3L)^2}{2EI} = \frac{9PL^3}{EI} - \frac{9PL^3}{EI} = 0$$

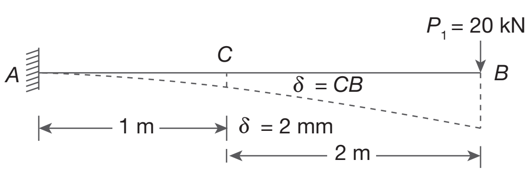
Hence, the correct option is (c).

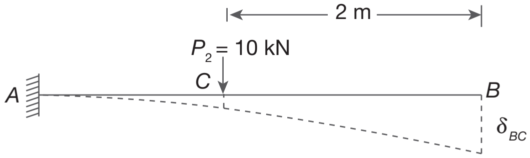
Two-marks Questions

1. For a cantilever beam of a span 3 m (shown below), a concentrated load of 20 kN applied at the free end causes a vertical displacement of 2 mm at a section located at a distance of 1 m from the fixed end. If a concentrated vertically downward load of 10 kN is applied at the section located at a distance of 1 m from the fixed end (with no other load on the beam), the maximum vertical displacement in the same beam (in mm) is ________. **[2014]**



Solution: 1



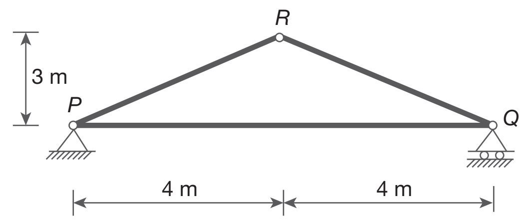


According to Maxwell-Betti's theorem, for a linearly elastic structure in equilibrium subjected to two systems of forces, the virtual workdone by first system of forces through the displacements caused by the second system of forces is equal to the virtual workdone by the second system of forces through the displacements caused by the first system of forces.

$$P_1\delta_{BC} = P_2\delta_{CB}$$
$$20\times\delta_{BC} = 10\times 2$$
$$\delta_{BC} = 1 \text{ mm}$$

Hence, the answer is 1.

2. For the truss shown below, the member PQ is short by 3 mm. The magnitude of vertical displacement of joint *R* (in mm) is _________. **[2014]**

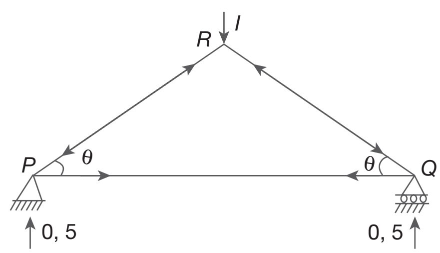


Solution: 2

Vertical displacement of joint *R*, $\delta_{VR} = \Sigma\frac{PkL}{AE}$

$$\delta_{VR} = \Sigma uk$$

u: Displacement of the member
k: Force in the member due to unit vertical force at *R*



$$\text{Tan}\,\theta = \frac{3}{4},\ \sin\theta = \frac{3}{5},\ \cos\theta = \frac{4}{5}$$

At joint *P*:

$$\Sigma V = 0 \Rightarrow 0.5 - k_{PR}.\sin\theta = 0$$

$$k_{PR} = 0.5\times\frac{5}{3} = 0.833(C)$$

$$\Sigma H = 0 \Rightarrow -0.833\cos\theta + k_{PQ} = 0$$

$$k_{PQ} = 0.833\times\frac{4}{5} = 0.667(T)$$

$$k_{QR} = 0.833(C)$$

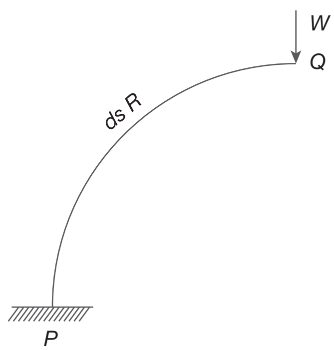
$$\delta_{VR} = (-3)\times 0.667 + 0\times(-0.833) + 0\times(-0.833)$$

$$= -2\,\text{mm} = 2\,\text{mm}\,(\text{upwards})$$

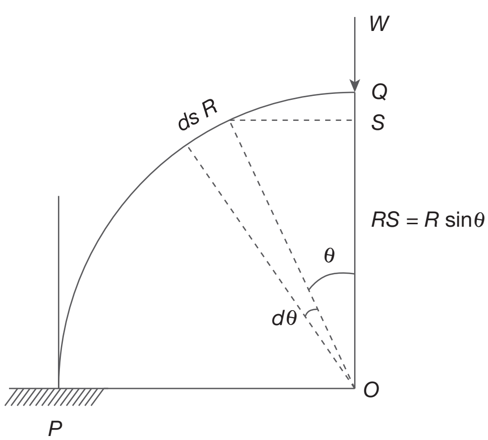
Hence, the answer is 2.

3. A uniform beam (EI = constant) PQ in the form of a quarter-circle of radius R is fixed at end P and free at the end Q, where a load W is applied as shown. The vertical downward displacement, σ_q at the loaded point Q is given by: **[2013]**

$\delta_q = \beta\left(\dfrac{WR^3}{EI}\right)$. Find the value of β (correct to 4-decimal places).



Solution: 0.7854



Vertically deflection at Q, $\delta_q = \dfrac{\int Mm\,dx}{EI}$

Let us consider a small element of curved length ds subtending $d\theta$ at the centre, which is at an angular distance of θ from the line OQ.
O being the centre of the quadrant of the circle

R: Radius of the circular curve

$$M = -WR\sin;\ m = -R\,\text{Sin}\theta$$

$$ds = Rd\theta$$

$$\delta_q = \frac{1}{EI}\int_0^{\pi/2}(-WR\sin\theta)(-R\sin\theta)Rd\theta$$

$$= \frac{WR^3}{EI}\int_0^{\pi/2}\sin^2\theta d\theta = \frac{WR^3}{EI}\int_0^{\pi/2}\frac{1-\cos 2\theta}{2}d\theta$$

$$= \frac{WR^3}{2EI}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

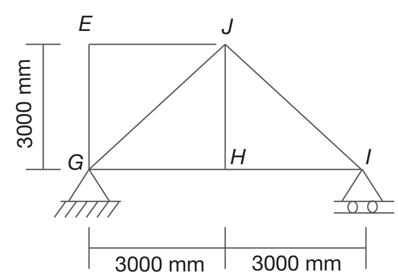
$$= \frac{WR^3}{2EI}\left[\frac{\pi}{2} - 0\right] = \pi\frac{WR^3}{4EI}$$

$$\delta_V = \frac{\pi}{4}\frac{WR^3}{EI}$$

$$\beta = \frac{\pi}{4} = 0.7854$$

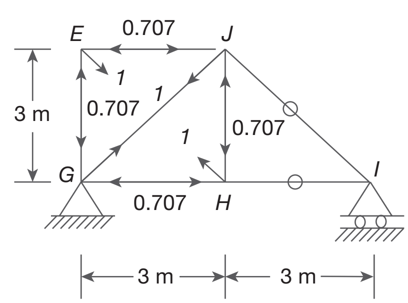
Hence, the answer is 0.7854.

4. The members EJ and IJ of a steel truss shown in the figure below are subjected to a temperature rise of 30°C. The coefficient of thermal expansion of steel is 0.000012 per °C per unit length. The displacement (mm) of joint E relative to joint H along the direction HE of the truss, is **[2008]**



(a) 0.255 (b) 0.589
(c) 0.764 (d) 1.026

Solution: (c)



Length of diagonal member = $\sqrt{3^2+3^2} = 4.243$ m

To find the relative displacement of joints E and H, apply unit loads at each joint as shown in figure.

Member	Temp. Rise, T	$\delta L = L\alpha T$	k	$k\delta L$
EJ	30	1.08	–0.707	–0.764
EG	0	0	–0.707	0
GH	0	0	–0.707	0
HJ	0	0	–0.707	0
HI	0	0	0	0
GJ	0	0	1.0	0
JI	30	1.53	0	0
			$\Sigma k\delta L$	–0.764

Coefficient of thermal expansion of steel, $\alpha = 12 \times 10^{-6}$ per °C per unit length.

For EJ, $L\alpha T = 3 \times 12 \times 10^{-6} \times 30 \times 10^3 = 1.08$ mm

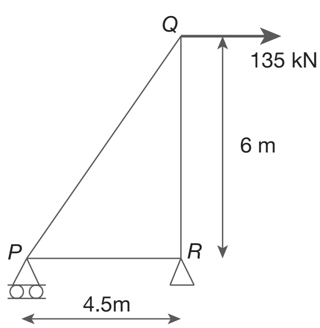
For JI, $L\alpha T = 4.243 \times 12 \times 10^{-6} \times 30 \times 10^3 = 1.53$ mm

$$\Sigma k.\delta.L = -0.764$$

–ve sign indicates the points H and E move away from each other. The displacement of joint E relative to joint H along the direction 0.764 m.

Hence, the correct option is (c).

5. The right triangular truss is made of members having equal cross sectional area of 1550 mm^2 and Young's modulus of 2×10^5 MPa. The horizontal deflection of the joint Q is **[2007]**

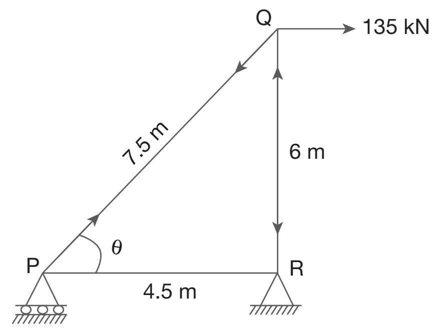


(a) 2.47 mm (b) 10.25 mm
(c) 14.31 mm (d) 15.68 mm

Solution: (d)

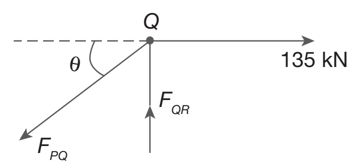
$$PQ = \sqrt{6^2 + 4.5^2} = 7.5 \text{ m}$$

$$\tan\theta = \frac{6}{4.5}; \theta = 53.13°$$



Joint Q,

$$\Sigma H = 0 \Rightarrow F_{PQ}\cos\theta = 135, F_{PQ} = 225 \text{ kN(T)}$$

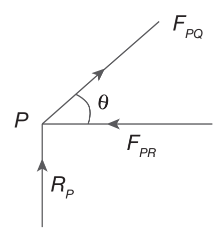


Joint Q

$$\Sigma V = 0 \Rightarrow F_{QR} - F_{PQ}.\sin\theta = 0,$$
$$F_{QR} = 225\sin\theta = 180 \text{ kN(C)}$$

Joint P, $\Sigma H = 0 \Rightarrow F_{PR} = F_{PQ}\cos\theta$

$$= 225\cos\theta = 135 \text{ kN(C)}$$



Joint P

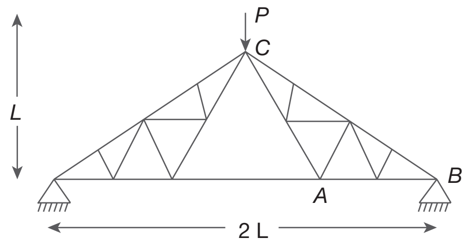
Member	P	k	L	PkL
PQ	225	1.67	7.5	2818.1
QR	–180	–1.33	6	1436.4
RP	–135	–1	4.5	607.5
				4862.0

$$\delta_Q = \Sigma\frac{Pkl}{AE} = \frac{4862 \times 10^6}{1550 \times 2 \times 10^5} = 15.68 \text{ mm}$$

Hence, the correct option is (d).

Statement for Linked Questions 6 and 7:

A truss is shown in the figure. Members are to equal cross section A and same modulus of elasticity E. A vertical force P is applied at point C. **[2005]**

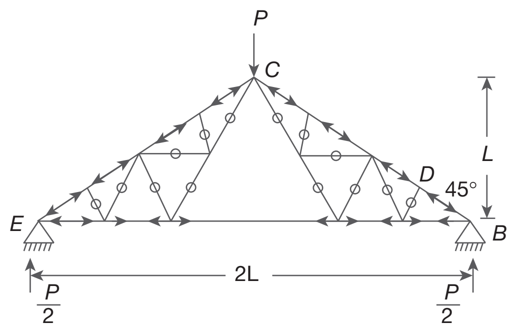


6. Force in the member *AB* of the truss is

(a) $P/\sqrt{2}$ (b) $P/\sqrt{3}$

(c) $P/2$ (d) P

Solution: (c)



Considering the vertical equilibrium of the joint *B*,

$$\frac{P}{2} - P_{BD}\sin 45° = 0;\ P_{BD} = \frac{P}{\sqrt{2}}$$

Considering the horizontal equilibrium of the joint *B*,

$$P_{AB} = \frac{P}{\sqrt{2}}\cos 45° = \frac{P}{2}$$

Hence, the correct option is (c).

7. Deflection of the point *C* is

(a) $\frac{(2\sqrt{2}+1)}{2}\frac{PL}{EA}$ (b) $\sqrt{2}\frac{PL}{EA}$

(c) $(2\sqrt{2}+1)\frac{PL}{EA}$ (d) $(\sqrt{2}+1)\frac{PL}{EA}$

Solution: (a)

The computations for deflection at C by unit load method are shown in table.

