1. Relativistic Mechanics

Relativistic Mechanics: Frame of reference, Inertial & non-inertial frames, Galilean transformations, Michelson-Morley experiment, Postulates of special theory of relativity, Lorentz transformations, Length contraction, Time dilation, Velocity addition theorem, Variation of mass with velocity, Einstein‶s mass energy relation, Relativistic relation between energy and momentum, Massless particle.

Introduction: Newtonian mechanics works very well at low speeds and it fails when applied to particles whose speed approaches c ie the velocity of light.In 1905, Albert Einstein published his special theory of relativity. With this theory one can predict experimental observations over the speeds ranging till velocity of light.

Frame of Reference: Motion of a body is described with respect to some well defined co-ordinate system. This coordinate system is known as frame of reference. There are two types of frames of reference

(i) Inertial or unaccelerated frame.
(ii) Non inertial or accelerated frame.

In **inertial frame**, bodies **obey Newtons laws of inertia**and laws of Newtonian mechanics, while in **non inertial framesNewtons laws are not valid**.

Differentiate between inertial and non inertial frames.(AKTU March 2022)2 Marks

Inertial Frame of Reference	Non-Inertial Frame of reference
A frame of reference which is at a constant velocity with respect to other frames of reference is called Inertial frame of reference.	A frame of reference which is at a constant acceleration with respect to other frame of reference is called non inertial or accelerated frame of reference.
Acceleration of the frame is zero.	Acceleration of the frame is nonzero.
Inertial frame of reference obeys Newton's Laws of motion.	Non-Inertial frame of reference does not obey Newton's Laws of motion.
Example: A train moving with uniform velocity	Example: A freely falling lift

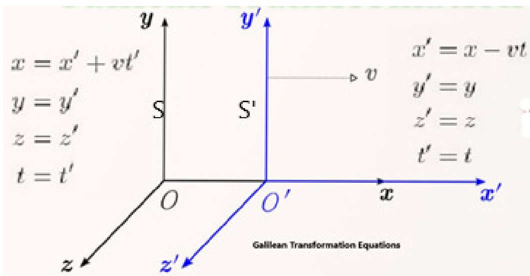
Is earth an inertial or non-inertial frame of reference? Justify your answer.(AKTU 2017)

Earth is **an Inertial Reference Frame** as it revolves around the Sun at a constant velocity. But Earth rotating and at the same time revolving at a constant velocity is also due to a centripetal acceleration. So we can conclude that Earth in accordance with Sun is a Non Inertial Reference Frame

Obtain Galilean transformation equations. Show that length and acceleration are invariant under Galilean transformations.(AKTU2017)

Galilean Transformation:

Taking two frames of reference S and S' as shown in the figure, with their axes parallel to each other, let frame S' move with velocity v, with respect to frame S in the positive x direction. The origins O and O'coincide at t = 0.



In frame S , let (x,y,z) be coordinates of a point P . as measured from O. In frame S' , let (x'.y'.z') be coordinates of point P at time t, then

Galilean Transformation Equations	$x' = x - vt$ $y' = y$ $z' = z$ $t' = t$

Differentiating the above equation with respect to t, we get

Since , , , we have

If a photon is oserved to move with velocity c, in reference frame S' , then

c =

Note: If $u_x = c + v$, then speed of light measured in frame S and S' will not be the same. It will exceed c in one of the frames. **These, two results contradict the results of Michelson Morley experiment**.

Also, acceleration of a particle is the time derivative of its velocity.

Differentiating velocity transformation and using the facts, that t' = t and v = constant.

, , , .

Hence, acceleration as measured by the two observers in the time frames are the same. It shows that acceleration is invariant under Galilean Transformation.

Failure of Galilean Relativity

Galilean relativity failed to explain

(i)both the postulates of special theory of relativity

(ii) results of Michelson Morley experiment

Hence, new sets of different transformation equations were required.

Example: Using Galilean transformation equations prove that, the distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is invariant in two inertial frames.

Solution: Let the coordinates of two points in frame S be (x_1, y_1, z_1) and (x_2, y_2, z_2) while those in frame S' be (x_1', y_1', z_1') and (x_2', y_2', z_2') .

Using Galilean transformations, we have

, ,

, ,

Distance between two points in moving frameS'

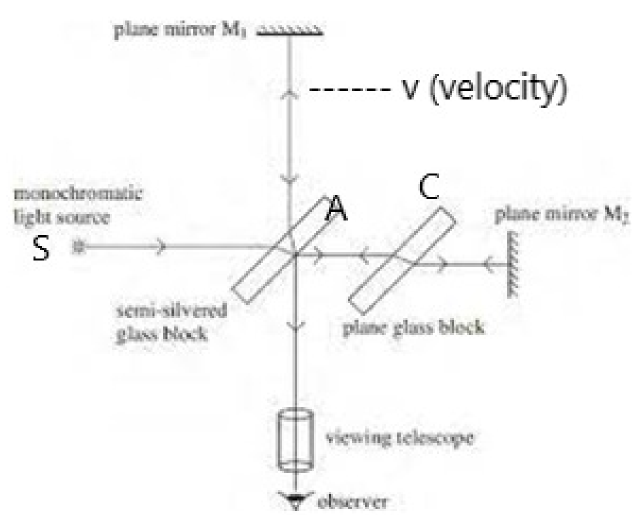
= distance between two points in stationary frame S.

Hence, **distance is invariant under Galilean transformation**.

