Chapter 1

Simple Stresses and Strains

One-mark Questions

1. The Poisson's ratio is defined as **[2012]**

(a) $\left|\dfrac{\text{axial stress}}{\text{lateral stress}}\right|$ (b) $\left|\dfrac{\text{lateral strian}}{\text{axial strian}}\right|$

(c) $\left|\dfrac{\text{lateral stress}}{\text{axial stress}}\right|$ (d) $\left|\dfrac{\text{axial strian}}{\text{lateral strian}}\right|$

Solution: (b)

Poisson's ratio is defined as the ratio of lateral strain to axial strain.

Hence, the correct option is (b).

2. The number of independent elastic constants for a linear elastic isotropic and homogeneous material is **[2010]**

(a) 4 (b) 3

(c) 2 (d) 1

Solution: (c)

For a linear elastic isotropic and homogeneous material,

$$E = 2G\left(1+\frac{1}{m}\right) = 3K\left(1-\frac{2}{m}\right)$$

All the three elastic constants can be found if any two of them are known. Hence, the number of independent elastic constants are 2.

Hence, the correct option is (c).

3. For an isotropic material, the relationship between the Young's modulus (E), shear modulus (G) and Poisson's ratio (μ) is given by **[2007]**

(a) $G = \dfrac{E}{2(1+\mu)}$ (b) $E = \dfrac{G}{2(1+\mu)}$

(c) $G = \dfrac{E}{(1+2\mu)}$ (d) $G = \dfrac{E}{2(1-\mu)}$

Solution: (a)

E = Young's modulus

G = Shear modulus

μ = Poisson's ratio

$$E = 2G(1+\mu)$$

$$G = \frac{E}{2(1+\mu)}$$

Hence, the correct option is (a).

4. U_1 and U_2 are the strain energies stored in a prismatic bar due to axial tensile forces P_1 and P_2, respectively. The strain energy U stored in the same bar due to combined action of P_1 and P_2 will be **[2007]**

(a) $U = U_1 + U_2$ (b) $U = U_1U_2$

(c) $U < U_1 + U_2$ (d) $U > U_1 + U_2$

Solution: (d)

Strain energy stored in a prismatic bar due to axial load $P_1, U_1 = \dfrac{P_1^2 L}{2AE}$.

Strain energy stored in a prismatic bar due to axial load $P_2, U_2 = \dfrac{P_2^2 L}{2AE}$.

Strain energy stored in a prismatic bar due to combined axial load P_1 and P_2,

$$U = \frac{(P_1+P_2)^2 L}{2AE} = \frac{P_1^2 L}{2AE} + \frac{P_2^2 L}{2AE} + \frac{2P_1P_2L}{2AE}$$

$$U = U_1 + U_2 + \frac{2P_1P_2L}{2AE}$$

Therefore, $U > U_1 + U_2$

Hence, the correct option is (d).

5. For linear elastic systems, the type of displacement function for the strain energy is **[2004]**
(a) linear (b) quadratic
(c) cubic (d) quartic

Solution: (b)

Strain energy, $U = \frac{1}{2} \times \text{Stress} \times \text{Strain}$

$U = \frac{1}{2}\sigma e = \frac{1}{2} E\, e.e = \frac{1}{2} E.e^2$

Therefore, strain energy is a function of the square of displacement. The displacement function for strain energy is quadratic.
Hence, the correct option is (b).

6. The shear modulus (G), modulus of elasticity (E)and the Poisson's ratio (v) of a material are related as, **[2002]**
(a) $G = E/[2(1+v)]$
(b) $E = G/[2(1+v)]$
(c) $G = E/[2(1-v)]$
(d) $G = E/[2(v-1)]$

Solution: (a)
G = Shear modulus
E = Modulus of elasticity
v = Poisson's ratio
Relationship between the above parameters is $E = 2G\,(1 + v)$
Hence, the correct option is (a).

7. The material that exhibits the same elastic properties in all directions at a point is said to be **[1995]**
(a) homogeneous (b) orthotropic
(c) viscoelastic (d) isotropic

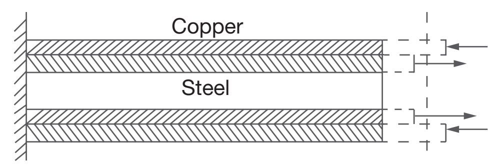
Solution: (d)
Homogeneous material is uniform in composition and character.
Orthotropic material has different material properties or strengths in different orthogonal directions.
Isotropic material has the same properties in all directions.
Viscoelasticity is the property of the material that exhibit both viscous and elastic characteristics when undergoing deformation.
Hence, the correct option is (d).

8. The maximum value of Poisson's ratio for an elastic material is **[1991]**
(a) 0.25 (b) 0.5
(c) 0.75 (d) 0.1

Solution: (b)
The maximum Poisson's ratio is 0.5 for an ideal elastic incompressible material whose volumetric strain is 0.
Hence, the correct option is (b).

9. A cantilever beam of tubular section consists of two materials, copper as outer cylinder and steel as inner cylinder. It is subjected to a temperature rise of 20°C and it is given that $\alpha_{copper} > \alpha_{steel}$. The stresses developed in the tubes will be **[1991]**
(a) compression in steel and tension in copper.
(b) tension in steel and compression in copper.
(c) no stress in both.
(d) tension in both the materials.

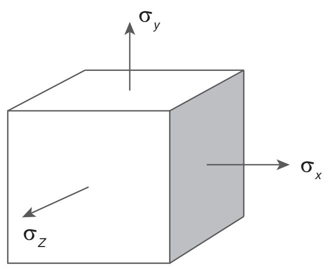
Solution: (b)



Rise of temperature, $T = 20°C$
$\alpha_{copper} > \alpha_{steel}$
Extension of beam due to rise of temperature, $\delta l = L\alpha T$
L = Length of the beam
α = Coefficient of linear expansion
T = Change of temperature
Since, $\alpha_{copper} > \alpha_{steel}$, the free expansion of copper is more than the steel, $(\delta l)_{copper} > (\delta l)_{steel}$
Since, the two materials are brazed together, copper tube try to pull the steel tube and steel tube push the copper tube. Therefore, tensile stress induced in steel tube and compressive stress is induced in copper tube.
Hence, the correct option is (b).

Two-marks Questions

1. An elastic isotropic body is in a hydrostatic state of stress as shown in the figure. For no change in the volume to occur, what should be its Poisson's ratio? **[2016]**



(a) 0.00 (b) 0.25
(c) 0.50 (d) 1.00

Solution: (c)
For no change in the volume
Volumetric strain (ε_v) = 0

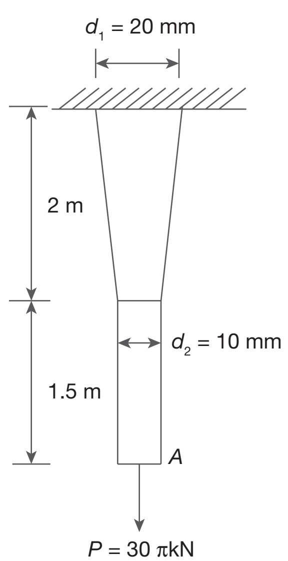
$$\left(\frac{\varepsilon_x+\varepsilon_y+\varepsilon_z}{3}\right)(1-2\mu)=0$$

$1 - 2\mu = 0$
$1 = 2\mu$

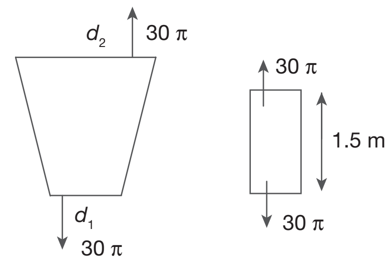
Poissions ratio $\mu=\frac{1}{2}=0.5.$

Hence, the correct option is (c).

2. A tapered circular rod of diameter varying from 20 mm to 10 mm is connected to another uniform circular rod of diameter 10 mm as shown in the following figure. Both bars are made of same material with the modulus of elasticity, $E = 2 \times 10^5$ MPa. When subjected to a load $P = 30\ \pi$kN, the section at point A is _______ mm. **[2015]**



Solution: 15



For tappered bar

$$\Delta_1=\frac{4PL}{\pi d_1 d_2 E}$$

$$=\frac{4\times 30\pi\times 2\times 10^6}{\pi\times 20\times 10\times 2\times 10^5}=6\text{ mm}$$

$$\Delta_2=\frac{PL}{AE}$$

$$=\frac{30\pi\times(1.5\times 10^6)\times 4}{\pi\times 10\times 10\times 2\times 10^5}=9\text{ mm}$$

∴ Total deflection at $A = \Delta_1 + \Delta_2 = 15$ mm.

Hence, the answer is 15 mm.

3. A mild steel specimen is under uni-axial tensile stress. Young's modulus and yield stress for mild steel are 2×10^5 MPa and 250 MPa, respectively. The maximum amount of strain energy per unit volume that can be stored in this specimen without permanent set is **[2008]**
(a) 156 Nmm/mm^3
(b) 15.6 Nmm/mm^3
(c) 1.56 Nmm/mm^3
(d) 0.156 Nmm/mm^3

Solution: (d)
Modulus of elasticity of mild steel, $E = 2 \times 10^5$ MPa
Yield stress of mild steel, σ_y = 250 MPa

Strain energy per unit volume, $U=\frac{\sigma^2}{2E}$

Maximum strain energy occurs at yield stress.

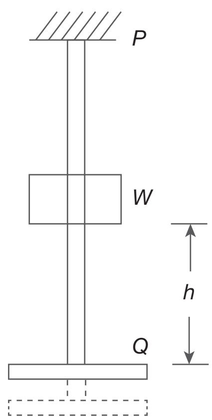
$$U_{\max}=\frac{(250)^2}{2\times 2\times 10^5}=0.156\text{ N mm/mm}^3$$

Hence, the correct option is (d).

4. A vertical rod PQ of length L is fixed at its top end P and has a flange fixed to the bottom end Q. A weight W is dropped vertically from a height $h < L$ on to the flange. The axial stress in the rod can be reduced by **[2008]**

(a) increasing the length of the rod.
(b) decreasing the length of the rod.
(c) decreasing the area of cross-section of the rod.
(d) increasing the modulus of elasticity of the material.

Solution: (a)



The kinetic energy of the weight W is stored in the form of strain energy in the rod. Strain energy stored in the rod, $U = \frac{\sigma^2}{2E} \times AL$

σ = Axial stress in the rod
A = Cross sectional area of the rod
L = Length of the rod
E = Modulus of elasticity of the rod

The strain energy remains constant. The axial stress in rod can be reduced by

(i) increasing the length of rod.
(ii) increasing cross sectional area of the rod.
(iii) decreasing the modulus of elasticity of the rod.

Hence, the correct option is (a).

5. A metal bar of length 100 mm is inserted between two rigid supports and its temperature is increased by 10°C. If the coefficient of thermal expansion is 12×10^{-6} per °C and the Young's modulus is 2×10^5 MPa, the stress in the bar is **[2007]**

(a) 0
(b) 12 MPa
(c) 24 MPa
(d) 2400 MPa

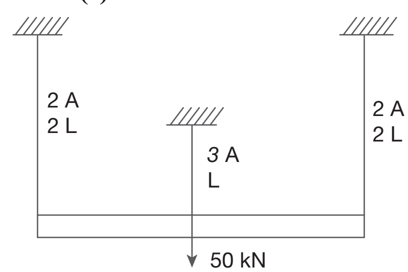
Solution: (c)

Length of the bar, L = 100 mm
Temperature increase = ΔT
Coefficient of thermal expansion,
$\alpha = 12 \times 10^{-6}$ / °C
Young's modulus, $E = 2 \times 10^{-5}$ MPa
Stress in the bar due to change of temperature,
$\sigma = \alpha \mathrm{TE} = 12 \times 10^{-6} \times 10 \times 2 \times 10^5 = 24$ MPa
Hence, the correct option is (c).

6. A rigid bar is suspended by three rods made of the same material as shown in the figure. The area and length of the central rod are $3A$ and L, respectively while that of the two outer rods are $2A$ and $2L$, respectively. If a downward force of 50 kN is applied to the rigid bar, the forces in the central and each of the outer rods will be **[2007]**

(a) 16.67 kN each
(b) 30 kN and 15 kN
(c) 30 kN and10 kN
(d) 21. 4 kN and 14.3 kN

Solution: (c)



Let
P_1 = Force in the central rod
P_2 = Force in each outer rods

$$P_1 + 2P_2 = 50 \qquad (1)$$

Since, the rigid bar is symmetric, the elongation of central rod and outer rod is same.

$$\frac{P_1L_1}{A_1E} = \frac{P_2L_2}{A_2E}; \quad \frac{P_1L}{3AE} = \frac{P_2 2L}{2AE}; \quad P_1 = 3P_2 \qquad (2)$$

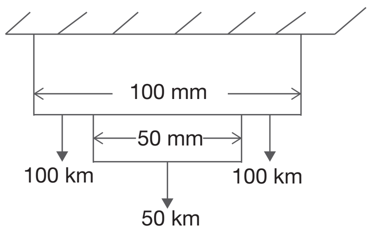
$$3P_2 + 2P_2 = 50$$

$$P_2 = 10 \text{ kN}$$

$$P_1 = 30 \text{ kN}$$

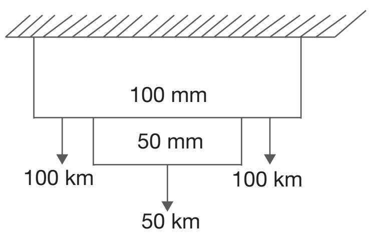
Force in central rod, P_1 = 30 kN
Force in each of outer rod, P_2 = 10 kN
Hence, the correct option is (c).

7. A bar of varying square cross-section is loaded symmetrically as shown in the figure. Loads shown are placed on one of the axes of symmetry of cross-section. Ignoring self weight, the maximum tensile stress in N/mm^2 anywhere is **[2003]**



(a) 16.0 (b) 20.0
(c) 25.0 (d) 30.0

Solution: (c)



Tensile stress, $\sigma = \dfrac{P}{A}$

Load in lower bar, $P_1 = 50$ kN
Load on upper bar, $P_2 = 100 + 100 + 50 = 250$ kN
Cross sectional area of lower bar, $A_1 = 50 \times 50 = 2500$ mm^2
Cross sectional area of upper bar, $A_2 = 100 \times 100 = 1 \times 10^4$ mm^2
Tensile stress in lower bar,

$$\sigma_1 = \frac{50 \times 10^3}{250} = 20 \text{ N/mm}^2$$

Tensile stress in upper bar,

$$\sigma_2 = \frac{250 \times 10^3}{1 \times 10^4} = 25 \text{ N/mm}^2$$

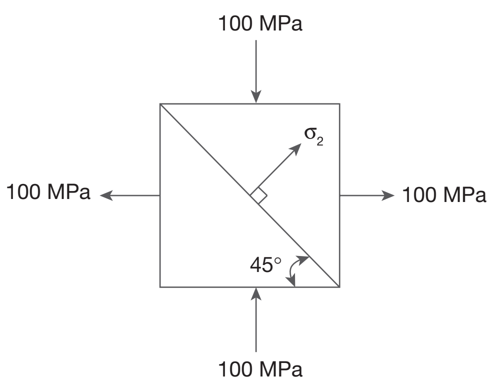
Maximum tensile stress = 25 N/mm^2
Hence, the correct option is (c).

Chapter 2

Principal Stresses and Strains

One-mark Questions

1. Two triangular wedges are glued together as shown in the figure. The stress acting normal to the interface, σ_n is ______ MPa. **[2015]**



Solution: 0

Normal stress

$$J_n = \frac{J_1 + J_2}{2} + \left(\frac{J_1 - J_2}{2}\right)\cos 2\theta$$

$J_1 = 100$ (tension)

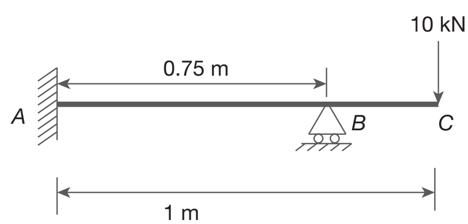
$J_2 = -100$ (compression)

$$J_n = \frac{100 - 100}{2} + \left(\frac{100 + 100}{2}\right)\cos 90$$

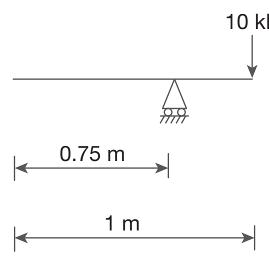
$= 0$

Hence, the answer is 0.

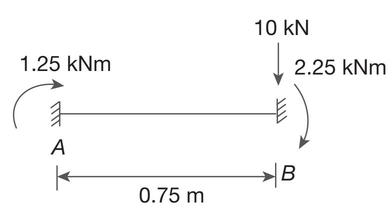
2. A horizontal beam ABC is loaded as shown in the figure below. The distance of the point of contraflexure from end A (in m) is __________. **[2015]**



Solution: 0.25



Equivalent beam



Due to moment, $M = 2.25$ kNm

Carryover moment of 1.25 kNm

acts on support A

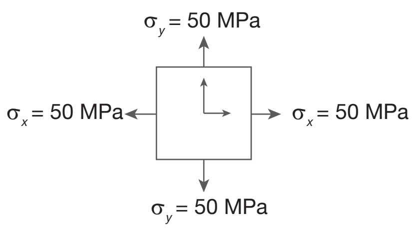
$$\Sigma M_A = 10 \times 1 - R_B \times 0.75 + 1.25 = 0$$
$$R_B = 15 \text{ kN}$$
$$R_N = -5 \text{ kN}$$

Point of contraflexure

$$M_A + R_A x = 0$$
$$1.25 - 5 \times x = 0$$
$$x = 0.25 \text{ m}$$

Hence, the answer is 0.25.

3. For the plane stress situation shown in the figure, the maximums shear stress and the plane on which it acts are: **[2015]**



(a) −50 MPa, on a plane 45º clockwise w.r.t. x-axis
(b) −50 MPa, on a plane 45º anti-clockwise w.r.t. x-axis
(c) 50 MPa, at all orientations
(d) Zero, at all orientations

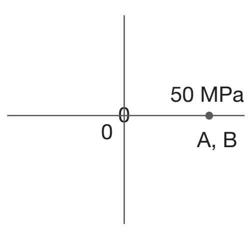
Solution: (d)

$$\sigma_1 = 50 \text{ MPa } (-\text{ Tensile})$$
$$\sigma_2 = 50 \text{ MPa } (-\text{ Tensile})$$

Maximum share stress

$$\sigma_{\max} = 0$$

(radius of mohr circle)



And it acts in all directions
Hence, the correct option is (d).

4. Consider the following statements. **[2009]**
(I) On a principal plane, only normal stress acts.
(II) On a principal plane, both normal and shear stresses act.
(III) On a principal plane, only shear stress acts.
(IV) Isotropic state of stress is independent of frame of reference.

The TRUE statements are
(a) I and IV (b) II
(c) II and IV (d) II and III

Solution: (a)
Principal planes are those in which only normal stresses act and no shear stress.
Isotropic state of stress is independent of frame of reference.
Hence, the correct option is (a).

5. The necessary and sufficient condition for a surface to be called as a free surface is **[2006]**
(a) no stress should be acting on it.
(b) tensile stress acting on it must be zero.
(c) shear stress acting on it must be zero.
(d) no point on it should be under any stress.

Solution: (c)
Free surface is the surface subjected to constant-normal stress and zero-tangential stress. The necessary and sufficient condition for a surface to be called as 'free surface' is shear stress acting on it must be 0.
Hence, the correct option is (c).

6. Mohr's circle for the state of stress defined by $\begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix}$ MPa is a circle with **[2006]**
(a) center at (0, 0) and radius 30 MPa
(b) center at (0, 0) and radius 60 MPa
(c) center at (30, 0) cand radius 30 MPa
(d) center at (30, 0)and zero radius

Solution: (d)

$$I = \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix}$$

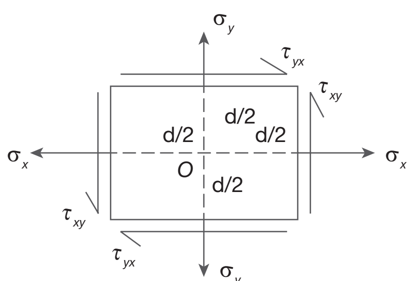
$$\sigma_1 = 30 \text{ MPa} \quad \sigma_2 = 30 \text{ MPa}$$

Radius of the Mohr's circle = 0
Centre of Mohr's circle = (30, 0)

Hence, the correct option is (d).

7. The symmetry of stress tensor at a point in the body under equilibrium is obtained from **[2005]**
(a) conservation of mass.
(b) force equilibrium equations.
(c) moment equilibrium equations.
(d) conservation of energy.

Solution: (c)
The symmetry of stress tensor at a point in the body under equilibrium is obtained from moment equilibrium equations.



Taking moments of all forces about the centre O,

$$\tau_{yx}\frac{d}{2}+\tau_{yx}\frac{d}{2}=\tau_{xy}\frac{d}{2}+\tau_{xy}\frac{d}{2}$$

$$\tau_{xy}=\tau_{yx}$$

Hence, the correct option is (c).

8. The components of strain tensor at a point in the plane strain case can be obtained by measuring longitudinal strain in which of the following directions? **[2005]**
(a) Along any two arbitrary directions
(b) Along any three arbitrary directions
(c) Along two mutually orthogonal directions
(d) Along any arbitrary direction

Solution: (b)
The components of strain tensor at a point in the plane strain case can be obtained by measuring longitudinal strain along any three arbitrary directions.
Hence, the correct option is (b).

9. Pick the incorrect statement from the following four statements. **[2000]**
(a) On the plane which carries maximum normal stress, the shear stress is 0.
(b) Principal planes are mutually orthogonal.
(c) On the plane which carries maximum shear stress, the normal stress is 0.
(d) The principal stress axes and principal strain axes coincide for an isotropic material.

Solution: (c)
Maximum normal stress is equal to the major principal stress. On the plane in which major principal stress acts, the shear stress is zero. Option 'a' is true.
Principal planes are mutually orthogonal. $\theta_1 = \theta_2 \pm 90°$. Option 'b' is true

$$\text{Maximum shear stress, } \tau_{max}=\frac{\sigma_1-\sigma_2}{2}$$

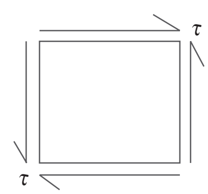
On the plane of maximum shear stress, the normal stress need not be zero. Option 'c' is false.
For an isotropic material, principal stress axes and principal strain axes coincide. Option 'd' is true.
Hence, the correct option is (c).