Member	A	L	P	k	PkL
EC	A	$\sqrt{2}L$	$\frac{P}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{PL}{\sqrt{2}}$
BC	A	$\sqrt{2}L$	$\frac{P}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{PL}{\sqrt{2}}$
EB	A	2 L	$-\frac{P}{2}$	$-\frac{1}{2}$	$\frac{PL}{2}$

Vertical deflection at *C*,

$$\delta_{VC} = \sum\frac{PkL}{AE} = \left(\frac{2PL}{\sqrt{2}} + \frac{PL}{2}\right)\frac{1}{AE} = \frac{2\sqrt{2}+1}{2}\frac{PL}{AE}$$

Hence, the correct option is (a).

8. The unit load method used in structural analysis is **[2004]**

(a) applicable only to statistically indeterminate structures

(b) another name for stiffness method

(c) an extension of Maxwell's reciprocal theorem

(d) derived from Castigliano's theorem

Solution: (d)

Unit load method used in structural analysis is derived from Castigliano's theorem.

It is applicable to determine the deflection and slope of a statically determinate structures and the analysis of statically indeterminate structures.

$$\delta = \frac{\partial U}{\partial P} = \int M\frac{\partial M}{\partial P}\frac{dx}{EI} = \int\frac{M.m}{EI}dx$$

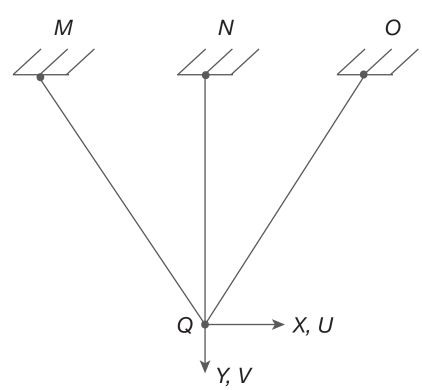
Hence, the correct option is (d).

9. In a redundant joint model, three bar members are pin connected at *Q* as shown in the figure. Under some load placed at *Q*, the elongation of the members *MQ* and *OQ* are found to be 48 mm and 35 mm respectively. Then the horizontal displacement *u* and the vertical displacement *v* of the node *Q*, in mm, will be respectively. **[2003]**

$MN = 400$ mm

$NO = 500$ mm

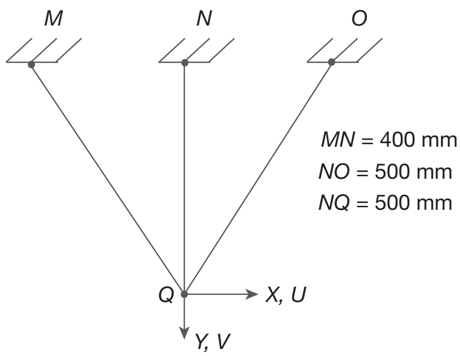
$NQ = 500$ mm



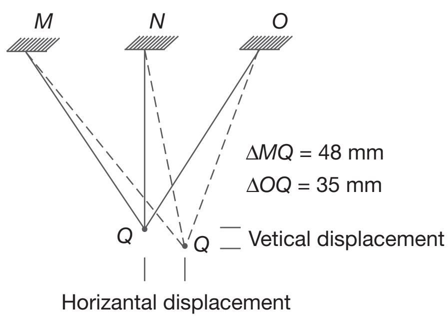
(a) −6.64 and 56.14 (b) 6.64 and 56.14

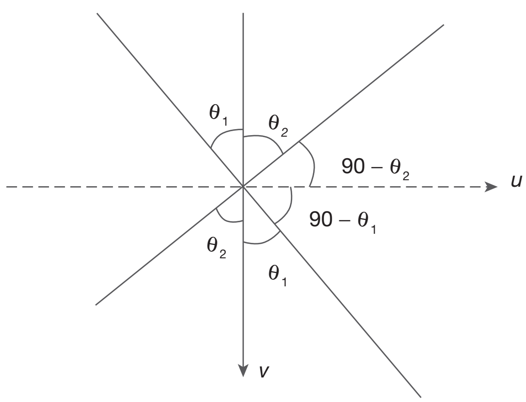
(c) 0.0 and 59.41 (d) 59.41 and 0.0

Solution: (b)



The position of members after displacements is shown below.





$$\tan\theta_1 = \frac{400}{500} = 0.8;\ \sin\theta_1 = 0.625,\ \cos\theta_1 = 0.781$$

$$\tan\theta_2 = \frac{500}{500} = 1;\ \sin\theta_2 = 0.707,\ \cos\theta_2 = 0.707$$

The elongation of member MQ in terms of u and v will be

$$u\sin\theta_1 + v\cos\theta_1 = 48$$

$$0.625u + 0.781v = 48 \qquad (1)$$

The elongation of member OQ in terms of u and v will be

$$-u\sin\theta_2 + v\cos\theta_2 = 35$$

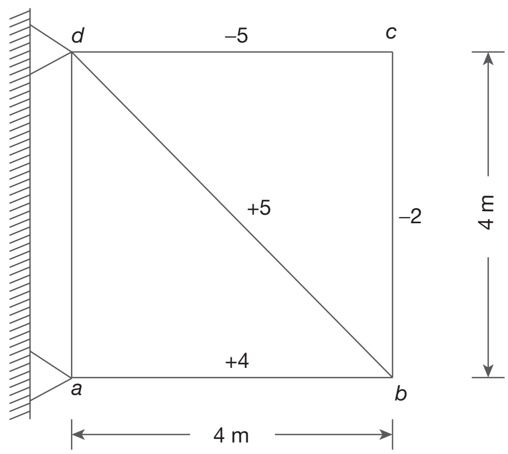
$$-0.707u + 0.707v = 35 \qquad (2)$$

Solving (1) and (2)

$$u = 6.64\text{ mm} \qquad v = 56.14\text{ mm}$$

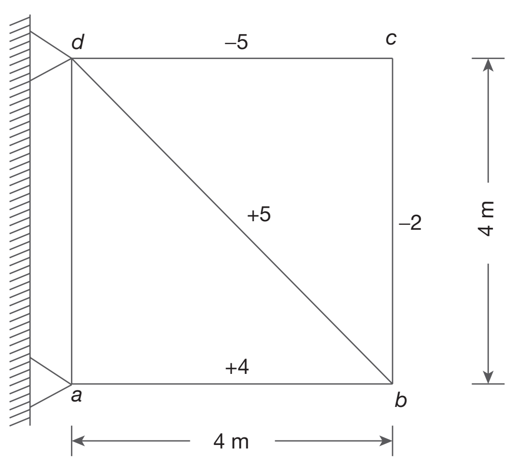
Hence, the correct option is (b).

10. If the deformation of the truss members are as shown in parantheses, the rotation of the member '*bd*' is **[1996]**



(a) 0.5×10^{-3} radian (b) 1.0×10^{-2} radian
(c) 1.5×10^{-2} radian (d) 2.0×10^{-2} radian

Solution: (b)



Vertical displacement of joint B, $\delta_{vb} = \Sigma\left(\frac{PkL}{AE}\right)$

$$= \Sigma k\Delta$$

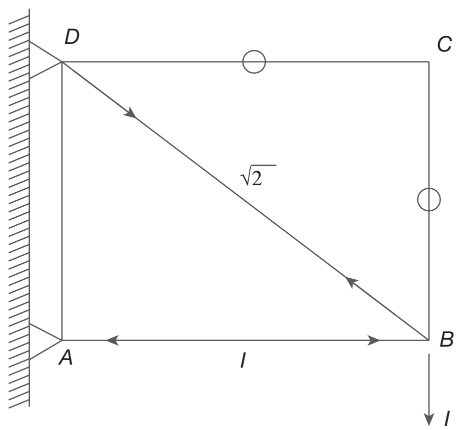
Horizontal displacement of joint B, $\delta_{hB} = \Sigma\left(\frac{Pk'L}{AE}\right)$

$$= \Sigma k'\Delta$$

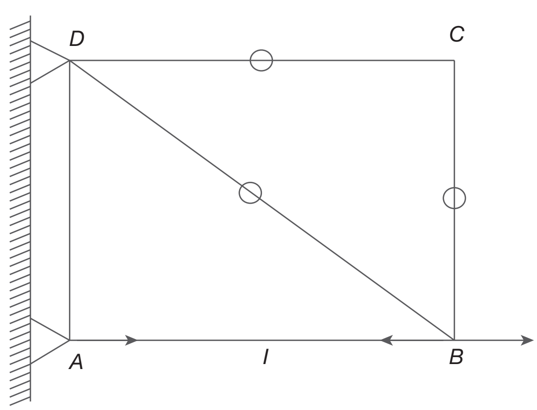
k: Force in a member due to unit vertical force at B
k': Force in a member due to unit horizontal force at B

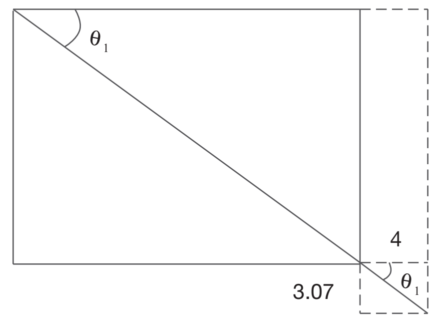
Member	Δ	k	k'	k.Δ	k'Δ
AB	4	–1	1	–4	4
BC	–2	0	0	0	0
CD	–5	0	0	0	0
AD	0	0	0	0	0
BD	5	$\sqrt{2}$	0	7.07	0
				3.07	4

Forces in the members of the truss due to unit vertical force at B



Forces in the members of the truss due to unit horizontal force at B





$$\text{Tan}\,\theta_1 = \frac{4003.07}{4004}$$

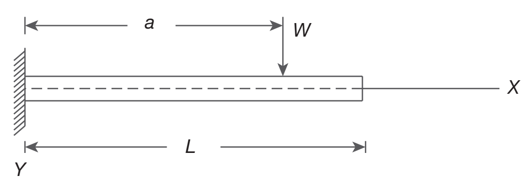
$$\theta_1 = 44.993°$$

Rotation of the member 'bd' = 45 – 44.993 = 0.007°

$$= 0.007 \times \frac{\pi}{180} = 1.22 \times 10^{-4} \text{ radians}$$

Hence, the correct option is (b).

11. A cantilever beam of span L is subjected to a load W at a distance 'a' from support. It is desired to obtain the vertical displacement at the free end by unit load method. The expression for deflection is **[1992]**



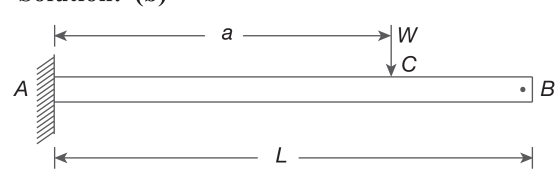
(a) $y = \int_0^a \frac{W(a-x)(a-x)}{EI}dx$

(b) $y = \int_0^a \frac{W(a-x)(L-x)}{EI}dx$

(c) $y = \int_0^a \frac{W(x-a)(L-x)}{EI}dx$

(d) $y = \int_0^a \frac{W(L-x)(L-x)}{EI}dx$

Solution: (b)



Deflection at a point in a beam using unit load method is given by

$$\delta = \int \frac{Mm}{EI} dx$$

δ: Vertical deflection at the free end = y

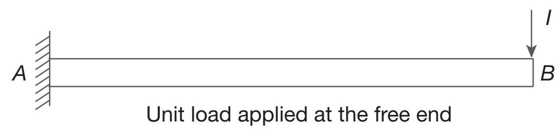
M: Bending moment at a distance x from fixed end due to applied load.

= 0 between B and C.

= $-W(a - x)$ between C and A.

m: Bending moment at a distance x from fixed end due to unit load applied at B.

= $-(L - x)$ between B and A



Unit load applied at the free end

$$y = 0 + \int_0^L \frac{W(a-x)(L-x)}{EI} dx$$

$$= \int_0^L \frac{W(a-x)(L-x)}{EI} dx$$

Hence, the correct option is (b).

Chapter 6

Influence Lines

ONE-MARK QUESTIONS

1. Muller Breslau principle in structural analysis is used for **[2003]**

(a) drawing influence line diagram for any force function
(b) writing virtual work equation
(c) super-position of load effects
(d) none of these

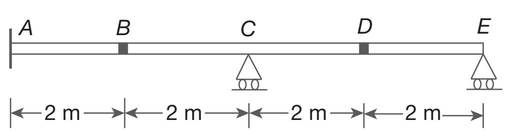
Solution: (a)

Muller Breslau principle in structural analysis is used for drawing influence line diagram for any force function.

According to Muller Breslau principle, the influence line for any stress function of a structure, such as *SF*, *BM* or any reactive force or moment is given by imposing a unit distortion in the direction of the stress function.

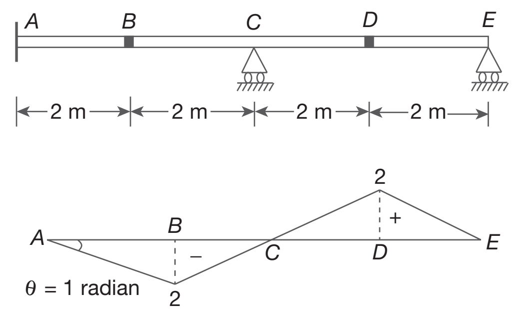
Hence, the correct option is (a).