Michelson Morley Experiment

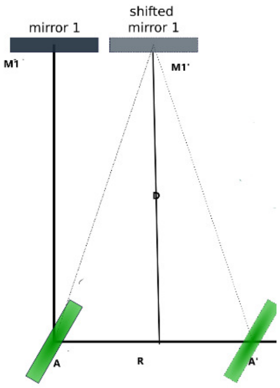
Concept of Ether

Existence of a medium, that fills all space and penetrates all matter is essential for the propagation of light and other electromagnetic waves in free space. Hence, it was assumed by physicists, that all bodies including earth, move freely through this hypothetical medium called ether, without disturbing it.It was contrcdicted , that if the ether hyposhesis is correct , then , it should be possible to determine the absolute velocity of the earth. With respect to stationary ether frame. Keeping, this view in mind Michelson-Morley experiment was performed, using Michelson Interferometer. The experimental set up of the apparatus is as follows



S = monochromatic source of light, A = half sivered plate, M_1, M_2 = Mirrors

Monochromatic source S is taken and light is made to fall from it on lens combination. The half silvered plate A splits it into two components, one parallel and one perpendicular. The reflected component travels to the mirror M_1 and is reflected back, while the transmitted component is reflected back from M_2.



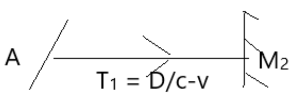
The reflected beams from M_1 and M_2 interfere and the interference fringes are observed through the telescope T.The appararus is arranged to move along the direction of the earths orbit around the sun, with velocity of earth v, this whole apparatus was floated on mercury and was adjusted such that it is always moving along the direction of the earths orbit round the sun.

Let $AM_1 = AM_2 = D$. Also,

velocity of light in the direction of the motion of the earth = c – v

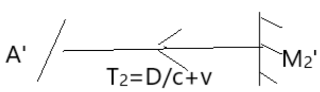
Velocity in opposite direction = c + v , where c is speed of light .

As shown in figure



T_1 = time taken by light to travel from A to M_2

T_2 = time taken from M_2' to A'.



Since, light ray travels to and fro, hence total time is

$T = T_1 + T_2$

----------------- (1)

Total distance travelled by light is

x_1 = T x c

x_1 = (Note : Higher power terms are neglected) ------------ (2)

Let T' be time taken by light to travel from A to M1'. In this time M_1 is shifted to M_1' and A is shifted to R.(Refer figure)

Distance AR = vT' ------------------------ (3)

From right angled triangle AM_1'R we have

(Taking approximation)

Total time taken by light in going from A to M_1' and back to A'.

t = 2T' =

Total distance travelled $x_2 = ct$

$x_2 = 2D$ -------------------------- (4)

From equations 3 and 4 the path difference is given as

$x_2 - x_1 = 2D$ -

$\Delta x =$ ------------------------------ (5)

Let, this path difference correspond to shifting of n fringes, then

$\Delta x = n\lambda$ ---------------------- (6)

Equating equation 5 and 6 we get

In the actual experiment performed by Michelson and Morley, D was made about 10m in effective length, through the use of multiple reflections.

Wavelength of light used = 500nm

Expected fringe shift, in each path when apparatus is rotated through 90^0 is

Putting D = 10m, $v = 3 \times 10^4$ m/s, $c = 3 \times 10^8$m/s , $\lambda = 5 \times 10^{-7}$m we get

= 0.2fringe

Both paths experience this fringe shift, hence total shift = 2n = 2 x 0.2 = 0.4 fringe

Experimentally this amount of shift is readily observable, hence Michelson Morley looked forward to establishing the existence of ether. When this experiment was performed at different seasons of the year and in different locations, no motion through ether was detected.

Negative Results of Michelson Morley Experiment

1. Hypothesis of ether was unatenable.
2. Regardless of any motion of source and observer, it suggests that the speed of light in free space is the same everywhere.

Explanations of the Negative Results

In order to explain, the negative results of MM experiment, following interpretations were given:

(i) Ether drag hypothesis: Michelson stated that moving earth drags the ether along with it , and hence there is no relative motion between the earth and ether.

(ii) Lorentz Fitzgerald contraction hypothesis: From, this hypothesis a material body moving through ether is contracted in the direction of motion by a factor of .
Hence length *l* in the direction of motion is shortened to *l*,This equalizes the times along and perpendicular direction in MM experiment and hence no fringe shift can be expected.

(iii) Constancy of speed of light: As stated by Einstein, speed of light is invariant, and it does not depend upon the motion of the source, observer or medium. Hence, time taken by light to travel the two paths in MM experiment would be same, hence no fringe shift is expected.

Conclusion: These negative results meant, that it was impossible to measure the absolute velocity of the earth with respect to the ether frame.

What are the postulates of special theory of relativity and hence derive Lorentz transformation equations (AKTU2020)

Einstein's Special Theory of Relativity is based on the following two **postulates**

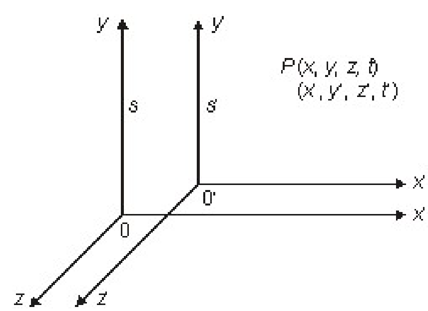
(i) Laws of Physics are same in all inertial frames.

(ii) Speed of light is the same for all observers, independent of the motion of the source or observer.