10. Two perpendicular axes x and y of a section are called principal axes when **[1999]**
(a) Moments of inertia about the axes are equal ($I_x = I_y$)
(b) Product moment of inertia (I_{xy}) is 0
(c) Product moments of inertia ($I_x \times I_y$) is 0
(d) Moments of inertia about one of the axes is greater than the other

Solution: (b)
Principal axes are the two mutually perpendicular axes in which the product of inertia is equal to zero ($I_{xy} = 0$). Along the principal axes, one of the moment of inertia is maximum and the other is minimum.
Hence, the correct option is (b).

11. If an element of a stressed body is in a state of pure shear with a magnitude of 80 N/mm^2, the magnitude of maximum principal stress at that location is **[1999]**
(a) 80 N/mm^2 (b) 113.14 N/mm^2
(c) 120 N/mm^2 (d) 56.57 N/mm^2

Solution: (a)



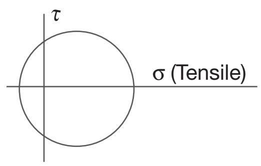
Shear stress, $\tau = 80$ N/mm^2
Normal stress in x direction, $\sigma_x = 0$
Normal stress in y direction, $\sigma_y = 0$
Maximum principle stress,

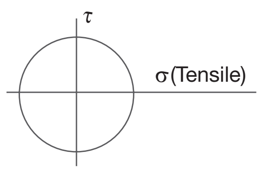
$$\sigma_1=\frac{\sigma_x+\sigma_y}{2}+\frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}$$

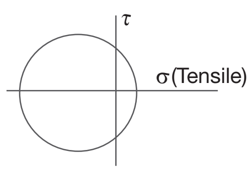
$$\sigma_1 = 80\text{ N/mm}^2$$

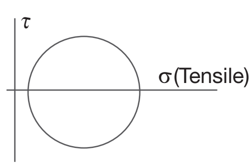
Hence, the correct option is (a).

12. Which of the following Mohr's circles qualitatively correctly represents the state of plane stress at a point in a beam above the neutral axis, where it is subjected to combined shear and bending compressive stresses? **[1993]**

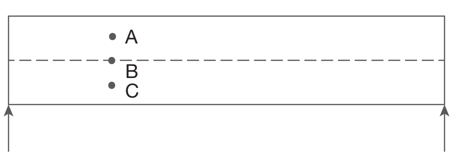
(a)


(b)


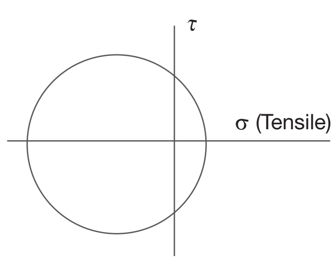
(c)


(d)


Solution: (c)

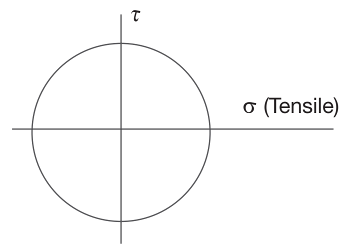


Point A lies above the neutral axis which is subjected to combined shear and bending compressive stress.



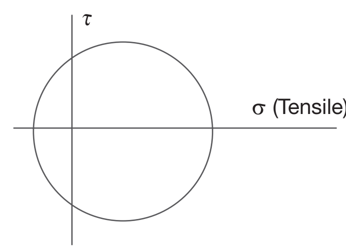
Mohr crcle for point A

Point B lies on the neutral axis which is subjected to only shear stress.



Mohr crcle for point B

Point C lies below the neutral axis which is subjected to combined shear and bending tensile stresses.

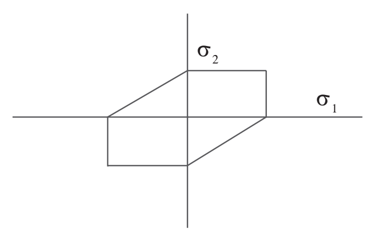


Mohr crcle for point C

Hence, the correct option is (c).

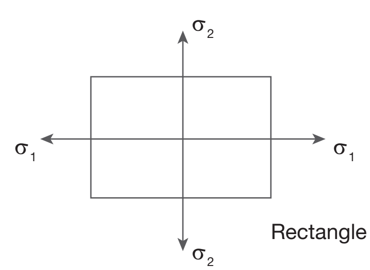
13. A failure theory postulated for metals is shown in a two dimensional stress plane. The theory is called **[1991]**

(a) Maximum Distortion Energy Theory
(b) Maximum Normal Stress Theory
(c) Maximum Shear Stress Theory
(d) Maximum Strain Theory

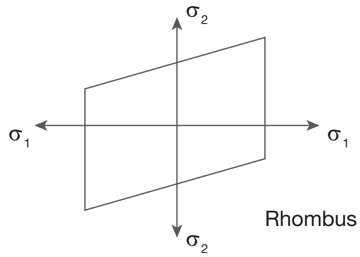


Solution: (c)
The failure theory and associated failure planes are shown in figures.

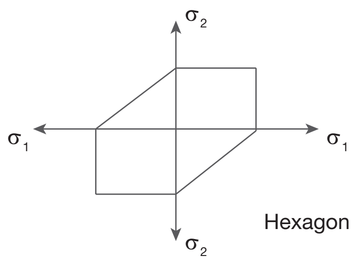
1. Maximum normal stress theory



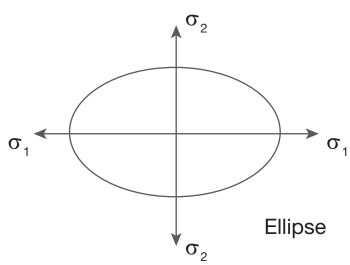
2. Maximum strain theory



3. Maximum shear stress theory



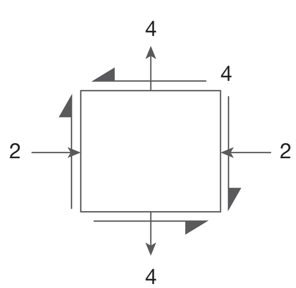
4. Maximum strain energy theory/maximum shear strain energy theory/distortion energy theory.



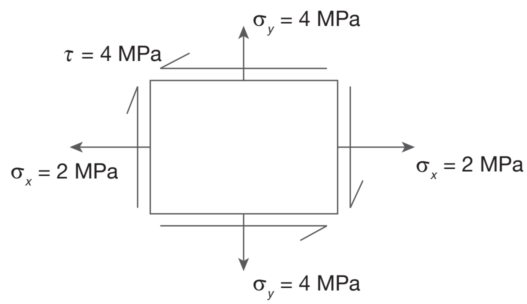
Hence, the correct option is (c).

Two-marks Questions

1. For the state of stresses (in MPa) shown in the figure, the maximum shear stress (in MPa) is **[2014]**



Solution: 5



The stresses acting on an element are:

$$\sigma_x = 2\text{ MPa}\,(C), \sigma_y = 4\text{ MPa}\,(T), \tau = 4\text{ MPa}$$

Maximum shear stress, $\tau_{\max} = \dfrac{\sigma_1 - \sigma_2}{2}$

$$= \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{(-2-4)^2 + 4\times 4^2} = \frac{1}{2}\sqrt{36+64} = 5\text{ MPa}$$

Hence, the answer is 5.

2. The state of 2D-stress at a point is given by the following matrix of stress **[2013]**

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix} = \begin{bmatrix} 100 & 30 \\ 30 & 20 \end{bmatrix} \text{MPa}$$

What is the magnitude of maximum shear stress in MPa?

(a) 50 (b) 75
(c) 100 (d) 110

Solution: (a)

The state of 2D stress at a point is given by

$$\begin{bmatrix} \sigma_{\text{xx}} & \sigma_{\text{xy}} \\ \sigma_{\text{xy}} & \sigma_{\text{yy}} \end{bmatrix} = \begin{bmatrix} 100 & 30 \\ 30 & 20 \end{bmatrix} \text{MPa}$$

Maximum shear stress,

$$\tau_{\max} = \pm\frac{\sigma_1 - \sigma_2}{2} = \frac{\sqrt{(\sigma_x - \sigma_y)^2 4\tau^2}}{2}$$

$$= \frac{1}{2}\sqrt{(100-20)^2 + 4\times 30^2} = \frac{1}{2}\sqrt{6400+3600}$$

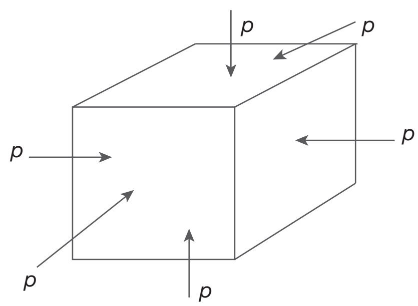
$$= \frac{1}{2}\times 100 = 50\text{ N/mm}^2$$

Hence, the correct option is (a).

3. If a small concrete cube is submerged deep in still water in such a way that the pressure exerted on all faces of the cube is p, then maximum shear stress developed inside the cube is **[2012]**

(a) 0 (b) $\frac{p}{2}$
(c) p (d) $2p$

Solution: (a)



$$\sigma_x = \sigma_y = \sigma_z = p$$

Since, only normal forces are acting, the shear stress $\tau = 0$.

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma - \sigma}{2} = 0$$

Hence, the correct option is (a).

4. The major and minor principal stresses at a point are 3 MPa and –3 MPa, respectively. The maximum shear stress at the point is **[2010]**

(a) 0 (b) 3 MPa
(c) 6 MPa (d) 9 MPa

Solution: (b)

Major principal stress, $\sigma_1 = 3$ MPa
Minor principal stress, $\sigma_3 = -3$ MPa
Maximum shear stress,

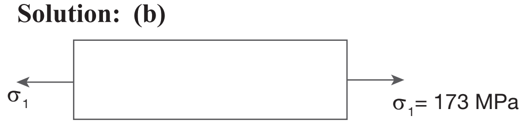
$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{3-(-3)}{2} = 3 \text{ MPa}$$

Hence, the correct option is (b).

5. An axially loaded bar is subjected to a normal stress of 173 MPa. The shear stress in the bar is **[2007]**

(a) 75 MPa (b) 86.5 MPa
(c) 100 MPa (d) 122.3 MPa

Solution: (b)



Normal stress in x direction, $\sigma_1 = 173$ MPa

Normal stress in y direction, $\sigma_1 = 0$
Maximum shear stress,

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{173-0}{2} = 86.5 \text{ MPa}$$

Hence, the correct option is (b).

6. If principal stresses in a two-dimensional case are −10 MPa and 20 MPa respectively, then maximum shear stress at the point is **[2005]**

(a) 10 MPa (b) 15 MPa
(c) 20 MPa (d) 30 MPa

Solution: (b)

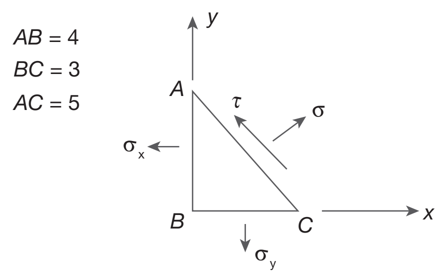
Major Principal stress, $\sigma_1 = 20$ MPa
Minor Principal stress, $\sigma_3 = -10$ MPa

Maximum shear stress, $\tau_{\max} = \dfrac{\sigma_1 - \sigma_3}{2}$

$$\tau_{\max} = \frac{20-(-10)}{2} = 15 \text{ MPa}$$

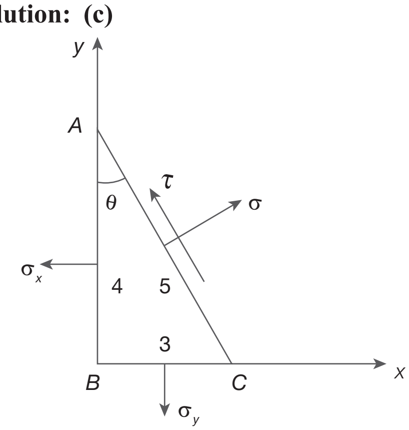
Hence, the correct option is (b).

7. In a two dimensional stress analysis, the state of stress at a point is shown below. If $s = 120$ MPa and $t = 70$ MPa, then s_x and s_y are respectively **[2004]**



(a) 26.7 MPa and 172.5 MPa
(b) 54 MPa and 128 MPa
(c) 67.5 MPa and 213.3 MPa
(d) 16 MPa and 138 MPa

Solution: (c)



$\sigma = 120$ MPa $\quad \tau = 70$ MPa

$\sin\theta = \dfrac{3}{5}, \quad \cos\theta = \dfrac{4}{5}, \quad \tan\theta = \dfrac{3}{4}$

Considering the horizontal equilibrium,

$$\sigma_x AB = AC(\sigma\cos\theta - \tau\sin\theta)$$

$$\sigma_x \times 4 = 5\left(120\times\frac{4}{5} - 70\times\frac{3}{5}\right)$$

$$\sigma_x = 67.5 \text{ MPa}$$

Considering the vertical equilibrium,

$$\sigma_y BC = AC(\sigma\sin\theta + \tau\cos\theta)$$

$$\sigma_y \times 3 = 5\left(120\times\frac{3}{5} + 70\times\frac{4}{5}\right)$$

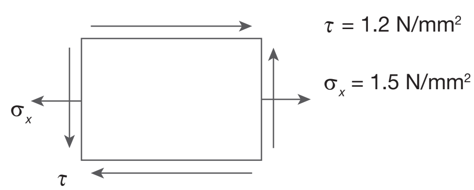
$$\sigma_y = 213.3 \text{ MPa}$$

Hence, the correct option is (c).

8. The state of two dimensional stress acting on a concrete lamina consists of a direct tensile stress, σ_x = 1.5 N/mm^2, and shear stress τ = 1.20 N/mm^2, which cause cracking of concrete. Then the tensile strength of the concrete in N/mm is **[2003]**

(a) 1.5 (b) 2.08
(c) 2.17 (d) 2.29

Solution: (c)



Direct tensile stress, σ_x = 1.5 N/mm^2
Shear stress, τ = 1.20 N/mm^2
The major and minor principle stresses are given by

$$\sigma_{1,3} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

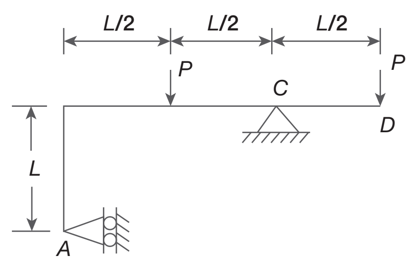
$$= \frac{1.5}{2} \pm \frac{1}{2}\sqrt{(1.5)^2 + 4(1.2)^2}$$

$$= 0.75 \pm \frac{1}{2}\sqrt{2.25 + 5.76} = 0.75 \pm 1.42$$

$$\sigma_1 = 0.75 \pm 1.42 = 2.17 \text{ N/mm}^2$$

Hence, the correct option is (c).

9. A frame ABCD is supported by a roller at A and is on a hinge at C as shown in the figure: **[2000]**



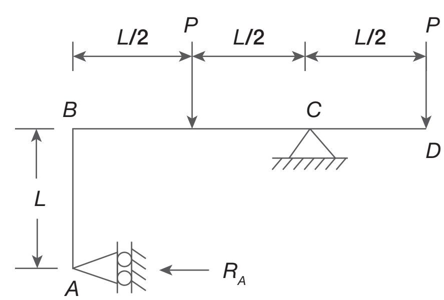
The reaction at the roller end A is given by

(a) P (b) $2P$
(c) $\dfrac{P}{2}$ (d) 0

Solution: (d)
Let R_A = Reaction of the roller support A.
Taking moments of all forces about the hinge C,

$$\sum M_c = 0 \Rightarrow R_A L - P\frac{L}{2} + P\frac{L}{2} = 0$$

$$R_A = 0$$



Hence, the correct option is (d).

Chapter 3

Shear Force and Bending Moment

ONE-MARK QUESTIONS

1. If the shear force at a section of beam under bending is equal to 0 then the bending moment at the section is _______________. **[1995]**

(a) zero (b) maximum
(c) minimum (d) constant

Solution: (d)
The relationship between shear force and bending moment is

$$F = \frac{dM}{dx}$$

For zero, shear force at a section,

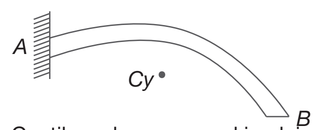
$$\frac{dM}{dx} = 0$$

Integrating wrt x, M = constant.
Hence, the correct option is (d).

2. A cantilever beam curved in plan is subjected to lateral loads will develop at any section **[1994]**

(a) bending moment and shearing force.
(b) bending moment and twisting moment.
(c) twisting moment and shearing force.
(d) bending moment, twisting moment and shearing force.

Solution: (d)



Cantilever beam curved in plain

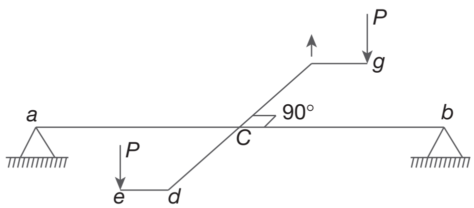
When the cantilever beam cured in plan is subjected to transverse load, the longitudinal axis of the beam does not coincides with the centre of gravity of the transverse loads and hence the beam at any section is subjected to torsion in addition to shear force and bending moment.
Hence, the correct option is (d).

3. In a real-beam, at an end, the boundary condition of zero-slope and zero-vertical displacement exists. In the corresponding conjugate beam, the boundary conditions at this end will be **[1992]**

(a) shear force = 0 and bending moment = 0.
(b) slope = 0 and vertical displacement = 0.
(c) slope = 0 and bending moment = 0.
(d) shear force = 0 and vertical displacement = 0.

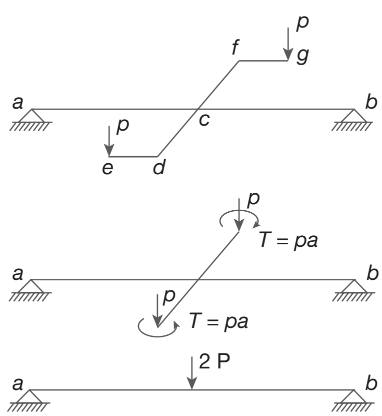
Solution: (a)
The slope in a real-beam at a point is equal to the shear force in the conjugate beam at the same point. The vertical deflection in a real-beam at a point is equal to the bending moment in conjugate beam at the same point.
Hence, the correct option is (a).

4. A beam having a double cantilever attached at mid span is shown in the figure. The nature of force in beam ab is **[1991]**



(a) Bending and shear
(b) Bending, shear and torsion
(c) Pure torsion
(d) Torsion and shear

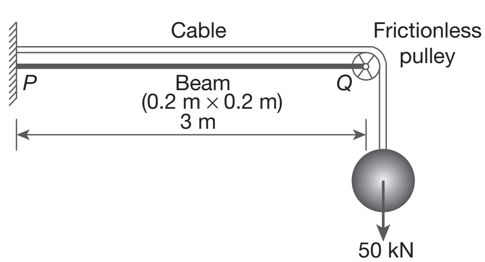
Solution: (a)



The beam ab is subjected to shear force and bending moment.
Hence, the correct option is (a).

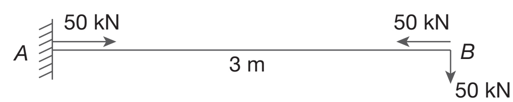
Two-marks Questions

1. The values of axial stress (σ) in kN/m^2, bending moment (M) in kNm, and shear force (V) in kN acting at point P for the arrangement shown in the figure are respectively, **[2014]**



(a) 1000, 75, and 25.
(b) 1250, 150, and 50.
(c) 1500, 225, and 75.
(d) 1750, 300, and 100.

Solution: (b)



Free body diagram of the beam

Size of the beam $= 0.2\text{ m} \times 0.2\text{ m}$

Axial force in beam,

$$F = 50\text{ kN (C)}$$

Axial stress at point P,

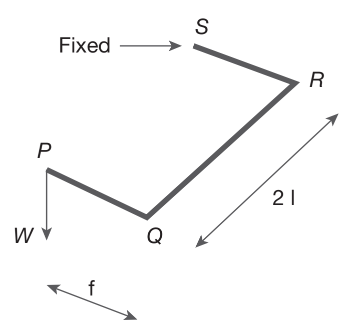
$$\sigma = \frac{F}{A} = \frac{50}{0.2 \times 0.2} = 1250\text{ kN/m}^2$$

Bending moment at P,

$$M_P = 50 \times 3 = 150\text{ kNm}$$

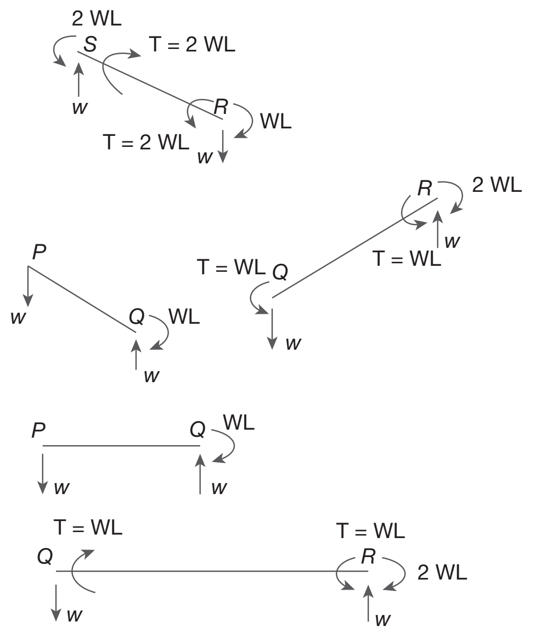
Shear force at P, Vp = 50 kN
Hence, the correct option is (b).

2. For the cantilever bracket, PQRS loaded as shown in the figure ($PQ = RS = L$, and, $QR = 2\,L$), which of the following statements is FALSE? **[2011]**



(a) The portion RS has a constant twisting moment with a value of 2 WL.
(b) The portion QR has a varying twisting moment with a maximum value of WL.
(c) The portion PQ has a varying bending moment with a maximum of WL.
(d) The portion PQ has no twisting moment.

Solution: (b)



Bar PQ: No twisting moment, varying BM with maximum value of WL.

Bar QR: Constant twisting moment of WL, varying bending moment with a maximum value of 2 WL.

Bar RS: Constant twisting moment of 2 WL, varying bending moment with a maximum value of 2 WL.

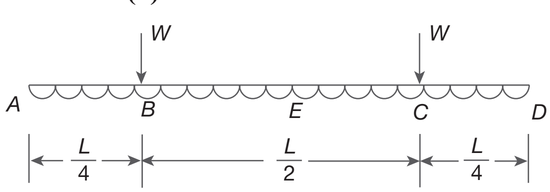
Hence, the correct option is (b).