2. Identify, from the following, the correct value of the bending moment M_A (in kNm units) at the fixed end *A* in the statically determinate beam shown below (with internal hinges at *B* and *D*), when a uniformly distributed load of 10 kN/m is placed on all spans. (Hint: Sketching the influence line for M_A or applying the Principle of Virtual Displacements makes the solution easy) **[2001]**



(a) –80 (b) –40
(c) 0 (d) +40

Solution: (c)



The influence line for *BM* at *A* will be obtained on releasing the moment at *A* by providing an artificial hinge and rotate through 1 radian clockwise. The resulting shape of the beam is the influence line for bending moment at *A*.

When a udl is acting over the entire span,

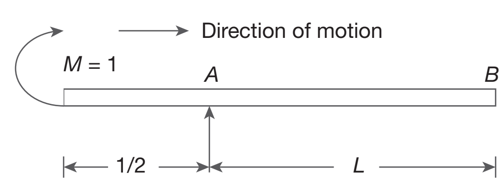
Bending moment at *A* = Area of ILD × intensity of load

$$= \left[-\frac{1}{2}\times 4\times 2+\frac{1}{2}\times 4\times 2\right]\times 10 = 0\times 10 = 0$$

Therefore, bending moment at $A = 0$

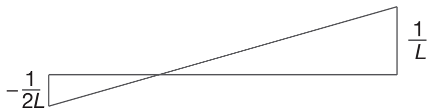
Hence, the correct option is (c).

3. A simply supported beam with an overhang is traversed by a unit concentrated moment from the left to the right as shown below: **[2000]**



The influence line for reaction at B is given by

(a)

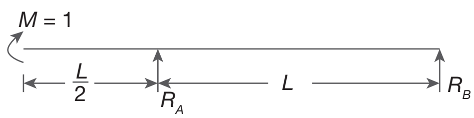


(b)

(c)

(d) zero every where

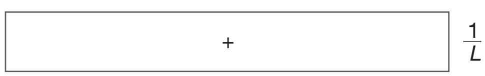
Solution: (c)



Taking moments of all forces about the support A,

$$\Sigma M_A = 0 \Rightarrow 1 - R_B.L = 0 \Rightarrow R_B = \frac{1}{L}$$

Irrespective of the position of concentrated Moment, the reaction of support B is same.



IL for reaction at B

Hence, the correct option is (c).

4. Influence line for redundant structures can be obtained by **[1994]**

(a) Castigliano's theorem
(b) Muller Breslau principle
(c) Unit load theorem
(d) Maxwell-Betti's reciprocal theorem

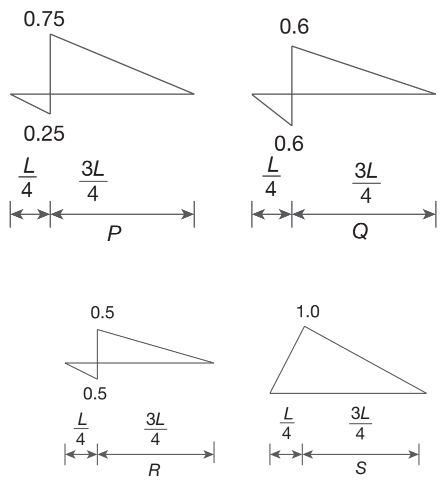
Solution: (b)

Muller Breslau principle is used to determine the influence line diagrams for arious structural parameters of statically indeterminate structures. Castigliano's theorem, Unit load theorem, Maxwell's- Betti's reciprocal theorem are used for determining the deflection in the structure.

Hence, the correct option is (b).

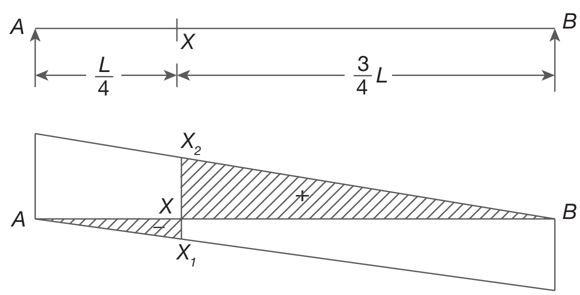
Two-marks Questions

1. In a beam of length L, four possible influence line diagrams for shear force at a section located at a distance of $\frac{L}{4}$ from the left end support (marked as P, Q, R and S) are shown below. The correct influence line diagram is **[2014]**



(a) P (b) Q
(c) R (d) S

Solution: (a)



The influence line for shear force at X is a resulting diagram obtained by releasing the shear force by cutting the beam at X and keeping them at a unit distance such that the two members on either side of X are parallel to each other.

The shaded portion of the above diagram is the influence line for shear force at X.

Hence, the correct option is (a).

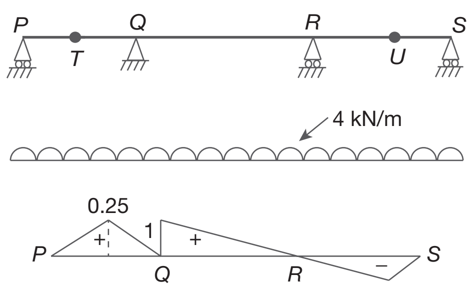
2. Beam $PQRS$ has internal hinges in spans PQ and RS as shown. The beam may be subjected to a moving distributed vertical load of maximum intensity 4 kN/m of any length any where on the beam. The maximum absolute value of the shear force (in kN) that can occur due to this loading just to the right of support Q shall be **[2013]**

(a) 30 (b) 40
(c) 45 (d) 55

Solution: (c)

The influence line for shear force to the right of support Q is obtained by cutting the section to the right of Q and lift the point Q by 1 unit in beam QS, and also lift the hinge at T such that TQ and QR are parallel.

The absowwlute maximum value of the shear force at Q will be obtained by placing the load between P and R.

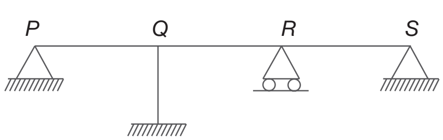


IL for shear to the right 4Q

$$V_Q = \left[\frac{1}{2}\times 10\times 0.25+\frac{1}{2}\times 20\times 1\right]4$$
$$= 11.25\times 4 = 45 \text{ kN}$$

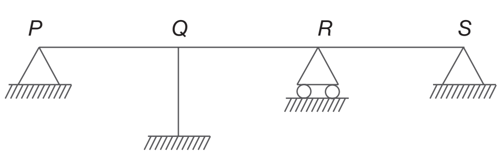
Hence, the correct option is (c).

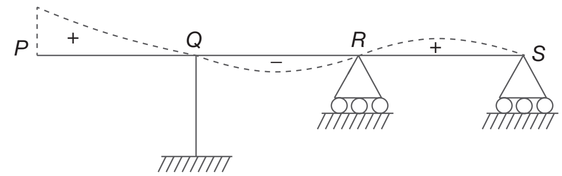
3. The span(s) to be loaded uniformly for maximum positive (upward) reaction at support P, as shown in the figure below, is (are) **[2008]**



(a) PQ only (b) PQ and QR
(c) QR and RS (d) PQ and RS

Solution: (d)

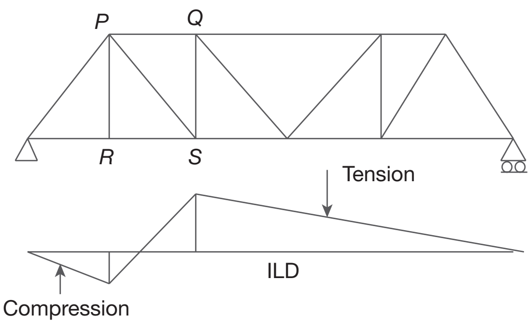




According to Muller-Breslau principle, the influence line for reaction at P is obtained by removing the support at P and apply unit load in the direction of the reaction and the corresponding shape of deflected curve is the shape of IL for reaction at A. When the positive influence line diagram is loaded, then the reaction will be maximum. Hence the spans PQ and RS should be loaded uniformly for maximum positive reaction at P.

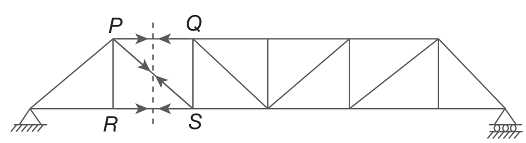
Hence, the correct option is (d).

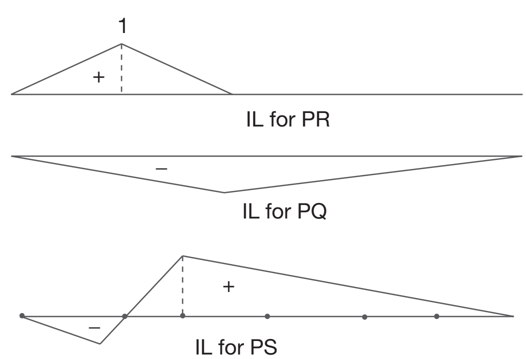
4. The influence line diagram (ILD) shown is for the member is **[2007]**

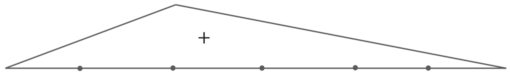


(a) PS (b) RS
(c) PQ (d) QS

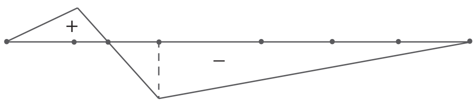
Solution: (a)







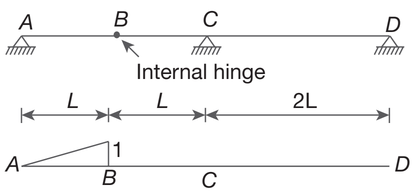
IL for RS



IL for QS

Hence, the correct option is (a).

5. Consider the beam ABCD and the influence line as shown below. The influence line pertains to **[2006]**



(a) reaction at A, R_A

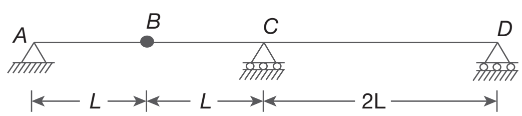
(b) shear force at B, V_B

(c) shear force on the left of C, V_C^-

(d) shear force on the right of C, V_C^+

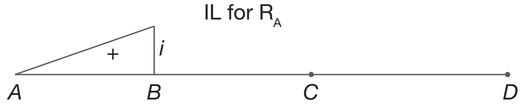
Solution: (b)

The influence line diagrams for various structural parameters in the given options are shown in figures.

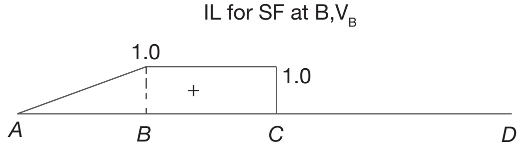


IL for R_A

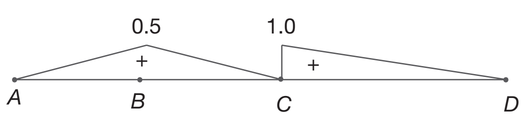
IL for R_A



IL for SF at B,V_B



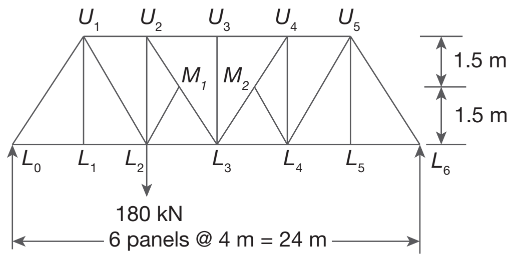
IL for SF on the left at C



IL for SF on the left at C

Hence, the correct option is (b).

6. A truss, as shown in figure, is carrying 180 kN load at node L_2. The force in the diagonal member M_2U_4 will be **[2003]**



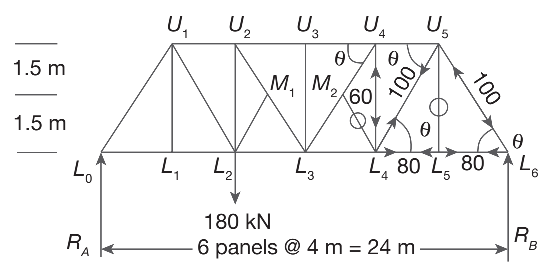
(a) 100 kN tension

(b) 100 kN compression

(c) 80 kN tension

(d) 80 kN compression

Solution: (a)

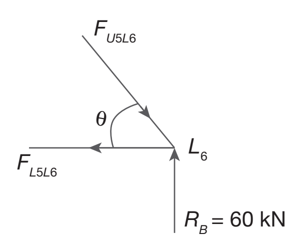


Taking moments of all forces about left support,

$$R_B \times 24 - 180 \times 8 = 0$$

$$R_B = 60 \text{ kN } R_A = 120 \text{ kN}$$

Joint L_6:



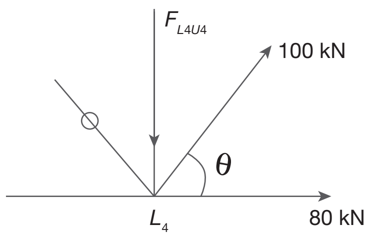
$$\Sigma V = 0 \Rightarrow 60 - F_{U_5L_6} \sin\theta = 0;$$

$$F_{U_5L_6} = \frac{60}{0.6} = 100 \text{ kN (C)}$$

$$\Sigma H = 0 \Rightarrow F_{U_5L_6} \cos\theta - F_{L_5L_6} = 0;$$

$$F_{L_5L_6} = 100 \times 0.8 = 80 \text{ kN (T)}$$

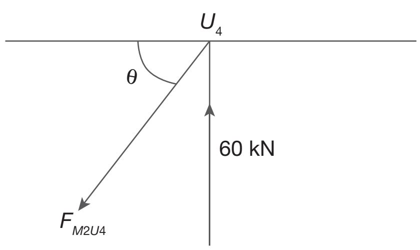
Joint L_4:



$$\Sigma V = 0 \Rightarrow 100\sin\theta - F_{L_4U_4} = 0;$$

$$F_{L_4U_4} = 100 \times 0.6 = 60\,\text{kN (C)}$$

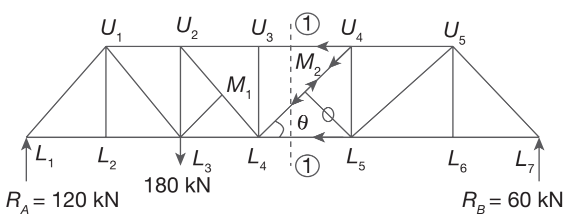
Joint U_4:



$$\Sigma V = 0 \Rightarrow 60 - F_{M_2U_4}.\sin\theta = 0;$$

$$F_{M_2U_4} = \frac{60}{0.6} = 100\,\text{kN (T)}$$

(OR)



Consider the right part of the section in equilibrium,

$$60 - F_{L_4M_2}\sin\theta = 0;\ F_{L_4M_2} = \frac{60}{0.6} = 100\,\text{kN};$$

$$F_{L_4M_2} = 100\,\text{kN(T)}$$

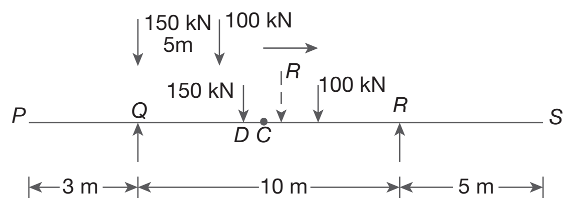
Hence, the correct option is (a).