State the postulates of special theory of relativity and derive the Lorentz transformation equations. When Lorentz transformation equations get reduced to Galilean transformation equations.(AKTU March 2022) [10 marks]

Lorentz Transformations

H.A. Lorentz in 1890, developed transformation equations which apply to all speeds in the range $0 < v < c$.



S, S' are frames of reference, where frame S' is moving with velocity v.

Let two frmaes of reference S and S' be taken. the frame S' moves with velocity v relative to stationary frame of reference along x – axis. At point P having coordinates x, y, z, t for an observer O instationary frame S. While, for frame S' the coordinates are x'.y',z',t.

Transformation for x' in terms of x is
x ' = k(x – vt) --------------------(1)
Where k is proportionality constant independent of x and t.
From Einstein's first postulate, the above equation for frame S' is

x = k(x' + vt') ------------ (2)
Putting the value of x' from equation 1 in 2 we get
x = k[k(x – vt) + vt']

vt' =

-------------- (3)
From Einsteins second postulate , that velocity of light is same in both frames of reference S and S'. we get
x = ct and x' = ct'
Putting the values of x and x' we get
ct'= kt(c – v) ------------------- (4)
ct = kt'(c+v) ---------------------- (5)

Multiplying equaton's 4 and 5 we get

$c^2tt' = k^2tt' (c^2– v^2)$
$c^2 = k^2(c^2 – v^2)$

---------------------- (6)
Putting this value in equation 3, we get

Putting the value of , we get
x' = k(x – vt)

Since there is no motion along y and z axes, hence
y'=y
$z^1=z$

These four equation are called **LorentzTransformation equation**.

$$y' = y$$
$$z' = z$$

Note : At **low velocities**, when $v<<c$, these equation reduce to Galilean transformations.

When $v <<c$, then .

∴ $$x' = x - ut$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

INVERSE LORENTZ TRANSFORMATIONS

Invense Lorentz transformation equation are obtained by interchanging, the primed and unprimedco-ordinates in Lorentz transformation equations and replacing v by $-v$.

$$y = y'$$

$$z = z'$$

Proper Length: The length of L_0 of the rod by an observer in the frame in which the rod is at rest is called Proper Length (L_0) or actual length.

Example: What will the length of a meter rod appear to be for a person travelling parallel to the length of the rod at a speed of 0.8c relative to the rod.

Solution: From the expression

Given $u = 0.8c, L_0 = 1m$

Example: How fast would a rocket have to go relative to an observer for its length to be contacted to 99% of its length at rest.

Solution: From the expression

Given

Putting $c = 3 \times 10^8 m/s$, we have

$v = 3 \times 10^8 \times = 42.32 \times 10^6 ms^{-1}$

Example: A rocket ship is 50*m*long. When it is on flight its length appears to be for 49.5 meters to an observer on the ground. Find the speed of the rocket.

Solution: From the expression

Putting $c = 3 \times 10^8 m/s$, $L = 49.5\ m$, $L_0 = 50\ m$

Example: Calculate the percentage contraction of a rod moving with a velocity of 0.8c in a direction at 60° to its own length.

Solution: From the formula of length contraction we have

The components of Lalong and perpendicular to direction of motion are $L_0 \cos 60°$ and $L_0 \sin 60°$.

Putting v=0.8c, we get

In perpendicular direction, the apparent length will be same

Hence, length of the moving rod in the moving frame is

Percentage Contraction in length =

=

= 8.41%

Example: A rod has a length of 100*cm*. When the rod is in a satellite moving with a velocity that is one half of the velocity of light relative to the laboratory, what is the length of the rod as determined by a and observer ***(i)*** in the satellite ***(ii)*** in the laboratory.

Solution:

(i) An observer in the satellite observer the proper length of the rod.

(ii) From the expression

Given $L_0 = 100cm,\ u = 0.5c$

SIMULTANEITY OF EVENTS

The relativity of simultaneity is the concept that distant simultaneity – **whether two spatially separated events occur at the same time** – is not absolute, but depends on the observer's reference frame.

Take two events occurring at different locations x_1 and x_2 ($x_1 \neq x_2$) which are simultaneous in reference frame *s*. Let, t_1 and t_2 be the times of occurrence of the two events, then

$$t_1 = t_2$$

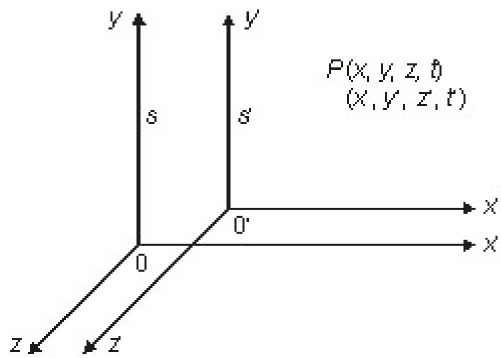
In a reference frame S', moving with velocity *v* relative to S, the times at which these events occur are given by Lorentz transformation equation as

As $t_1 = t_2$ but $x_1 \neq x_2$,

Hence, events which are simultaneous in S are not simultaneous in S'.

Example: Show that velocity of light, is an absolute constant.