3. Two people weighing W each are sitting on a plank of length L floating on water at L/4 from either end. Neglecting the weight of the plank, the bending moment at the centre of the plank is **[2010]**

(a) $\dfrac{WL}{8}$ (b) $\dfrac{WL}{16}$

(c) $\dfrac{WL}{32}$ (d) 0

Solution: (d)



The plank will be in equilibrium due to the buoyant force acting from the bottom. Taking the vertical equilibrium of plank,

$$W + W = wL; \qquad w = \frac{2W}{L}(\uparrow)$$

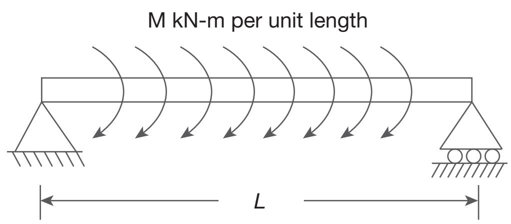
BM at the centre of the plank,

$$BM_E = \frac{2W}{L}\frac{L}{2}\frac{L}{4} - W\frac{L}{4},$$

$$= \frac{WL}{4} - \frac{WL}{4} = 0$$

Hence, the correct option is (d).

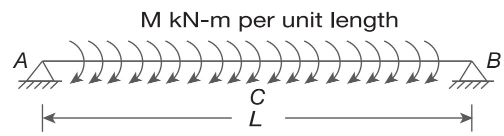
4. For the simply supported beam of length L, subjected to a uniformly distributed moment M kN-m per unit length as shown in the figure, the bending moment (in kN-m) at the mid-span of the beam is **[2010]**



(a) Zero (b) M

(c) ML (d) M/L

Solution: (a)



Taking moments of all forces about the hinge A,

$$\sum M_A = 0 \Rightarrow M.L - R_B.L = 0; \quad R_B = M(\uparrow)$$

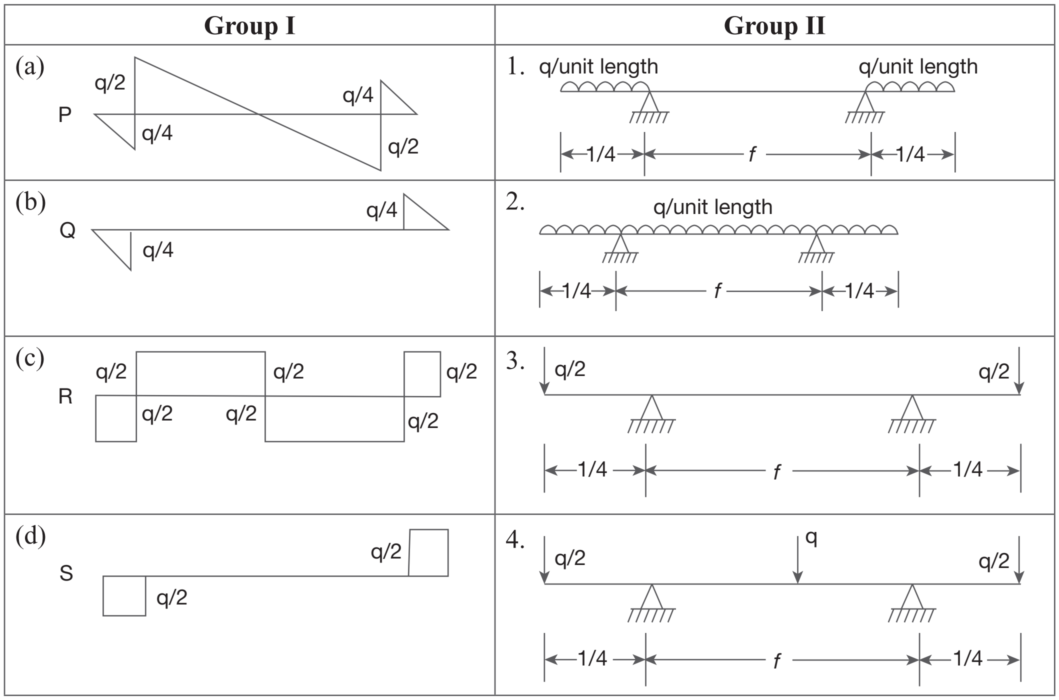
$$\sum V = 0 \Rightarrow R_A + R_B = 0; \qquad R_A = -M = M(\downarrow)$$

Bending moment at mid span,

$$M_C = -M\frac{L}{2} + M\frac{L}{2} = 0$$

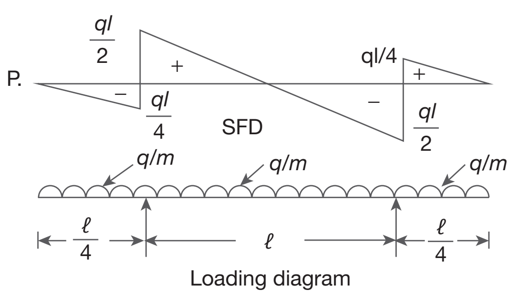
The bending moment of any point in the beam is 0.
Hence, the correct option is (a).

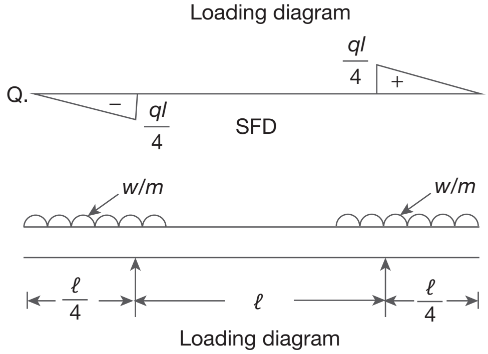
5. Group-I gives the shear force diagrams and Group-II gives the diagrams of beams with supports and loading shown in the figure. Match the Group-I with Group-II **[2009]**

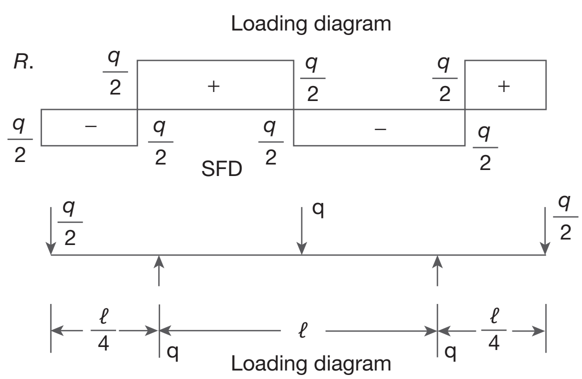


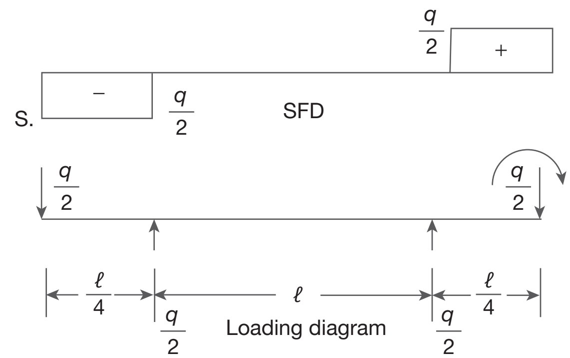
(a) P: 3; Q: 1; R: 2; S: 4
(b) P: 3; Q: 4; R: 2; S: 1
(c) P: 2; Q: 1; R: 4; S: 3
(d) P: 2; Q: 4; R: 3; S: 1

Solution: (a)



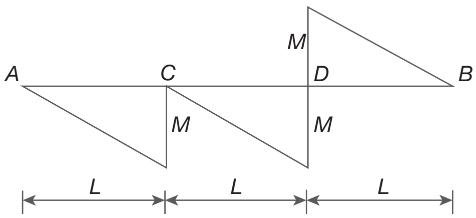






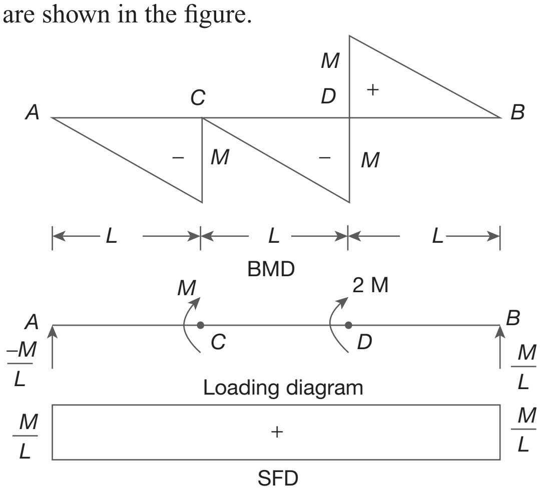
Hence, the correct option is (a).

6. A simply supported beam AB has the bending moment diagram as shown in the figure. The beam is possibly under the action of following loads: **[2006]**



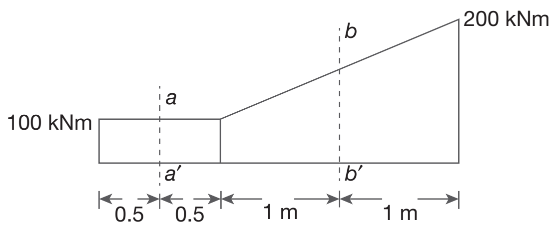
(a) Couples of M at C and $2M$ at D.
(b) Couples of $2M$ at C and M at D.
(c) Concentrated loads of M/L at C and $2M/L$ at D.
(d) Concentrated load of M/L at C and couple of $2\,M$ at D.

Solution: (a)
The shear force and bending loading diagrams corresponding to the given bending moment diagram are shown in the figure.



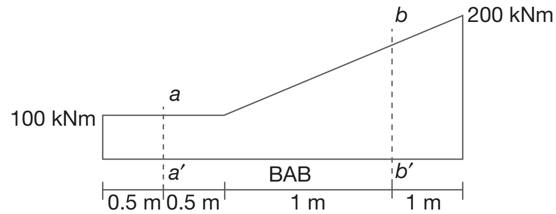
Hence, the correct option is (a).

7. The bending moment diagram for a beam shown in the figure. The shear force at sections 'aa' and 'bb' respectively are of the magnitude **[2005]**



(a) 100 kN, 150 kN (b) zero, 100 kN
(c) zero, 50 kN (d) 100 KN, 100 kN

Solution: (c)

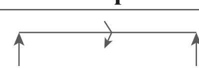
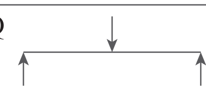
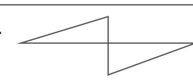
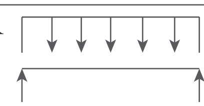
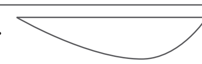
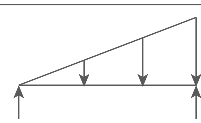
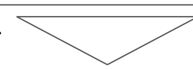
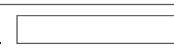


Shear force is the rate of change of bending moment. At section aa', the bending moment to the left and right is constant. Hence, shear force at aa' is 0.

Shear force at $bb' = \dfrac{200-100}{2} \times 1 = 50$ kN

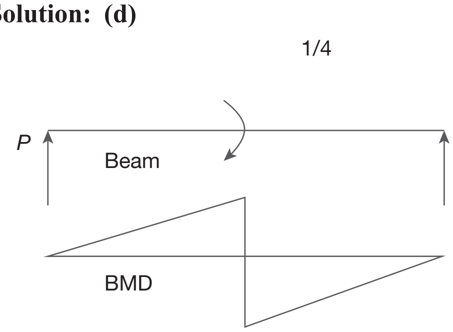
Shear force at $aa' = 0$
Shear force at $bb' = 50$ kN
Hence, the correct option is (c).

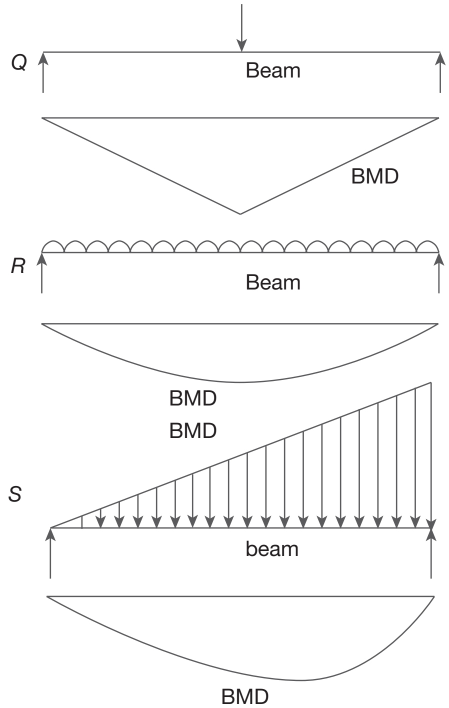
8. Group-I shows different loads acting on a beam and Group-II shows different bending moment distributions in the figure. Match the load with the corresponding bending moment diagram. **[2003]**

Group I	Group II
P 	1. 
Q 	2. 
R 	3. 
S 	4. 
	5. 

(a) P: 4; Q: 2; R:1; S: 3 (b) P: 5; Q: 4; R: 1; S: 3
(c) P: 2; Q: 5; R: 3; S: 1 (d) P: 2; Q: 4; R: 1; S: 3

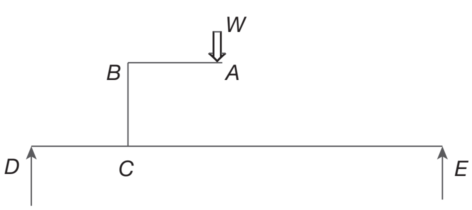
Solution: (d)





Hence, the correct option is (d).

9. For the loading given in the figure, two statements (I and II) are made. **[2002]**



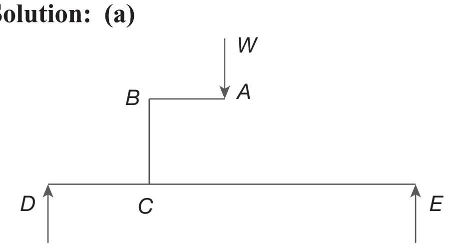
I. Member AB carries shear force and bending moment.

II. Member BC carries axial load and shear force.

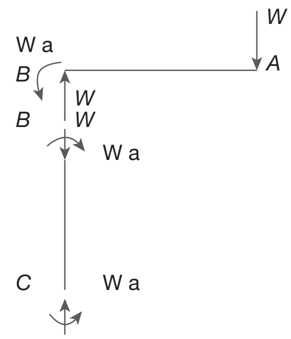
Which of the following is true?

(a) Statement I is true but II is false.
(b) Statement I is false but II is true.
(c) Both statements I and II are true.
(d) Both statements I and II are false.

Solution: (a)



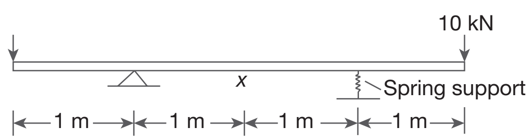
Free body diagrams



Free body diagrams

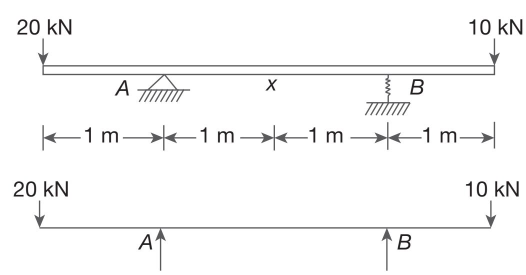
Member AB is subjected to shear force and bending moment.
Member BC is subjected to axial load and Bending moment.
Hence, the correct option is (a).

10. The bending moment (in kNm units) at the mid-span location X in the beam with overhangs shown in the figure is equal **[2001]**



(a) 0 (b) –10
(c) –15 (d) –20

Solution: (c)



Taking moments of all forces about the support B,

$$\sum M_B = 0 \Rightarrow -20 \times 3 + R_A \times 2 + 10 \times 1 = 0$$

$$R_A = 25 \text{ kN}(\uparrow)$$

Bending moment at X = –20 × 2 + 25 × 1 = –15 kNm (Hogging)
Hence, the correct option is (c).

Chapter 4

Simple Bending Theory

One-mark Questions

1. The first-moment of area about the axis of bending for a beam cross-section is ______________. **[2014]**

(a) section modulus
(b) moment of inertia
(d) shape factor
(c) polar moment of inertia

Solution: (a)

First moment of area is based on mathematical construct moments in metric spaces. First moment of area is equal to the summation of the product of area and its distance from an axis. $\sum_i A_i x_i$. It is a measure of the distribution of the area of a shape in relationship to an axis.

Section Modulus: It is a direct measure of the strength of the beam.

$$Z = \frac{I}{y}$$

I: Moment of inertia of the section about the neutral axis.

y: Distance of extreme fibre from the neutral axis.

Section modulus is the first moment of area about the axis of bending for a beam cross section. **Moment of Inertia:** Moment of Inertia about an axis is defined as the sum of the product of the area and square of its distance from the axis under consideration.

$$I_x = \int y^2\, dt \quad \text{(or)} \quad I = \sum_i A_i x_i^2$$

Moment of Inertia is a measure of an objects resistance to changes to in rotation direction.

Moment of inertia is the capacity of a cross section to resist bending.

Polar moment of Inertia: Polar moment of Inertia about an axis perpendicular to two mutually perpendicular axes is equal to the sum of moments of inertia about the axes.

Shape factor: Shape factor is defined as the ratio of plastic moment to yield moment.

$$S = \frac{M_P}{M_y} = \frac{\sigma_y Z_P}{\sigma_y Z} = \frac{Z_P}{Z}$$

Hence, the correct option is (a).

2. The dimensions for the flexural rigidity of a beam element in mass (M), length (L) and time (T) is given by **[2000]**

(a) MT^{-2} (b) ML^3T^{-2}
(c) $ML^{-1}T^{-2}$ (d) $ML^{-1}T^2$

Solution: (b)

Flexural rigidity = EI kN-m^2

Dimensions for $EI = MLT^{-2}L^2$

$= ML^3T^{-2}$

Hence, the correct option is (b).

3. The basic assumption of plane sections normal to the neutral axis before bending remaining plane and normal to the neutral axis after bending, leads to **[1995]**

(a) uniform strain over the beam cross-section.
(b) uniform stress over the beam cross-section.
(c) linearly varying strain over the cross-section.
(d) stresses, which are proportional to strains at the cross-section.

Solution: (c)

The basic assumption "plane sections normal to the neutral axis before bending remaining plane and normal to the neutral axis after bending" leads to linearly varying strain over the cross section.

Hence, the correct option is (c).

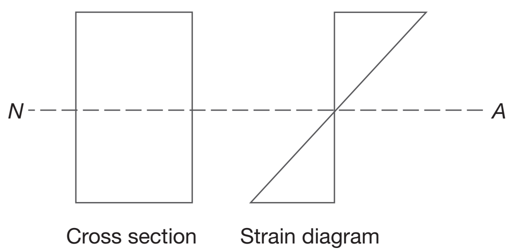
Two-marks Questions

1. The "plane section remains plane" assumption in bending theory implies **[2013]**

(a) strain profile is linear
(b) stress profile is linear
(c) both strain and stress profile are linear
(d) shear deformations are neglected

Solution: (a)

The plane section remains plane assumption in bending theory implies strain profile is linear.



Hence, the correct option is (a).

2. The following statements are related to bending of beams: **[2012]**

The slope of the bending moment diagram is equal to the shear force.

The slope of the shear force diagram is equal to the load intensity.

The slope of the curvature is equal to the flexural rotation.

The second derivative of the deflection is equal to the curvature.

The only FALSE statement is

(a) I (b) II
(c) III (d) IV

Solution: (c)

Slope of bending moment diagram is equal to shear force.

$$\frac{dM}{dx} = F$$

I. Statement I is True

II. Slope of shear diagram = load intensity,

$$\frac{dF}{dx} = w.$$

Statement II is True

III. Slope of the deflected curve is not equal to the flexural rotation.

Slope of the deflected curve is equal to slope. Statement III is False

IV. $$EI\frac{d^2y}{dx^2} = -M \qquad R.d\theta = dx$$

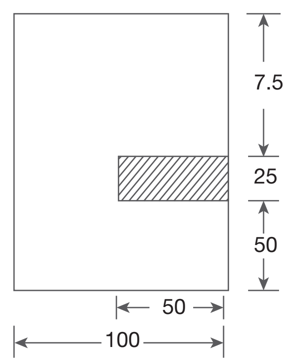
$$\frac{d\theta}{dx} = \frac{d^2y}{dx^2} = \frac{1}{R}$$

The second derivative of the deflection is equal to the curvature.

Statement IV is True

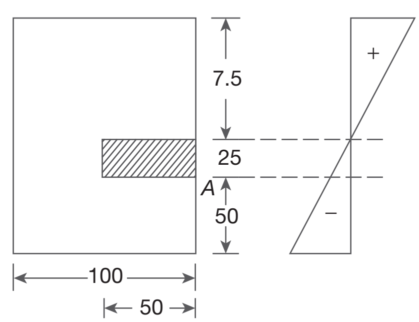
Hence, the correct option is (c).

3. A beam with the cross-section shown in the figure, is subjected to appositive bending moment (causing compression at the top) of 16 kN-m acting around the horizontal axis. The tensile force acting on the hatched area of the cross-section is **[2006]**



(a) 0 (b) 5.9 kN
(c) 8.9 kN (d) 17.8 kN

Solution: (c)



Maximum bending stress,

$$\sigma_{\max} = \frac{M}{I} y_{\max}$$

Maximum bending moment,

$$M = 16 \text{ kNm}$$

Moment of inertia of the section about NA,

$$I = \frac{100(150)^3}{12} = 28.125 \times 16^6 \text{ mm}^4$$

Extreme distance from NA, $y = 75$ mm

$$\sigma_{\max} = \frac{16 \times 10^6}{28.125 \times 10^6} \times 75 = 42.67 \text{ N/mm}^2$$

Bending tensile stress at A,

$$\sigma = \frac{42.67}{75} \times 25 = 14.22 \text{ N/mm}^2$$

Tensile force on the hatched area,

T = Cross sectional area × Average stress

$$= 50 \times 25 \times \frac{14.22}{2} = 8.888 \times 10^3 \text{ N} \approx 8.9 \text{ kN}$$

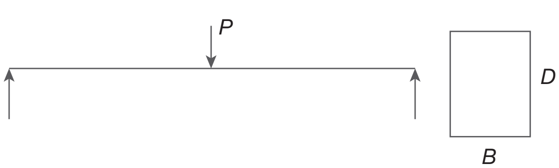
Hence, the correct option is (c).

5. A homogeneous simply supported prismatic beam of width B, depth D and span L is subjected to a concentrated load of magnitude P. The load can be placed anywhere along the span of the beam. The maximum flexural stress developed in beam is **[2004]**

(a) $\frac{2}{3}\frac{PL}{BD^2}$ (b) $\frac{3}{4}\frac{PL}{BD^2}$

(c) $\frac{4}{3}\frac{PL}{BD^2}$ (d) $\frac{3}{2}\frac{PL}{BD^2}$

Solution: (d)



Maximum bending moment will be developed at midspan, when the concentrated load P is placed at midspan.