Common Data for Questions 7 to 9:
A beam PQRS is 18 m long and is simply supported at points Q and R 10 m. Overhangs PQ and RS are 3 m and 5 m respectively. A train of two point loads of 150 kN and 100 kN, 5 m apart, crosses this beam from left to right with 100 kN load leading. **[2003]**

7. The maximum sagging moment under the 150 kN load anywhere is

(a) 500 kNm (b) 45 kNm
(c) 400 kNm (d) 375 kNm

Solution: (c)



Maximum sagging bending moment occurs under the load under consideration when the centre of the beam lies midway between the resultant and the load under consideration.

Maximum bending moment under 150 kN load occurs when 150 kN load and the resultant of loads are equidistant from the centre of span.

Let $\bar{x}$: Resultant of loads from 150 kN load

$$= \frac{150 \times 0 + 100 \times 5}{150 + 100} = 2\text{ m}$$

$$CD = 1\text{ m},\ QD = 4\text{ m},\ DR = 6\text{ m}.$$

Ordinate of *ILD* for *BM* under 150 kN load

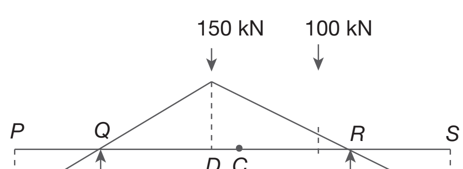
$$= \frac{QDDR}{QR}$$

$$= \frac{4 \times 6}{10} = 2.4\text{ m}$$

Ordinate of ILD under 100 KN load $= \frac{2.4}{6} \times 1$

$$= 0.4\text{ m}$$

Maximum sagging BM under 150 kN load = 2.4 × 150 + 0.4 × 100 = 400 kNm



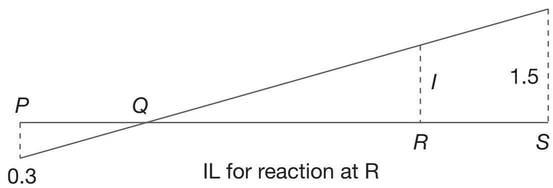
ILD for BM

Hence, the correct option is (c).

8. During the passage of the loads, the maximum and the minimum reactions at support R, in kN, are respectively **[2003]**

(a) 300 and –30 (b) 300 and –25
(c) 225 and –30 (d) 225 and –25

Solution: (a)



IL for reaction at R

The influence line diagram for reaction of support R is shown in fig.
Maximum reaction at R will occur when 100 kN load is at S.
Maximum reaction at $R = 100 \times 1.5 + 150 \times 1.0 = 300$ kN
Minimum reaction at R will occur when 100 kN load is at P.
Minimum reaction at $R = -100 \times 0.3 = -30$ kN
Hence, the correct option is (a).

9. The maximum hogging moment in the beam anywhere is **[2003]**

(a) 300 kNm (b) 450 kNm
(c) 500 kNm (d) 750 kNm

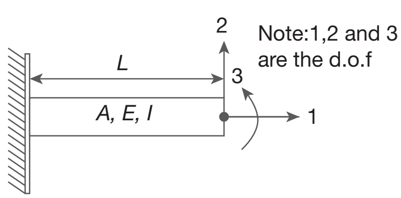
Solution: (c)
Maximum hogging moment in the beam occurs at either Q or R.
Maximum hogging BM at Q will occur when 150 KN load is at P.
Maximum BM at $Q = -150 \times 3 = -450$ kNm
Maximum hogging BM at R will occur when 100 KN load is at S.
Maximum BM at $R = -100 \times 5 = -500$ kNm
Hence, the maximum hogging moment anywhere in the beam is 500 kNm.
Hence, the correct option is (c).

Chapter 7

Arches and Cables

One-mark Questions

1. For the beam shown below, the stiffness coefficient K 22 can be written as **[2015]**



(a) $\frac{6EI}{L^2}$ (b) $\frac{12EI}{L^3}$

(c) $\frac{3EI}{L}$ (d) $\frac{EI}{6L^2}$

Solution: (b)

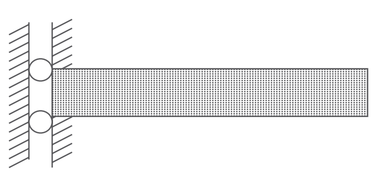
The stiffness coefficient K_{22} will be

$K_{22} = (6EI/L^2) + (6EI/L^2)/L$

$= 12EI/L3$

Hence, the correct option is (b).

2. A guided support as shown in the figure below is represented by three springs (horizontal, vertical and rotations) with stiffness, k_x, k_y and k_θ respectively. The limiting values of k_x, k_y and k_θ are: **[2015]**



(a) $\infty, 0, \infty$ (b) ∞, ∞, ∞

(c) $0, \infty, \infty$ (d) $\infty, \infty, 0$

Solution: (a)

Stiffness $K = \frac{\text{Force}}{\text{deflection}}$

$\because$ Restricted in x and rotational direction $\Rightarrow$ deflection in those directions = 0

$$\because \quad K_x = K_\theta = \frac{\text{Force}}{0} = \infty$$

In vertical direction,
As rollers are there, no force develops

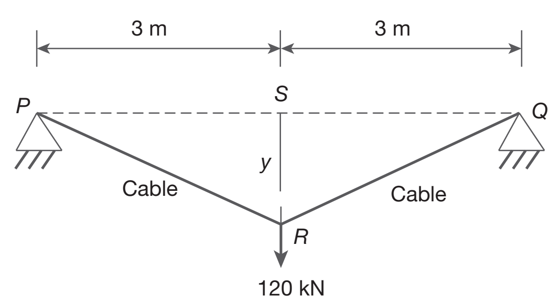
$$\because \quad K_y = \frac{0}{\text{deflection}} = 0$$

$$K_x, K_y, K_\theta = \infty, 0\ \infty$$

Hence, the correct option is (a).

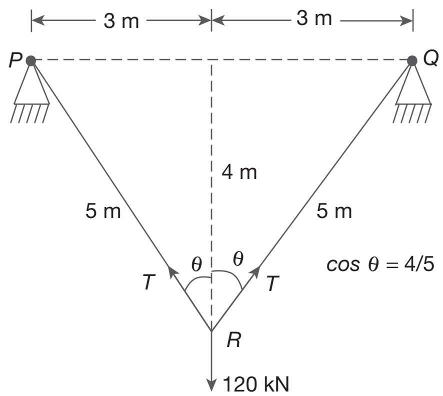
Two-marks Questions

1. The tension (in kN) in a 10 m long cable, shown in figure, neglecting its self weight is **[2014]**



(a) 120 (b) 75
(c) 60 (d) 45

Solution: (b)



Let T be the tension in the cable.
Considering the vertical equilibrium of all forces at joint R,

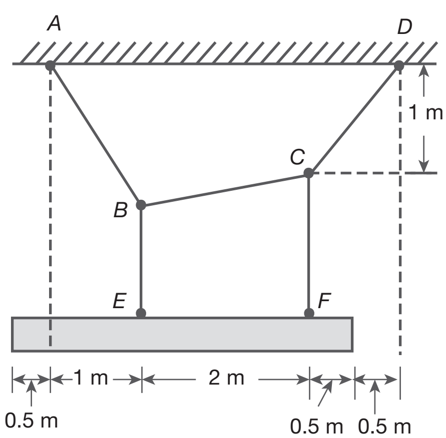
$$\Sigma V = 0 \Rightarrow 2T\cos\theta = 120$$

$$2T \times \frac{4}{5} = 120$$

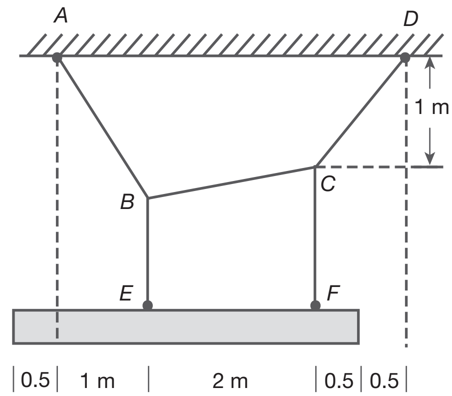
$$T = 75 \text{ kN}$$

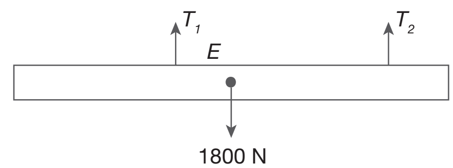
Hence, the correct option is (b).

2. A uniform beam weighing 1800 N is supported at E and F by cable ABCD. Determine the tension (in N) in segment AB of this cable (correct to 1-decimal place). Assume the cables ABCD, BE and CF to be weightless. **[2013]**



Solution: 1311.9





Forces active an bean

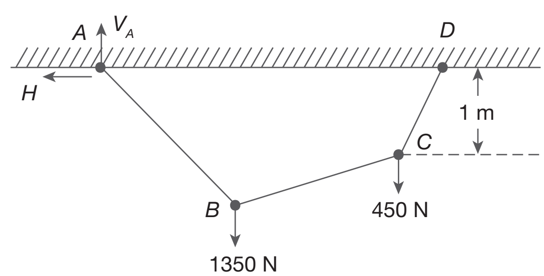
Taking moments of all forces about F,

$$\Sigma M_F \Rightarrow T_1 \times 2 = 1800 \times 1.5$$

$$T_1 = 1350 \text{ N}$$

$$\Sigma V = 0 \Rightarrow T_1 + T_2 = 1800$$

$$T_2 = 1800 - 1350 = 450 \text{ N}$$



The shape of the cable represents bending moment diagram to some scale.

$$\frac{y_B}{y_C} = \frac{1125 \times 1}{1125 \times 3 - 1350 \times 2}$$

$$y_B = \frac{1125}{675} \times 1 = 1.667 \text{ m}$$

$$\Sigma M_D = 0 \Rightarrow V_A \times 4 - 1350 \times 3 - 450 \times 1 = 0$$

$$V_A = 1125 \text{ kN}$$

$$\Sigma M_B = 0 \Rightarrow 1125 \times 1 - H \times 1.667 = 0$$
$$H = 674.86\ \text{N}$$

T: Tension in the cable AB

$$= \sqrt{V_A^2 + H^2} = \sqrt{1125^2 + 674.86^2} = 1311.9\ \text{N}$$

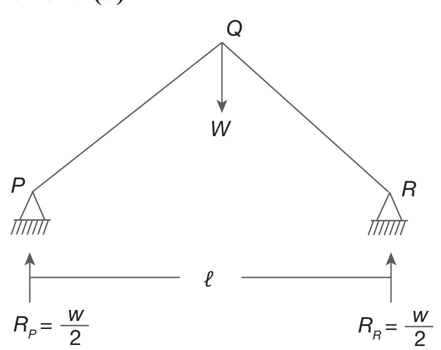
Hence, the answer is 1311.9.

3. A symmetric frame PQR consists of two inclined members PQ and QR, connected at 'Q' with a rigid joint, and hinged at 'P' and 'R'. The horizontal length PR is l. If a weight w is suspended at 'Q', the bending moment at 'Q' is **[2012]**

(a) $\frac{Wl}{2}$ (b) $\frac{Wl}{4}$

(c) $\frac{Wl}{8}$ (d) zero

Solution: (d)



The given frame can be treated as a linear arch. Linear arch is subjected to only axial forces and no shear force and bending moment. Hence, bending moment at every point including at Q is zero.