Solution: Let two fames of reference *S* and S' be taken which coincide at $t = 0$. Let frame S' move with constant speed *u* along positive *x*-direction with respect to S. Let a flash of light be produced at $t = 0$, which spreads out like a sphere. Let velocity of light in the two fames be *c* and c'



In frame S, equation of sphere after time ‘t’ is

$x^2 + y^2 + z^2 = c^2t^2$ —— 1

Equation of sphere after time t' in s' is

$$(x')^2 + (y')^2 + (z')^2 = (c')^2(t')^2 \qquad \text{—— 2}$$

—— 3

From Lorentz transformation equations, we have

From equation,

$$(c')^2 = c^2 \text{ Hence Proved}$$

Example: Show that space time interval between two events remains invariant under Lorentz transformations.

Solution: The space time interval is

$$x^2 + y^2 + z^2 = c^2t^2$$

In order to prove that

$$x^2 + y^2 + z^2 - c^2t^2 = (x')^2 + (y')^2 + (z')^2 - c^2(t')^2$$

From Lorentz tranformations:

Putting these values in equation we get

=

=

=

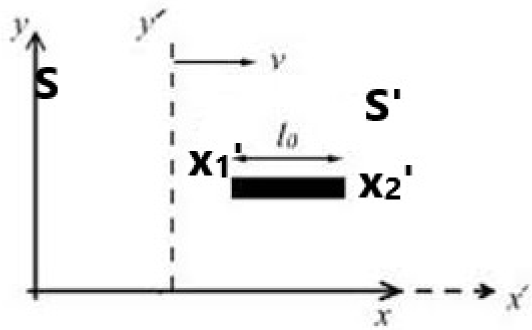
=

=

LENGTH CONTRACTION:

Length contraction is **the phenomenon that a moving object's length is measured to be shorter than its proper length**, which is the length as measured in the object's own rest frame.

Take a rod reference frame S^1, which is moving with constant speed u, relative to another reference frame S. Taking end points of the rod in S^1frame as .



Then, length measured by observe who is stationary with respect to the rod is known as proper length L_0.

—— 1

From Lorentz transformation equations, if x_1 and x_2 are co-ordinates of the end points of the rod in frame $s_1$then they are related to as

—— 2

Putting these values in equation 1 we get

—— 3

Let L be the measured by the observe in reference frame S, then

$$L = x_2 - x_1 \qquad —— 4$$

Putting these values in equation 3 we get

—— 5

Length contraction.

Note: Since is less than 1, L less than L_0.

Hence, **length of a moving rod appears to be smaller** than the length, when it is at rest with respect to the observer. This is known as length contraction.

Proper Length: Length L_0 of the rod measured by an observer in the frame in which the rod is at rest is called proper length.

Example: Find the length of rod moving with a velocity of 0.8c in a direction inclined at 45° to its own length. Proper length of rod is given to be 60*cm*.

Solution: Let proper length of rod be L' and apparent length be L.

Length of rod in x-direction

Length of rod in *y*-direction

$$l = 60cm, l^{\prime} = 48cm$$

Example: If a spaceship is 50*m* long and it were to pass the earth travelling at 2.4×10 *m/s*, what would be its apparent length, assuming Lorentz - Fitzgerald contraction.

Solution:

$$50 \times 0.6 = 30m$$

Example: A rocket ship is 50 meter long. When it is on flight, its length appears to be for 49.5 metres to an observer on the ground. Find the speed of the rocket.

Solution:

Example: Obtain the volume of a cube, the proper length of each edge of which is l_0. When it is moving with velocity *v* along one of its edge.

Solution: From the concept of length contraction, the length of a moving object is contracted by a

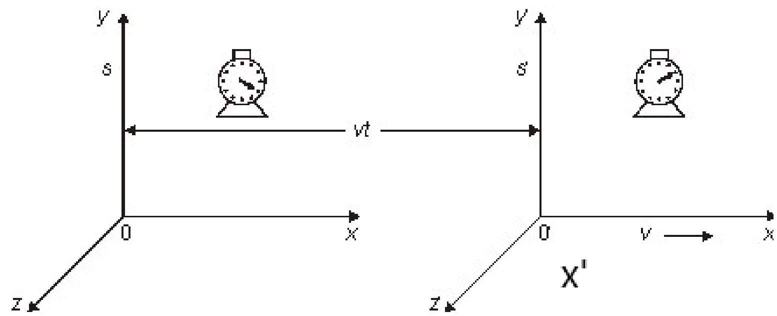
factor along its direction of motion, whereas its dimension perpendicular to the direction of motion remains unchanged.

$$\text{And } L_x = L_z = L_0$$

TIME DILATION

Relative motion effects the measurement of time in travel. A clock in a stationary frame measures longer time interval between two events occurring in a moving frame of reference, than does a clock in the moving frame.Taking two frames of reference *S* and S' where S' is moving with velocity relative to *S* along positive *x*-direction.

WHEN THE CLOCK IS IN FRAME S':



Let the clock be situated at x' in frame s' and the time interval between two ticks is measured as t_o by an observer in the same frame (real time interval) whereas observer in another frame s (say stationary) notices an apparent change in this time interval 't'.

The position of clock is continuously changing for an observer in the frame s.

Using time coordinates of inverse

Lorentz transformations

—— 1

The two timeticksas observed by an observer in s-frame are

—— 2

—— 3

Subtracting equation 2 from equation 3 we get

Note: $t > t_0$, that is time is dilating for a moving observer.

Hence, a clock moving with a uniform velocity v, relative to an observer appear to him to go slow by a factor , than when at rest, relative to him.

This effect is called time dilation.

Time dilation can also be stated as "slowing down" of a clock as determined by an observer who is in relative motion with respect to that clock.