Maximum bending moment,

$$M = \frac{PL}{4}$$

Maximum flexural stress,

$$\sigma = \frac{M}{I} y$$

Moment of inertia about NA,

$$I = \frac{BD^3}{12}$$

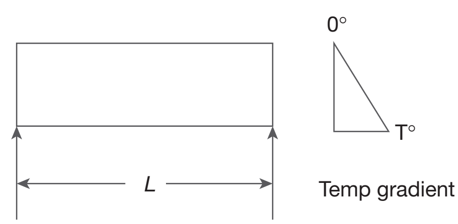
Extreme fibre distance from NA,

$$y = \frac{D}{2}$$

$$\sigma_{\max} = \frac{\frac{PL}{4}}{\frac{BD^3}{12}} \frac{D}{2} = \frac{3PL}{2BD^2}$$

Hence, the correct option is (d).

6. A simply supported beam of uniform rectangular cross-section of width b and depth h is subjected to linear temperature gradient, 0° at the top and T° at the bottom, as shown in the figure. The coefficient of linear expansion of the beam material is α. The resulting vertical deflection at the mid span of the beam is **[2003]**



(a) $\frac{\alpha Th^2}{8L}$ upward

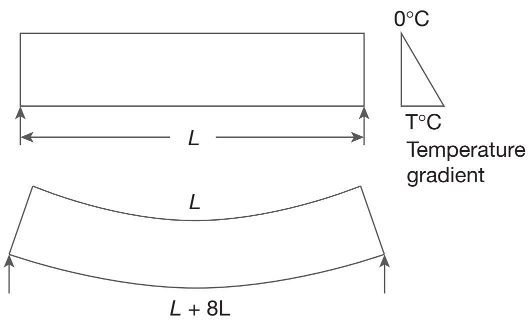
(b) $\frac{\alpha TL^2}{8h}$ upwards

(c) $\frac{\alpha Th^2}{8L}$ downwards

(d) $\frac{\alpha TL^2}{8h}$ downwards

Solution: (d)

Average change in temperature = $T/2$



Compression in the top fibre $= L\alpha \dfrac{T}{2}$

Elongation in the bottom fibre $= L\alpha \dfrac{T}{2}$

Temperature strain, $e_0 = \dfrac{L\alpha T}{L2} = \dfrac{\alpha T}{2}$

The deflection at mid point is downward.

The bending equation is $\dfrac{M}{I} = \dfrac{\sigma}{y} = \dfrac{E}{R}$

Curvature, $\dfrac{1}{R} = \dfrac{\sigma}{Ey}$

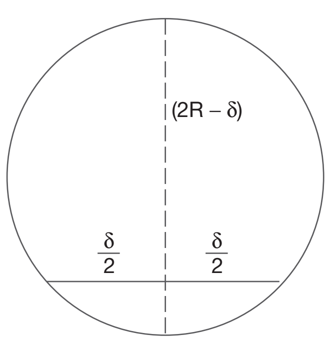
$$= \frac{\text{Strain}}{y} = \frac{2e_0}{h} = \frac{\alpha . T}{h} \quad \left(\because y = \frac{h}{2}\right)$$

From property of circle,

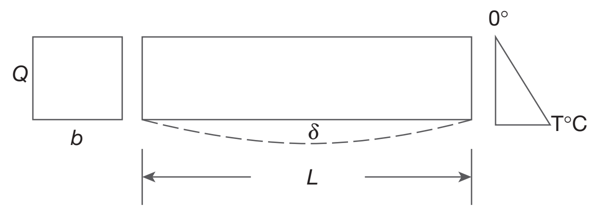
$$\frac{L}{2}\frac{L}{2} = \delta(2R - \delta)$$

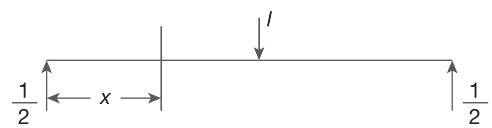
$$\frac{L^2}{4} = 2\delta R - \delta^2$$

$$\delta = \frac{L^2}{8R} = \frac{L^2}{8}\frac{\alpha T}{h} = \frac{\alpha T L^2}{8h} \text{(downwards)}$$



(OR)





Vertical deflection at midspan,

$$\delta = \int \frac{Mm}{EI} dx$$

Bending equation is

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\frac{1}{R} = \frac{M}{EI} = \frac{\sigma}{Ey} = \frac{\text{strain}}{y} = \frac{2e_0}{h}$$

Temperature strain,

$$e_0 = \frac{L.\alpha.T}{L.2} = \frac{\alpha.T}{2}$$

Therefore,

$$\frac{1}{R} = \frac{M}{EI} = \frac{\alpha T}{h}$$

Moment at a distance x due to unit load,

$$m = \frac{1}{2}x$$

$$\delta = 2\int_0^{d/2} \frac{\alpha T}{h}\frac{x}{2} dx = \frac{\alpha T}{2h}\left[x^2\right]_0^{d/2} = \frac{\alpha T}{2h} d^2 = \frac{\alpha T L^2}{8h}$$

Hence, the correct option is (d).

7. The maximum bending stress induced in a steel wire of modulus of elasticity 200 kM/mm^2 and diametre 1 mm, when wound on a drum of diametre 1 m is approximately equal to **[1992]**

(a) 50 N/mm^2 (b) 100 N/mm^2

(c) 200 N/mm^2 (d) 400 N/mm^2

Solution: (c)

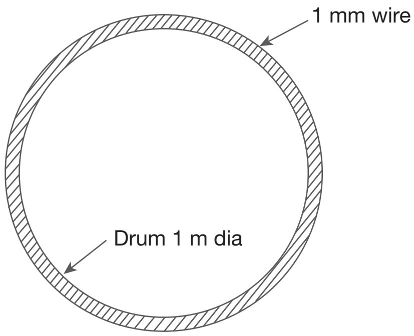
Modulus of elasticity of steel, $E = 200$ kN/mm^2

Diameter of the wire, $d = 1$ mm^2

Diameter of drum over which wire is winding = 1 m

Radius of the bent wire, $R = 0.5$ m

Extreme fibre distance, $y = 0.5$ mm



Bending equation is

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Maximum bending stress,

$$\sigma = \frac{E}{R}.y$$

$$\sigma = \frac{200 \times 10^3}{0.5 \times 10^3} \times 0.5 = 200 \text{ N/mm}^2$$

Hence, the correct option is (c).

Chapter 5

Shear Stresses in Beams

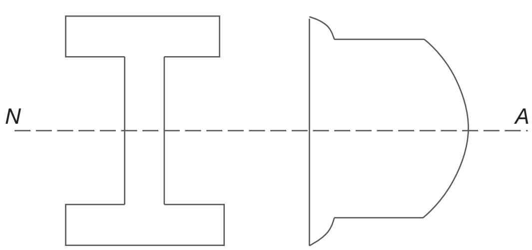
ONE-MARK QUESTION

1. For a given shear force across a symmetrical I section, the intensity of shear stress is maximum at the **[1991, 1994]**

(a) extreme fibres.
(b) centroid of the section.
(c) at the junction of the flange and the web, but on the web.
(d) at the junction of the flange and the web, but on the flange.

Solution: (b)

The shear stress distribution across the cross section of a symmetrical I section is shown in figure. The maximum shear stress occurs at centroid of the section.



Cross section   Shear stress distribution

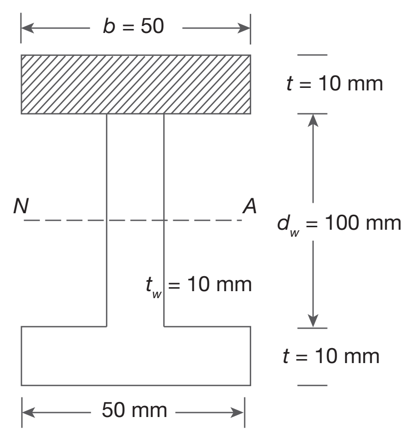
Hence, the correct option is (b).

TWO-MARKS QUESTIONS

1. A symmetric I-section (with width of each flange = 50 mm, thickness of each flange = 10 mm, depth of web = 100 mm, and thickness of web = 10 mm) of steel is subjected to a shear force of 100 kN. Find the magnitude of the shear stress (in N/mm²) in the web at its junction with the top flange ________. **[2013]**

Solution: 71.12

Width of flange, b = 50 mm
Thickness of flange, t = 10 m
Depth of web, d_w = 100 mm
Thickness of web, t_w = 10 mm
Shear force, F = 100 kN



Cross section

Shear stress at any distance from neutral axis,

$$\tau = \frac{F(A\bar{y})}{Ib}$$

Moment of the area above the point considered about neutral axis,

$$(A\bar{y}) = 50 \times 10 \times 55 = 275 \times 10^2 \text{ mm}^3$$

Moment of inertia of the section about neutral axis,

$$I = \frac{10\times100^3}{12} + 2\left[\frac{50\times10^3}{12} + 50\times10\times55^2\right]$$
$$= 833.3\times10^3 + 3033.3\times10^3 = 3866.6\times10^3\ \text{mm}^4$$

Width of section under consideration, $b = 10$ mm

$$\tau = \frac{100\times10^3\times275\times10^2}{3866.6\times10^3\times10} = 71.12\ \text{N/mm}^2$$

Hence, the answer is 71.12 N/mm².

2. The shear stress at the neutral axis in a beam of triangular section with a base of 40 mm and height 20 mm, subjected to a shear force of 3 kN is **[2007]**

(a) 3 MPa (b) 6 MPa
(c) 10 MPa (d) 20 MPa

Solution: (c)

Shear stress at neutral axis, $\tau_{NA} = ?$

Base of triangular section, $B = 40$ mm
Height of triangular section, $H = 20$ mm
Shear force, $F = 3$ kN
Distance of centroid from the apex

$$= \frac{2}{3}\times20 = 13.33\ \text{mm}$$

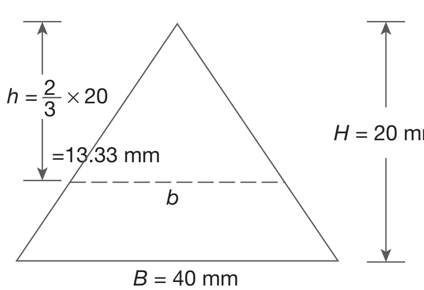
Width of the section at the level where shear stress is desired is given by

$$\frac{40}{20} = \frac{b}{13.33}; \qquad b = 26.67\ \text{mm}$$

Shear stress: $\tau = \dfrac{FA\bar{y}}{I.b}$

Moment of the area above the section under consideration about NA,

$$A\bar{y} = \frac{1}{2}\times26.67\times13.33\times\frac{1}{3}\times13.33 = 789.8\ \text{mm}^3$$



Moment of inertia of the section about NA,

$$I = \frac{40\times20^3}{36} = 8888.9\ \text{mm}^3$$

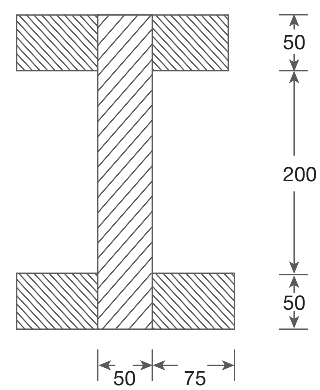
$$\tau_{NA} = \frac{3\times10^3\times789.8}{8888.9\times26.67} = 10\ \text{N/mm}^2$$

or

$$\tau_{NA} = \frac{4}{3}\tau_{avg}$$
$$= \frac{4}{3}\frac{F}{A} = \frac{4}{3}\frac{3\times10^3}{\frac{1}{2}\times40\times20} = 10\ \text{N/mm}^2$$

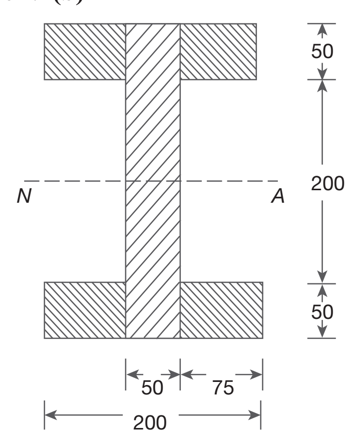
Hence, the correct option is (c).

3. T-section of a beam is formed by gluing wooden planks as shown in the figure. If this beam transmits a constant vertical shear force of 3000 N, the glue at any of the four joints will be subjected to a shear force (in kN per metre length) of **[2006]**



(a) 3.0 (b) 4.0
(c) 8.0 (d) 10.7

Solution: (b)



Shear stress, $\tau = \dfrac{FA\bar{y}}{Ib}$

Shear flow, $q = \tau b = \dfrac{F(A\bar{y})}{I}$

Shear force, $F = 30000$ N

Moment of inertia of the section about NA,

$$I = \frac{50\times 300^3}{12} + 2\left[\frac{150\times 50^3}{12} + 150\times 50\times 125^2\right]$$

$$= (1.125 + 2.375)\times 10^8 = 3.5\times 10^8 \text{ mm}^4$$

For any of the four joints,

$$A\bar{y} = 75\times 50\times 125 = 468750 \text{ mm}^3$$

$$q = \frac{3000\times 468750}{3.5\times 10^8} = 4.0 \text{ N/mm} = 4.0 \text{ kN/m}$$

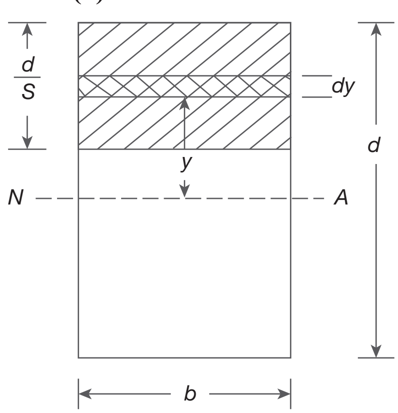
Hence, the correct option is (b).

4. If a beam of rectangular cross-section is subjected to a vertical shear force V, the shear force carried by the upper one-third of the cross-section is **[2006]**

(a) zero (b) $\dfrac{7V}{27}$

(c) $\dfrac{8V}{27}$ (d) $\dfrac{V}{3}$

Solution: (b)



Let y be the distance of point under consideration from the neutral axis.

Shear stress,

$$\tau = \frac{FA\bar{y}}{Ib}$$

$$\tau = \frac{V\left(\frac{d}{2} - y\right)b\left(\frac{d/2 + y}{2}\right)}{Ib} = \frac{V}{2I}\left(\frac{d^2}{4} - y^2\right)$$

Shear force carried beyond y distance from NA,

$$dF = \tau b dy$$

$$= \frac{Vb}{2I}\left(\frac{d^2}{4} - y^2\right)dy$$

$$F = \frac{Vb}{2I}\int_{d/6}^{d/2}\left(\frac{d^2}{4} - y^2\right)dy = \frac{Vb}{2I}\left[\frac{d^2}{4}y - \frac{y^3}{3}\right]_{d/6}^{d/2}$$

$$= \frac{Vb}{2I}\left[\frac{d^3}{8} - \frac{d^3}{24} - \frac{d^3}{24} + \frac{d^3}{648}\right] = \frac{Vb}{2.bd^3}\times 12\frac{28}{648}.d^3$$

$$F = \frac{7}{27}V$$

Hence, the correct option is (b).

Chapter 6

Deflection of Beams

One-mark Questions

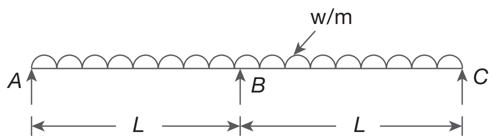
Statement for Linked Questions 1 and 2:
A two span continuous beam having equal spans each of length L is subjected to a uniformly distributed load w per unit length. The beam has constant flexural rigidly.

1. The reaction at the middle support is __________. **[2007]**

(a) wL (b) $\frac{5wL}{2}$

(c) $\frac{5wL}{4}$ (d) $\frac{5wL}{8}$

Solution: (c)



Since, the supports at A, B, and C are at the same level, the deflection at supports is equal to 0.
Downward deflection at B due to u.d.l removing the support at B,

$$\delta_{B1} = \frac{5}{384}\frac{w(2L)^4}{EI}$$

Upward deflection due to support reaction at B removing the u.d.l on the Beam,

$$\delta_{B2} = \frac{1}{48}\frac{R_B(2L)^4}{EI}$$

$$\delta_B = \delta_{B1} - \delta_{B2} = 0$$

$$\frac{5}{384}\frac{w(2L)^4}{EI} = \frac{1}{48}\frac{R_B.(2L)^3}{EI}$$

$$R_B = \frac{5}{8}w2L;\ R_B = \frac{5}{4}wL\ (\uparrow)$$

Hence, the correct option is (c).

2. The bending moment at the middle support is

(a) $\frac{wL^2}{4}$ (b) $\frac{wL^2}{8}$

(c) $\frac{wL^2}{12}$ (d) $\frac{wL^2}{16}$

Solution: (b)
Since, the beam is loaded symmetrically, the reactions at A and C is given by

$$R_A = R_C = \frac{w2L - \frac{5}{4}wL}{2} = \frac{3wL}{8}$$

Bending moment at B,

$$M_B = \frac{3}{8}wLL - wL\frac{L}{2} = -\frac{wL^2}{8}$$

$$M_B = \frac{wL^2}{8}\ \text{(Hogging)}$$

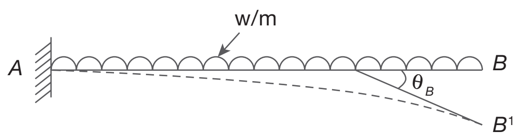
Hence, the correct option is (b).

3. The slope of the elastic curve at the free end of a cantilever beam of span L and with flexural rigidity EI, subjected to uniformly distributed load of intensity w is **[1999]**

(a) $\dfrac{wL^3}{6EL}$ (b) $\dfrac{wL^3}{3EL}$

(c) $\dfrac{wL^4}{8EL}$ (d) $\dfrac{wL^3}{2EL}$

Solution: (a)

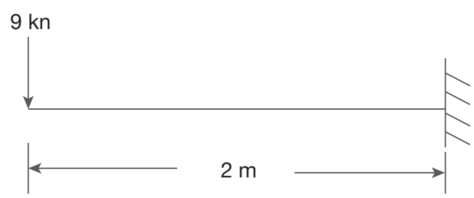


Slope at the free end of cantilever,

$$\theta_B = \frac{wL^3}{6EI}$$

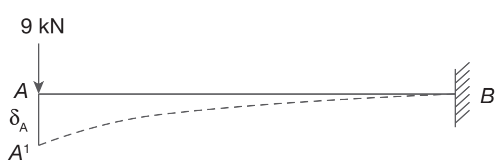
Hence, the correct option is (a).

4. A cantilever beam is shown in the figure. The moment to be applied at free end for 0 vertical deflection at that point is **[1998]**



(a) 9 kNm clockwise
(b) 9 kNm anticlockwise
(c) 12 kNm clockwise
(d) 12 kNm anticlockwise

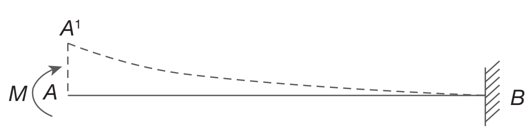
Solution: (c)



Downward deflection due to load at A,

$$\delta_A = \frac{W \times L^3}{3EI} = \frac{9 \times 2^3}{3EI} = \frac{24}{EI}$$

A moment M is to be applied at the free end so as to give upward deflection as shown in the figure.



Upward deflection at A due to clockwise moment M,

$$\delta'_A = \frac{ML^2}{2EI}$$

$$= \frac{M \times 2^2}{2EI} = \frac{2M}{EI}$$

Given

$$\delta_A = 0 \Rightarrow \delta_A + \delta'_A = 0; \quad \frac{24}{EI} - \frac{2M}{EI} = 0$$

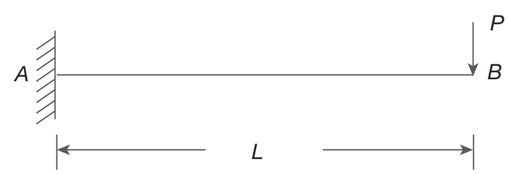
$M = 12$ kNm clockwise.
Hence, the correct option is (c).

5. A cantilever beam of span L is loaded with a concentrated load P at the free end. Deflection of the beam at the free end is **[1997]**

(a) $\dfrac{PL^3}{48EI}$ (b) $\dfrac{5PL^3}{384EI}$

(c) $\dfrac{PL^3}{3EI}$ (d) $\dfrac{PL^3}{6EI}$

Solution: (c)

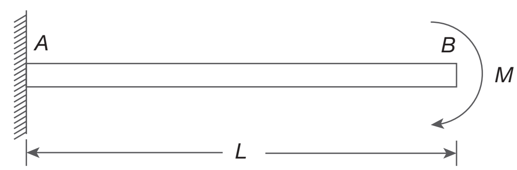


Deflection at free end,

$$\delta_B = \frac{PL^3}{3EI}$$

Hence, the correct option is (c).

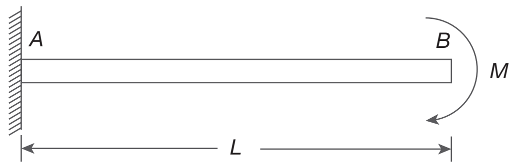
6. The deflection of cantilever beam at free end B applied with a moment M at the same point is **[1996]**



(a) $\dfrac{Ml^2}{EI}$ (b) $\dfrac{Ml^2}{2EI}$

(c) $\dfrac{Ml^2}{3EI}$ (d) $\dfrac{Ml^2}{4EI}$

Solution: (b)



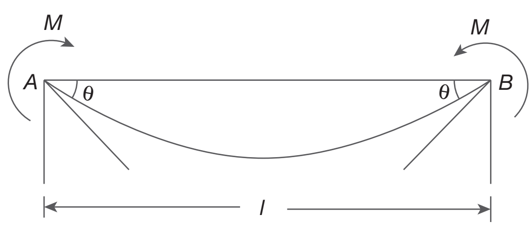
$\frac{M}{EI}$ diagram

Deflection at B = Moment of the area of $\frac{M}{E}$ diagram between A and B about B

$$\delta_B = \frac{M}{EI} L \frac{L}{2} = \frac{ML^2}{2EI}$$

Hence, the correct option is (b).