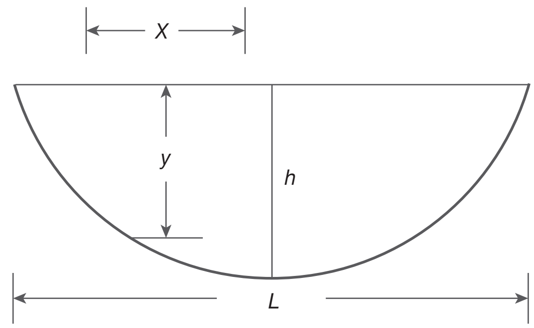
Hence, the correct option is (d).

4. A parabolic cable is held between two supports at the same level. The horizontal span between the supports is L. The sag at the mid-span is h. The equation of the parabola is $y = 4h\frac{x^2}{L^2}$, where x is the horizontal coordinate and y is the vertical coordinate with the origin at the centre of the cable. The expression for the total length of the cable is **[2010]**

(a) $\int_0^L \sqrt{1 + 64\frac{h^2x^2}{L^4}}dx$ (b) $2\int_0^{L/2} \sqrt{1 + 64\frac{h^3x^2}{L^4}}dx$

(d) $\int_0^{L/2} \sqrt{1 + 64\frac{h^2x^2}{L^4}}dx$ (d) $2\int_0^{L/2} \sqrt{1 + 64\frac{h^2x^2}{L^4}}dx$

Solution: (d)



Horizontal span between supports = L

Sag at the mid span = h

Equation of parabola, $y = 4h\frac{x^2}{L^2}$

x: Horizontal coordinate from the centre

y: Vertical coordinate from the centre.

Length of curve between $x = a$ and $x = b$ is given by

$$f(x) = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = 8h\frac{x}{L^2}$$

At $x = 0$, $y = 0$

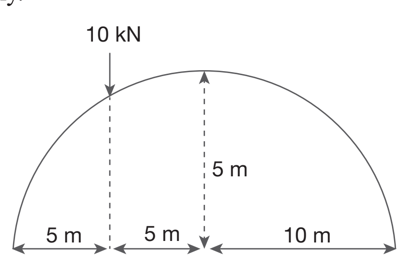
At $x = L/2$, $y = h$

$$f(x) = 2\int_0^{L/2} \sqrt{1 + \left(\frac{8hx}{L^2}\right)^2}\, dx$$

$$f(x) = 2\int_0^{L/2} \sqrt{1 + \frac{64h^2x^2}{L^4}}dx$$

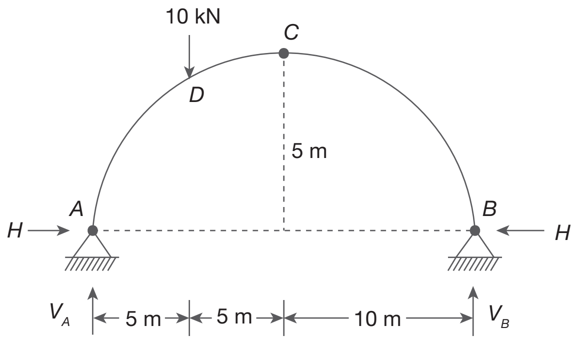
Hence, the correct option is (d).

5. A three hinged parabolic arch having a span of 20 m and a rise of 5 m carries a point load of 10 kN at quarter span form the left end as shown in the figure. The resultant reaction at the left support and its inclination with the horizontal are respectively. **[2010]**



(a) 9.01 kN and 56.31°
(b) 9.01 kN and 33.69°
(c) 7.50 kN and 56.31°
(d) 2.50 kN and 33.69°

Solution: (a)



Let V_A and V_B be the vertical reaction at support A and B respectively

Taking moments of all forces about support A,

$$\Sigma M_A = 0 \Rightarrow V_B \times 20 - 10 \times 5 = 0;\ V_B = 2.5\ \text{kN}(\uparrow)$$

$$\Sigma V = 0 \Rightarrow V_A + V_B = 10;\ V_A = 10 - 2.5 = 7.5\ \text{kN}(\uparrow)$$

Let H be the horizontal thrust at the supports.

Taking moments of forces about the hinge C,

$$\Sigma M_C = 0 \Rightarrow 7.5 \times 10 - H \times 5 - 10 \times 5 = 0;\ H = 5\ \text{kN}$$

Resultant reaction at left support, $R_A = \sqrt{V_A^2 + H^2}$

$$R_A = \sqrt{(7.5)^2 + (5)^2} = 9.01\ \text{kN}$$

Let θ be the inclination of the resultant with horizontal

$$\tan\theta = \frac{V_A}{H};\ \tan\theta = \frac{7.5}{5.0};\ \theta = 56.31°$$

Hence, the correct option is (a).

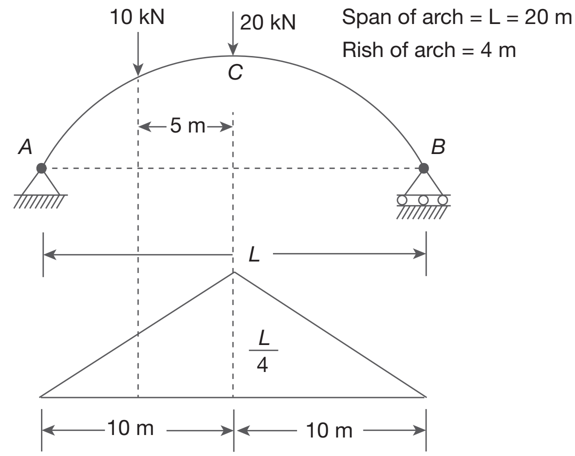
6. A three-hinged parabolic arch ABC has a span of 20 m and a central rise of 4 m. The arch has hinges at the ends and at the centre. A train of two point loads of 20 KN and 10 KN, 5 m apart, crosses this arch from left to right, with 20 KN load leading. The maximum thrust induced at the supports is **[2004]**

(a) 25.00 kN (b) 28.13 kN
(c) 31.25 kN (d) 32.18 kN

Solution: (c)

For a three hinged parabolic arch, the influence line diagram for horizontal thrust is linear.

Maximum thrust will be induced at the supports when 20 kN load is at the crown.



$$\text{Ordinate of ILD under 10 kN load} = \frac{5}{10}\frac{L}{4h}$$

$$= \frac{1}{2}\frac{20}{4 \times 4} = 0.625\ \text{kN}$$

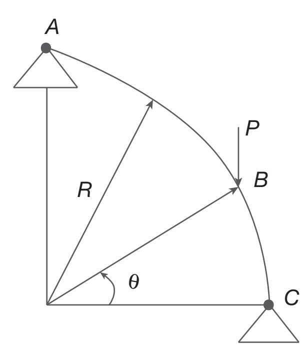
$$\text{Ordinate of ILD under crown} = \frac{L}{4h}$$

$$= \frac{20}{4 \times 4} = 1.25\ \text{kN}$$

Horizontal thrust, $H = 10 \times 0.625 + 20 \times 1.25$
$= 6.25 + 25 = 31.25$ kN

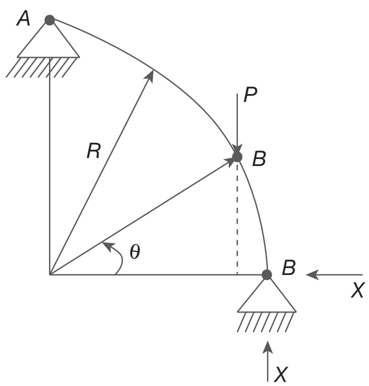
Hence, the correct option is (c).

7. A three hinged arch shown in figure is quarter of a circle. If the vertical and horizontal components of reaction at A are equal, the value of θ is **[1998]**



(a) 60° (b) 45°
(c) 30° (d) None in 0°–90°.

Solution: (d)



Let X be the vertical and horizontal reaction at A. Taking moments of all forces about the hinge A,

$$XR - XR + PR\cos\theta = 0$$

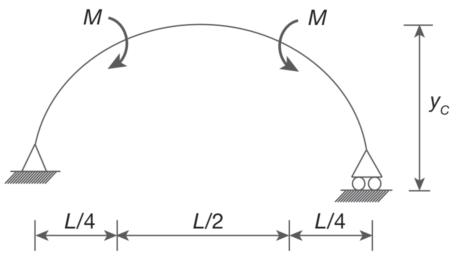
$$PR\cos\theta = 0$$

$$\cos\theta = 0 \Rightarrow \theta = 90^\circ$$

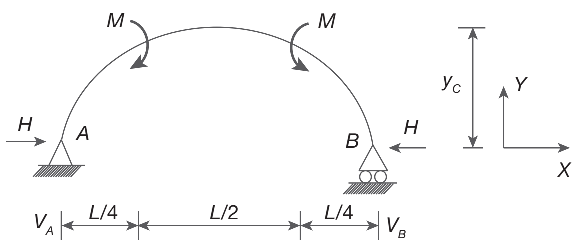
Therefore, option '*d*' is correct.
Hence, the correct option is (d).

FIVE-MARKS QUESTIONS

1. A two hinged parabolic arch carries two concentrated moments as shown in the figure below. The moment of inertia of the arch at any particular cross-section is equal to the moment of inertia at the crown multiplied by the secant of the angle θ, where θ is the angle between the horizontal and the tangent to the arch axis at that particular section. Determine the support reactions. **[2000]**



Solution:



We know that Equation of parabolic arch is

$$y = \frac{4y_c}{L^2}x(L-x) \quad (1)$$

Also horizontal reaction will be

$$H = -\frac{\int Mydx}{\int y^2 dx} \quad (2)$$

∴ Numerator

$$= \int Mydx = \int_{L/4}^{3L/4} Mydx$$

$$= \frac{4y_c}{L^2}M\int_{L/4}^{3L/4}(Lx - x^2)dx$$

$$= \frac{4y_c}{L^2}M\left[\frac{L}{2}x^2 - \frac{x^3}{3}\right]_{L/4}^{3L/4}$$

$$= \frac{4y_c}{L^2}M\left[\left(\frac{9}{32}L^3 - \frac{27}{192}L^3\right) - \left(\frac{L^3}{32} - \frac{L^3}{64}\right)\right]$$

$$= \frac{11}{24}My_cL$$

Denominator

$$= \int_0^L y^2 dx = \int_0^L\left[\frac{4y_c}{L^2}x(L-x)\right]^2 dx$$

$$= \frac{16y_c^2}{L^4}\int_0^L x^2(L-x)^2 dx$$

$$= \frac{16y_c^2}{L^4}\int_0^L (x^2L^2 + x^4 - 2Lx^3)dx$$

$$= \frac{16y_c^2}{L^4}\left[L^2\frac{x^3}{3} + \frac{x^5}{5} - 2L\frac{x^4}{4}\right]_0^L$$

$$= \frac{16y_c^2}{L^4}\left[\frac{L^5}{3} + \frac{L^5}{5} - \frac{L^5}{2}\right]$$

$$= \frac{16}{30}Ly_c^2$$

Substituting the value of numerator and denominator in eq. (2), we get

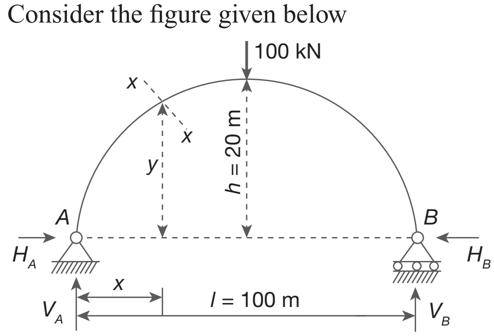
$$H = -\left(\frac{11}{24}My_cL\right) \times \left(\frac{30}{16}\frac{1}{y_c^2 L}\right)$$

Or $\quad H = -\dfrac{55}{64}\dfrac{M}{y_c}$ and $V = 0$

2. A two-hinged parabolic arch of span 100 m and rise 20 m carries a central concentrated load of 100 kN. The moment of inertia of any section is $I_c \sec\theta$ where θ is the slope at the section and I_c is the moment of inertia at the crown. Compute the reactions at support by the strain energy method. Neglect the effect of rib shortening. **[1997]**

Solution:

Consider the figure given below



Now, from equation of equilibrium $\sum F_y = 0$

$$\Rightarrow \quad V_A + V_B = 100 \text{ kN} \quad (1)$$

$$\Rightarrow \quad \sum F_x = 0$$

$$\Rightarrow \quad H_A = H_B \quad (2)$$

$$\sum M_B = 0$$

$$\Rightarrow \quad V_A \times 100 - 100 \times 50 = 0$$

$$\Rightarrow \quad V_A = 50 \text{ kN}$$

From equation (1)

$$V_B = 50 \text{ kN}$$

Since the two hinged arch is a indeterminate structure with one degree of indeterminacy therefore to find the horizontal reaction, we have to use compatability equation.
Now take $H_A = H$ as redundant.