From the expression

Note:

- If $\boldsymbol{v} \ll \boldsymbol{c}$, then can be neglected hence $t = t_0$ hence time interval measured by an observer in a moving clock is same as when the clock is at rest. If $v=c$, then $t = \infty$. It shows that a clock moving with speed of light appears to be completely stopped to an observer in stationary frame of reference.
- t_0 is proper time

  t is relativistic or non proper time.

- **Twin Paradox:** A moving twin in space estimates the age of his twin on earth to be shorter as he treats himself in an inertial frame and says that twin brother on earth is moving in opposite directions, so that the time should dilate for him. Similar observation is given by twin brother on earth.

Existence of μ–mesons on earth surface: μ-mesons are cosmic ray particles produced in the atmosphere at the height of 10 *km*, from the surface of the earth.

Their life time is found to be $t = 2\times10^{-6}s$ and their speed is, $v = 2.994\times 10^{8} m/s$.

Hence, distance travelled by these particles is $d = vt = 2.994 \times 10^{8} \times 2\times 10^{-6} = 600m$.

But, these particles are found on the surface of earth, which indicates that they are actually travelling a distance of 10*km* not only 600*m*.

This will be possible if time dilation is considered. Due to time dilation, lifetime of the particle is obtained as

$$t = 3.33\ \mu s$$

Hence, distance travelled is $d = u \times t$

$$d = 2.994 \times 10^{8} \times 3.33\times10^{-5}$$

$$= 10km.$$

But, these particles actually travel around 10*km* to reach earth surface before diminishing. This is another example of **time dilation as real effect**.

Example: At what speed should a clock be moved so that it may appear to lose 1 minute in each hour.

Solution: Since clock is losing 1 minute in every hour or 60 minutes, hence, it must record 59 minutes for 60 minutes recorded by stationary clock w.r. to observer.

Using

Putting $t_0 = 59$ minutes, $t = 60$ minutes

$$v = 5.5 \times 10^{7} m/s$$

Example: A certain particle called μ meson has mean life of $2.5\times10^{-7} sec$. What is the velocity of μ meson, if its proper life is 2.5×10^{-8} second.

Solution: Using

Given $\Delta t = 2.5\times10^{-7}$ sec, $\Delta t' = 2.5\times10^{-8}$ sec

Example: A clock keeps correct time, with what speed should it be moved relative to an observer so that it may appear to lose 4 minutes in 24 hours.

Solution: Using

Given $t_0 = 24\times60 = 1440$ minute is the proper time-interval measured by an observer moving with the clock and $t = 1444$ minutes is the time interval measured by a stationary observer.

Hence,

When, the higher order terms may be neglected, hence

$$v^2 = 0.003 \times 2 \times (3 \times 10^8)^2$$

$$v = 2.3 \times 10^7 ms^{-1}$$

Example: Pilot of a spaceship wears a wrist watch, which keeps correct time on earth. How much will it appear to lose per day with respect to an observer on the earth. When spaceship leaves the earth with a constant velocity of $10^7 m/s$.

Solution: Using

Given $t = 24$ hours, $v = 10^{+7} m/s, c = 3 \times 10^8 m/s$.

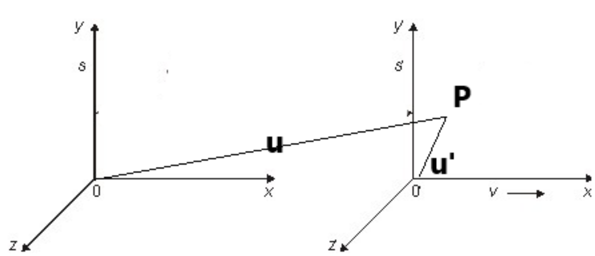
∴ Loss of time per day = 24 – 23.9 = 0.1 hrs.

State and prove the velocity addition theorem . Show that the theorem is consistent with Einstein's second postulate. (AKTU March 2022) [10 marks]

VELOCITY ADDITION THEOREM

The velocity addition theorem gives a three-dimensional equation that relates the velocities of objects in different reference frames.

Let frame S^1 move with velocity v relative to frame S, along positive x-direction. Let u and u^1 be the velocities of a particle P as observed in frames S and S`.



—— 1

—— 2

Also

Using inverse Lorentz transformations

—— 3

Independently differentiating the coordinates from equation 3 we get

—— 4

Velocity is obtained by , hence, we have

Putting ,we get

—— 5

Similarly from *y* and *z* frame, we get

—— 6

Similarly for an observer in frame S', velocity co-ordinates can be written using Lorentz transformations:

—— 7

Similarly for *y* and *z* frame, we get

Note:

1. Let a particle is moving with velocity *c* in frame *s'* is also moving with speed *c* relative to frame *s* along *x*-direction then, its velocity is given by

   This is in consistence with Einstein's second postulate.

2. If a light single is travelling at velocity *c* in a stationary frame *s'*, then its velocity is estimated by an observer in frame *s* will also be *c*.

Example: Two particles come towards each other each with speed 0.9c with respect to the laboratory. What is their relative speed.

Solution: Using

Given

Example: A particle has a velocity in a co-ordinate system moving with velocity 0.8c relative to laboratory along +*ve* direction of *x*-axis. Find *u* in laboratory frame.

Solution: Using

Example: An experimenter observes a radius active atom moving with a velocity of 0.25c. The atom then emits a beta particle which has a velocity of 0.9c relative to the atom in the direction of its motion. What is the velocity of the beta particle as observed by the experimenter.