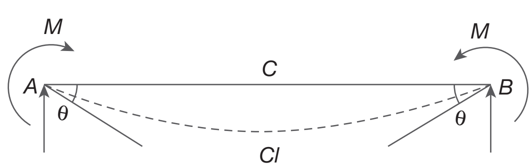
7. $M - \theta$ relationship for a simply supported beam is given by (shown in the figure) **[1996]**

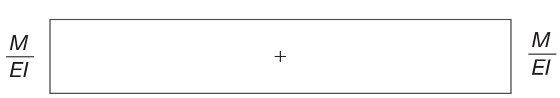


(a) $\frac{Ml}{EI} = 2\theta$ (b) $\frac{Ml}{EI} = 3\theta$

(c) $\frac{Ml}{EI} = 4\theta$ (d) $\frac{Ml}{EI} = 6\theta$

Solution: (a)





$\frac{M}{EI}$ diagram

According to moment area theorem, the angle in radians between two tangents drawn from the points on the elastic curve is equal to area of $\frac{M}{EI}$ diagram between those two points under consideration.

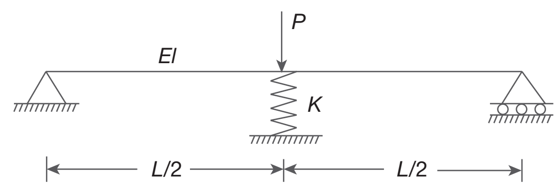
$$\theta = \text{Area of } \frac{M}{EI} \text{ diagram between A and C}$$

$$= \frac{M}{EI}\frac{L}{2} = \frac{ML}{2EI}$$

$$\frac{ML}{EI} = 2\theta$$

Hence, the correct option is (a).

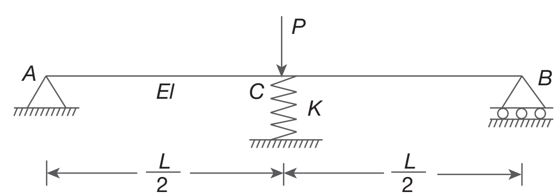
8. A simply supported beam of span length L and flexural stiffness EI has another spring support at the centre of stiffness K as shown in figure. The central deflection of the beam due to a central concentrated load of P would be **[1993]**



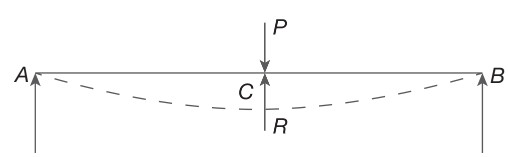
(a) $\frac{PL^3}{48EI + KL^3}$ (b) $\frac{P}{(48EI / L^3)} - K$

(c) $\left(\frac{PL^3}{48EI}\right)\left(\frac{P}{K}\right)$ (d) $\frac{P}{48EI / L^3} + K$

Solution: (a)



Let R be the reaction of the spring.
K: Stiffness of spring, which is equal to the load corresponding to unit deflection. $= \frac{R}{\delta}$



δ: Compression of the spring, which is equal to the downward deflection of beam at C.

$$\frac{R}{K} = \frac{PL^3}{48\,EI} - \frac{RL^3}{48\,EI}$$

$$R\left[\frac{1}{K} + \frac{L^3}{48\,EI}\right] = \frac{PL^3}{48\,EI}$$

$$R\left[\frac{48\,EI + KL^3}{48\,EI\,K}\right] = \frac{PL^3}{48\,EI}$$

$$R = \frac{PL^3 K}{48\,EI + KL^3}$$

Deflection of the beam,

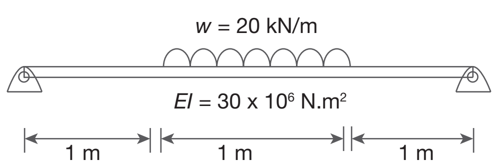
$$\delta = \frac{R}{K}$$

$$\delta = \frac{PL^3}{48EI + KL^3}$$

Hence, the correct option is (a).

Two-marks Questions

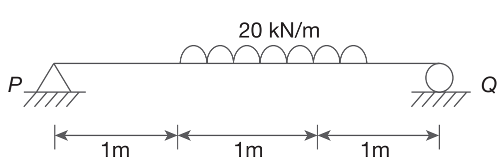
1. A 3 m long simply supported beam of uniform cross section is subjected to a uniformly distributed load of w = 20 kN/m in the central 1 m as shown in the figure. **[2016]**



If the flexural rigidity (EI) of the beam is 30×10^6 N-m^2, the maximum slope (expressed in radians) of the deformed beam is

(a) 0.681×10^{-7} (b) 0.943×10^{-7}
(c) 4.310×10^{-7} (d) 5.910×10^{-7}

Solution:



$$EI = 30 \times 10^6 \text{ N-m}^2$$
$$R_p = R_Q = 10 \text{ kN}$$

$$M(x) = -EI\frac{d^2y}{dx^2} = 10x \qquad (0 \le x \le 1)$$
$$= 10(1+y) - 20y(0.5 - y/2) \qquad (0 \le y \le 0.5)$$

$0 \le x \le 1$

$$-EI\frac{dy}{dx} = \frac{10x^2}{2} + C_1$$

$0 \le y \le 0.5$

$$-EI\frac{dy}{dx} = 10y - \frac{10}{3}y^3 + C_1$$

at $\quad y = 0.5; \dfrac{dy}{dx} = 0$

$$\Rightarrow 0 = 10 \times 0.5 - \frac{10}{3} \times (0.5)^3 + C_1$$
$$\Rightarrow C_1 = -4.583$$
$$\left.\frac{dy}{dx}\right|_{x=1} = \left.\frac{dy}{dx}\right|_{y=0}$$
$$\Rightarrow 5 + C_1 = -4.583$$
$$\Rightarrow C_1 = -9.583$$

So, $\quad \left.\dfrac{dy}{dx}\right|_{\max} = \dfrac{C_1}{EI} = \dfrac{-9.583}{30 \times 10^6} = -3.19 \times 10^{-7}$

Hence, none of the options given in question is correct.

2. Two beams PQ (fixed at P and with a roller support at Q, as shown in Figure I, which allows vertical movement) and XZ (with a hinge at Y) are shown in the Figures I and II, respectively. The spans of PQ and XZ are L and $2L$, respectively. Both the beams are under the action of uniformly distributed load (W) and have the same flexural stiffness, EI (where, E and I, respectively denote modulus of elasticity and moment of inertia about axis of bending). Let the maximum deflection and maximum rotation be $\delta_{\max1}$ and $\theta_{\max1}$, respectively, in the case of beam PQ and the corresponding quantities for the beam XZ be $\delta_{\max2}$ and $\theta_{\max2}$, respectively. **[2016]**

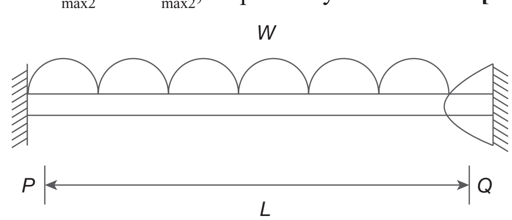


Figure 1

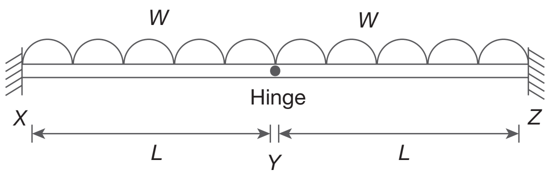


Figure 2

Which one of the following relationships is true?

(a) $\delta_{\max 1} \neq \delta_{\max 2}$ and $\theta_{\max 1} \neq \theta_{\max 2}$

(b) $\delta_{\max 1} = \delta_{\max 2}$ and $\theta_{\max 1} \neq \theta_{\max 2}$

(c) $\delta_{\max 1} \neq \delta_{\max 2}$ and $\theta_{\max 1} = \theta_{\max 2}$

(d) $\delta_{\max 1} = \delta_{\max 2}$ and $\theta_{\max 1} = \theta_{\max 2}$

Solution: (d)

By principal of superposition,

$$g_{\max 1} = g_{\max 2};\ \theta_{\max 1} = \theta_{\max 2}$$

Hence, the correct option is (d).

3. A simply supported reinforced concrete beam of length 10 m sags while undergoing shrinkage. Assuming a uniform curvature of 0.004 m^{-1} along the span, the maximum deflection (in m) of the beam at mid-span is __________ **[2015]**

Solution: 0.05 m

Curvature = 1/R = 0.004 m^{-1}

Radius of curvature $R = 1/0.004 = 250$ m

From geometry of circles we get

$$(L/2)(L/2) = (AB)(BC)$$
$$L^2/4 = (2R - y)y$$
$$L^2/4 = 2Ry - y^2$$

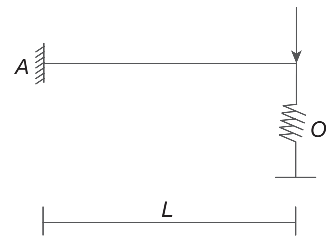
neglecting y^2 in above equation we get

$$Y = L^2/8R = (10)^2/(8 \times 250) = 0.05 \text{ m}$$

Hence, the answer is 0.05 m.

4. A steel strip of length, $L = 200$ mm is fixed at end A and rests at B on a vertical spring of stiffness, $k = 2$ N/mm. The steel strip is 5 mm wide and 10 mm thick. A vertical load, $P = 50$ N is applied at B, as shown in the figure. Considering $E = 200$ GPa, the force (in N) developed in the spring is __________. **[2015]**

Solution: 3.2



For a cantilever, with point load at free end, deflection

$$S_c = \frac{PL^3}{3EI}$$

Deflection of spring $= S_s = \dfrac{F}{k}$

But at point B, $S_c = S_s$

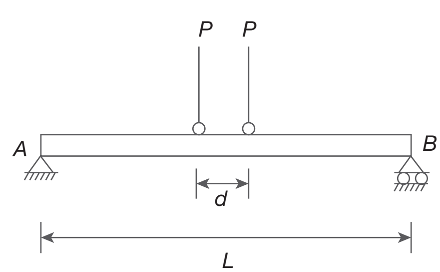
$$\frac{PL^3}{3EI} = \frac{F}{K}$$

$$\frac{50 \times (200)^3}{3 \times 200 \times 10^3 \times \dfrac{5 \times 10^3}{12}} = \frac{F}{2}$$

$$F = 3.2 \text{ N}$$

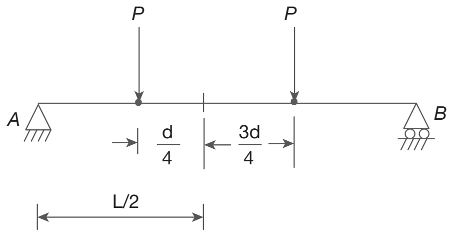
Hence, the answer is 3.2.

5. A simply supported beam AB of span $L = 24$ m is subjected to two wheel loads acting at a distance, $d = 5$ m apart as shown in the figure below. Each wheel transmits a load, $P = 3$ kN and may occupy any position along the beam. If the beam is an I-section having section modulus. $S = 16.2$ cm^3, the maximum bending stress (in GPa) due to the wheel loads is ____________. **[2015]**



Solution: 1.783

For maximum bending moment, the maximum load point and resultant should be at equal distance from the centre of beam



$L = 24$ m, $P = 3$ kN, $S = 16.2$ cm³, $d = 5$ m

$$\Rightarrow \quad R_B \times 24 = P \times \left(\frac{L}{2} + \frac{3d}{4}\right) + p\left(L - \frac{d}{4}\right)$$

$$R_B \times 24 = 3 \times \left(12 + \frac{15}{4}\right) + 3\left(12 - \frac{5}{4}\right)$$

$$R_B = 3.3125 \text{ kN}$$

$$R_A = (6 - 3.3.125)$$

$$= 2.6875 \text{ kN}$$

Maximum *BM* occur at point *X* under the maximum load

$$M_x = R_A \times x$$

$$= 2.6875 \times (12 - 1.25)$$

$$= 28.89 \text{ kNm}$$

From Bending Equation,

$$\frac{E}{R} = \frac{M}{I} = \frac{f}{y}$$

Bending stress $f = \dfrac{M}{I} \times y$

$$\frac{I}{y} = S = \text{section Modulus}$$

$$F_{\max} = \frac{M_{\max}}{S}$$

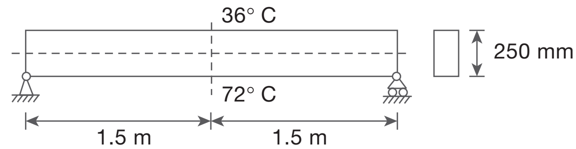
$$= \frac{28.89 \times 10^3}{16.2 \times 10^{-6}}$$

$$= 1.783 \times 10^9 \text{N/m}^2$$

$$= 1.783 \text{ GPa.}$$

Hence, the answer is 1.783.

6. The beam of an overall depth 250 mm (shown in the figure) is used in a building subjected to two different thermal environments. The temperatures at the top and bottom surfaces of the beam are 36°C and 72°C, respectively. Considering coefficient of thermal expansion (α) as 1.50 × 10^{-5}/°C, the vertical deflection of the beam (in mm) at its mid span due to temperature gradient is ____________. **[2014]**



Solution: 2.43

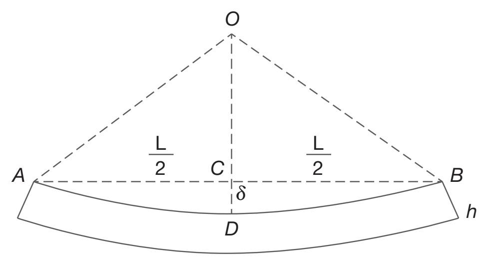
Overall depth of beam, $h = 250$ mm

Temperature at top of the beam, $T_1 = 36$°C

Temperature at bottom of the beam, $T_2 = 72$°C

Coefficient of thermal expansion, α = 1.5 × 10^{-5}/°C

Vertical deflection at mid span of the beam = ?



$$R = \frac{h}{\alpha T}$$

From the property of circle,

$$(2R - \delta)\delta = \frac{L}{2}\frac{L}{2}$$

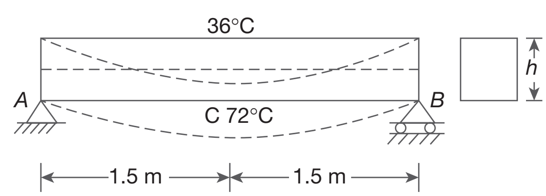
Neglecting the term of δ^2,

$$2R\delta = \frac{L^2}{4}$$

$$\delta = \frac{L^2}{8R} = \frac{L^2 \alpha T}{8h}$$

$$\delta = \frac{(3)^2 \times 1.5 \times 10^{-5}(72 - 36)}{8 \times 250 \times 10^{-3}}$$

$$= 2.43 \times 10^{-3} \text{ m} = 2.43 \text{ mm}$$



Hence, the answer is 2.43.

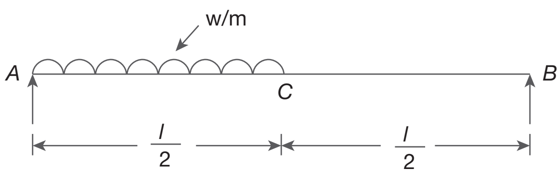
7. A simply supported beam is subjected to a uniformly distributed load of intensity *w* per unit length on half of the span form one end. The length of the span and the flexural stiffness are denoted as

l and EI, respectively. The deflection at mid-span of the beam is **[2012]**

(a) $\dfrac{5}{6144}\dfrac{Wl^4}{EI}$ (b) $\dfrac{5}{768}\dfrac{Wl^4}{EI}$

(c) $\dfrac{5}{384}\dfrac{Wl^4}{EI}$ (d) $\dfrac{5}{192}\dfrac{Wl^4}{EI}$

Solution: (b)



Taking moment of all forces about A,

$$\Sigma M_A = 0 \Rightarrow R_B l - w\frac{l}{2}\frac{l}{4} = 0;\ R_B = \frac{wl}{8}$$

$$\Sigma V = 0 \Rightarrow R_A + R_B = \frac{wl}{2};\ R_A = \frac{wl}{2} - \frac{wl}{8} = \frac{3}{8}wl$$

BM at any distance x from B,

$$M_x = \frac{wl}{8}x - \frac{w}{2}\left(x - \frac{l}{2}\right)^2$$

$$EI\frac{d^2y}{dx^2} = -M_x = -\frac{wl}{8}x \,\vdots + \frac{w}{2}\left(x - \frac{l}{2}\right)^2$$

Integrating wrt x,

$$EI\frac{dy}{dx} = -\frac{wl}{16}x^2 + c_1 \,\vdots + \frac{w}{6}\left(x - \frac{l}{2}\right)^3$$

Integrating wrt x,

$$EI\,y = \frac{wl}{48}x^3 + c_1 x + c_2 \,\vdots + \frac{w}{24}\left(x - \frac{l}{2}\right)^4$$

At $\quad x = 0,\ y = 0 \quad \Rightarrow \quad c_2 = 0$

At $\quad x = l,\ y = 0 \quad \Rightarrow \quad 0 = \dfrac{wl^4}{48} + c_1 l + \dfrac{wl^4}{384};$

$$c_1 = \frac{wl^3}{48} - \frac{wl^3}{384} = \frac{7}{384}wl^3$$

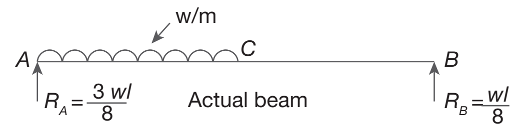
$$EI\,y = -\frac{wl}{48}x^3 + \frac{7}{384}wl^4 x/l + \frac{w}{24}\left(x - \frac{l}{2}\right)^4$$

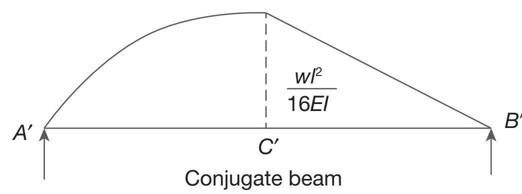
At $\quad x = \dfrac{l}{2},\ EI\,y_c = -\dfrac{wl^4}{384} + \dfrac{7}{384}wl^3\dfrac{l}{2};$

$$y_c = \frac{wl^4}{EI}\frac{1}{768}[-2+7] = \frac{5}{768}\frac{wl^4}{EI}$$

or

By Conjugate beam method:





$$\Sigma M'_A = 0 \Rightarrow R'_B l - \frac{1}{2}\frac{wl^2}{16EI}\frac{l}{2}\left(\frac{l}{2} + \frac{1}{3}\frac{l}{2}\right)$$
$$-\frac{2}{3}\times\frac{wl^2}{16EI}\frac{l}{2}\frac{3}{4}\frac{l}{2} = 0$$

$$R'_B = \frac{wl^3}{96EI} + \frac{wl^3}{128EI} = \frac{wl^3}{EI}\left(\frac{1}{96} + \frac{1}{128}\right) = \frac{wl^3}{EI}\frac{7}{384}$$

BM at C′ in conjugate beam = Deflection at C in real beam

$$= \frac{7}{384}\frac{wl^3}{EI}\frac{l}{2} - \frac{1}{2}\frac{wl^2}{16EI}\frac{l}{2}\frac{1}{3}\frac{l}{2}$$

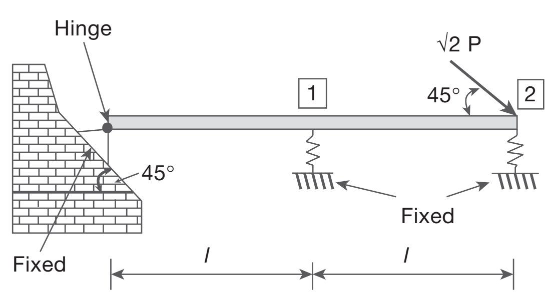
$$= \frac{wl^4}{EI}\left[\frac{7}{768} - \frac{1}{384}\right]$$

$$\delta_c = \frac{5}{768}\frac{wl^4}{EI}$$

Hence, the correct option is (b).

Statement for Linked Questions 8 and 9:

A rigid beam is hinged at one end and supported on linear elastic springs (both having a stiffness of 'k') at points '1' and '2' and an inclined load acts at '2', as shown in the figure. **[2011]**



8. Which of the following options represents the deflection δ_1 and δ_2 at points '1' and '2'?

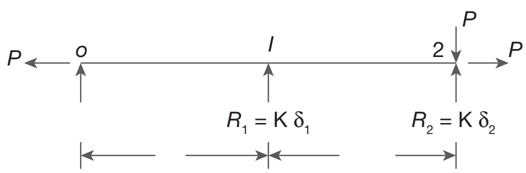
(a) $\delta_1 = \frac{2}{5}\left(\frac{2P}{k}\right)$ and $\delta_2 = \frac{4}{5}\left(\frac{2P}{k}\right)$

(b) $\delta_1 = \frac{2}{5}\left(\frac{P}{k}\right)$ and $\delta_2 = \frac{4}{5}\left(\frac{P}{k}\right)$

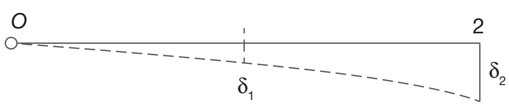
(c) $\delta_1 = \frac{2}{5}\left(\frac{P}{\sqrt{2k}}\right)$ and $\delta_2 = \frac{4}{5}\left(\frac{P}{\sqrt{2k}}\right)$

(d) $\delta_1 = \frac{2}{5}\left(\frac{\sqrt{2}P}{k}\right)$ and $\delta_2 = \frac{4}{5}\left(\frac{\sqrt{2}P}{k}\right)$

Solution: (b)



Forces acting on rigid body



Deflection of the rigid beam

From the deflection of the rigid beam,

$$\frac{\delta_1}{l} = \frac{\delta_2}{2l} \Rightarrow \delta_2 = 2\delta_1 .$$

Taking moments of all forces about the hinge O,

$$\Sigma M = 0 \Rightarrow P2l - k\delta_2 2l - k\delta_1 l = 0$$

$$2Pl - 4k\delta_1 l - k\delta_2 l = 0$$

$$2P - 5k\delta_1 l = 0$$

$$\delta_1 = \frac{2}{5}\left(\frac{P}{k}\right)$$

$$\delta_2 = \frac{4}{5}\left(\frac{P}{k}\right)$$

Hence, the correct option is (b).

9. If the load P equals 100 kN, which of the following options represents forces R_1 and R_2 in the springs at points '1' and '2'?

(a) R_1 = 20 kN and R_2 = 40 kN
(b) R_1 = 50 kN and R_2 = 50 kN
(c) R_1 = 30 kN and R_2 = 60 kN
(d) R_1 = 40 kN and R_2 = 80 kN

Solution: (d)

P = 100 kN

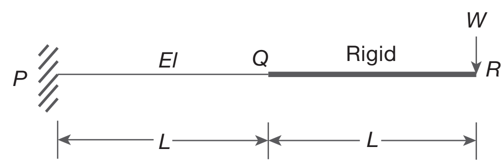
$$\delta_1 = \frac{2}{5} \times \frac{100}{k} = \frac{40}{k},\ \delta_2 = \frac{4}{5} \times \frac{100}{k} = \frac{80}{k}$$

$$R_1 = k\delta_1 = k\frac{40}{k} = 40 \text{ kN},$$

$$R_2 = k.\delta_2 = k\frac{80}{k} = 80 \text{ kN}$$

Hence, the correct option is (d).