As per compatability equation

$$\frac{\partial U}{\partial H} = \Delta_{AB} = 0$$

∴ Equation of parabolic arch is

$$y = \frac{4h}{l^2} \times (l - x)$$

$$\Rightarrow \quad y = \frac{4 \times 20}{(100)^2} \times (100 - x) = 0.008x(100 - x)$$

$$= 0.8x - 0.008x^2$$

$$\frac{dy}{dx} = \frac{4h}{l^2}(l - 2x)$$

$$\Rightarrow \quad \tan\theta = \frac{4 \times 20}{(100)^2}(100 - 2x)$$

$$= 0.008 \times 2(50 - x) = 0.016(50 - x)$$

Bending moment at section *x*-*x*

$$M_x = V_A x - Hy = 50x - Hy$$

$$\frac{\partial M_x}{\partial H} = -y$$

$$\because \quad U = \int \frac{M_x^2 dS}{2EI}$$

$$\Rightarrow \frac{\partial U}{\partial H} = 2\int_0^{50} M \frac{\partial M}{\partial H}\frac{\partial S}{EI}$$

$$= \int_0^{50} (50x - Hy)(-y)\frac{dS}{EI}$$

$$\Rightarrow \quad 0 = -\int_0^{50} 50xy \frac{dS}{EI} + H\int y^2 \frac{dS}{EI}$$

$$\Rightarrow \quad H = \frac{\int_0^{50} 50xy \frac{dS}{EI}}{\int_0^{50} y^2 \frac{dS}{EI}}$$

$$H = \frac{\int_0^{50} 50x(0.8x - 0.008x^2)\frac{dx}{EI}}{\int_0^{50} (0.8x - 0.008x^2)^2 \frac{dx}{EI}}$$

$$= \frac{\int_0^{50} (40x^2 - 0.4x^3)dx}{\int_0^{50} (0.64x^2 + 6.4 \times 10^{-5}x^4 - 0.0128x^3)dx}$$

$$= \frac{\left[40 \times \frac{x^3}{3} - 0.4\frac{x^4}{4}\right]_0^{50}}{\left[0.64\frac{x^3}{3} + 6.4 \times 10^{-5}\frac{x^5}{5} - 0.0128\frac{x^4}{4}\right]_0^{50}}$$

$$= \frac{1041666.67}{10666.67} = 97.656 \text{ kN}$$

Chapter 8

Matrix Methods of Structural Analysis

One-mark Questions

1. The stiffness coefficient k_{ij} indicates **[2007]**

(a) force at I due to a unit deformation at j
(b) deformation at j due to a unit force at i
(c) deformation at i due to a unit force at j
(d) force at j due to a unit deformation at i

Solution: (a)

Stiffness is the force required to produce unit deformation (displacement)

$$P_i = k_{ij}\delta_j$$

where P_i: Force at point *i*,
δ_j: Deformation at joint *j*,
k_{ij}: Stiffness coefficient, and
k_{ij} denotes force required at I due to unit displacement at *j*.
Hence, the correct option is (a).

2. For a linear elastic frame, if stiffness matrix is doubled, the existing stiffness matrix, the deflection of the resulting frame will be **[2005]**

(a) twice the existing value
(b) half the existing value
(c) the same as existing value
(d) indeterminate value

Solution: (b)

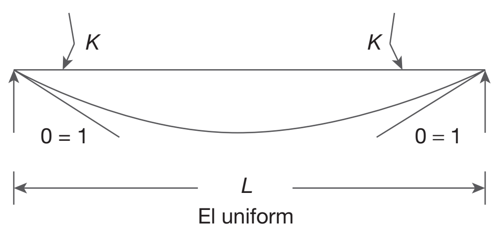
The stiffness matrix and deflection are related as $P = k\delta$

Stiffness matrix is independent of the load acting on the structure.

Deflection is inversely proportional to stiffness.

When the stiffness matrix is doubled, then the deflection will reduce to half of the existing value. Hence, the correct option is (b).

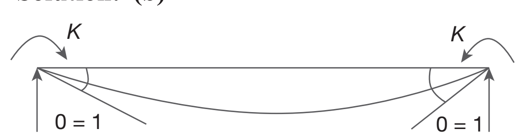
3. The stiffness K of a beam deflecting in a symmetric mode, as shown in the figure, is **[2003]**



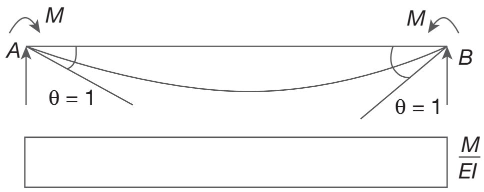
(a) $\dfrac{EI}{L}$ (b) $\dfrac{2EI}{L}$

(c) $\dfrac{4EI}{L}$ (d) $\dfrac{6EI}{L}$

Solution: (b)



The beam will deflect in a symmetric mode when a constant moment M is applied at both ends.



$\frac{M}{EI}$ diagram

Slope at either ends, $\theta_A = \theta_B = \frac{M}{EI}\frac{L}{2} = \frac{ML}{2EI}$

$$\frac{M}{\theta} = \frac{2EI}{L}$$

Stiffness is the moment required to produce unit rotation.

$$K = \frac{2EI}{L}$$

Hence, the correct option is (b).

4. The stiffness matrix of a beam element is given as $(2EI/L)\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$. Then the flexibility matrix is **[1998]**

(a) $\left(\frac{L}{2EI}\right)\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ (b) $\left(\frac{L}{6EI}\right)\begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$

(c) $\left(\frac{L}{3EI}\right)\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ (d) $\left(\frac{L}{5EI}\right)\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

Solution: (d)

Stiffness matrix, $K = \frac{2EI}{L}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

Flexibility matrix, = $[F]$

$$[K][F] = I \Rightarrow [F] = [K]^{-1}$$

Flexibility matrix is equal to the inverse of stiffness.

$$|K| = \left(\frac{2EI}{L}\right)^2 (4-1) = 3\left(\frac{2EI}{L}\right)^2$$

$$\text{Adj}\,K = \frac{2EI}{L}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$[K]^{-1} = \frac{\text{Adj}\,\text{K}}{|K|}$$

$$= \frac{1}{3}\left(\frac{L}{2EI}\right)^2 \frac{2EI}{L}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$[F] = \frac{L}{6EI}\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$$

Hence, the correct option is (d).

5. The order or the flexibility for a structure is **[1997]**

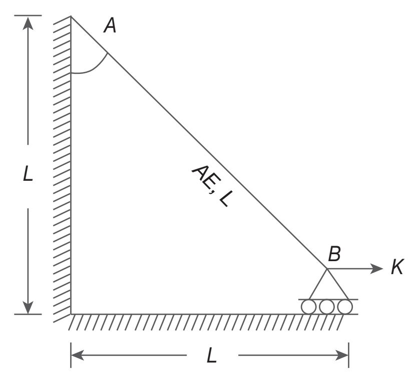
(a) equal to the number of redundant forces
(b) more than the number of redundant forces
(c) less than the number of redundant forces
(d) equal of the number of redundant forces plus three

Solution: (a)

The order of the flexibility matrix for a structure is equal to the number of redundant forces.

Hence, the correct option is (a).

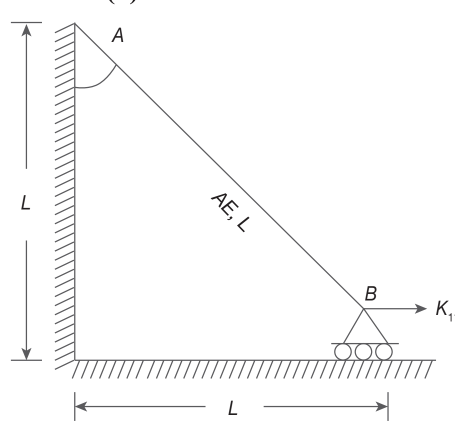
6. Horizontal stiffness coefficient k_{11} of bar '*ab*' is given by **[1996]**

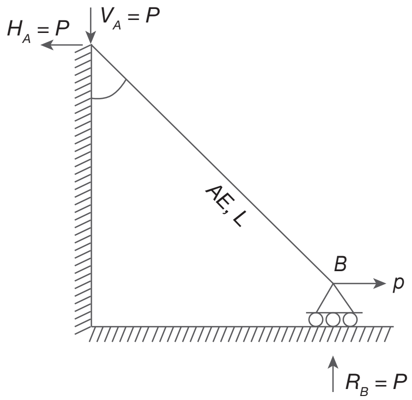


(a) $\frac{AE}{L\sqrt{2}}$ (b) $\frac{AE}{2L}$

(c) $\frac{AE}{L}$ (d) $\frac{2AE}{L}$

Solution: (b)

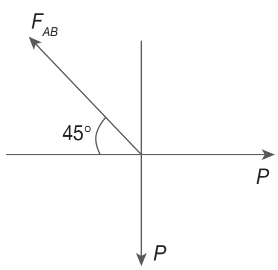




Taking moments of all forces about A,

$$\sum M_A = 0 \Rightarrow -PL + R_B L = 0 \Rightarrow R_B = P$$
$$\sum V = 0 \Rightarrow -V_A + R_B = 0 \Rightarrow V_A = P$$
$$\sum H = 0 \Rightarrow P - H_A = 0 \Rightarrow H_A = P$$

Consider the joint B



$$\sum H = 0 \Rightarrow F_{AB} \cos 45 - P = 0 \Rightarrow F_{AB} = \sqrt{2}P$$

Deflection at joint B in horizontal direction,

$$\delta_{11} = \frac{F_{AB} k_{AB} L}{AE}$$
$$= \frac{\sqrt{2}P\sqrt{2}L}{AE} = \frac{2PL}{AE}$$

Stiffness coefficient, $k_{11} = \dfrac{P}{\delta_{11}}$

$$= \frac{P}{\frac{2PL}{AE}} = \frac{AE}{2L}$$

Hence, the correct option is (b).

7. Which one of the following statements is true with regard to the flexibility method of analysis? **[1995]**

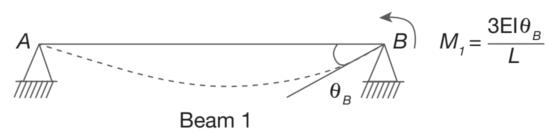
(a) The method is used to analyze determinate structures
(b) The method is used only for manual analysis of indeterminate structures
(c) The method is used for analysis of flexible structures
(d) The method is used for analysis of indeterminate structure with lesser degree of static indeterminacy

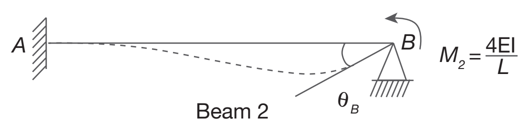
Solution: (d)
Flexibility method is used for analysis of redundant structure with lesser degree of static indeterminacy.
Hence, the correct option is (d).

8. The ratio of the stiffness of a beam at the near end when the far end is hinged to the stiffness of the beam at the near end when the far end is fixed is **[1994]**

(a) 1/2 (b) 3/4
(c) 1 (d) 4/3

Solution: (b)





$$\frac{\text{Stiffness of Beam 1}}{\text{Stiffness of Beam 2}} = \frac{K_1}{K_2} = \frac{3EI/L}{4EI/L} = \frac{3}{4}$$

Hence, the correct option is (b).

9. In flexibility method, the unknown quantities are ________ whereas in stiffness method the unknown quantities are ________. **[1994]**

Solution:
Force / Moment ; Displacement / Rotation
In flexibility method, the unknown quantities are force / moment, whereas in stiffness method, the unknown quantities are displacement / rotation.

10. Flexibility of structure may be defined as the displacement caused for ……. force and stiffness of structure may be defined as the force caused for…….. displacement **[1994]**

Solution:
unit ; unit
Flexibility of structure may be defined as the displacement caused for unit force.
Stiffness of structure may be defined as the force caused for unit displacement.

11. In a linear elastic structural element **[1991]**
(a) stiffness is directly proportional to flexibility
(b) stiffness is inversely proportional to flexibility
(c) stiffness is equal to flexibility
(d) stiffness and flexibility are not related

Solution: (b)
Stiffness is defined as the force required to produce unit deformation.
Flexibility is defined as the displacement required to produce unit force.

$$S = \frac{P}{\delta}; F = \frac{\delta}{P}$$

S: Stiffness
F: Flexibility
P: Load
δ: Deflection

$$[S] = \frac{1}{[F]}$$

Stiffness is inversely proportional to flexibility.
Hence, the correct option is (b).

12. In a linear structural element **[1991]**
(a) stiffness is directly proportional to flexibility
(b) stiffness is inversely proportional to flexibility
(c) stiffness is equal to flexibility
(d) stiffness and flexibility are not related

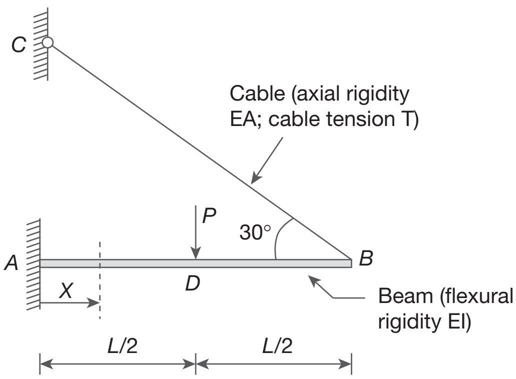
Solution: (b)

FIVE-MARKS QUESTIONS

1. The figure below shows a cable supported cantilever beam of span L subjected to a concentrated load P at mid-span. **[2001]**

(a) Express the bending momentat any section of the beam AB located at a distance x from the fixed end A, in terms of P, L and the cable tension T.

(b) Applying the Theorem of Least Work, derive an expression for T in terms of P, assuming. Consider only the flexural strain energy in the beam and the axial strain energy in the cable.