Solution: From law of addition of relativistic velocities we have

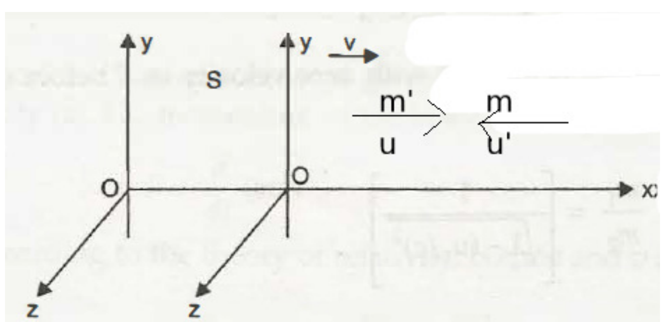
Given $u^{`} = 0.9c$, $v = 0.25c$

Deduce an expression for the variation of mass with velocity.(AKTU2019)

RELATIVISTIC VARIATION OF MASS WITH VELOCITY

There is variation of mass with velocity in relativity , that is mass varies with the velocity when the velocity is comparable with velocity of light. This is known as relativistic variation of mass with velocity. We now derive an expression for same.

Taking to fames s and $s^{`}$ which is moving with uniform velocity v relative to frame S along positive x-direction. The velocities of these bodies according to the fame s areu_1 and u_2 in frame S. Let masses of the bodies in the frame s be $m_1$and m_2 and since after collision the velocity of coalesced mass m is zero relative to frame $s^{`}$ from law of conversation of momentum in frame s, for comparing the net momentum before collision and after collision, using velocity addition theorem we have,



$$m_1 u_1 + m_2 u_2 = (m_2 + m_2) v \qquad \text{—— 1}$$

—— 2

From equation 1 we have

Dividing whole equation bym_2,

—— 3

Using squared form of velocities u_1 and u_2.

=

=

=

=

—— 4

Similarly for u_2

—— 5

Dividing equation 5 by equation 4 we get

—— 6

Taking square root of equation 6 and putting the value in equation 3 we get

—— 7

If body is at rest or moving with zero velocity in stationary frame S that is $u_2 = 0$ before collision and if $m_2 = m_0$, the rest mass of the body, then

In commonly used notations (m_1=m) and (u_1=v), the relativistic formula is now expressed as

Required relativistic formula for variation of mass with velocity.

Note:

(i) as the velocity of the body relative to the observer increases, the mass of the body increases.
(ii) as velocity exceeds c, mass becomes imaginary, which is not possible. This means nobody can have velocity greater than the velocity of light.
(iii) When $v \ll c$, then $m = m_0$. This means that at ordinary velocities, the difference between m and m_0 is very small to detection.

RELATIVISTIC MOMENTUM AND FORCE

Relativistic momentum is defined in such a way that the conservation of momentum will hold in all inertial frames. **Whenever, the net external force on a system is zero, relativistic momentum is conserved**, just as is the case for classical momentum.

From relativistic mechanics

Using $p = mv$, we have

Force is defined as **rate of change of momentum**, hence

Example: At what speed will the mass of a body be 2.25 times its rest mass.

Solution: Using

Given $m = 2.25m_0$, we have

Example: A man weighs 50kg on the earth. When, he is it in rocket ship in flight, his mass is 50.5 kg as measured by an observer on earth. What is the speed of the rocket.

Solution: Using

Given $m_0 = 50$kg, $m = 50.5$kg, $c = 3\times10^8 m/s$

Example: How fast must an electron move in order that its mass equals the rest mass of the proton.

Solution: Using

Putting $m_0 = m_e = 9.11 \times 10^{-31}$kg, $m = m_p = 1.67\times10^{-27}$kg, $c = 3 \times 10^8 m/s$.

$$v = 2.98 \times 10^8 ms^{-1}$$

Example: The rest mass of an electron is m_0. what will be its mass if it's speed is

Solution:

Given $m_0 = 9.1\times10^{-31}$kg v =

MASS ENERGY EQUIVALENCE : EINSTEIN'S MASS ENERGY RELATION

Mass-energy equivalence states that mass is concentrated energy. In his theory of special relativity Einstein formulated the equation **E=mc²**.

For Newton's law, where *p* is momentum. Also —— 1

If this body is forced to move by some displacement '*dx*', then, the increase in total energy is an increase in thefrom of kinetic energy and is equal to work done.

Hence, —— 2

Work done

$\therefore dW = v\, d\,(mv) = v^2 dm + mvdv$ —— 3

Usingsquaring the equation we get

—— 4

Differentiating equation 4 we get

$$2mdmc^2 - 2mdmv^2 - 2m^2vdv = 0 \quad —— 5$$

$$dm \times c^2 - dm \times v^2 - mvdv = 0$$

$$v^2dm = dm \times c^2 - mvdv \quad —— 6$$

Putting this value in

$$dW = v^2dm + mvdv$$ we get

$$dW = c^2dm$$

From work energy theorem, work done in displacing a particle is the kinetic energy given to the particle.

$$K = (m - m_0)c^2 = mc^2 - m_0c^2$$

$$K + m_0c^2 = mc^2 \quad —— 7$$

Here, $\boldsymbol{m_0c^2}$ is the **rest mass energy**.

Total energy is $\mathbf{E} = \boldsymbol{mc^2}$ —— 8

This is **universal mass energy equation equivalence**.