Statement for Linked Questions 10 and 11:
In the cantilever beam PQR shown in figure, the segment PQ has flexural rigidity EI and the segment QR has infinite flexural rigidity. **[2009]**

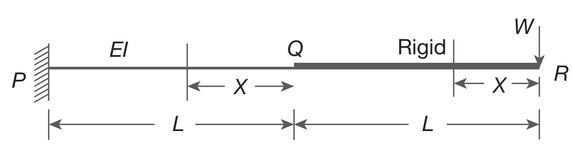


10. The deflection and slope of the beam at 'Q' are respectively

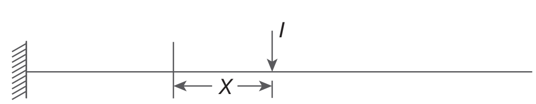
(a) $\frac{5WL^3}{6EI}$ and $\frac{3WL^3}{2EI}$ (b) $\frac{WL^3}{3EI}$ and $\frac{WL^2}{2EI}$

(c) $\frac{WL^3}{2EI}$ and $\frac{WL^2}{EI}$ (d) $\frac{WL^3}{3EI}$ and $\frac{3WL^2}{2EI}$

Solution: (a)



Deflection at Q, (δ_{vQ}):
Let us apply a unit load at point where the vertical deflection is to be found



Vertical deflection at

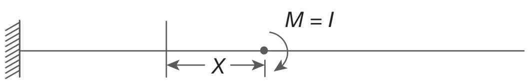
$$Q, = \delta_{VQ} = \int \frac{Mm}{EI} dx$$

$$= \int_{\infty} \frac{(-Wx)0dx}{\infty} + \int_0^L \frac{-W(L+x)(-x)}{EI}dx$$

$$= \frac{W}{EI}\int_0^L (Lx+x^2)dx = \frac{W}{EI}\left[\frac{L^3}{2}+\frac{L^3}{3}\right]$$

$$\delta_{vQ} = \frac{5}{6}\frac{WL^3}{EI}$$

Slope at Q (θ_Q):
Apply unit clockwise moment at Q



Slope at Q,

$$\theta_Q = \int \frac{Mm}{EI}dx$$

$$= 0 + \int_0^L \frac{-W(L+x)(-1)}{EI}dx = \frac{W}{EI}\int_0^L (L+x)dx$$

$$= \frac{W}{EI}\left[L^2 + \frac{L^2}{2}\right]$$

$$\theta_Q = \frac{3}{2}\frac{WL^2}{EI}$$

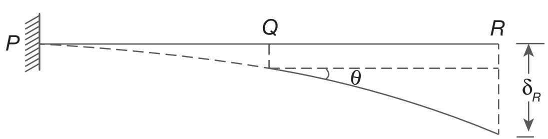
Hence, the correct option is (a).

11. The deflection of the beam at 'R' is

(a) $\dfrac{8WL^3}{EI}$ (b) $\dfrac{5WL^3}{6EI}$

(c) $\dfrac{7WL^3}{3EI}$ (d) $\dfrac{8WL^3}{6EI}$

Solution: (c)



Deflection at R,

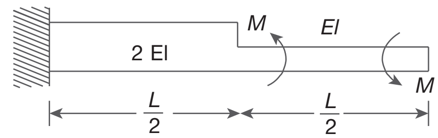
$$\delta_R = \delta_Q + \theta_Q L$$

$$= \frac{5}{6}\frac{WL^3}{EI} + \frac{3}{2}\frac{WL^2}{EI}L = \frac{WL^3}{EI}\left[\frac{5}{6}+\frac{3}{2}\right] = \frac{14}{6}\frac{WL^3}{EI}$$

$$\delta_R = \frac{7}{3}\frac{WL^3}{EI}$$

Hence, the correct option is (c).

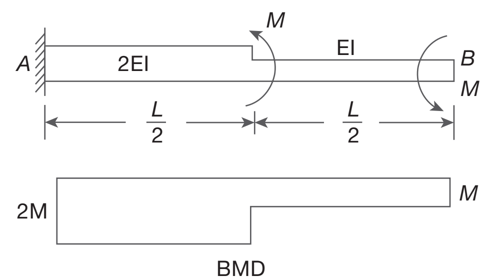
12. The stepped cantilever is subjected to moments, M as shown in the figure. The vertical deflection at the free end (neglecting the self weight) is **[2008]**

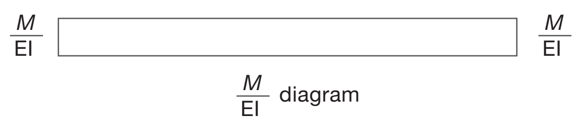


(a) $\dfrac{ML^2}{8EI}$ (b) $\dfrac{ML^2}{4EI}$

(c) $\dfrac{ML^2}{2EI}$ (d) 0

Solution: (c)





Using moment area method:

Deflection at B, δ_B = Moment of the area of $\dfrac{M}{EI}$ diagram between A and B about B

$$= \frac{M}{EI}L\frac{L}{L} = \frac{ML^2}{2EI}$$

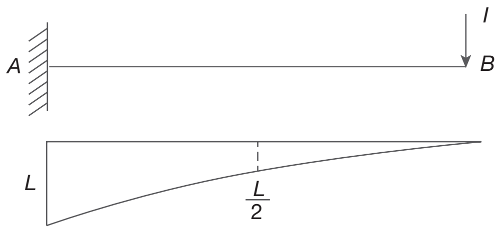
(or)

Using unit load method:

$$\delta_B = \int \frac{Mm}{EI}dx$$

= Area of $\dfrac{M}{EI}$ diagram × ordinate of m diagram at c.g. of $\dfrac{M}{EI}$ diagram

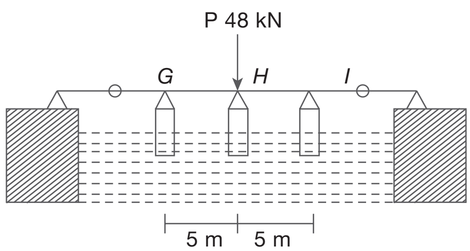
$$= \frac{M}{EI}L\frac{L}{2} = \frac{ML^2}{2EI}$$



M diagram

Hence, the correct option is (c).

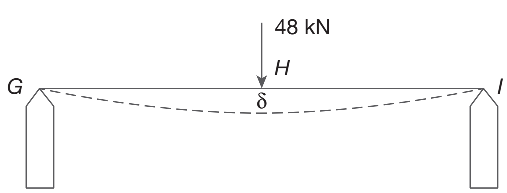
Statement for Linked Questions 13 and 14:
Beam GHI is supported by three pontoons as shown in the figure. The horizontal cross-sectional area of each pontoon is 8 m^2, the flexural rigidity of the beam is 10000 kN-m^2 and the unit weight of water is 10 kN/m^3. **[2008]**

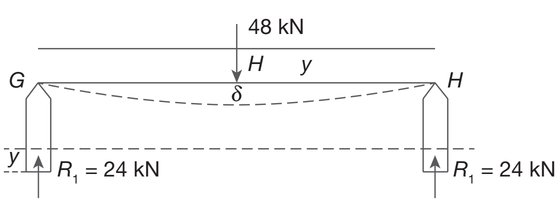


13. When the middle pontoon is removed, the deflection at H will be

(a) 0.2 m (b) 0.4 m
(c) 0.6 m (d) 0.8 m

Solution: (b)
The reactions at the extreme ends of the supports are zero as there are hinges to the left of G and to the right of I. When the middle pontoon is removed, the beam GHI acts as a simply supported beam.





The deflection at H will be due to the load at H as well as due to the displacement of pantoons at G and I in water.

Since, the loading is symmetrical, both pantoons will be immersed to the same depth.

Let y be the immersed depth of pantoons

$$R_G = R_I = R_1$$

Using the principle of buoyancy,
$y \times$ cross-sectional area of pontoon $\times \gamma_w = R_1$

$$y \times 8 \times 10 = R_1$$

$$y = \frac{R_1}{80} = \frac{24}{80} = 0.3 \text{ m}$$

δ = deflection due to 48 kN load

$$= \frac{wl^3}{48\,EI} = \frac{48 \times 10^3}{48 \times 10000} = 0.1 \text{ m}$$

Total deflection at H,

$$\delta_H = y + \delta$$

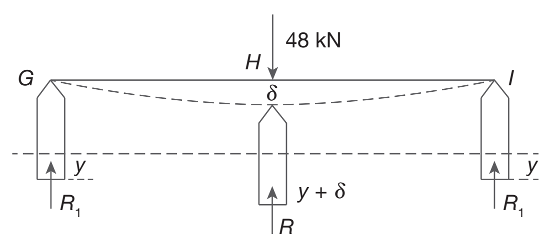
$$= 0.3 + 0.1 = 0.4 \text{ m}$$

Hence, the correct option is (b).

14. When the middle pontoon is brought back to its position as shown in the figure, the reaction at H will be **[2008]**

(a) 8.6 kN (b) 15.7 kN
(c) 19.2 kN (d) 24.4 kN

Solution: (c)



As the beam is loaded symmetrically, the reaction at G and I will be same.

$$\Sigma V = 0 \Rightarrow 2R_1 + R = 48$$

Elastic deflection at H,

$$\delta = \frac{(P - R)l^3}{48\,EI}$$

Also, using principle of buoyancy,
$(y + \delta) \times$ cross sectional area of pontoon $\times\ \gamma_w = R$

$$(y+\delta)\times 8\times 10 = R$$

$$y+\delta = \frac{R}{80};\ \frac{R_1}{80}+\delta = \frac{R}{80}$$

$$\delta = \frac{1}{80}(R-R_1) = \frac{1}{80}(R-24+0.5R)$$

$$= \frac{1}{80}(1.5R-24)$$

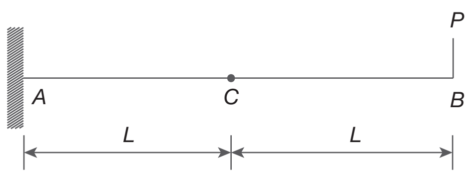
$$\frac{(48-R)10^3}{48\times 10000} = \frac{1}{160}(3R-48)$$

$$48-R = 9R-144$$

$$10R = 192;\ R = 19.2\text{ kN}$$

Hence, the correct option is (c).

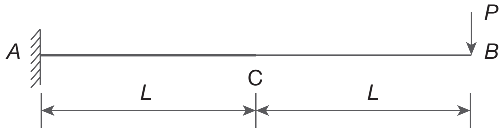
15. Consider the beam AB shown in the figure. Part AC of the beam is rigid while Part CB has the flexural rigidity EI. Identify the correct combination of deflection at end B and bending moment at end A, respectively **[2006]**



(a) $\frac{PL^3}{3EL}, 2PL$ (b) $\frac{PL^3}{3EL}, PL$

(c) $\frac{8PL^3}{3EL}, 2PL$ (d) $\frac{8PL^3}{3EL}, PL$

Solution: (a)



Part AC of the beam is rigid, so end C will act as a fixed end.

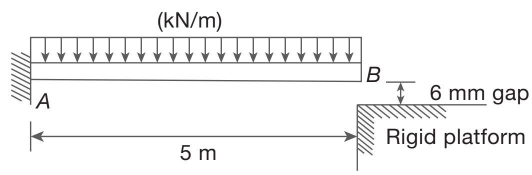
$$\text{Deflection at } B = \frac{PL^3}{3EI}$$

Bending moment does not depend on the rigidity or flexibility of the beam.

$$BM \text{ at } A = P2L = 2PL$$

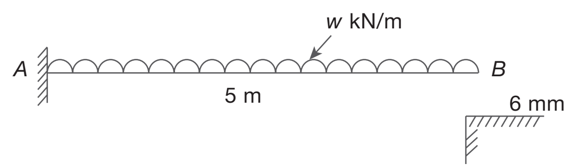
Hence, the correct option is (a).

16. For the linear elastic beam shown in the figure, the flexural rigidity, EI, is 781250 kN-m^2. When w = 10 kN/m, the vertical reaction R_A at A is 50 kN. The value of R_A for w = 100 kN/m is **[2004]**



(a) 500 kN (b) 425 kN
(c) 250 kN (d) 75 kN

Solution: (b)



Deflection at the free end B due to w = 10 kN/m load

$$\delta_{B_1} = \frac{wl^4}{8EI} = \frac{10\times 5^4\times 10^3}{8\times 781250} = 1\text{ mm}$$

The gap available between the beam and rigid platform is 6 mm. Therefore, no reaction will be developed at the end B.

Deflection at the free end B due to w = 100 kN/m load, δ_{B_2} = 10 mm

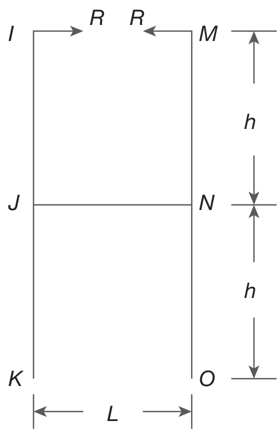
But the end B can undergo a deflection of 6 mm and for the remaining 4 mm, the reaction at B induced.

$$\frac{R_B l^3}{3EI} = 4;\quad \frac{R_B(5)^3}{3\times 781250} = 4\times 10^{-3};\quad R_B = 75\text{ kN}$$

$$R_A = 5\times 100 - 75 = 425\text{ kN}$$

Hence, the correct option is (b).

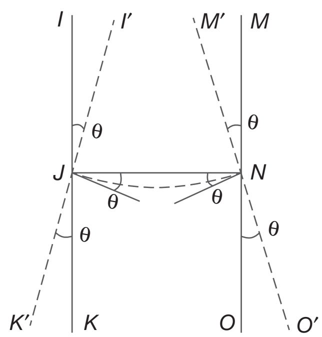
17. A "H" shaped frame of uniform flexural rigidity EI is loaded as shown in the figure. The relative outward displacement between points K and O is. **[2003]**



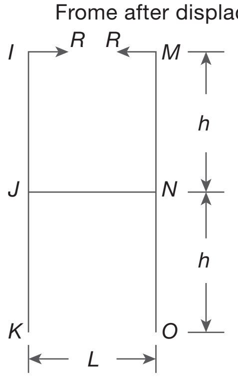
(a) $\dfrac{RLh^2}{EI}$ (b) $\dfrac{RL^2h}{EI}$

(c) $\dfrac{RLh^2}{3EI}$ (d) $\dfrac{RL^2h}{3EI}$

Solution: (a)



Frome after displacement



Frame

BM at the ends of the member $JN = R.h$(sagging)
Slope at J or N,

$$\theta = \frac{Rh}{EI}\frac{L}{2} = \frac{RhL}{2EI}$$

Displacement at K,

$$KK' = \theta.h$$

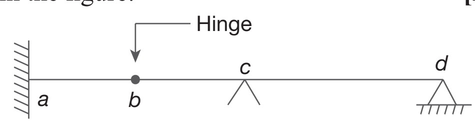
$$= \frac{RhL}{2EI}h = \frac{RLh^2}{2EI}$$

Outward displacement between joints K and O

$$= 2\frac{RLh^2}{2EI} = \frac{RLh^2}{EI}$$

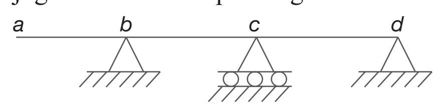
Hence, the correct option is (a).

18. A two span beam with an internal hinge is shown in the figure. **[2000]**

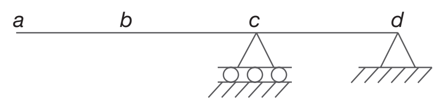


Conjugate beam corresponding to this beam is

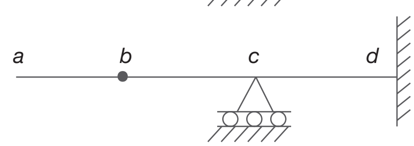
(a)



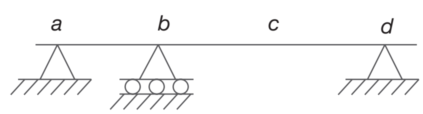
(b)



(c)

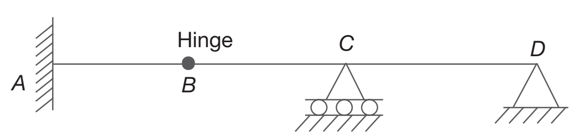


(d)

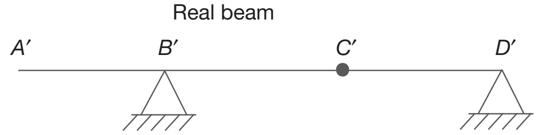


Solution: (a)

Real beam	Conjugate beam
Fixed support	Free end
Free end	Fixed support
External Hinged support	External Hinged support
Internal hinge	Hinged support
Hinged support	Internal hinge



Real beam

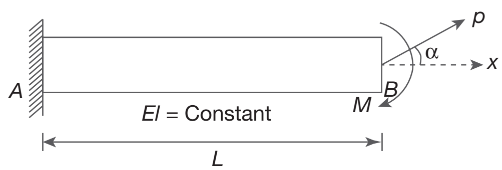


Conjugate Beam

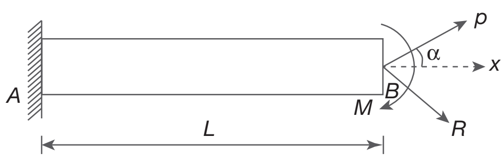
Hence, the correct option is (a).

Five-marks Questions

1. A cantilever beam AB carries a concentrated force P and a moment $M = PL/3$ at its tip as shown below. Show, using Castigliano's theorem that, if the angle of inclination α of the line of action of the force P is such that $\alpha = 1/2$, then the displacement of the point B, due to bending, will be in the direction of force P. **[2000]**



Solution:



Deflection perpendicular to $P = 0$ because load is given along P

Hence, by Castiglino's first theorem

$$\int_0^L \frac{M}{EI}\left(\frac{\partial M}{\partial R}\right)dx = 0 \tag{1}$$

Then Moment in X-direction

$$M_x = P\sin\alpha x - R\cos\alpha x - M = 0$$

So $$\frac{\partial M_x}{\partial R} = -x\cos\alpha$$

Hence from eq. (1)$[-Px^2 \sin\alpha\cos\alpha + R\cos^2\alpha x + Mx\cos\alpha]dx = 0$

Put $R = 0$ and integrating, we get

$$-\frac{P}{2}\sin 2\alpha L^3 + M\frac{L^2}{2}\cos\alpha = 0$$

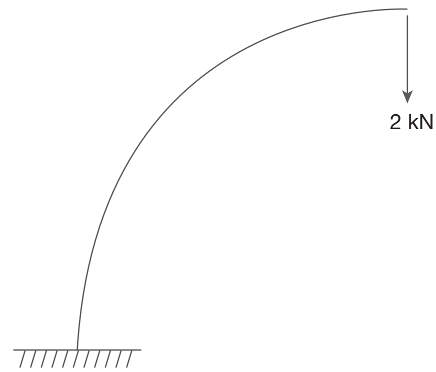
In above equation, putting $M = \frac{PL}{3}$, we get

$$\frac{PL^3}{6}\cos\alpha = \frac{P}{6}L^3\sin 2\alpha \text{ or } \cos\alpha = 2\sin\alpha\cos\alpha$$

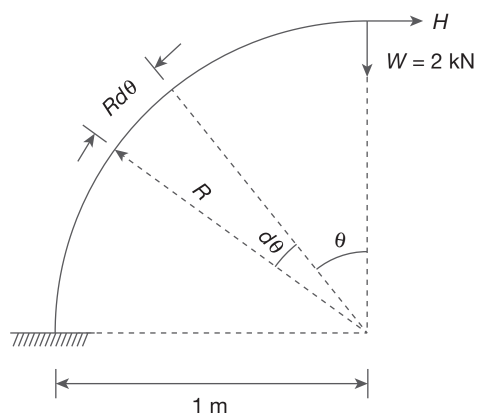
$$\therefore \sin\alpha = \frac{1}{2}$$

Hence, the answer is $\frac{1}{2}$.

2. The given figure shows a cantilever member bent in the form of a quadrant of a circle with a radius of 1.0 m up to the centre of the cross-section. The member is subjected to a load of 2 kN as shown. The member is having circular cross-section with a diameter of 50 mm. Modulus of elasticity (E) of the material is 2.0×10^5 MPa. Calculate the horizontal displacement of the tip. **[1999]**



Solution:



Apply a Pseudo load, H as shown in the figure.

$$\therefore M = WR\sin\theta + H(R - R\cos\theta)$$

$$\therefore U = \int \frac{M^2 dS}{2EI}$$

$$U = \int_0^{\pi/2} \frac{2[WR\sin\theta + HR(1-\cos\theta)]^2 R}{2EI}d\theta$$

$$\therefore \frac{\partial U}{\partial H} = \int_0^{\pi/2} \frac{2[WR\sin\theta + HR(1-\cos\theta)][R(1-\cos\theta)]R}{2EI}d\theta$$

$$\Rightarrow \frac{\partial U}{\partial H} = \int_0^{\pi/2} \frac{WR\sin\theta + HR(1-\cos\theta)[R(1-\cos\theta)]R}{EI}d\theta$$

Now, putting $H = 0$

$$\therefore \frac{\partial U}{\partial H} = WR^3 \int_0^{\pi/2} \frac{\sin(1-\cos\theta)}{EI} d\theta$$

$$\Delta_H = \frac{WR^3}{EI} \int_0^{\pi/2} \left(\sin\theta - \frac{\sin 2\theta}{2}\right) d\theta$$

$$\Delta_H = \frac{WR^3}{EI}\left[-\cos\theta + \frac{\cos 2\theta}{4}\right]_0^{\pi/2}$$

$$\Delta_H = \frac{WR^3}{EI}\left[\left(0-\frac{1}{4}\right) - \left(-1+\frac{1}{4}\right)\right]$$

$$\Delta_H = \frac{WR^3}{EI}\left[-\frac{1}{4}+\frac{3}{4}\right]$$

$$\Delta_H = \frac{WR^3}{2EI}$$

Given load in vertical direction,
Now using the formula given below

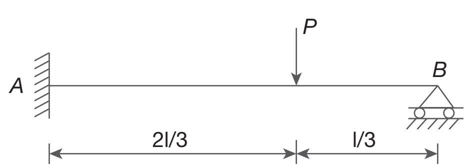
$$I = \frac{\pi d^2}{64} = \frac{\pi \times 50^4}{64}$$

$$\therefore \Delta_H = \frac{2\times 10^3 \times (1000)^3 \times 64}{2 \times 2 \times 10^5 \times \pi \times (50)^4}$$

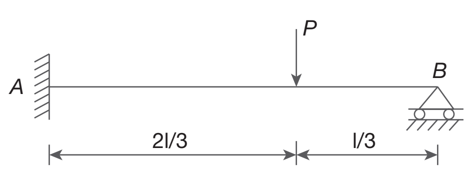
$$= 16.2975 \text{ mm}$$

Hence, the answer is 16.2975 mm.