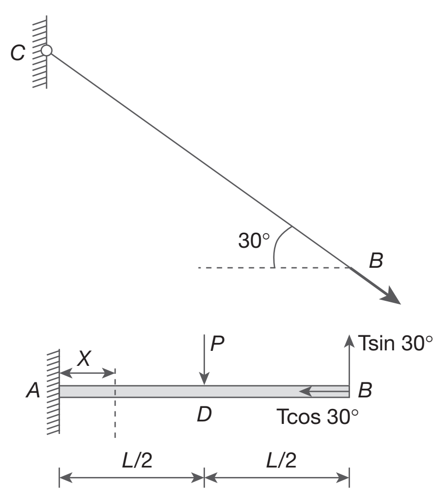


Solution:

(a)

$$M_x = P\left(\frac{L}{2} - x\right) - T\sin 30°(L-x)$$

$$= P\left(\frac{L}{2} - x\right) - \frac{T}{2}(L-x)$$



(b)

$$U = \int \frac{M^2 dx}{2EI} + \int \frac{T^2 dx}{2AE}$$

$$= \int \frac{M^2 dx}{2EI} + \int \frac{T^2 L^2 dx}{2\sqrt{3}EI}$$

$$= \int_0^{L/2} \frac{\left\{P\left(\frac{L}{2} - x\right) - \frac{T}{2}(L-x)\right\}^2 dx}{2EI}$$

$$+ \int_{L/2}^{L} \frac{\left(-\frac{T}{2}(L-x)\right)^2 dx}{2EI} + \int_0^{2L/\sqrt{3}} \frac{T^2 L^2 dx}{2\sqrt{3}EI}$$

By least work theorem,

$$\frac{\partial U}{\partial x} = 0$$

Or

$$2\int_0^{L/2} \left[P\left(\frac{L}{2} - x\right) - \frac{T}{2}(L-x)\right]\left(-P + \frac{T}{2}\right)dx$$

$$+2\int_{L/2}^{L} -\frac{T}{2}(L-x)\frac{T}{2}dx + \frac{T^2 L^2}{2\sqrt{3}EI} = 0$$

Or

$$2\left(\frac{T}{2} - P\right)\left[P\frac{L}{2}x - \frac{Px^2}{2} - \frac{T}{2}Lx + \frac{T}{4}Lx^2\right]_0^{L/2}$$

$$-\frac{2T^2}{4}\left[Lx - \frac{x^2}{2}\right]_{L/2}^{L} + \frac{T^2 L^2}{2\sqrt{3}EI} = 0$$

Or

$$2\left(\frac{T}{2} - P\right)\left[\frac{PL^3}{4} - \frac{PL^2}{8} - \frac{TL^2}{4} + \frac{TL^2}{16} - 0\right]$$

$$-\frac{T^2}{2}\left[L^2 - \frac{L^2}{2} - \frac{L^2}{2} + \frac{L^2}{8}\right] + \frac{T^2 L^2}{2\sqrt{3}EI} = 0$$

Or

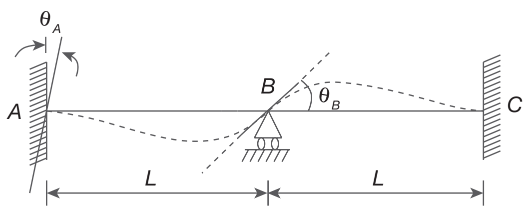
$$2\left(\frac{T}{2} - P\right)\left[\frac{PL^2}{8} - \frac{3}{16}TL^2\right] - \frac{T^2 L^2}{2}\left[\frac{8-4-4+1}{8}\right]$$

$$+\frac{T^2 L^2}{2\sqrt{3}} = 0$$

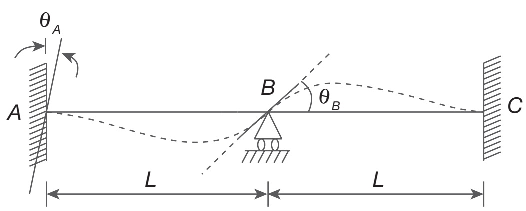
Or

$$2\left(\frac{T}{2}-P\right)\left[\frac{PL^2}{8}-\frac{3}{16}TL^2\right]-\frac{T^2L^2}{16}+\frac{T^2L^2}{2\sqrt{3}}=0$$

2. The two-span continuous beam shows below is subjected to a clockwise rotational slip $\theta_A = 0.004$ radian at the fixed end A. Applying the slope deflection method of analysis, determine the slope θ_B at B. Given that the flexural rigidity $EI = 25000$ kNm^2 and span $L = 5$ m, determine the end moments (in kNm units) in the two spans, and draw the bending moment diagram. **[2001]**



Solution:



Using slope deflection equation, we get

$$M_{AB}=\frac{2EI}{L}(2\theta_A+\theta_B)$$

$$M_{BA}=\frac{2EI}{L}(\theta_A+2\theta_B)$$

$$M_{BC}=\frac{2EI}{L}(2\theta_B+0)$$

$$M_{CB}=\frac{2EI}{L}(\theta_B)$$

At joint B, we can write

$$M_{BA}+M_{BC}=0$$

Or $$\frac{2EI}{L}(\theta_A+2\theta_B+2\theta_B)=0$$

Or $$4\theta_B+\theta_A=0$$

Or $\theta_B=-\dfrac{\theta_A}{4}=-\dfrac{0.004}{4}$ radians $=-0.001$ radians

$\Rightarrow$ $\theta_B = 0.001$ radian (anti-clockwise)

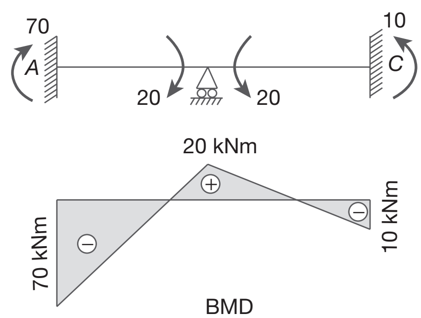
$$M_{AB}=\frac{2\times 25000}{5}(2\times 0.004-0.001)$$

$$=70\text{ kNm}$$

$$M_{BA}=\frac{50000}{5}(0.004-0.002)=20\text{ kNm}$$

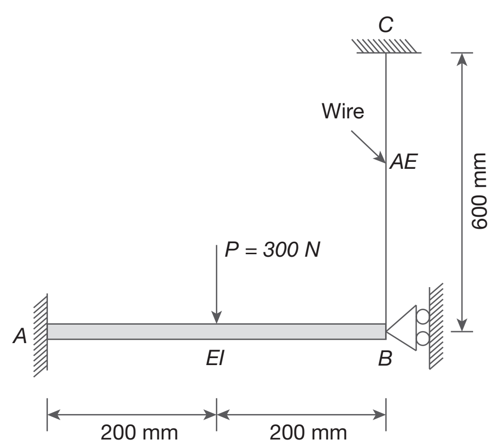
$$M_{BC}=10000(-2\times 0.001)=-20\text{ kNm}$$

$$M_{CB}=10000\times -0.001=-10\text{ kNm}$$

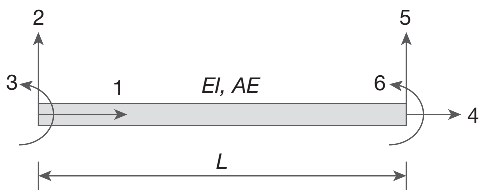


BMD

3. (a) A beam AB is suspended from a wire CB as shown in figure below. The beam carries a central concentrate load P. It may be assumed that $E = 7 \times 10^{10}$ N/m^2. $I = 800$ mm^4, $A = 1.2$ mm^2 and $P = 300$ N. Determine, using stiffness matrix approach, the deflection of point B. what would be this deflection if the load P were to be applied upwards, instead of downward? The axial deformation of the beam AB may be ignored. **[2000]**



(b) The stiffness matrix for a beam element is given to be as follows:



$$K=\frac{EI}{L^3}\begin{bmatrix} \frac{AL^2}{0} & & & & & \text{Symmetric} \\ 0 & 12 & & & & \\ 0 & 6L & 4L^2 & & & \\ -\frac{AL^2}{I} & 0 & 0 & \frac{AL^2}{I} & & \\ 0 & -12 & -6L & 0 & 12 & \\ 0 & 6L & 2L^2 & 0 & -6L & 4L^2 \end{bmatrix}$$

Solution:

Let structural stiffness Matrix = K

$$K=\frac{EI}{L^3}\begin{bmatrix} \frac{AL^2}{0} & & & & & \text{Symmetric} \\ 0 & 12 & & & & \\ 0 & 6L & 4L^2 & & & \\ -\frac{AL^2}{I} & 0 & 0 & \frac{AL^2}{I} & & \\ 0 & -12 & -6L & 0 & 12 & \\ 0 & 6L & 2L^2 & 0 & -6L & 4L^2 \end{bmatrix}$$

Fixed end moments at point B will be

$$B=\frac{wl}{8}=\frac{-300\times 400}{8}$$
$$=-15000\ \text{Nmm}$$

Reaction at $B=\frac{w}{2}=\frac{300}{2}=150\ \text{N}$

Since the wire will stretch, so it will take load, and if it is P_w

Then the Load taken by wire = P_w

Load vector

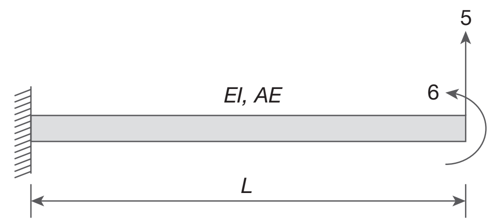
$$[JL_u]=\begin{bmatrix}-150tP_w\\ 15000\end{bmatrix}$$

We know that

$$[S_{uu}][\Delta_u]=[JL_u]+[R_c]$$

Since the upward reaction and rotation at B are restrained, so

$$[S_{uu}]=\frac{EI}{L^3}\begin{array}{c} \begin{array}{cc}(5) & (6)\end{array} \\ \begin{bmatrix}12 & -6L\\ -6L & 4L^2\end{bmatrix}\end{array}\begin{array}{c} \\ (5)\\ (6)\end{array}$$



deformation matrix will be

$$[\Delta_u]=\begin{bmatrix}\delta_B\\ \theta_B\end{bmatrix}$$

using eq. (1), we get

$$\frac{EI}{L^3}\begin{bmatrix}12 & -6L\\ -6L & 4L^2\end{bmatrix}\begin{bmatrix}\delta_B\\ \delta_\theta\end{bmatrix}=\begin{bmatrix}-150+P_w\\ 1500\end{bmatrix}+[0]$$

$$\Rightarrow \begin{bmatrix}\delta_B\\ \delta_\theta\end{bmatrix}=\frac{L^3}{EI}\begin{bmatrix}12 & -6L\\ -6L & 4L^2\end{bmatrix}\begin{bmatrix}-150+P_w\\ 1500\end{bmatrix}$$

$$=\frac{L^3}{EI}\begin{bmatrix}\frac{1}{3} & \frac{1}{2L}\\ \frac{1}{2L} & \frac{1}{L^2}\end{bmatrix}\begin{bmatrix}-150+P_w\\ 1500\end{bmatrix}$$

$$\Rightarrow \delta_B=\frac{L^3}{EI}\left[-50+\frac{15000}{2L}\right]$$

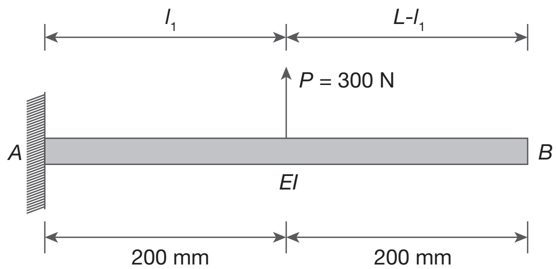
$$\text{Or}\ \ \delta_B=\frac{L^3}{EI}\left[-50+\frac{AE\delta_B}{3L}+\frac{15000}{2\times 400}\right]$$

$$\Rightarrow\qquad \delta_B\left[\frac{1.2\times 7\times 10^4}{3\times 600}-\frac{7\times 10^4\times 800}{(400)^3}\right]$$

$$=50-18.75$$

$$\Rightarrow\qquad \delta_B=0.6824\ \text{mm}$$

Now when the 300 N load acts upward, wire will be buckle it can resist any load.
Hence beam will act as a cantilever.
If the position of load from point A is l_1, then deflection at B is given by



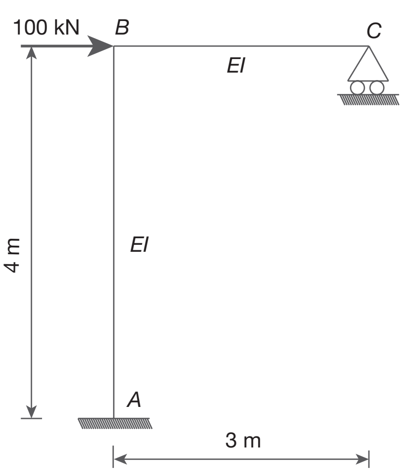
$$\delta_B = \frac{PL^3}{3EI} + \frac{ML^2}{2EI}$$

$$\delta_B = \frac{300 \times (200)^3}{3 \times 7 \times 10^4 \times 800} + \frac{300(200)^2(200)}{2 \times 7 \times 10^4 \times 800}$$

$$= \frac{1}{7 \times 10^4 \times 800}(800 \times 10^6 + 1200 \times 10^6)$$

Or $\delta_B = 35.71$ mm

4. Analyse the frame shown in the figure by the method of moment distribution. Draw the bending moment diagram on the tension side of the members. **[1997]**



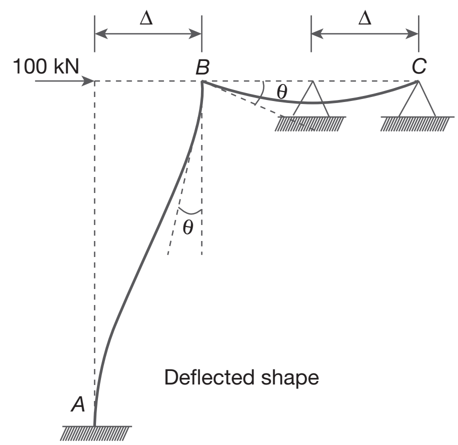
Solution:
There is no need of non-sway analysis because load is acting on the joint only

The frame will sway towards right with a sway force of 100 kN.