Note:

(i) If $v << c$, then kinetic energy expression is converted into non-relativistic mechanics.

$$K = (m - m_0)c^2$$

Putting we have

Using binomial expansion

(ii) If $v > c$, >1, the relativistic kinetic energy K tends to infinity. Hence, concluded speed of light is the highest speed in all inertial frames.

RELATION BETWEEN MOMENTUM AND TOTAL ENERGY FOR A RELATIVISTIC MOVING PARTICLE.

For a relativistically moving particle, total energy and momentum are

$$E = mc^2 \qquad \text{—— 1}$$

$$p = mv \qquad \text{—— 2}$$

Multiplying above equations by c^2, squaring and subtracting we get

$$E^2 - p^2c^2 = m^2c^4 - m^2v^2c^2 \qquad \text{—— 3}$$

Also —— 4

Putting this value in equation 3 we get

The equation $E^2 = (pc)^2 + (mc^2)^2$ relates the relativistic total energy E and the relativistic momentum p. At extremely high velocities, the rest energy mc^2 becomes negligible, and E = pc.

Einstein's relation between momentum and energy.(AKTU 2022)

Example: The mass of a moving electron is 11 times its rest mass. Calculate its kinetic energy and momentum.

Solution: $K = (m - m_0)c^2$

Given $m = 11m_0$, $m_0 = 9.1 \times 10^{-31}$kg

$K = (11m_0 - m_0)c^2 = 10m_0c^2$

Putting the value we get

$m_0 = 9.1 \times 10^{-31}$kg, $c = 3 \times 10^8 m/s$

$K = 10 \times 9.1 \times 10^{-31} \times (3 \times 10^8)^2$

$K = 8.19 \times 10^{-13}$ J.

Example: The total energy of a moving meson is exactly twice its rest energy. Find the speed of meson.

Solution: $E = mc^2$

Given E = 2 × rest energy = $2m_0c^2$

$$2m_0c^2 = mc^2$$

Example: How much does a proton gain in mass when accelerated to a kinetic energy of 500MeV.

Solution: Relativistic kinetic energy is

$$k = (m - m_0)c^2$$

$$mc^2 = k + m_0c^2$$

Gain is mass

Given k = 500MeV = $500 \times 10^6 \times 1.6 \times 10^{-19}$ J

$m_0 = 2.67 \times 10^{-27}$ kg, $c = 3 \times 10^8 m/s$

Example: Find the rest mass, relativistic mass and momentum of a photon of energy 5eV.

Solution: E = 5eV = $5 \times 1.6 \times 10^{-19}$J = 8×10^{-19}J

Momentum

Relativistic mass of photon

Also

Velocity of photon $v = c$, rest mass of photon $m_0 = 0$.

Example: Show that the momentum of particle of rest mass $m_0$and kinetic energy K_E is given by

Solution: Using $K_E = E - m_0c^2$

Where

Squarring both sides, we get

What do you understand by length contraction? Calculate the percentage contraction and orientation of a rod moving with the velocity of 0.8c in a direction inclined at 60^0 to its own length.(AKTU 2020)

When an object moves with a velocity v (comparable with the velocity of light) relative to a stationary observer, it measured length appears to be contracted in the direction of its motion by a factor , whereas its other dimensions perpendicular to the direction of motion remain unaffected.

The apparent length of the rod along the direction of motion

Putting $l_0 = l_0 cos60^0$, $v = 0.8c$

$l = 0.3l_0$

Apparent length in a direction perpendicular to the direction of motion

$= l_0 sin60^0 = \sqrt{3}/2 l_0$

Length of the moving rod in moving frame

$l = \ = 0.916 l_0$

Percentage in contraction in length =

Deduce the relativistic velocity addition theorem. Show that it is consistent with Einstein's second postulate.(AKTU 2017)
What do you mean by time dilation? Establish a relation for it. At what speed should a clock be moved so that it may appear to lose 1 min each hour?(AKTU2017)
Obtain the volume of a cube, the proper length of each edge of which is l_0 when it is moving with velocity v along one edge of the cube.(AKTU 2019)

The length of a moving object is contracted by a factor $(1 - v^2/c^2)^{1/2}$along its direction of motion, whereas its dimension perpendicular to the direction of motion remains unchanged. If $l_0$is the proper length of side of cube, then its contracted side would be

$l_x = l_0(1 - v^2/c^2)$

The length of other two sides remains unchanged, that is they are l_0.

Volume of the cube, $V = l_x l_y l_z = l_0(1 - v^2/c^2)^{1/2} l_0 \, x l_0$

$V = l_0^3(1 - v^2/c^2)^{1/2}$

QUESTION BANK

1. **What are Inertial and Non inertial frames of reference**
   A Inertial frame of reference is a reference frame in which an object stays either at rest or at a constant velocity unless another force acts upon it. When a body does not seem to be acting in accordance with inertia, it is in a non-inertial frame of reference or accelerating.
2. **What are Galilean Transformation equations.**
   A.Galilean transformations, also called Newtonian transformations, set of equations in classical physics that relate the space and time coordinates of two systems moving at a constant velocity relative to each other
3. **What are the postulates of special theory of Relativity.**
   The first postulate of special relativity is the idea that the laws of physics are the same and can be stated in their simplest form in all inertial frames of reference. The second postulate of special relativity is the idea that the speed of light c is a constant, independent of the relative motion of the source What is Length contraction and Time Diltion.
4. **What is concept of simultaneity**

A The relativity of simultaneity is the concept that distant simultaneity – whether two spatially separated events occur at the same time – is not absolute, but depends on the observer's reference frame

5. **What is significance of mass energy relation**

A The equation $E = mc^2$ states that the amount of energy possessed by an object is equal to its mass multiplied by the square of the speed of light. In other words, energy can be converted to mass and mass to energy.