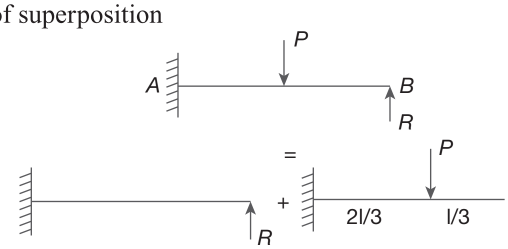
3. Compute the slope at the support B of the propped cantilever beam shown in the figure. The value of EI is constant **[1998]**

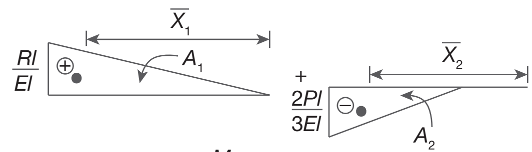


Solution:



If 'R' be the prop reaction at B then using method of superposition





$\frac{M}{EI}$ diagram

As end B is confined by prop reaction 'R'.
Therefore deflection at $B = 0$

$$\Rightarrow \quad \delta_B = 0$$

Now $\delta_B = A_1\bar{x}_1 + A_2\bar{x}_2$

$$\Rightarrow 0 = \frac{1}{2}\frac{Rl}{EI} l \frac{2}{3} l - \frac{1}{2}\frac{2Pl}{3EI}\frac{2l}{3}\left(\frac{2}{3}\frac{2l}{3} + \frac{l}{3}\right)$$

$$\Rightarrow 0 = \frac{Rl^3}{3EI} - \frac{2Pl^2}{9EI}\left(\frac{7}{9}l\right)$$

$$\Rightarrow \frac{Rl^3}{3} = \frac{14Pl^3}{81}$$

$$\Rightarrow R = \frac{14P}{27}$$

Now,

Slope of support B = Area of $\frac{M}{EI}$ diagram.

$$= \frac{1}{2}\frac{Rl}{EI} l - \frac{1}{2}\frac{2Pl}{3EI}\frac{2l}{3}$$

$$= \frac{14P}{27}\left(\frac{l^2}{2EI}\right) - \frac{2Pl^2}{9EI}$$

$$= \frac{14Pl^2}{54EI} - \frac{2Pl^2}{9EI} = \frac{Pl^2}{27EI}$$

Chapter 7

Torsion

One-mark Questions

1. A long shaft of diameter d is subjected to twisting moment T at its ends. The maximum normal stress acting at its cross-section is equal to **[2006]**

(a) zero (b) $\frac{16T}{\pi d^3}$

(c) $\frac{32T}{\pi d^3}$ (d) $\frac{64T}{\pi d^3}$

Solution: (b)

Let d, Diameter of the shaft
T, Twisting moment
σ, Maximum normal stress
τ, Maximum shear stress
Torsion formulais

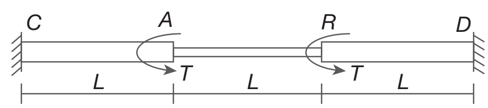
$$\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{R}$$

$$\tau_{\max} = \frac{T}{J}R$$

$$= \frac{T}{\frac{\pi d^4}{32}}\frac{d}{2} = \frac{16T}{\pi d^3}$$

Maximum normal stress is zero.
Due to twisting, the shaft is subjected to only shear stress.
Hence, the correct option is (b).

2. A circular shaft shown in the figure is subjected to torsion T at two points A and B. The torsional rigidity of portions CA and BD is GJ_1 and that of portion AB is

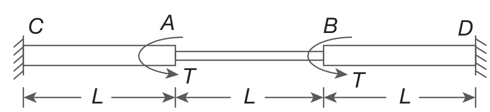


GJ_2. The rotations of shaft at point A and B are θ_1 and θ_2. The rotation θ_1 is **[2005]**

(a) $\frac{TL}{GJ_1 + GJ_2}$ (b) $\frac{TL}{GJ_1}$

(c) $\frac{TL}{GJ_2}$ (d) $\frac{TL}{GJ_1 - GJ_2}$

Solution: (b)



By symmetry, the shaft shows that there is no torsion on the portion AB. Hence, Torque T is acting on each of the end portions AC and BD.

$$\frac{T}{J} = \frac{G\theta}{l} \Rightarrow \theta = \frac{TL}{GJ}$$

For portion AC or BD, $\theta_1 = \frac{TL}{GJ_1}$

Hence, the correct option is (b).

3. A circular solid shaft of span L = 5 m is fixed at one end and free at the other end.

A twisting moment T = 100 kN-m is applied at the free end. The torsional rigidity GJ is 50000 kN-m^2/rad. Following statements are made for this shaft.

I. The maximum rotation is 0.01 rad
II. The torsional strain energy is 1 kN-m

With reference to the above statements, which of the following applies? **[2004]**

(a) Both statements are true
(b) Statement I is true but II is false
(c) Statement II is true but I is false
(d) Both the statements are false

Solution: (b)

Length of shaft, $L = 5$ m
Twisting moment, $T = 100$ kNm
Torsion rigidity, $GJ = 50000$ kN-m^2/rad
Torsion formula,

$$\frac{T}{J} = \frac{G\theta}{l} = \frac{\tau}{R}$$

Maximum rotation,

$$\theta = \frac{Tl}{GJ} = \frac{100 \times 5}{50000} = 0.01 \text{ rad}$$

Torsional strain energy,

$$U = \frac{1}{2}T\theta$$

$$= \frac{1}{2} \times 100 \times 0.01 = 0.5 \text{ kNm}$$

Statement I is true but II is false.
Hence, the correct option is (b).

Two-marks Questions

1. Polar moment of inertia (I_p) in cm^4,of a rectangular section having width, $b = 2$ cm and depth, $d = 6$ cm is ___________ **[2014]**

Solution: 40

Width of the beam, $b = 2$ cm
Depth of the beam, $d = 6$ cm
Polar moment of Inertia,

$$I_p = I_x + I_y$$

$$= \frac{bd^3}{12} + \frac{db^3}{12}$$

$$= \frac{2 \times 6^3}{12} + \frac{6 \times 2^3}{12} = 36 + 4 = 40 \text{ cm}^4$$

Hence, the answer is 40 cm^4.

2. A sold circular shaft of diameter d and length L is fixed at one end and free at the other end. A torque T is applied at the free end. The shear modulus of the material is G. The angle of twist at the free ends is **[2010]**

(a) $\dfrac{16TL}{\pi d^4 G}$ (b) $\dfrac{32TL}{\pi d^4 G}$

(c) $\dfrac{64TL}{\pi d^4 G}$ (d) $\dfrac{128TL}{\pi d^4 G}$

Solution: (b)

Diameter of solid circular shaft, d
Length of shaft, L
Applied torque, T
Shear modulus, G
Polar moment of inertia, $J = \dfrac{\pi d^4}{32}$

Torsion formula, $\dfrac{T}{J} = \dfrac{G\theta}{l} = \dfrac{\tau}{R}$

Angle of twist, $\theta = \dfrac{Tl}{GJ}$

$$\theta = \frac{TL}{G\dfrac{\pi d^4}{32}} = \frac{32TL}{\pi d^4 G}$$

Hence, the correct option is (b).

3. A hollow circular shaft has an outer diameter of 100 mm and a wall thickness of 25 mm. The allowable shear stress in the shaft is 125 MPa. The maximum torque the shaft can transmit is **[2009]**

(a) 46 kNm (b) 24.5 kNm
(c) 23 kNm (d) 11.5 kNm

Solution: (c)

Outer diameter of the shaft, $D = 100$ mm
Thickness of the shaft, $t = 25$ mm
Internal diameter of the shaft, $d = 50$ mm
Allowable shear stress in the shaft, $\tau = 125$ MPa
Torque transmitted by the shaft, $T = ?$

Torsion formula, $\dfrac{T}{J} = \dfrac{G\theta}{l} = \dfrac{\tau}{r}$

Outer radius of the shaft, $R = 50$ mm

Polar moment of Inertia, $J = \dfrac{\pi}{32}(D^4 - d^4)$

$$J = \frac{\pi}{32}(100^4 - 50^4) = 920.3 \times 10^4 \text{ mm}^4$$

$$T = \frac{\tau}{R}J = \frac{125}{50} \times 920.3 \times 10^4 = 23.0 \times 10^6 \text{ Nmm}$$

$$= 23.0 \text{ kNm}$$

Hence, the correct option is (c).

4. The maximum shear stress in a solid shaft of circular cross-HE =section having diameter d subjected to a torque T is τ. If the torque is increased by four times and the diameter of the shaft is increased by two times, the maximum shear stress in the shaft will be **[2008]**

(a) 2τ (b) τ
(c) $\tau/2$ (d) $\tau/4$

Solution: (c)

T, Torque
τ, Maximum shear stress
d, Diameter of the solid circular shaft

$$T_1 = 4T\ ;\ d_1 = 2d;\ R_1 = 2R;\ \tau_1 : ?$$

Torsion formula is,

$$\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{R};\ \tau = \frac{T}{J}R$$

Polar moment of inertia,

$$J = \frac{\pi d^4}{32} = \frac{\pi R^4}{2}$$

$$\frac{\tau_1}{\tau} = \left(\frac{T_1}{T}\right)\left(\frac{J}{J_1}\right)\left(\frac{R_1}{R}\right) = \frac{T_1}{T}\left(\frac{D}{D_1}\right)^4 \frac{R_1}{R}$$

$$= 4 \times \frac{1}{(2)^4} \times 2 = \frac{1}{2}$$

$$\tau_1 = \frac{\tau}{2}$$

Hence, the correct option is (c).

5. The maximum and minimum shear stresses in a hollow circular shaft of outer diameter 20 mm and thickness 2 mm, subjected to a torque of 92.7 N-m will be **[2007]**

(a) 59 MPa and 47.2 MPa
(b) 100 MPa and 80 MPa
(c) 118 MPa and 160 MPa
(d) 200 MPa and 160 MPa

Solution: (b)

Outer diameter of shaft, $D = 20$ mm
Thickness of the shaft, $t = 2$ mm
Inner diameter of shaft, $d = 20 - 2 \times 2 = 16$ mm
Torque, $T = 92.7$ Nm

Torsion equation is $\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{r}$

Polar moment of inertia, $J = \frac{\pi}{32}(D^4 - d^4)$

$$= \frac{\pi}{32}(20^4 - 16^4) = 9274 \text{ mm}^4$$

Maximum shear stress occurs at the outer surface and minimum shear stress occurs at inner surface of the shaft.

$$\tau_{max} = \frac{92.7 \times 10^3}{9274} \times 10 = 100 \text{ N/mm}^2$$

$$\tau_{min} = \frac{92.7 \times 10^3}{9274} \times 8 = 80 \text{ N/mm}^2$$

Hence, the correct option is (b).

Chapter 8

Columns and Struts

One-mark Questions

1. A long structural column (length= L) with both ends hinged is acted upon by an axial compressive load, P. The differential equation governing the bending of column is given by

$$EI\frac{d^2y}{dx^2} = -Py$$

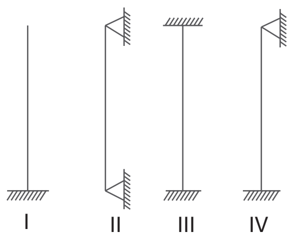
where y is the structural lateral deflection and EI is the flexural rigidity. The first critical load on column responsible for its buckling is given by **[2003]**

(a) $\pi^2 EI/L^2$ (b) $\sqrt{2}\pi^2 EI/L^2$
(c) $2\pi^2 EI/L^2$ (d) $4\pi^2 EI/L^2$

Solution: (a)
Length of column = L
End condition: Both ends hinged
Compressive load = P
Flexural rigidity = EI
Critical load on column, $P_c = \dfrac{\pi^2 EI}{L^2}$

Hence, the correct option is (a).

2. Four column of the same material and having identical geometric properties are supported in different ways as shown in the figure. **[2000]**



It is required to order these four beams in the increasing order of their respective first buckling loads. The correct order is given by

(a) I, II, III, IV (b) III IV, I, II
(c) II, I, IV, III (d) I, II, IV, III

Solution: (d)

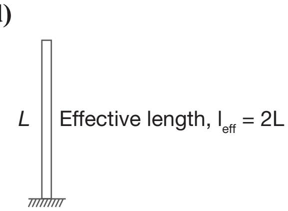
	End condition	Buckling load
I	One end fixed and the other free	$\dfrac{\pi^2 EI}{4L^2}$
II	Both ends hinged	$\dfrac{\pi^2 EI}{L^2}$
III	Both ends fixed	$\dfrac{4\pi^2 EI}{L^2}$
IV	One end fixed and the other hinged	$\dfrac{2\pi^2 EI}{L^2}$

Increasing order of buckling loads: I, II, IV and III
Hence, the correct option is (d).

3. The effective length of a circular electric pole of length L and constant diametres erected on ground is **[1996]**

(a) 0.80 L (b) 1.20 L
(c) 1.50 L (d) 2.00 L

Solution: (d)



Hence, the correct option is (d).

4. When a column is fixed at both ends, corresponding Euler's critical load is **[1994]**

(a) $\frac{\pi^2 EI}{L^2}$ (b) $\frac{2\pi^2 EI}{L^2}$

(c) $\frac{3\pi^2 EI}{L^2}$ (d) $\frac{4\pi^2 EI}{L^2}$

where L is the length of the column.

Solution: (d)

Eulers buckling load for a column hinged at both ends, $P_E = \frac{\pi^2 EI}{L^2}$

If both ends of the column are fixed, $L_{eff} = \frac{L}{2}$

Eulers buckling load for a column fixed at both ends, $P_E = \frac{4\pi^2 EI}{L^2}$

Hence, the correct option is (d).

5. The axial load carrying capacity of a long column of given material, cross-sectional area A and length L is governed by **[1992]**

(a) strength of its material only
(b) its flexural rigidity only
(c) its slenderness ratio only
(d) both flexural rigidity and slenderness ratio

Solution: (d)

Eulers buckling load,

$$P_E = \frac{\pi^2 EI}{L^2}$$

$$P_E = \frac{\pi^2 EAk^2}{L^2} = \frac{\pi^2 EA}{\lambda^2}$$

λ, Slenderness ratio $= \frac{L}{k}$

k, Radius of gyration

For a given material, E is constant.

For a given cross sectional area, A is constant and for a given length, L is constant.

P_E depends on least radius of gyration, which in turn depends on slenderness ratio / flexural rigidity.

Hence, the correct option is (d).

Two-marks Questions

1. Two steel columns P(length L and yield strength f_y = 250 MPa) and Q(length $2L$ and yield strength f_y = 500 MPa) have the same cross-sections and end-conditions. The ratio of buckling load of column P to that of column Q is **[2013]**

(a) 0.5 (b) 1.0
(c) 2.0 (d) 4.0

Solution: (d)

Column *P*:

Length = L

Yield strength, f_y = 250 MPa

Column *Q*:

Length = 2 L

Yield strength, f_y = 500 MPa

Columns P and Q have same cross sectional area and end conditions.

Buckling load, $P_{cr} = \frac{\pi^2 EI}{L^2}$

$$\frac{(P_{cr})_{\text{columm P}}}{(P_{cr})_{\text{columm Q}}} = \frac{\left(\frac{\pi^2 EI}{L^2}\right)_P}{\left(\frac{\pi^2 EI}{L^2}\right)_Q} = \frac{(2L)^2}{L^2} = 4$$

Hence, the correct option is (d).

2. The ratio of the theoretical critical bucking load for a column with fixed ends to that of another column with the same dimensions and material, but with pinned ends, is equal to **[2012]**

(a) 0.5 (b) 1.0
(c) 2.0 (d) 4.0

Solution: (d)

Column with fixed ends,

$$P_{cr} = \frac{4\pi^2 EI}{l^2}$$

Column with pinned ends,

$$P_{cr} = \frac{\pi^2 EI}{l^2}$$

$$\frac{P_{cr}\text{fixed}}{P_{cr}\text{hinged}} = 4$$

Hence, the correct option is (d).

3. The effective length of a column of length L fixed against rotation and transition at one end and free at the other end is **[2010]**

(a) 0.5 L (b) 0.7 L
(c) 1.414 L (d) 2 L

Solution: (d)
End condition: Fixed at one end and free at the other end
Effective length, $l_e = 2\text{ L}$
Hence, the correct option is (d).

4. Consider the following s statements for a compression member **[2009]**

I. The elastic critical stress in compression increases with decrease in slenderest ratio
II. The effective length depends on the boundary conditions at its ends
III. The elastic critical stress in compression is independent of the slenderness ratio
IV. The ratio of the effective length to its radius of gyration is called as slenderness ratio

The true statements are

(a) II and III (b) III and IV
(c) II, III and IV (d) I, II and IV

Solution: (d)
Effective length of a compression member depends on the boundary condition at its ends.
Effective length, $l_e = L$ Both ends hinges

$= 2\text{ L}$ One end fixed and other free

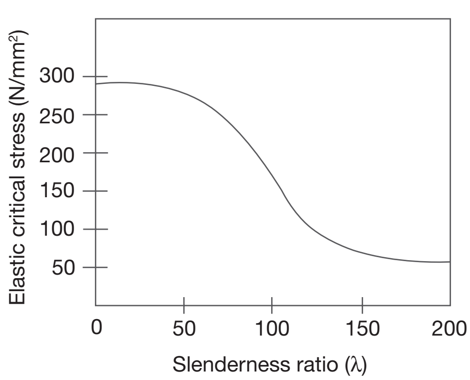
$= \dfrac{L}{2}$ Both ends fixed

$= \dfrac{L}{\sqrt{2}}$ One end fixed and other hinged.

Slenderness ratio is the ratio of the effective length to its radius of gyration.

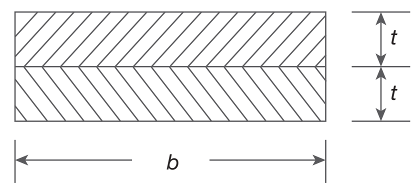
$$\lambda = \frac{l_e}{r}$$

The elastic critical stress in compression increases with decrease in slenderness ratio.



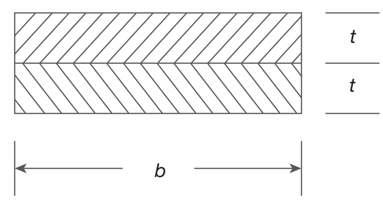
Hence, the correct option is (d).

5. Cross-section of a column consisting of two steel strips, each of thickness t and width b is shown in the figure below. The critical loads of the column with perfect bond and without bond between the strips are P and P_0,respectively. The ratio P/P_0 is **[2008]**



(a) 2 (b) 4
(c) 6 (d) 8

Solution: (b)



Crippling load, $P_c = \dfrac{\pi^2 EI}{L^2}$

When the bond between the strips is perfect,

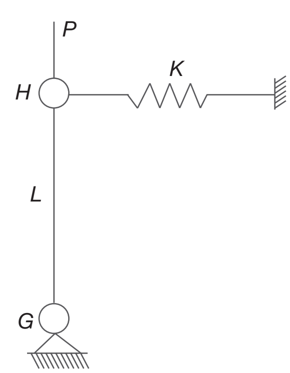
$$P = \frac{\pi^2 E}{L^2}\frac{1}{12}b(2t)^3 = \frac{\pi^2 E}{12\,L^2}\times 8\,bt^3$$

When the bond between the strips is not perfect,

$$P_0 = \frac{\pi^2 E}{L^2}2\frac{1}{12}bt^3 = \frac{\pi^2 E}{12\,L^2}\times 2bt^3,\ \frac{P}{P_o} = 4$$

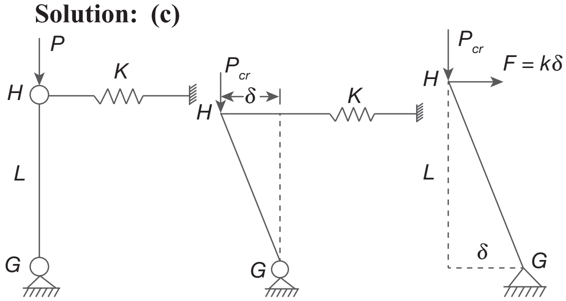
Hence, the correct option is (b).

6. A rigid bar GH of length L is supported by a hinge and a spring of stiffness k as shown in the figure below. The buckling load, P_{cr}, for the bar will be **[2008]**



(a) 0.5 KL (b) 0.8 KL
(c) 1.0 KL (d) 1.2 KL

Solution: (c)



Let δ be the deflection of the spring and F be the force in the spring.

Taking moments about the hinge G,

$$P_{cr}\delta = FL$$

$$P_{cr} = \frac{K\delta L}{\delta}$$

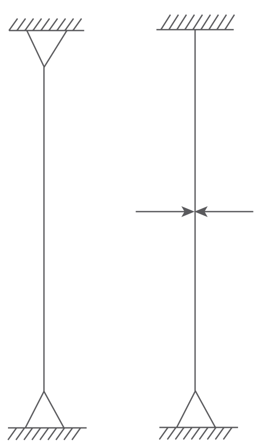
$$P_{cr} = KL$$

Hence, the correct option is (c).

7. A steel column, pinned at both ends, has a buckling load of 200 kN. If the column is restrained against lateral movement at its mid height, its buckling load will be **[2007]**

(a) 200 kN (b) 283 kN
(c) 400 kN (d) 800 kN

Solution: (d)



Case (a) Case (b)

Buckling load when the column ends hinged, P_E = 200 kN

For case a, when the ends of the column are hinged $l_e = L$

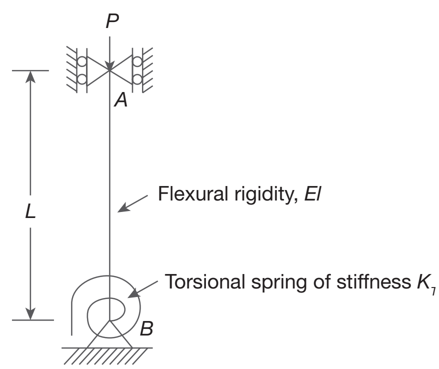
$$P_E = \frac{\pi^2 EI}{L^2} \Rightarrow 200 = \frac{\pi^2 EI}{L^2}$$

For case b, when the lateral movement at mid height is restrained, $l_e = \frac{L}{2}$

$$P_E = \frac{\pi^2 EI}{\left(\frac{L}{2}\right)^2} = \frac{4\pi^2 EI}{L^2} = 4 \times 200 = 800 \text{ kN}$$

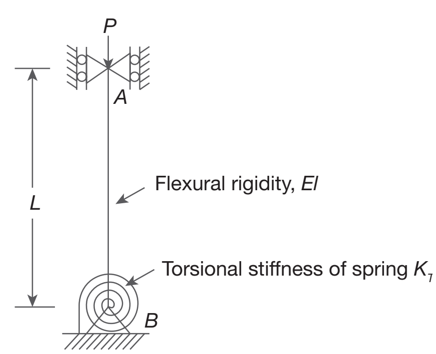
Hence, the correct option is (d).