Now, calculation of Distribution factor

Joint	Member	Relative stiffners	Total relative stiffners	Distribution factor
	BA	$\frac{I}{4}$		$\frac{1}{2}$
B			$\frac{I}{2}$	
	BC	$\frac{3}{4} \times \frac{I}{3} = \frac{I}{4}$		$\frac{1}{2}$

If the frame will sway towards right with 'Δ'



Ratio of fixed end moment.

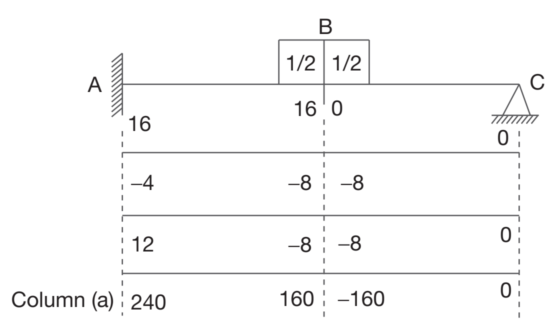
$$\overline{M}_{AB} : \overline{M}_{BA} : \overline{M}_{BC} : \overline{M}_{CB}$$

$$:: -\frac{6EI\Delta}{(4)^2} : -\frac{6EI\Delta}{(4)^2} : 0 : 0$$

$$:: 1 : 1 : 0 : 0$$

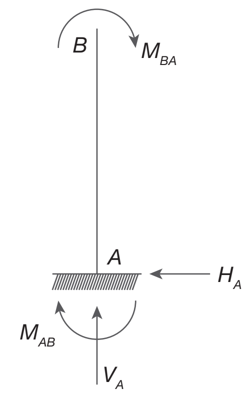
$$:: 16 : 16 : 0 : 0 \text{ (say)}$$

Distribution table



$$\text{Actual sway moment} = \frac{\text{Column(a)}}{S'} \times S$$

Calculation of reaction at support $\Sigma M_B = 0$



$$\Rightarrow \qquad -H_A \times (4) + M_{AB} + M_{BA} = 0$$

$$\Rightarrow \qquad H_A = \frac{M_{AB} + M_{BA}}{4}$$

$$\frac{12+8}{4} = 5 \text{ kN}$$

We know that the moment obtained in columb a are due to some sway force S' . Which is not actual sway force.

The sway force S' can be expressed as

$$\Sigma F_x = H_A = 5 \text{ kN}$$

$$\therefore \qquad S' = 5 \text{ kN}$$

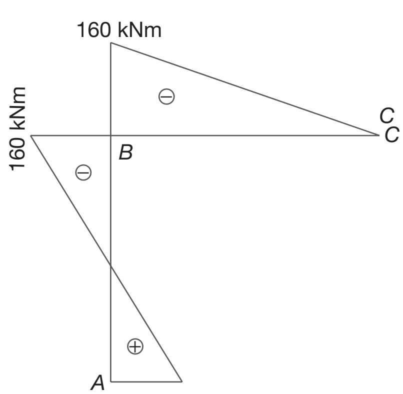
actual sway moments due to actual sway force will be

$$S = 100 \text{ kN is } \frac{\text{Column(a)}}{S'} \times S$$

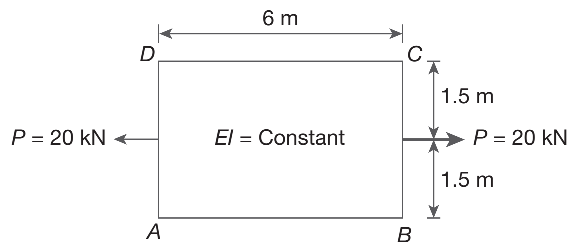
$$\Rightarrow \text{Actual sway moment} = \frac{\text{Column(a)}}{5} \times 100$$

$$= \text{Column (a)} \times 20$$

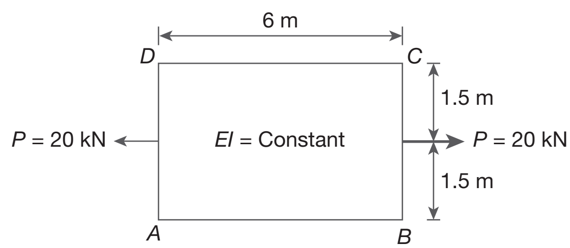
Consider the Bending moment diagram given below



5. Analyse the box frame by moment distribution method. Plot bending moment diagram. **[1996]**



Solution:



Fixed end moment:

$$\overline{M_{AB}} = 0, \overline{M_{BA}} = 0$$

$$\overline{M_{BC}} = -\frac{20 \times 3}{8} = -7.5 \text{ kNm}$$

$$\overline{M_{CB}} = \frac{20 \times 3}{8} = 7.5 \text{ kNm}$$

$$\overline{M_{CD}} = \overline{M_{DC}} = 0$$

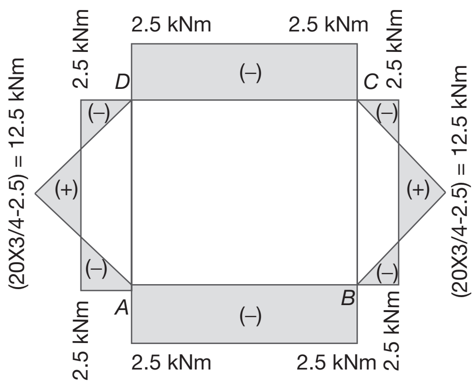
$$\overline{M_{DA}} = -\frac{20 \times 3}{8} = -7.5 \text{ kNm}$$

$$\overline{M_{AD}} = \frac{20 \times 3}{8} = 7.5 \text{ kNm}$$

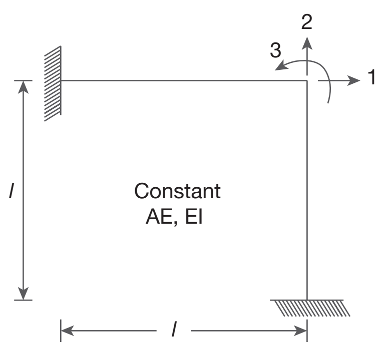
Joint	Member	Relative stiffness	Total relative stiffness	D.F
A	AD	1/3	$\frac{1}{2}$	2/3
	AB	1/6		1/3
B	BA	1/6	$\frac{1}{2}$	1/3
	BC	1/3		2/3
C	CB	1/3	$\frac{1}{2}$	2/3
	CD	1/6		1/3
D	DC	1/6	$\frac{1}{2}$	1/3
	DA	1/3		2/3

Distribution table:

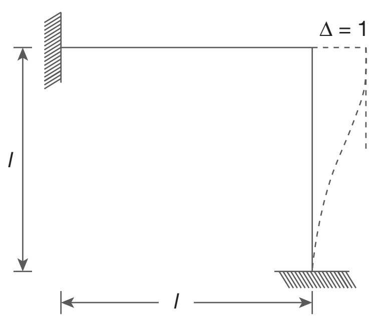
A		B		C		D	
2/3	1/3	1/3	2/3	2/3	1/3	1/3	2/3
7.5	0	0	–7.5	7.5	0	0	–7.5
–5	–2.5	2.5	5	–5	–2.5	2.5	5
+2.5	–2.5	2.5	–2.5	2.5	–2.5	2.5	–2.5



6. Generate the stiffness matrix for the frame corresponding to three degrees of freedom 1,2,3. **[1996]**



Solution:



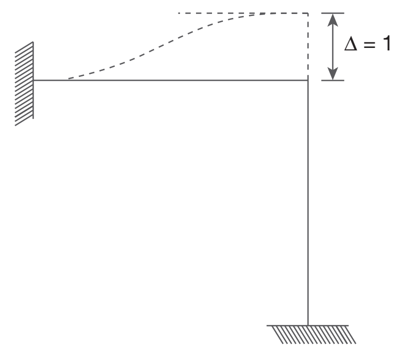
To generate the first column of stiffness matrix apply unit displacement in direction (1)

$$\therefore \quad k_{11} = \frac{12EI}{l^3} + \frac{AE}{l}$$

$$k_{21} = 0$$

$$k_{31} = \frac{6EI}{l^2}$$

To generate the second column of stiffness matrix, apply unit displacement in direction (2)

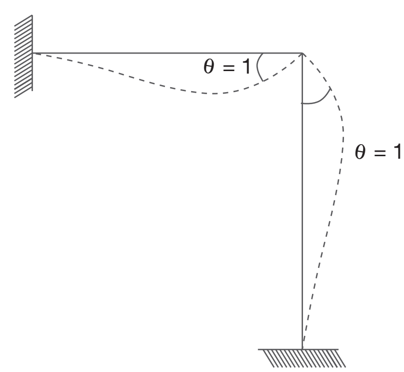


$$k_{12} = 0$$

$$k_{22} = \frac{AE}{l} + 12\frac{EI}{l^3}$$

$$k_{32} = 0$$

To generate the 3rd column of stiffness matrix, apply unit rotation in direction (3)



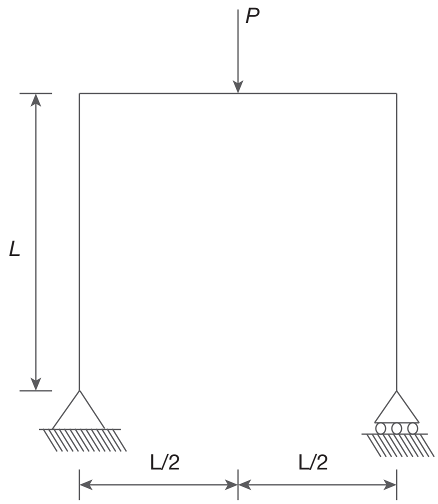
$$k_{13} = \frac{6EI}{l^2}$$

$$k_{23} = 0$$

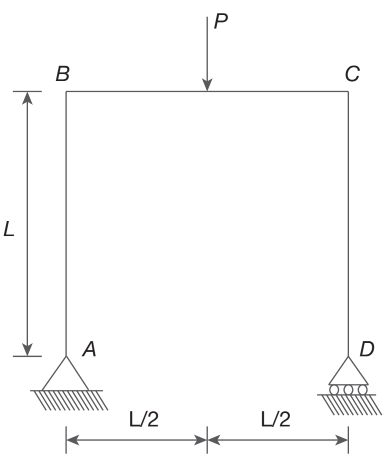
$$k_{33} = \frac{4EI}{l} + \frac{4EI}{l} = \frac{8EI}{l}$$

∴ Required stiffness matrix

7. Draw the bending moment diagram and the deflected shape of the elastic curve of the frame shown in the figure, assuming elastic and small deflection. **[1993]**



Solution:



Distribution factor

Joint	Member	RS	TRS	DF
	BA	$\frac{3}{4}\frac{I}{L}$		$\frac{3}{7}$
B			$\frac{7}{4}\frac{I}{L}$	
	BC	$\frac{I}{L}$		$\frac{4}{7}$
	CB	$\frac{I}{L}$		$\frac{4}{7}$
C			$\frac{7}{4}\frac{I}{L}$	
	CD	$\frac{3}{4}\frac{I}{L}$		$\frac{3}{7}$

Fixed end moment

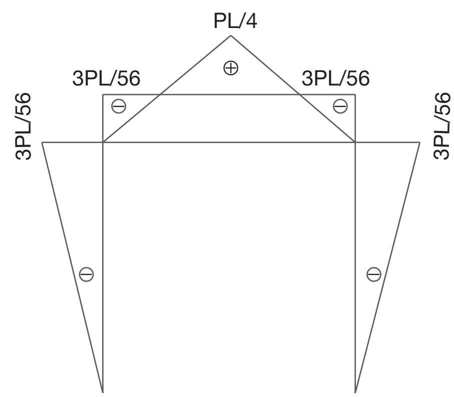
$$\overline{M_{AB}} = \overline{M_{BA}} = \overline{M_{CD}} = \overline{M_{DC}} = 0$$

$$\overline{M_{BC}} = -\frac{PL}{8}$$

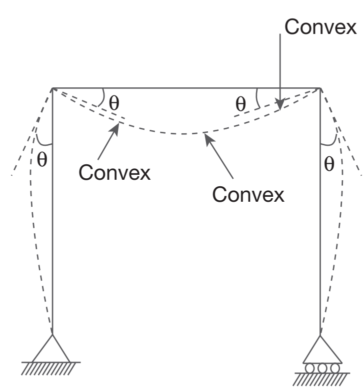
$$\overline{M_{CB}} = \frac{PL}{8}$$

Distribution table

A					D
	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{4}{7}$	$\frac{3}{7}$	
0	0	$-\frac{PL}{8}$	$\frac{PL}{8}$	0	0
	$\frac{3PL}{56}$	$\frac{4PL}{56}$	$-\frac{4PL}{56}$	$-\frac{3PL}{56}$	
0	$\frac{3PL}{56}$	$-\frac{3PL}{56}$	$\frac{3PL}{56}$	$-\frac{3PL}{56}$	0



Bending moment diagram



Deflected shape