6. **What are Lorentz transformation equations**

Lorentz transformations, set of equations in relativity physics that relate the space and time coordinates of two systems moving at a constant velocity relative to each other. What is the maximum limit of velocity a particle can achieve.

7. **What are massless particles**

A. A massless particle is an elementary particle whose invariant mass is zero.

8. **What is proper length and proper time.**

A Proper length or rest length refers to the length of an object in the object's rest frame. The difference is that the proper distance is definedbetween two spacelike-separated events (or along a spacelike path), while the proper time is defined between two timelike-separated events (or along a timelike path).

9**. Show that no signal can travel faster than velocity of light**

A If we put $v > c$, then l become imaginary but length can not be imaginary. Therefore it prove that no signal can travel faster than the speed of light

10 What is an example of mass energy?

A Mass-energy equivalence entails that the total mass of a system may change, although the total energy and momentum remain constant; for example, the collision of an electron and a proton annihilates the mass of both particles, but creates energy in the form of photons.

<u>Numerical Problems</u>

1. In Michelson Morley experiment, an expected fringe shift was found to be 0.4 when light of wavelength 6000 Å was used. The distance between the inclined mirror and silvered glass plate was found to be 11 m. Calculate the speed of earth.

   Hint :$\Delta n = 2lv^2/c^2\lambda$

2. Two particles came towards each other with speed 0.7c with respect to laboratory.What is the relative speed .

   Hint :$u = u' + v/(1 + u'v/c^2)$

3. Obtain the volume of a cube, the proper length of each edge which is l_0 when it is moving with velocity v along one of it's edge.

   Hint :$L_x = L_0(1 - v^2/c^2)$

4. Show that the circle $x^2+y^2=a^2$ in frame S appears to be an ellipse in frame S' which is moving with velocity v relative to S.

Hint :$x = x'(1 - v^2/c^2)$

5. The mean life of a meson is 2×10^{-8} sec. Calculate the mean life of a meson moving with a velocity 0.8c.

Hint :$t = t_0/(1 - v^2/c^2)$

6. Calculate the amount of work to be done to increase the speed of an electron from 0.6 c to 0.8 c.

   Hint :$K = (m - m_o)c^2$

7. Show that the momentum of a particle of rest mass m0 and kinetic energy KE is given by the expression:

$$p = \sqrt{\left(\frac{K_E^2}{c^2} + 2m_0 K_E\right)}$$

Hint : $K_E=E-m_oc^2$

8. Show that therelativistic form of Newton's Second law, when $\vec{F}$ is parallel to $\vec{v}$, is

$$\vec{F} = m_0 \frac{d\vec{v}}{dt}\left(1-\frac{v^2}{c^2}\right)^{-\frac{3}{2}}$$

Hint:F=ma, m=m_0**/**$(1-v^2/c^2)$

9 Show that $x^2+y^2+z^2-c^2t^2$ is invariant under Lorentz transformation

Hint :$x^2+y^2+z^2-c^2t^2= x'^2+y'^2+z'^2-c^2t'^2$

10 Find the mass and speed of 2MeV electron

Hint:KE= $mc^2-m_oc^2$

11 Show that the particle which travelswithnspeed of light must have a zero rest mass

Hint:m=m_0**/**$(1-v^2/c^2)$

12 Show that the circle $x^2+y^2=a^2$ in frame S appears to be an ellipse in frame S' which is moving with velocity v relative to S.

Hint:x=x'**/**$(1-v^2/c^2)$

13 A particle of rest mass m_0 moves with speed c/√2. Calculate its mass, momentum, total energy and kinetic energy.

Hint:m=m_0**/**$(1-v^2/c^2)$, KE= $mc^2-m_oc^2$

14 In an inertial frame S, a red light and a blue light are separated by a distance Δx=2.45 m, with the red light at the larger value of x. The blue light flashes, and 5.35μs later the red light flashes. Frame S' is moving in the direction of increasing x with speed of u=0.855 c. What is the distance between the two flashes and the time between them as measured in S'?

Hint:$x'=x-vt$**/√**$(1-v^2/c^2)$,$t'=t-xv/c^2$**/√**$(1-v^2/c^2)$

15 Calculate the percentage contraction in the length of rod in a frame of reference, moving with velocity 0.8 c in a direction at an angle of 300 with its length.

Hint:=L=L_0 cos 60+L_0 sin 60

<u>Long Answer Question-</u>

16 Discuss briefly the Michelson-Morley experiment and mention its outcome. Discuss the negative result of this experiment.

17 Deduce the relativistic velocity addition theorem. Show that it is consistent with Einstein's Second Postulate.

18 Deduce an expression of Time Dilation from Lorentz transformation equations.Prove that time dilation is a real effect.

19 Deduce the expression for variation of mass with velocity.

20 Deduce the Einstein's mass energy relation.

21 Discuss Galilean transformation for position,velocity and acceleration of the particle

22 State the fundamental postulates of Special theory of relativity. Deduce the Lorentz transformation equations and discuss how these account for the phenomenon of length contraction. What is Proper length.

23 Is there any condition at which the Lorentz transformation reduces to Galilian transformation? Explain it by taking suitable example