8. The buckling load $P = P_{cr}$ for the column AB in the figure, as K_T approaches infinity, become $\alpha \frac{\pi^2 EI}{L^2}$, where α is equal to **[2006]**



(a) 0.25 (b) 1.00
(c) 2.05 (d) 4.00

Solution: (d)



Both the ends behave as fixed supports because K_T approaches infinity.

$$P_{cr} = \frac{4\pi^2 EI}{L^2}$$

Therefore, $\alpha = 4.0$

Hence, the correct option is (d).

Chapter 9

Thin Cylinders

One-mark Question

1. A thin cylindrical vessel of mean diameter D and of length L closed at both ends is subjected to a water pressure p. The value of hoop stress and longitudinal stress in the shell shall be respectively **[1991]**

(a) $\frac{pD}{2t}, \frac{pD}{4t}$ (b) $\frac{pD}{4t}, \frac{pD}{8t}$

(c) $\frac{pD}{8t}, \frac{pD}{8t}$ (d) $\frac{pD}{t}, \frac{pD}{2t}$

Solution: (a)

D: Diameter of cylindrical vessel

L: Length of the cylindrical vessel

p: Internal fluid pressure

t: Thickness of the wall

Hoop stress,

$$\sigma_n = \frac{pD}{2t}$$

Longitudinal stress,

$$\sigma_l = \frac{pD}{4t}$$

Hence, the correct option is (a).

Two-marks Questions

1. A thin walled cylindrical pressure vessel having a radius of 0.5 m and wall thickness of 25 mm is subjected to an internal pressure of 700 kPa. The hoop stress developed is **[2009]**

(a) 14 MPa (b) 1.4 MPa

(c) 0.14 MPa (d) 0.014 MPa

Solution: (a)

Radius of the vessel, $r = 0.5$ m

Thickness of the vessel, $t = 25$ mm

Internal fluid pressure, $p = 700$ N/mm^2

Hoop stress

$$\sigma_h = \frac{pr}{t} = \frac{0.7 \times 500}{25} = 14 \text{ N/mm}^2 = 14 \text{ MPa}$$

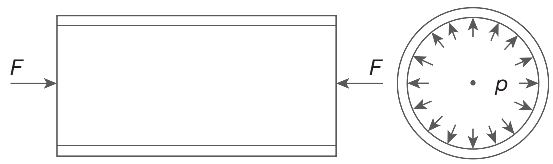
Hence, the correct option is (a).

2. A thin-walled long cylindrical tank of inside radius r is subjected simultaneously to internal gas pressure p and axial compressive force F at its ends. In order to produce 'pure shear' state of stress in the wall of the cylinder, F should be equal to **[2006]**

(a) πpr^2 (b) $2\pi pr^2$

(c) $3\pi pr^2$ (d) $4\pi pr^2$

Solution: (c)



Hoop stress,

$$\sigma_c = \frac{pr}{t}$$

Longitudinal stress,

$$\sigma_l = \frac{pr}{2t} - \frac{F}{2\pi rt}$$

For pure shear state, σ_l should be compressive and is equal to σ_c

$$\sigma_c = -\sigma_l$$

$$\frac{pr}{t} = -\frac{pr}{2t} + \frac{F}{2\pi rt}; \frac{3pr}{2t} = \frac{F}{2\pi rt}; F = 3\pi pr^2$$

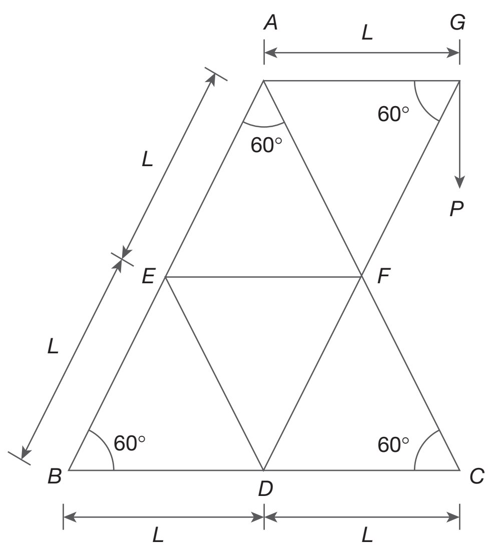
Hence, the correct option is (c).

Chapter 10

Miscellaneous Topics

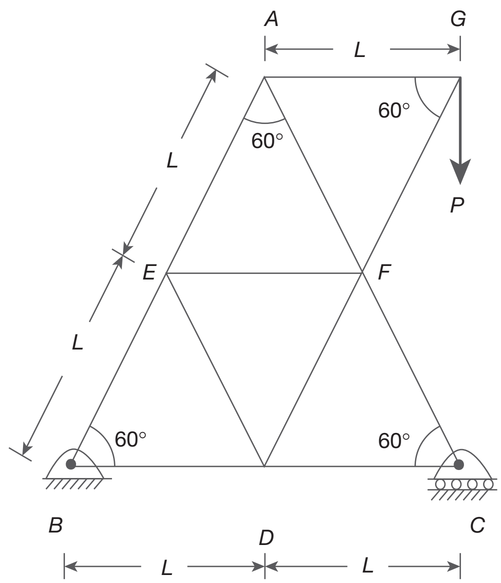
One-mark Questions

1. Consider the plane truss with load P as shown in the figure. Let the horizontal and vertical reactions at the joint B be H_B and V_B, respectively and V_C be the vertical reaction at the joint C. **[2016]**



Which one of the following sets gives the correct values of VB, HB and VC?

(a) $V_B = 0$; $H_B = 0$; $V_C = P$
(b) $V_B = P/2$; $H_B = 0$; $V_C = P/2$
(c) $V_B = P/2$; $H_B = P$ (sin 60°); $V_C = P/2$
(d) $V_B = P$; $H_B = P$ (cos 60°); $V_C = 0$



Solution: (a)

Exp: $\sum F_H = 0 \Rightarrow H_B = 0$

$\sum M_c = 0 \Rightarrow V_B \times 2L = 0 \Rightarrow V_B = 0$

$\sum V = 0 \Rightarrow V_c = P$

Hence, the correct option is (a).

2. The point within the cross sectional plane of a beam through which the resultant of the external loading on the beam has to pass through to ensure pure bending without twisting of the cross-section of the beam is called **[2009]**

(a) Moment centre (b) Centroid
(c) Shear centre (d) Elastic centre

Solution: (c)
Shear centre is the point within the cross sectional plane of a beam through which the resultant of the external loading on the beam has to pass through to ensure pure bending without twisting of cross section of the beam.
Hence, the correct option is (c).

3. The square root of the ratio of moment of inertia of the cross section to its cross sectional area is called **[2009]**

(a) Second moment of area
(b) Slenderness ratio
(c) Section modulus
(d) Radius of gyration

Solution: (d)
Radius of gyration,

$$k = \sqrt{\frac{I}{A}}$$

I: Moment of Inertia of the cross section
A: Cross-sectional area
Hence, the correct option is (d).

4. In section, shear centre is a point through which, if the resultant load passes, the section will not be subjected to any **[1999]**

(a) Bending (b) Tension
(c) Compression (d) Torsion

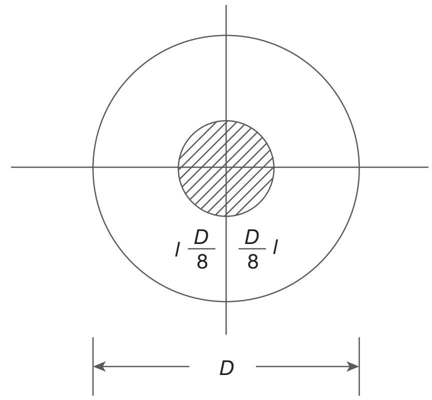
Solution: (d)
Shear center is a point on a line parallel to the axis of a beam through which any transverse force must be applied to avoid twisting of the section.
Hence, the correct option is (d).

5. The kern area (core) of a solid circular section column of diameter D is a concentric circle of diameter d equal to **[1992]**

(a) $\frac{D}{8}$ (b) $\frac{D}{6}$
(c) $\frac{D}{4}$ (d) $\frac{D}{2}$

Solution: (c)
Let D : Diametre of the solid circular section
d : Diametre of the core
Core or kernel of a section is the area within which the resultant load passes so that no part of the section is under tension.

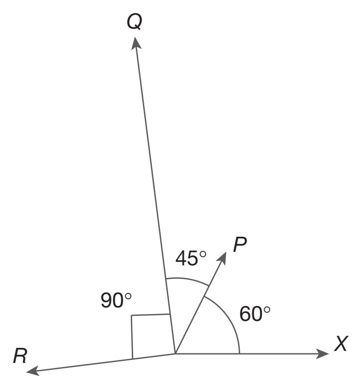
For solid circular section of diameter D, the diameter of the core is $\frac{D}{4}$



Hence, the correct option is (c).

Two-marks Questions

1. The magnitudes of vectors P, Q, and R are 100 kN, 250 kN and 150 kN, respectively as shown in the figure. **[2016]**



The respective values of the magnitude (in kN) and the direction (with respect to the x-axis) of there-sultant vector are

(a) 290.9 and 96.0° (b) 368.1 and 94.7°
(c) 330.4 and 118.9° (d) 400.1 and 113.5°

Solution: (c)
Resolving components w.r.t x-axis

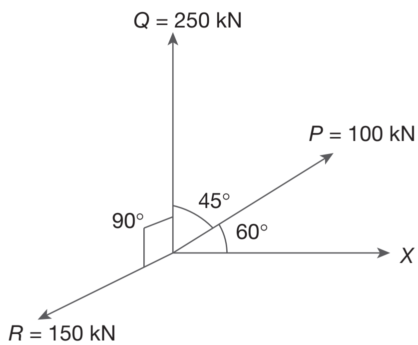
$$\sum F_x \Rightarrow P\cos 60° + \cos(60+45) + R\cos(90+45+60)$$
$$\sum F_x = 100\cos 60° + 250\cos(95) + 100\cos(195)$$
$$\sum F_x = -159.6 \text{ kN}$$
$$\sum F_y = P\sin 60 + Q\sin(60+45) + R\sin(90+45+60)$$
$$= 100\sin 60 + 250\sin(95) + 100\sin(195)$$

$$\sum F_y = 289.3 \text{ kN}$$

$$|F| = \sqrt{F_x^2 + F_y^2} = \sqrt{(-159.6)^2 + (289.3)^2} = 330.4 \text{ kN}$$

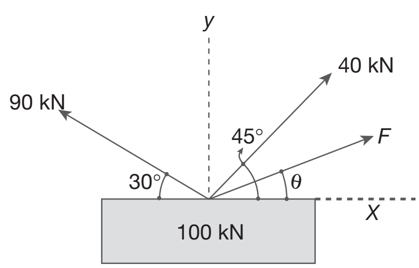
$$\tan\theta = \frac{F_y}{F_x} = \frac{289.3}{-159.6} \Rightarrow \theta = -61.1^\circ$$

$$\theta \text{ wrt } x\text{-axis} = 180 - 61.1 = 118.9^\circ$$



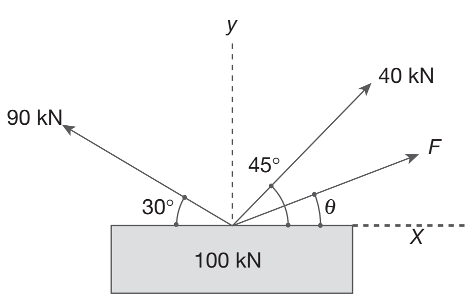
Hence, the correct option is (c).

2. A box of weight 100 kN shown in the figure is to be lifted without swinging. If all the forces are coplanar, the magnitude and direction (θ) of the force (F) with respect to x-axis should be **[2014]**



(a) $F = 56.389$ kN and $\theta = 28.28°$
(b) $F = -56.389$ kN and $\theta = -28.28°$
(c) $F = 9.055$ kN and $\theta = 1.1414°$
(d) $F = -9.055$ kN and $\theta = -1.1414°$

Solution: (a)



For no swinging of the box,

$$\Sigma F_x = 0$$

$$-90\cos 30° + 40\cos 45° + F\cos\theta = 0$$

$$F\cos\theta = 49.658 \tag{1}$$

$$\Sigma F_y = 0 \Rightarrow 90\sin 30° + 40\sin 45° + F\sin\theta - 100 = 0$$

$$F\sin\theta = 26.72 \tag{2}$$

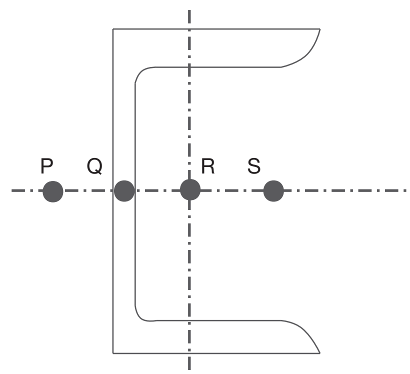
From (1) and (2), $\tan\theta = 0.538 \Rightarrow \theta = 28.8°$

$$F\cos 28.28° = 49.658$$

$$F = 56.389 \text{ kN}$$

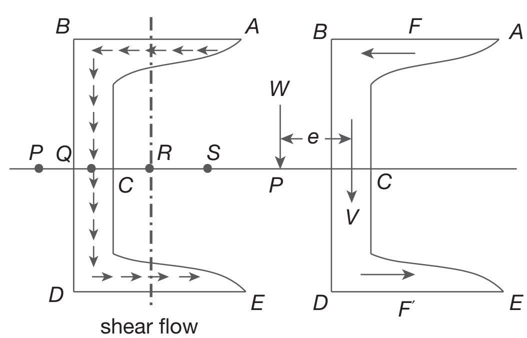
Hence, the correct option is (a).

3. The possible location of shear centre of the channel section, shown in the figure, is **[2014]**



(a) P (b) Q
(c) R (d) S

Solution: (a)



When a force W is applied at a distance e to the left of the centre line of web BD, the member bends in a vertical plane without twisting.

$$We = Fh$$

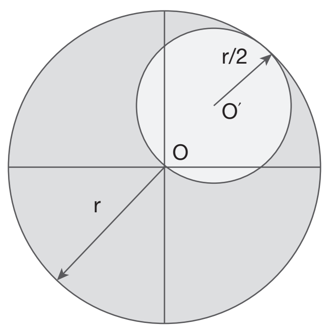
Distance of shear center from the cg of section,

$$e = \frac{Fh}{W}$$

Shear centre is the point P, where the line of action of force W intersects the axis of symmetry of the end section.

If the oblique load is applied through the shear centre, the member will also be free from twisting, since it can resolve into W_x and W_y.
Hence, the correct option is (a).

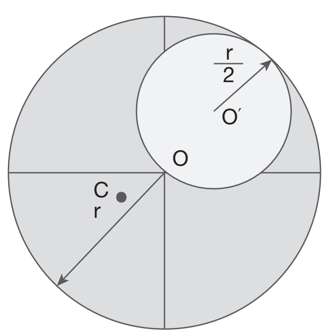
4. A disc of radius r has a hole of radius $r/2$ cut-out as shown. The centroid of the remaining disc (shaded portion) at a radial distance from the centre 'O' is **[2010]**



(a) $\frac{r}{2}$ (b) $\frac{r}{3}$

(c) $\frac{r}{6}$ (d) $\frac{r}{8}$

Solution: (c)



Centroid of the shaded area,

$$\bar{x} = \frac{A_1x_1 - A_2x_2}{A_1 - A_2}$$

$\bar{x}$: Radial distance from O

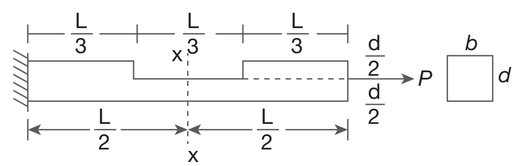
$$A_1 = \pi r^2 \qquad x_1 = 0$$

$$A_2 = \pi\left(\frac{r}{2}\right)^2 = \frac{\pi r^2}{4} \qquad x_2 = \frac{r}{2}$$

$$\bar{x} = \frac{\pi r^2 \times 0 - \frac{\pi r^2}{4} \times \frac{r}{2}}{\pi r^2 - \frac{\pi r^2}{4}} = -\frac{r}{6}$$

Centroid of the shaded area is at a distance of $\frac{r}{6}$ from O in radial direction.
Hence, the correct option is (c).

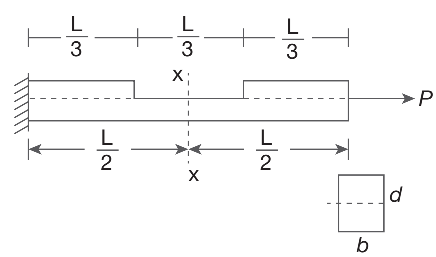
5. The maximum tensile stress at the section X-X shown in the figure is **[2008]**



(a) $\frac{8P}{bd}$ (b) $\frac{6P}{bd}$

(c) $\frac{4P}{bd}$ (d) $\frac{2P}{bd}$

Solution: (a)



At section XX, the load P is acting at an eccentricity of $d/4$. It causes direct and bending stresses.

Maximum tensile stress at XX, $\sigma_{max} = \frac{P}{A} + \frac{Pe}{Z}$

Cross sectional area at XX, $A = b\frac{d}{2}$

Section modulus at XX,

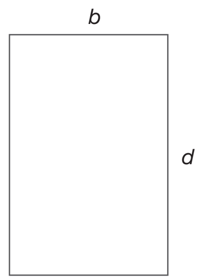
$$Z = \frac{I}{y} = \frac{1}{12}b\left(\frac{d}{2}\right)^3 \Big/ \frac{d}{4} = \frac{1}{24}bd^2$$

Eccentricity, $e = \frac{d}{4}$

$$\sigma_{\max} = \frac{P}{\frac{bd}{2}} + \frac{P\frac{d}{4}}{\frac{bd^2}{24}} = \frac{2P}{bd} + \frac{6P}{bd} = \frac{8P}{bd}$$

Hence, the correct option is (a).

6. For the section shown below, second moment of the area about an axis $d/4$ distance above the bottom of the area is **[2006]**



(a) $\frac{bd^3}{48}$ (b) $\frac{bd^3}{12}$

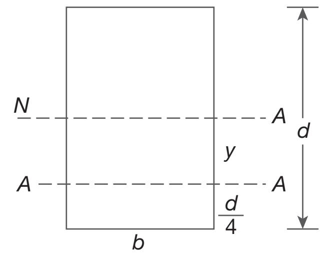
(c) $\frac{7bd^3}{48}$ (d) $\frac{bd^3}{3}$

Solution: (c)

Using parallel axis theorem,

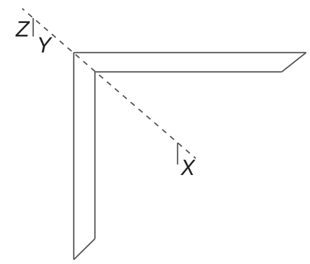
$$I_{AA} = I_{NA} + A\bar{y}^2$$

$$= \frac{bd^3}{12} + bd\left(\frac{d}{4}\right)^2 = \frac{bd^3}{12} + \frac{bd^3}{16} = \frac{7}{48}bd^3$$



Hence, the correct option is (c).

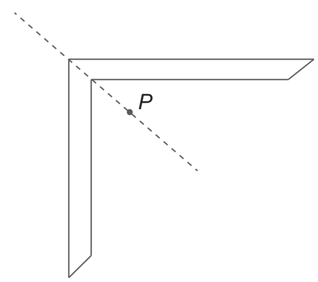
7. Shear centre for an angle purlin is located at **[1996]**



(a) X (b) Y

(c) Z (d) None

Solution: (a)

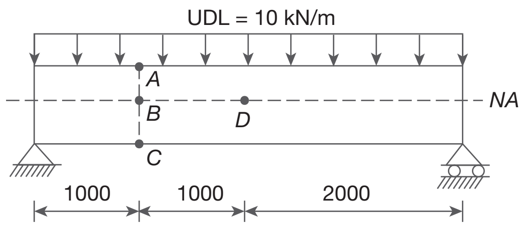


Location of shear centre for angle section.

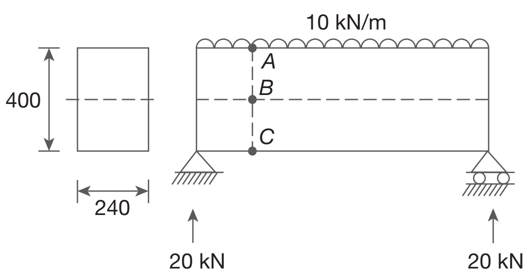
Hence, the correct option is (a).

Five-marks Question

1. The following figure shows a simply supported beam carrying a uniformly distributed load of 10 kN/m. Assuming the beam to have a rectangular cross-section of 240 mm (b) × 400 mm (h), calculate stress at infinitesimal element A, B, C and D as shown in the figure. **[2002]**



Solution:



We know

$$f' = \frac{M}{I} y$$

Where

$$M = 20 \times 1 - 10 \times 1 \times \frac{1}{2} = 15 \text{ kNm}$$

$$I = \frac{0.24 \times 0.4^3}{12} = 0.00128 \text{ m}^4$$

Stress at A:
Normal stress

$$f_A = \frac{M}{I} y_A \text{ where } y_A = 0.2 \text{ m}$$

$$= \frac{15 \times 0.2}{1.28 \times 10^{-3}} = 2343.75 \text{ kN/m}^2$$

Shear stress:

$$\tau_B = 0$$

Stress at B:
Normal stress

$$f_B = 0$$

Shear stress:

$$F_o = 20 - 10 \times 1 = 10 \text{ kN}$$

$$I = 0.00128 \text{ m}^4$$

$$b = 0.24 \text{ m}$$

$$A = 0.24 \times \frac{0.4}{2} = 0.048 \text{ m}^2$$

$$\bar{y} = \frac{0.2}{2} = 0.1 \text{ m}$$

$$\tau_B = \frac{F_o}{Ib} A\bar{y}$$

$$= \frac{10}{0.00128 \times 0.24} \times 0.048 \times 0.1$$

$$= 156.24 \text{ kN/m}^2$$

Stress at C:
Normal stress

$$f_C = -\frac{15 \times 0.2}{1.28 \times 10^{-3}} = -2343.75 \text{ kN/m}^3$$

Shear stress

$$\tau_C = 0$$

Stress at D:
Normal stress

$$f_D = 0$$

Shear stress

$$\tau_D = 0$$